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Research Article

New Approach on Robust and Reliable Decentralized H_∞ Tracking Control for Fuzzy Interconnected Systems with Time-Varying Delay

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This paper investigates the robust and reliable decentralized H_∞ tracking control issue for the fuzzy large-scale interconnected systems with time-varying delay, which are composed of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then, according to the Lyapunov direct method and the decentralized control theory of large-scale interconnected systems, the new linear matrix inequalities- (LMIs-) based conditions with some free variables are derived to guarantee the H_∞ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Moreover, there is no need for the precise failure parameters of the actuators, rather than the lower and upper bound. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Large-scale interconnected systems, such as electrical power systems, computer communication systems, economic systems, and process control systems, have attracted great interests from many researchers in recent years. Takagi-Sugeno (T-S) fuzzy model has become a popular and effective approach to control complex systems, and a lot of significant results on stabilization and H_∞ control via linear matrix inequality (LMI) approach have been reported; see [1–4]. Compared with the centralized control, the decentralized scheme is preferred in the control design issue of the large-scale interconnected systems [5]. Recently, there are some works about stability and stabilization of fuzzy large-scale systems [6–9]. It is well known that delays appear in many dynamic systems, which are potential causes of system instability [10, 11]. The tasks of stabilization and tracking are two typical control problems. In general, tracking problems are more difficult than stabilization problems especially for nonlinear systems [12]. Reference [13] has given decentralized H_∞ fuzzy model reference tracking control design, and the stable conditions in the sense of Lyapunov are given. The technology of descriptor model transformation is used in [14, 15]. A T-S fuzzy descriptor tracking control design for nonlinear systems with a guaranteed H_∞ model reference tracking performance is discussed [16]. However, in practical situations, failure of actuators often occurs. Thus, an important requirement is to design a reliable controller such that the stability and performance of the closed-loop system can tolerate actuator failures [17–20].

In this paper, we investigate the robust and reliable decentralized H_∞ tracking control issue for the fuzzy interconnected systems with time delay, which are composed of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then, according to the Lyapunov direct method and the decentralized control theory of large-scale interconnected systems, the new LMIs-based conditions with some free variables are derived to guarantee the H_∞ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

The innovation of this paper can be summarized as follows: (1) the more practical H_∞ performance index is used,

which considers not only the effect of tracking error $e(t)$ but also the effect of control $u(t)$; (2) utilizing the descriptor model transformation, the new LMIs-based reliable H_∞ performance conditions with some free variables are derived; (3) there is no need for the precise failure parameters of the actuators, rather than the lower and upper bound of failure parameters.

In the following sections, the identity matrices and zero matrices are denoted by I and 0, respectively. X^T denotes the transpose of matrix X. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The standard notation $>$ $(<)$ is used to denote the positive (negative-) definite ordering of matrices. Inequality $X > Y$ shows that the matrix $X - Y$ is positively definite. The symbol of $*$ denotes the transposed element in the symmetric position.

2. Systems Description

Suppose there are the interconnected systems consisting of J interconnected subsystems S_i, $i = 1, \ldots, J$. Each rule of the subsystem S_i is represented by a T-S fuzzy model as follows:

$$S_i^l : \begin{cases} \text{If } \xi_{i1} \text{ is } M_{i1}^l \text{ and} \cdots \text{and } \xi_{ig_i} \text{ is } M_{ig_i}^l, \\ \quad \text{Then } \dot{x}_i(t) = A_i^l x_i(t) + B_{1i}^l w_i(t) + B_{2i}^l u_i(t) \\ \qquad\qquad + \sum_{j=1, j\neq i}^{J} C_{jil} x_j\left(t - \tau_{ij}(t)\right) \\ x_i(t) = \phi_i(t), \quad t \in [-\overline{\tau}, 0] \end{cases} \tag{1}$$
$$i = 1, \ldots, J, \; l = 1, \ldots, r_i,$$

where $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{m_i}$, and $w_i(t)$ denote the state, input, and disturbance vector, respectively. $A_i^l, B_{1i}^l, B_{2i}^l$ denote system matrices and C_{jil} denote the interconnection matrices between the ith and jth subsystem of the lth rule. $\tau_{ij}(t)$ are the time-varying delays, $\tau_{ij}(t) \le \tau_{ij0}$, $\dot{\tau}_{ij}(t) \le \widetilde{H}_{ij}$, $\overline{\tau} = \max_{i,j}\tau_{ij0}$, and r_i is the number of IF-THEN rules of the subsystem S_i. The initial conditions $\phi_i(t)$ are the differentiable functions for $t \in [-\overline{\tau}, 0]$. Here, we denote $X_t^T = [x_1^T(t), \ldots, x_J^T(t)]$, $\Phi^T(t) = [\phi_1^T(t), \ldots, \phi_J^T(t)]$.

If we utilize the singleton fuzzifier, product fuzzy inference, and central-average defuzzifier, (1) can be inferred as

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_i^l\left(\xi_{iq}(t)\right)\Bigg(A_i^l x_i(t) + B_{1i}^l w_i(t) + B_{2i}^l u_i(t) + \sum_{j=1, j\neq i}^{J} C_{jil} x_j\left(t - \tau_{ij}(t)\right)\Bigg), \tag{2}$$

where

$$\lambda_i^l\left(\xi_{iq}(t)\right) = \prod_{q=1}^{g_i} M_{iq}^l\left(\xi_{iq}(t)\right),$$
$$\mu_i^l\left(\xi_{iq}(t)\right) = \frac{\lambda_i^l\left(\xi_{iq}(t)\right)}{\sum_{l=1}^{r_i}\lambda_i^l\left(\xi_{iq}(t)\right)}, \tag{3}$$

where $\xi_{i1}, \ldots, \xi_{ig_i}$ are the premise variables and $M_{iq}^l(\xi_{iq}(t))$ is the grade of membership of $\xi_{iq}(t)$ in M_{iq}^l. It can be seen that $\lambda_i^l(\xi_{iq}(t)) \ge 0$, $\mu_i^l(\xi_{iq}(t)) \ge 0$ and $\sum_{l=1}^{r_i}\mu_i^l(\xi_{iq}(t)) = 1$, $i = 1, \ldots, J$, $l = 1, \ldots, r_i$.

Consider the reference model for the ith subsystem as follows:

$$\dot{x}_{ri}(t) = A_{ri}x_{ri}(t) + B_{ri}v_i(t), \tag{4}$$

where $x_{ri}(t)$ denote the reference states, A_{ri} denote the specific asymptotically stable matrices, B_{ri} denote the system matrices with appropriate dimensions, and $v_i(t)$ denotes the bounded reference input.

Lemma 1 (see [10]). *For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M^T > 0$, scalar $\alpha > 0$, vector function $\zeta : [0, \alpha] \to \mathbb{R}^n$, such that the integrations in the following are well defined; then*

$$\alpha\int_0^\alpha \zeta^T(\beta) M\zeta(\beta)\, d\beta \ge \left(\int_0^\alpha \zeta(\beta)\, d\beta\right)^T M \int_0^\alpha \zeta(\beta)\, d\beta. \tag{5}$$

Lemma 2 (see [10]). *For any matrix $Q \in \mathbb{R}^{l\times n}$, any constant $\varepsilon > 0$, and any positive definite matrix T, all $\zeta \in \mathbb{R}^l$, $\eta \in \mathbb{R}^n$, the following result holds:*

$$2\zeta^T Q\eta \le \varepsilon\zeta^T QT^{-1}Q^T\zeta + \frac{1}{\varepsilon}\eta^T T\eta. \tag{6}$$

Lemma 3 (see [2]). *For any real matrix X_l, Y_l, $1 \le l \le n$, and $S > 0$ with appropriate dimensions, one has*

$$2\sum_{l=1}^{n}\sum_{m=1}^{n}\mu_l\mu_m X_l^T S Y_m \le \sum_{l=1}^{n}\mu_l\left(X_l^T S X_l + Y_l^T S Y_l\right), \tag{7}$$

where μ_l $(1 \le l \le n)$ are defined as $\mu_l(M(k)) \ge 0$, $\sum_{l=1}^{n}\mu_l(M(k)) = 1$.

3. H_∞ Tracking Control Design

According to the conventional parallel distributed compensation (PDC) concept, the fuzzy controllers corresponding to S_i are used as follows:

$$F_i^l : \begin{cases} \text{If } \xi_{i1} \text{ is } M_{i1}^l \text{ and} \cdots \text{and } \xi_{ig_i} \text{ is } M_{ig_i}^l, \\ \text{Then } u_i(t) = K_i^l\left(x_i(t) - x_{ri}(t)\right), \end{cases} \tag{8}$$

where K_i^l are the controllers gains of the lth rule for subsystem S_i.

And the final output of the fuzzy controllers for each subsystem S_i is

$$u_i(t) = \sum_{l=1}^{r_i}\mu_i^l K_i^l\left(x_i(t) - x_{ri}(t)\right). \tag{9}$$

Instead of actuator outage, a more general actuator failure model is adopted in this paper. Let $u_i^F(t)$ be the control input vector after failures have occurred. The following actuator failure model is adopted:

$$u_i^F(t) = \alpha_i u_i(t) = \alpha_i\sum_{l=1}^{r_i}\mu_i^l K_i^l\left(x_i(t) - x_{ri}(t)\right), \tag{10}$$

where $\alpha_i = \mathrm{diag}(\alpha_{i1}, \ldots, \alpha_{im_i})$ with $0 \le \underline{\alpha}_{ij} \le \alpha_{ij} \le \overline{\alpha}_{ij} \le 1$, $i = 1, \ldots, J$, $j = 1, \ldots, m_i$.

Note that the parameters $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$ characterize the admissible failures of the jth actuator in the ith subsystem.

Obviously, when $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 0$, the failure model (10) corresponds to the case of the jth actuator outage. When $0 \le \underline{\alpha}_{ij} \le \overline{\alpha}_{ij} \le 1$, it corresponds to the case of partial failure of the jth actuator. When $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = 1$, it implies that there is no failure in the jth actuator. Denote the matrix set $S_{\eta i} = \{\lambda_{Li} : \lambda_{Li} = \mathrm{diag}(\lambda_{Li1}, \ldots, \lambda_{Lim_i})\}$, where $\lambda_{Lij} = \underline{\alpha}_{ij}$ or $\overline{\alpha}_{ij}$, $i = 1, \ldots, J$, $j = 1, \ldots, m_i$, $L = 1, \ldots, 2^{m_i - l_i}$, $l_i \le m_i$ is the number of no failure actuators for subsystem S_i.

Lemma 4 (representation theorem [21]). *$S = \{x \mid Ax = b, x \ge 0\}$ is nonempty and bounded multiple-surface set, and there exist finite limit points $x^{(1)}, \ldots, x^{(k)}$; then, $x \in S$ if and only if $x = \sum_{j=1}^{k} \zeta_j x^{(j)}$, $\sum_{j=1}^{k} \zeta_j = 1$, $\zeta_j \ge 0$, $j = 1, \ldots, k$.*

Here, let $S_i = \{x_i \mid A_i x_i = b_i, x_i \ge 0\}$, and $A_i = I$; that is, $S_i = \{x_i \mid x_i = b_i, x_i \ge 0\}$, where $b_i = [\alpha_{i1} \cdots \alpha_{i(m_i - l_i)}]^T$. Then, there exist finite limit points $x_i^{(1)} = (\underline{\alpha}_{i1} \cdots \underline{\alpha}_{i(m_i - l_i)}), \ldots, x_i^{(2^{m_i - l_i})} = (\overline{\alpha}_{i1} \cdots \overline{\alpha}_{i(m_i - l_i)})$ for S_i.

Given any two points $X_i = (x_{ij}) \in S_i$, $Y_i = (y_{ij}) \in S_i$, $\underline{\alpha}_{ij} \le x_{ij}, y_{ij} \le \overline{\alpha}_{ij}$, $j = 1, \ldots, m_i - l_i$, for any real number $\lambda \in [0, 1]$, then $\underline{\alpha}_{ij} \le \lambda x_{ij} + (1 - \lambda) y_{ij} \le \overline{\alpha}_{ij}$; that is $\lambda X_i + (1 - \lambda) Y_i \in S_i$. Therefore, S_i is nonempty and bounded multiple-surface set.

Let the l_i no failure actuators are the last l_i actuators of ith subsystem, according to Lemma 4, one has

$$\begin{aligned} \alpha_i &= \mathrm{diag}\left(\alpha_{i1} \ \cdots \ \alpha_{i(m_i - l_i)} \ \ 1 \ \cdots \ 1\right) \\ &= \xi_{i1} \, \mathrm{diag}\left(\underline{\alpha}_{i1} \ \cdots \ \underline{\alpha}_{i(m_i - l_i)} \ \ 1 \ \cdots \ 1\right) \\ &\quad + \cdots + \xi_{i2^{m_i - l_i}} \, \mathrm{diag}\left(\overline{\alpha}_{i1} \ \cdots \ \alpha_{i(m_i - l_i)} \ \ 1 \ \cdots \ 1\right), \end{aligned} \tag{11}$$

where $\zeta_{iL} \ge 0$, $\sum_{L=1}^{2^{m_i - l_i}} \zeta_{iL} = 1$.

Therefore, $\alpha_i \in S'_{\alpha i}$, where $S'_{\alpha i} = \{\alpha_i : \alpha_i = \sum_{L=1}^{2^{m_i - l_i}} \zeta_{iL} \lambda_{Li}$, $\zeta_{iL} \ge 0$, $\sum_{L=1}^{2^{m_i - l_i}} \zeta_{iL} = 1\}$.

Denote the tracking error by

$$e_i(t) = x_i(t) - x_{ri}(t). \tag{12}$$

Then, the whole closed-loop fuzzy interconnected systems become

$$\begin{aligned} \dot{\overline{x}}_i(t) = \sum_{j=1, j \ne i}^{J} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \Big(&\overline{A}_{ilm} \overline{x}_i(t) + \overline{C}_{jil} \overline{x}_j\left(t - \tau_{ij}(t)\right) \\ &+ \overline{B}_{il} \overline{w}_i(t) \Big), \end{aligned} \tag{13}$$

where

$$\begin{gathered} \overline{x}_i(t) = \begin{bmatrix} x_i(t) \\ e_i(t) \end{bmatrix}, \qquad \overline{w}_i(t) \begin{bmatrix} w_i(t) \\ v_i(t) \end{bmatrix}, \\ \overline{A}_{ilm} = \sum_{L=1}^{2^{m_i - l_i}} \begin{bmatrix} A_i^l & B_{2i}^l \lambda_{Li} K_i^m \\ A_i^l - A_{ri} & A_{ri} + B_{2i}^l \lambda_{Li} K_i^m \end{bmatrix}, \\ \overline{C}_{jil} = \begin{bmatrix} C_{jil} & 0 \\ C_{jil} & 0 \end{bmatrix}, \qquad \overline{B}_{il} = \begin{bmatrix} B_{1i}^l & 0 \\ B_{1i}^l & -B_{ri} \end{bmatrix}. \end{gathered} \tag{14}$$

Equation (13) can be transformed to be the descriptor system form as follows:

$$E\dot{\xi}_i = \sum_{j=1, j \ne i}^{J} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \left(\widetilde{A}_{ilm} \xi_i + \widetilde{C}_{jil} \overline{x}_j\left(t - \tau_{ij}(t)\right) + \widetilde{B}_{il} \overline{w}_i \right), \tag{15}$$

where

$$\begin{gathered} \xi_i = \begin{bmatrix} \overline{x}_i \\ \dot{\overline{x}}_i \end{bmatrix}, \qquad E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \\ \widetilde{A}_{ilm} = \begin{bmatrix} 0 & I \\ \overline{A}_{ilm} & -I \end{bmatrix}, \\ \widetilde{C}_{jil} = \begin{bmatrix} 0 \\ \overline{C}_{jil} \end{bmatrix}, \quad \widetilde{B}_{il} = \begin{bmatrix} 0 \\ \overline{B}_{il} \end{bmatrix}. \end{gathered} \tag{16}$$

Definition 5. The H_∞ tracking control problem for the interconnected systems (13) is to design the controllers to minimize the prescribed level of disturbance attenuation $\sum_{i=1}^{J} \gamma_i > 0$, $i = 1, \ldots, J$, if the following two conditions are satisfied.

(1) When $\overline{w}_i = 0$, the whole fuzzy interconnected systems (13) are asymptotically stable.

(2) For the zero initially condition ($\Phi^T(t) = 0$, $t \in [-\overline{\tau}, 0]$), $\sum_{i=1}^{J} \int_0^\infty (e_i^T(t) Q_i e_i(t) + u_i^T(t) R_i u_i(t)) dt \le \sum_{i=1}^{J} \gamma_i^2 \int_0^\infty \overline{w}_i^T(t) \overline{w}_i(t) dt$.

Remark 6. From Definition 5, we can see that, compared with the condition $\sum_{i=1}^{J} \int_0^\infty e_i^T(t) Q_i e_i(t) dt \le \sum_{i=1}^{J} \gamma_i^2 \int_0^\infty \overline{w}_i^T(t) \overline{w}_i(t) dt$ commonly used for the H_∞ tracking control problem, the practical H_∞ performance index considers not only the effect of $e(t)$ but also the effect of $u(t)$. That is to say, the condition that the reducing of the tracking error $e(t)$ needs to cost the much bigger gain of the controller can be avoided effectively.

Theorem 7. *For the given constant δ_i, η_i, if there exist matrices $Y_{2i}^1, Y_{2i}^2, Y_{2i}^3, Y_{2i}^4, P_i^2, T_i^{12}, S_{ji}^{12}, S_{ii}^{12}, E_{i1}^{pq}, E_{i2}^{pq}, E_{ij3}^{pq}, E_{ij4}^{pq}, E_{i5}^{pq}$, $p, q = 1, 2$, and positive definite matrices $W_{2i}^m, P_i^1, P_i^3, T_i^{11}, T_i^{22}, S_{ji}^{11}, S_{ji}^{22}, S_{ii}^{11}, S_{ii}^{22}$, satisfying following eigenvalue problem (17), then the whole interconnected nonlinear systems are asymptotically stable with the H_∞ tracking performance index γ_i, $i, j = 1, \ldots, J$, $j \ne i$, $l, m = 1, \ldots, r_i$, $L = 1, \ldots, 2^{m_i - l_i}$*

$$\begin{aligned} &\min \quad \sum_{i=1}^{J} \gamma_i \\ &\textit{subject to} \quad (18) \ \textit{and} \ (19) \end{aligned} \tag{17}$$

$$\begin{bmatrix}
\widetilde{\Pi}^{11} & \widetilde{\Pi}^{12} & \widetilde{\Pi}^{13} & \widetilde{\Pi}^{14} & \widetilde{\Pi}^{15} & \widetilde{\Pi}^{16} & \widetilde{\Pi}^{17} & \widetilde{\Pi}^{18} & \delta_i B_{1i}^l & 0 & Y_{2i}^3 & \left(W_{2i}^l\right)^T & E_{i1}^{11} & E_{i1}^{12} \\
* & \widetilde{\Pi}^{22} & \widetilde{\Pi}^{23} & \widetilde{\Pi}^{24} & \widetilde{\Pi}^{25} & \widetilde{\Pi}^{26} & \widetilde{\Pi}^{27} & \widetilde{\Pi}^{28} & \delta_i B_{1i}^l & -\delta_i B_{ri} & \eta_i Y_{2i}^3 & \eta_i\left(W_{2i}^l\right)^T & E_{i1}^{21} & E_{i1}^{22} \\
* & * & \widetilde{\Pi}^{33} & \widetilde{\Pi}^{34} & \widetilde{\Pi}^{35} & \widetilde{\Pi}^{36} & \widetilde{\Pi}^{37} & \widetilde{\Pi}^{38} & B_{1i}^l & 0 & 0 & 0 & E_{i2}^{11} & E_{i2}^{12} \\
* & * & * & \widetilde{\Pi}^{44} & \widetilde{\Pi}^{45} & \widetilde{\Pi}^{46} & \widetilde{\Pi}^{47} & \widetilde{\Pi}^{48} & B_{1i}^l & -B_{ri} & 0 & 0 & E_{i2}^{21} & E_{i2}^{22} \\
* & * & * & * & \widetilde{\Pi}^{55} & \widetilde{\Pi}^{56} & \widetilde{\Pi}^{57} & \widetilde{\Pi}^{58} & \widetilde{\Pi}^{59} & \left(E_{i5}^{21}\right)^T & 0 & 0 & E_{ij3}^{11} & E_{ij3}^{12} \\
* & * & * & * & * & \widetilde{\Pi}^{66} & \widetilde{\Pi}^{67} & \widetilde{\Pi}^{68} & \widetilde{\Pi}^{69} & \left(E_{i5}^{22}\right)^T & 0 & 0 & E_{ij3}^{21} & E_{ij3}^{22} \\
* & * & * & * & * & * & \widetilde{\Pi}^{77} & \widetilde{\Pi}^{78} & \widetilde{\Pi}^{79} & -\left(E_{i5}^{21}\right)^T & 0 & 0 & E_{ij4}^{11} & E_{ij4}^{12} \\
* & * & * & * & * & * & * & \widetilde{\Pi}^{88} & \widetilde{\Pi}^{89} & -\left(E_{i5}^{22}\right)^T & 0 & 0 & E_{ij4}^{21} & E_{ij4}^{22} \\
* & * & * & * & * & * & * & * & -\gamma_i^2 I & 0 & 0 & 0 & E_{i5}^{11} & E_{i5}^{12} \\
* & * & * & * & * & * & * & * & * & -\gamma_i^2 I & 0 & 0 & E_{i5}^{21} & E_{i5}^{22} \\
* & * & * & * & * & * & * & * & * & * & -Q_i^{-1} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -R_i^{-1} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -\dfrac{T_i^{11}}{\tau_{ji0}} & -\dfrac{T_i^{12}}{\tau_{ji0}} \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\dfrac{T_i^{22}}{\tau_{ji0}}
\end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} T_i^{11} & T_i^{12} \\ * & T_i^{22} \end{bmatrix} > 0, \qquad \begin{bmatrix} P_i^1 & P_i^2 \\ * & P_i^3 \end{bmatrix} > 0,$$
$$\begin{bmatrix} S_{ji}^{11} & S_{ji}^{12} \\ * & S_{ji}^{22} \end{bmatrix} > 0, \qquad \begin{bmatrix} S_{ii}^{11} & S_{ii}^{12} \\ * & S_{ii}^{22} \end{bmatrix} > 0, \tag{19}$$

where

$$\begin{aligned}
\widetilde{\Pi}^{11} &= \delta_i A_i^l Y_{2i}^1 + \delta_i\left(A_i^l Y_{2i}^1\right)^T + \delta_i B_{2i}^l \lambda_{Li} W_{2i}^m \\
&\quad + \delta_i\left(B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T + S_{ii}^{11}, \\
\widetilde{\Pi}^{12} &= \delta_i A_i^l Y_{2i}^2 + \delta_i \eta_i B_{2i}^l \lambda_{Li} W_{2i}^m + \delta_i\left(\left(A_i^l - A_{ri}\right) Y_{2i}^1\right)^T \\
&\quad + \delta_i\left(A_{ri} Y_{2i}^3 + B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T + S_{ii}^{12}, \\
\widetilde{\Pi}^{22} &= \delta_i\left(A_i^l - A_{ri}\right) Y_{2i}^2 + \delta_i\left(\left(A_i^l - A_{ri}\right) Y_{2i}^2\right)^T \\
&\quad + \delta_i \eta_i\left(A_{ri} Y_{2i}^3 + B_{2i}^l \lambda_{Li} W_{2i}^m\right) \\
&\quad + \delta_i \eta_i\left(A_{ri} Y_{2i}^3 + B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T + S_{ii}^{22}, \\
\widetilde{\Pi}^{13} &= \left(A_i^l Y_{2i}^1\right)^T + \left(B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T + P_i^1 - \delta_i Y_{2i}^1, \\
\widetilde{\Pi}^{14} &= \left(\left(A_i^l - A_{ri}\right) Y_{2i}^1\right)^T + \left(A_{ri} Y_{2i}^3 + B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T \\
&\quad + P_i^2 - \delta_i Y_{2i}^2, \\
\widetilde{\Pi}^{23} &= \left(A_i^l Y_{2i}^2\right)^T + \eta_i\left(B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T + \left(P_i^2\right)^T - \delta_i Y_{2i}^3, \\
\widetilde{\Pi}^{24} &= \left(\left(A_i^l - A_{ri}\right) Y_{2i}^2\right)^T + \eta_i\left(A_{ri} Y_{2i}^3 + B_{2i}^l \lambda_{Li} W_{2i}^m\right)^T \\
&\quad + P_i^3 - \delta_i \eta_i Y_{2i}^3, \\
\widetilde{\Pi}^{33} &= -\left(Y_{2i}^1\right)^T - Y_{2i}^1 + \tau_{ji0} T_i^{11}, \\
\widetilde{\Pi}^{34} &= -\left(Y_{2i}^3\right)^T - Y_{2i}^2 + \tau_{ji0} T_i^{12}, \\
\widetilde{\Pi}^{44} &= -\eta_i\left(Y_{2i}^3\right)^T - \eta_i Y_{2i}^3 + \tau_{ji0} T_i^{22}, \\
\widetilde{\Pi}^{15} &= E_{i1}^{11}, \qquad \widetilde{\Pi}_{ijlm}^{16} = E_{i1}^{12}, \qquad \widetilde{\Pi}^{25} = E_{i1}^{21}, \\
\widetilde{\Pi}_{ijlm}^{26} &= E_{i1}^{22}, \qquad \widetilde{\Pi}^{35} = E_{i2}^{11}, \qquad \widetilde{\Pi}_{ijlm}^{36} = E_{i2}^{12}, \\
\widetilde{\Pi}^{45} &= E_{i2}^{21}, \qquad \widetilde{\Pi}_{ijlm}^{46} = E_{i2}^{22},
\end{aligned}$$

$$
\begin{aligned}
&\widetilde{\Pi}^{55} = \left(E_{ij3}^{11}\right)^T + E_{ij3}^{11},\\
&\widetilde{\Pi}^{56} = \left(E_{ij3}^{21}\right)^T + E_{ij3}^{12},\\
&\widetilde{\Pi}^{66} = \left(E_{ij3}^{22}\right)^T + E_{ij3}^{22}, \qquad \widetilde{\Pi}^{17} = -E_{i1}^{11} + \delta_i C_{jil} Y_{2i}^1,\\
&\widetilde{\Pi}^{27} = -E_{i1}^{21} + \delta_i C_{jil} Y_{2i}^1, \qquad \widetilde{\Pi}^{18} = -E_{i1}^{12} + \delta_i C_{jil} Y_{2i}^2,\\
&\widetilde{\Pi}^{28} = -E_{i1}^{22} + \delta_i C_{jil} Y_{2i}^2, \qquad \widetilde{\Pi}^{37} = -E_{i2}^{11} + C_{jil} Y_{2i}^1,\\
&\widetilde{\Pi}^{47} = -E_{i2}^{21} + C_{jil} Y_{2i}^1, \qquad \widetilde{\Pi}^{38} = -E_{i2}^{12} + C_{jil} Y_{2i}^2,\\
&\widetilde{\Pi}^{48} = -E_{i2}^{22} + C_{jil} Y_{2i}^2, \qquad \widetilde{\Pi}^{57} = -E_{ij3}^{11} + \left(E_{ij4}^{11}\right)^T,\\
&\widetilde{\Pi}^{58} = -E_{ij3}^{12} + \left(E_{ij4}^{12}\right)^T, \qquad \widetilde{\Pi}^{67} = -E_{ij3}^{21} + \left(E_{ij4}^{21}\right)^T,\\
&\widetilde{\Pi}^{68} = -E_{ij3}^{22} + \left(E_{ij4}^{22}\right)^T,\\
&\widetilde{\Pi}^{77} = -E_{ij4}^{11} - \left(E_{ij4}^{11}\right)^T - \left(1 - \widetilde{H}_{ij}\right) S_{ji}^{11},\\
&\widetilde{\Pi}^{78} = -E_{ij4}^{12} - \left(E_{ij4}^{12}\right)^T - \left(1 - \widetilde{H}_{ij}\right) S_{ji}^{12},\\
&\widetilde{\Pi}^{88} = -E_{ij4}^{22} - \left(E_{ij4}^{22}\right)^T - \left(1 - \widetilde{H}_{ij}\right) S_{ji}^{22},\\
&\widetilde{\Pi}^{59} = \left(E_{i5}^{11}\right)^T, \qquad \widetilde{\Pi}^{69} = \left(E_{i5}^{12}\right)^T,\\
&\widetilde{\Pi}^{79} = -\left(E_{i5}^{11}\right)^T, \qquad \widetilde{\Pi}^{89} = -\left(E_{i5}^{12}\right)^T.
\end{aligned}
\tag{20}
$$

Proof. We choose the following Lyapunov function for the whole interconnected system (13):

$$
\begin{aligned}
V\left(X_t\right) &= V_1\left(X_t\right) + V_2\left(X_t\right) + V_3\left(X_t\right),\\
V_1\left(X_t\right) &= \sum_{i=1}^{J} \xi_i^T(t) E^T \widetilde{P}_i \xi_i(t),\\
V_2\left(X_t\right) &= \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \int_{t-\tau_{ji}(t)}^{t} \overline{x}_i(s)^T S_i \overline{x}_i(s)\, ds,\\
V_3\left(X_t\right) &= \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \int_{-\tau_{ji0}}^{0} \int_{t+\theta}^{t} \dot{\overline{x}}_i(s)^T T_i \dot{\overline{x}}_i(s)\, ds\, d\theta,
\end{aligned}
\tag{21}
$$

where S_i, T_i are positive definite matrices, $E^T\widetilde{P}_i = \widetilde{P}_i^T E \geq 0$, and denote

$$
\widetilde{P}_i = \begin{bmatrix} P_{0i} & 0 \\ P_{1i} & P_{2i} \end{bmatrix}, \quad P_{0i} = P_{0i}^T. \tag{22}
$$

Computing the time derivative of $V(X_t)$, we have

$$
\begin{aligned}
\dot{V}_1\left(X_t\right) &= \sum_{i=1}^{J} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \left(\xi_i^T \left(\widetilde{A}_{ilm}^T \widetilde{P}_i + \widetilde{P}_i^T \widetilde{A}_{ilm}\right) \xi_i + \xi_i^T \widetilde{P}_i^T \left(\sum_{j=1,\, j\neq i}^{J} \widetilde{C}_{jil} \overline{x}_j\left(t - \tau_{ij}(t)\right) \right) \right.\\
&\qquad \left. + \left(\sum_{j=1,\, j\neq i}^{J} \overline{x}_j^T\left(t - \tau_{ij}(t)\right) \widetilde{C}_{jil}^T \right) \widetilde{P}_i \xi_i + \xi_i^T \widetilde{P}_i^T \widetilde{B}_{il} \overline{w}_i + \overline{w}_i^T \widetilde{B}_{il}^T \widetilde{P}_i \xi_i \right)\\
&= \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \mu_i^l \mu_i^m \begin{bmatrix} \xi_i \\ \overline{x}_j\left(t - \tau_{ij}(t)\right) \\ \overline{w}_i \end{bmatrix}^T \begin{bmatrix} \overline{A}_{ilm}^T P_{1i} + P_{1i}^T \overline{A}_{ilm} & \overline{A}_{ilm}^T P_{2i} + P_{0i}^T - P_{1i}^T & P_{1i}^T \overline{C}_{jil} & P_{1i}^T \overline{B}_{il} \\ * & -P_{2i} - P_{2i}^T & P_{2i}^T \overline{C}_{jil} & P_{2i}^T \overline{B}_{il} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}\\
&\qquad \times \begin{bmatrix} \xi_i \\ \overline{x}_j\left(t - \tau_{ij}(t)\right) \\ \overline{w}_i \end{bmatrix},\\
\dot{V}_2\left(X_t\right) &\leq \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \left(\overline{x}_i(t)^T S_i \overline{x}_i(t) - \left(1 - \widetilde{H}_{ij}\right) \overline{x}_j\left(t - \tau_{ij}(t)\right)^T S_j \overline{x}_j\left(t - \tau_{ij}(t)\right) \right),\\
\dot{V}_3\left(X_t\right) &= \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \left(\tau_{ji0} \dot{\overline{x}}_i^T T_i \dot{\overline{x}}_i - \int_{t-\tau_{ji0}}^{t} \dot{\overline{x}}_i^T(s) T_i \dot{\overline{x}}_i(s)\, ds \right)\\
&\leq \sum_{i=1}^{J} \sum_{j=1,\, j\neq i}^{J} \left(\tau_{ji0} \dot{\overline{x}}_i^T T_i \dot{\overline{x}}_i - \int_{t-\tau_{ij}(t)}^{t} \dot{\overline{x}}_j^T(s) T_j \dot{\overline{x}}_j(s)\, ds \right).
\end{aligned}
\tag{23}
$$

From the Leibniz-Newton formula, the following equation is considered:

$$
\begin{aligned}
&2\left(\overline{x}_i^T E_{i1} + \dot{\overline{x}}_i^T E_{i2} + \overline{x}_j^T E_{ij3} + \overline{x}_j^T\left(t - \tau_{ij}(t)\right) E_{ij4} + \overline{w}_i^T E_{i5} \right)\\
&\quad \times \left(\overline{x}_j - \overline{x}_j\left(t - \tau_{ij}(t)\right) - \int_{t-\tau_{ij}(t)}^{t} \dot{\overline{x}}_j(s)\, ds \right) = 0.
\end{aligned}
\tag{24}
$$

From (24), we have

$$
\begin{aligned}
&2\left(\overline{x}_i^T E_{i1} + \dot{\overline{x}}_i^T E_{i2} + \overline{x}_j^T E_{ij3} + \overline{x}_j^T\left(t - \tau_{ij}(t)\right) E_{ij4} \right.\\
&\quad \left. + \overline{w}_i^T E_{i5} \right) \left(\overline{x}_j - \overline{x}_j\left(t - \tau_{ij}(t)\right) \right) = \overline{\Gamma}_{ij}^T \Phi_{ij1} \overline{\Gamma}_{ij},
\end{aligned}
\tag{25}
$$

where

$$\Gamma_{ij} = \begin{bmatrix} \xi_i \\ \overline{x}_j \\ \overline{x}_j\left(t-\tau_{ij}\right) \end{bmatrix}, \qquad \overline{\Gamma}_{ij} = \begin{bmatrix} \xi_i \\ \overline{x}_j \\ \overline{x}_j\left(t-\tau_{ij}\right) \\ \overline{w}_i \end{bmatrix},$$

$$\begin{aligned} \Phi_{ij1} &= \begin{bmatrix} 0 & 0 & E_{i1} & -E_{i1} & 0 \\ * & 0 & E_{i2} & -E_{i2} & 0 \\ * & * & E_{ij3}+E_{ij3}^T & -E_{ij3}+E_{ij4}^T & E_{i5}^T \\ * & * & * & -E_{ij4}-E_{ij4}^T & -E_{i5}^T \\ * & * & * & * & 0 \end{bmatrix}, \\ &- 2\left(\overline{x}_i^T E_{i1} + \dot{\overline{x}}_i^T E_{i2} + \overline{x}_j^T E_{ij3} + \overline{x}_j^T\left(t-\tau_{ij}(t)\right) E_{ij4}\right. \\ &\quad \left. + \overline{w}_i^T E_{i5}\right) \int_{t-\tau_{ij}(t)}^{t} \dot{\overline{x}}_j(s)\, ds \\ &\le \tau_{ij0}\overline{\Gamma}_{ij}^T \overline{E}_i T_j^{-1} \overline{E}_i^T \overline{\Gamma}_{ij} + \int_{t-\tau_{ij}(t)}^{t} \dot{\overline{x}}_j^T(s)\, T_j \dot{\overline{x}}_j(s)\, ds, \end{aligned} \tag{26}$$

where $\overline{E}_i = [E_{i1} \quad E_{i2} \quad E_{ij3} \quad E_{ij4} \quad E_{i5}]^T$.

According to Lemma 3, we have

$$\begin{aligned} & e_i^T(t) Q_i e_i(t) + u_i^T(t) R_i u_i(t) \\ &\quad = \sum_{l=1}^{r_i}\sum_{m=1}^{r_i} \mu_i^l \mu_i^m e_i^T(t)\left(Q_i + \left(K_i^l\right)^T R_i K_i^m\right) e_i(t) \\ &\quad \le \sum_{l=1}^{r_i} \mu_i^l e_i^T(t)\left(Q_i + \left(K_i^l\right)^T R_i K_i^l\right) e_i(t). \end{aligned} \tag{27}$$

Letting $P_{1i} = \delta_i P_{2i}$ and using (15), we have

$$\begin{aligned} & \dot{V} + \sum_{i=1}^{J} e_i^T(t) Q_i e_i(t) + \sum_{i=1}^{J} u_i^T(t) R_i u_i(t) \\ &\quad - \sum_{i=1}^{J} \gamma_i^2 \overline{w}_i^T \overline{w}_i \\ &\quad + \sum_{i=1}^{J}\sum_{j=1,\, j\ne i}^{J} 2\left(\overline{x}_i^T E_{i1} + \dot{\overline{x}}_i^T E_{i2} + \overline{x}_j^T E_{ij3}\right. \\ &\qquad \left. + \overline{x}_j^T\left(t-\tau_{ij}(t)\right) E_{ij4} + \overline{w}_i^T E_{i5}\right) \\ &\quad \times \left(\overline{x}_j - \overline{x}_j\left(t-\tau_{ij}(t)\right) - \int_{t-\tau_{ij}(t)}^{t} \dot{\overline{x}}_j(s)\, ds\right) \\ &\quad \le \sum_{i=1}^{J}\sum_{j=1,\, j\ne i}^{J}\sum_{l=1}^{r_i}\sum_{m=1}^{r_i} \mu_i^l \mu_i^m \overline{\Gamma}_{ij}^T \Pi_{ijlm} \overline{\Gamma}_{ij}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \Pi_{ijlm} &= \begin{bmatrix} \Pi_{ijlm}^{11} & \Pi_{ijlm}^{12} & \Pi_{ijlm}^{13} & \Pi_{ijlm}^{14} & \Pi_{ijlm}^{15} \\ * & \Pi_{ijlm}^{22} & \Pi_{ijlm}^{23} & \Pi_{ijlm}^{24} & \Pi_{ijlm}^{25} \\ * & * & \Pi_{ijlm}^{33} & \Pi_{ijlm}^{34} & \Pi_{ijlm}^{35} \\ * & * & * & \Pi_{ijlm}^{44} & \Pi_{ijlm}^{45} \end{bmatrix} \\ &\quad + \tau_{ij0} \overline{E}_i T_j^{-1} \overline{E}_i^T. \end{aligned} \tag{29}$$

From (29), we can see that, if $\Pi_{ijlm} < 0$, then $\Pi'_{ijlm} < 0$, where

$$\begin{aligned} \Pi_{ijlm}^{11} &= \delta_i P_{2i}^T \overline{A}_{ilm} + \delta_i \overline{A}_{ilm}^T P_{2i} + \overline{Q}_i + S_i, \\ \Pi_{ijlm}^{12} &= \overline{A}_{ilm}^T P_{2i} + P_{0i}^T - \delta_i P_{2i}^T, \\ \Pi_{ijlm}^{22} &= -P_{2i}^T - P_{2i} + \tau_{ji0} T_i, \\ \Pi_{ijlm}^{13} &= E_{i1}, \qquad \Pi_{ijlm}^{23} = E_{i2}, \qquad \Pi_{ijlm}^{33} = E_{ij3} + E_{ij3}^T, \\ \Pi_{ijlm}^{14} &= -E_{i1} + \delta_i P_{2i}^T \overline{C}_{jil}, \qquad \Pi_{ijlm}^{24} = -E_{i2} + P_{2i}^T \overline{C}_{jil}, \\ \Pi_{ijlm}^{34} &= -E_{ij3} + E_{ij4}^T, \qquad \Pi_{ijlm}^{15} = \delta_i P_{2i}^T \overline{B}_{il}, \\ \Pi_{ijlm}^{44} &= -E_{ij4} - E_{ij4}^T - \left(1 - \widetilde{H}_{ij}\right) S_j, \qquad \Pi_{ijlm}^{25} = P_{2i}^T \overline{B}_{il}, \\ \Pi_{ijlm}^{35} &= E_{i5}^T, \qquad \Pi_{ijlm}^{45} = -E_{i5}^T, \qquad \Pi_{ijlm}^{55} = -\gamma_i^2 I, \\ \overline{Q}_i &= \operatorname{diag}\left[0 \quad Q_i + \left(K_i^l\right)^T R_i K_i^l\right], \\ \Pi'_{ijlm} &= \begin{bmatrix} \Pi_{ijlm}^{11} & \Pi_{ijlm}^{12} & \Pi_{ijlm}^{13} & \Pi_{ijlm}^{14} & E_{i1} \\ * & \Pi_{ijlm}^{22} & \Pi_{ijlm}^{23} & \Pi_{ilm}^{24} & E_{i2} \\ * & * & \Pi_{ijlm}^{33} & \Pi_{ijlm}^{34} & E_{ij3} \\ * & * & * & \Pi_{ijlm}^{44} & E_{ij4} \\ * & * & * & * & -\dfrac{T_j}{\tau_{ij0}} \end{bmatrix}. \end{aligned} \tag{30}$$

If $\overline{w}_i = 0$,

$$\dot{V} \le \sum_{i=1}^{J}\sum_{j=1,\, j\ne i}^{J}\sum_{l=1}^{r_i}\sum_{m=1}^{r_i} \mu_i^l \mu_i^m \Gamma_{ij}^T \Pi'_{ijlm} \Gamma_{ij} < 0, \tag{31}$$

then the whole nonlinear interconnected systems are asymptotically stable.

If $\overline{w}_i \ne 0$,

$$\begin{aligned} & \dot{V} + \sum_{i=1}^{J} e_i^T(t) Q_i e_i(t) + \sum_{i=1}^{J} u_i^T(t) R_i u_i(t) \\ &\quad - \sum_{i=1}^{J} \gamma_i^2 \overline{w}_i^T(t) \overline{w}_i(t) < 0. \end{aligned} \tag{32}$$

With zero initial condition ($\Phi(t) = 0$, $t \in [-\overline{\tau}, 0]$), hence $V(\Phi(t)) = 0$, $t \in [-\overline{\tau}, 0]$, and $V(X_\infty) \ge 0$, integrating both sides of (32) from 0 to ∞, we have

$$\begin{aligned} & V(X_\infty) - V(\Phi(0)) \\ &\quad \le \sum_{i=1}^{J} \gamma_i^2 \int_0^\infty \overline{w}_i^T(t) \overline{w}_i(t)\, dt \\ &\quad - \sum_{i=1}^{J} \int_0^\infty \left(e_i^T(t) Q_i e_i(t) + u_i^T(t) R_i u_i(t)\right) dt \end{aligned} \tag{33}$$

implying that $\sum_{i=1}^{J} \int_0^\infty (e_i^T(t) Q_i e_i(t) + u_i^T(t) R_i u_i(t)) dt \le \sum_{i=1}^{J} \gamma_i^2 \int_0^\infty \overline{w}_i^T(t) \overline{w}_i(t) dt$.

Let $Y_{2i} = P_{2i}^{-1}$, $\widetilde{E}_{i1} = Y_{2i}^T E_{i1} Y_{2i}$, $\widetilde{E}_{i2} = Y_{2i}^T E_{i2} Y_{2i}$, $\widetilde{E}_{ij3} = Y_{2i}^T E_{ij3} Y_{2i}$, $\widetilde{E}_{ij4} = Y_{2i}^T E_{ij4} Y_{2i}$, $\widetilde{E}_{i5} = E_{i5} Y_{2i}$. By Schur complement and premultiplying and postmultiplying to (29) by positive-define matrix $\mathrm{diag}[P_{2i}^{-T} \; P_{2i}^{-T} \; P_{2i}^{-T} \; P_{2i}^{-T} \; I \; P_{2i}^{-T}]$, $\mathrm{diag}[P_{2i}^{-1} \; P_{2i}^{-1} \; P_{2i}^{-1} \; P_{2i}^{-1} \; I \; P_{2i}^{-1}]$, respectively, we have

$$\widehat{\Pi}_{ijlm} = \begin{bmatrix} \widehat{\Pi}_{ijlm}^{11} & \widehat{\Pi}_{ijlm}^{12} & \widehat{\Pi}_{ijlm}^{13} & \widehat{\Pi}_{ijlm}^{14} & \widehat{\Pi}_{ijlm}^{15} & \widetilde{E}_{i1} \\ * & \widehat{\Pi}_{ijlm}^{22} & \widehat{\Pi}_{ijlm}^{23} & \widehat{\Pi}_{ijlm}^{24} & \widehat{\Pi}_{ijlm}^{25} & \widetilde{E}_{i2} \\ * & * & \widehat{\Pi}_{ijlm}^{33} & \widehat{\Pi}_{ijlm}^{34} & \widehat{\Pi}_{ijlm}^{35} & \widetilde{E}_{ij3} \\ * & * & * & \widehat{\Pi}_{ijlm}^{44} & \widehat{\Pi}_{ijlm}^{45} & \widetilde{E}_{ij4} \\ * & * & * & * & \widehat{\Pi}_{ijlm}^{55} & \widetilde{E}_{i5} \\ * & * & * & * & * & -\dfrac{\widetilde{T}_i}{\tau_{ji0}} \end{bmatrix}, \quad (34)$$

where

$$\begin{aligned}
&\widehat{\Pi}_{ijlm}^{11} = \delta_i \overline{A}_{ilm} Y_{2i} + \delta_i Y_{2i}^T \overline{A}_{ilm}^T + Y_{2i}^T \overline{Q}_i Y_{2i} + \widetilde{S}_{ii}, \\
&\widehat{\Pi}_{ijlm}^{12} = Y_{2i}^T \overline{A}_{ilm}^T + P_i - \delta_i Y_{2i}, \\
&\widehat{\Pi}_{ijlm}^{22} = -Y_{2i}^T - Y_{2i} + \tau_{ji0} \widetilde{T}_i, \\
&\widehat{\Pi}_{ijlm}^{13} = \widetilde{E}_{i1}, \qquad \widehat{\Pi}_{ijlm}^{23} = \widetilde{E}_{i2}, \qquad \widehat{\Pi}_{ijlm}^{33} = \widetilde{E}_{ij3} + \widetilde{E}_{ij3}^T, \\
&\widehat{\Pi}_{ijlm}^{14} = -\widetilde{E}_{i1} + \delta_i \overline{C}_{jil} Y_{2i}, \qquad \widehat{\Pi}_{ijlm}^{24} = -\widetilde{E}_{i2} + \overline{C}_{jil} Y_{2i}, \\
&\widehat{\Pi}_{ijlm}^{34} = -\widetilde{E}_{ij3} + \widetilde{E}_{ij4}^T, \qquad \widehat{\Pi}_{ijlm}^{15} = \delta_i \overline{B}_{il}, \quad \widehat{\Pi}_{ijlm}^{25} = \overline{B}_{il}, \\
&\widehat{\Pi}_{ijlm}^{44} = -\widetilde{E}_{ij4} - \widetilde{E}_{ij4}^T - \left(1 - \widetilde{H}_{ij}\right) \widetilde{S}_{ji}, \\
&\widehat{\Pi}_{ijlm}^{35} = \widetilde{E}_{i5}^T, \qquad \widehat{\Pi}_{ijlm}^{45} = -\widetilde{E}_{i5}^T, \qquad \widehat{\Pi}_{ijlm}^{55} = -\gamma_i^2 I, \\
&\overline{Q}_i = \mathrm{diag}\left[0 \;\; Q_i + \left(K_i^l\right)^T R_i K_i^l\right], \\
&\widetilde{T}_i = Y_{2i}^T T_i Y_{2i}, \qquad \widetilde{S}_{ji} = Y_{2i}^T S_j Y_{2i}.
\end{aligned} \quad (35)$$

Denoting $W_{2i}^m = K_i^m Y_{2i}^3$, $Y_{2i}^4 = \eta_i Y_{2i}^3$, $\widetilde{E}_{i1} = \begin{bmatrix} E_{i1}^{11} & E_{i1}^{12} \\ E_{i1}^{21} & E_{i1}^{22} \end{bmatrix}$, $\widetilde{E}_{i2} = \begin{bmatrix} E_{i2}^{11} & E_{i2}^{12} \\ E_{i2}^{21} & E_{i2}^{22} \end{bmatrix}$, $\widetilde{E}_{ij3} = \begin{bmatrix} E_{ij3}^{11} & E_{ij3}^{12} \\ E_{ij3}^{21} & E_{ij3}^{22} \end{bmatrix}$, $\widetilde{E}_{ij4} = \begin{bmatrix} E_{ij4}^{11} & E_{ij4}^{12} \\ E_{ij4}^{21} & E_{ij4}^{22} \end{bmatrix}$, $\widetilde{E}_{i5} = \begin{bmatrix} E_{i5}^{11} & E_{i5}^{12} \\ E_{i5}^{21} & E_{i5}^{22} \end{bmatrix}$, $P_i = \begin{bmatrix} P_i^1 & P_i^2 \\ * & P_i^3 \end{bmatrix}$, $\widetilde{T}_i = \begin{bmatrix} T_i^{11} & T_i^{12} \\ * & T_i^{22} \end{bmatrix}$, $\widetilde{S}_{ji} = \begin{bmatrix} S_{ji}^{11} & S_{ji}^{12} \\ * & S_{ji}^{22} \end{bmatrix}$, $\widetilde{S}_{ii} = \begin{bmatrix} S_{ii}^{11} & S_{ii}^{12} \\ * & S_{ii}^{22} \end{bmatrix}$, $Y_{2i} = \begin{bmatrix} Y_{2i}^1 & Y_{2i}^2 \\ Y_{2i}^3 & Y_{2i}^4 \end{bmatrix}$, using (13), and by Schur complement, the proof is completed. □

Remark 8. Firstly, utilizing the descriptor model transformation, LMIs-based conditions with some free variables are derived. Secondly, when $\overline{w}_i(t) = 0$, compared with the stable conditions in the sense of Lyapunov in [13], we obtain the asymptotically stable conditions. Finally, compared with [16], the time-varying delay fuzzy large systems are considered, and a more practical H_∞ performance index is considered. Therefore, obtained results are new and less conservative.

If $J = 1$, then the T-S model-based interconnected systems (13) can be transformed into a common T-S system as follows:

$$\dot{\overline{x}}(t) = \sum_{l=1}^{r} \sum_{m=1}^{r} \mu_l \mu_m \left(\overline{A}_{lm} \overline{x}(t) + \overline{B}_l \overline{w}(t)\right), \quad (36)$$

where

$$\overline{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \qquad \overline{w}(t) = \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}, \qquad \overline{B}_l = \begin{bmatrix} B_1^l & 0 \\ B_1^l & -B_r \end{bmatrix}, \quad (37)$$

$\overline{A}_{lm} = \sum_{L=1}^{2^{m_1 - l_1}} \begin{bmatrix} A_l & B_2^l \lambda_L K_m \\ A_l - A_r & A_r + B_2^l \lambda_L K_m \end{bmatrix}$, and l_1 denotes the number of no failure actuators.

Corollary 9. *For the given constant δ, η, if there exist matrices Y_1, Y_2, Y_3, Y_4, P_2 and positive definite matrices W_2^m, P_1, P_3, satisfy* (38), *then the T-S fuzzy system* (36) *is asymptotically stable with the H_∞ tracking performance index γ, and the gain of controllers $K_m = W_2^m Y_3^{-1}$, $l, m = 1, \ldots, r$, $L = 1, \ldots, 2^{m_i - l_1}$*

$$\begin{aligned} &\min \quad \gamma \\ &\textit{subject to} \quad (39) \textit{ and } (40), \end{aligned} \quad (38)$$

$$\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0, \quad (39)$$

$$\begin{bmatrix} \overline{\Pi}^{11} & \overline{\Pi}^{12} & \overline{\Pi}^{13} & \overline{\Pi}^{14} & \delta B_1^l & 0 & Y_3 & \overline{\Pi}^{18} \\ * & \overline{\Pi}^{22} & \overline{\Pi}^{23} & \overline{\Pi}^{24} & \delta B_1^l & -\delta B_r & \eta_i Y_3 & \overline{\Pi}^{28} \\ * & * & \overline{\Pi}^{33} & \overline{\Pi}^{34} & B_1^l & 0 & 0 & 0 \\ * & * & * & \overline{\Pi}^{44} & B_1^l & -B_r & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & -Q^{-1} & 0 \\ * & * & * & * & * & * & * & \overline{\Pi}^{88} \end{bmatrix} < 0, \quad (40)$$

where

$$\begin{aligned}
\overline{\Pi}^{11} &= \delta A_l Y_1 + \delta \left(A_l Y_1\right)^T + \delta B_2^l \lambda_L W_2^m + \delta \left(B_2^l \lambda_L W_2^m\right)^T, \\
\overline{\Pi}^{12} &= \delta A_l Y_2 + \delta \eta B_2^l \lambda_L W_2^m + \delta \left(\left(A_l - A_r\right) Y_1\right)^T \\
&\quad + \delta \left(A_r Y_3 + B_2^l \lambda_{Li} W_2^m\right)^T, \\
\overline{\Pi}^{22} &= \delta \left(A_l - A_r\right) Y_2 + \delta \left(\left(A_l - A_r\right) Y_2\right)^T \\
&\quad + \delta \eta \left(A_r Y_3 + B_2^l \lambda_L W_2^m\right) + \delta \eta \left(A_r Y_3 + B_2^l \lambda_L W_2^m\right)^T,
\end{aligned}$$

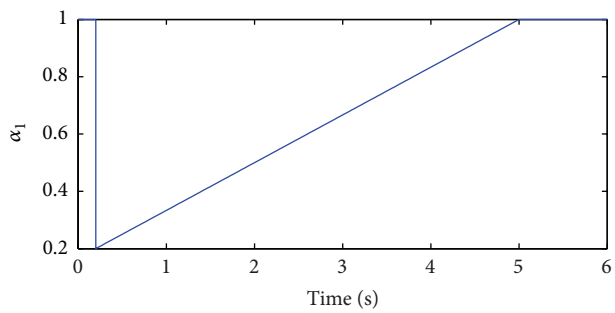

FIGURE 1: The actuator failure parameter.

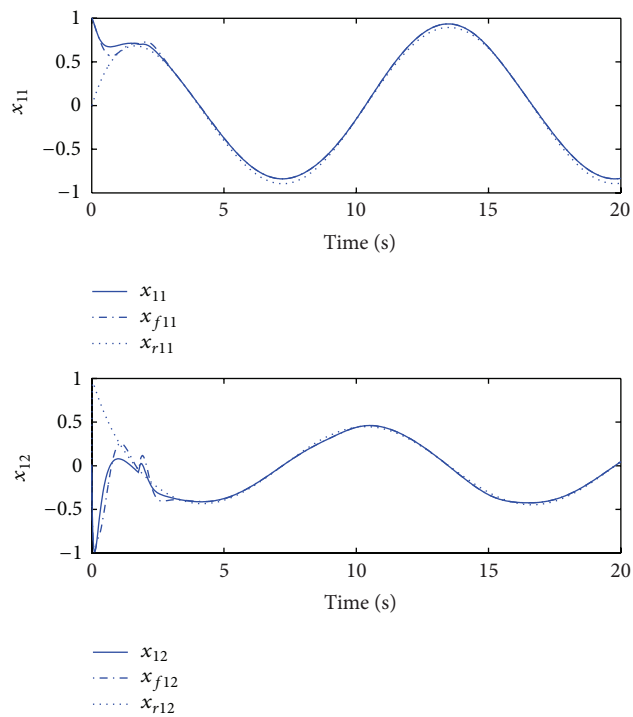

FIGURE 2: The states of subsystem 1.

$$\begin{aligned}
\overline{\Pi}^{13} &= (A_l Y_1)^T + \left(B_2^l \lambda_L W_2^m\right)^T + P_1 - \delta Y_1, \\
\overline{\Pi}^{14} &= ((A_l - A_r) Y_1)^T + \left(A_r Y_3 + B_2^l \lambda_L W_2^m\right)^T + P_2 - \delta Y_2, \\
\overline{\Pi}^{23} &= (A_l Y_2)^T + \eta\left(B_2^l \lambda_L W_2^m\right)^T + P_2^T - \delta Y_3, \\
\overline{\Pi}^{24} &= ((A_l - A_r) Y_2)^T + \eta\left(A_r Y_3 + B_2^l \lambda_L W_2^m\right)^T + P_3 - \delta\eta Y_3, \\
\overline{\Pi}^{33} &= -Y_1^T - Y_1, \qquad \overline{\Pi}^{34} = -Y_3^T - Y_2, \qquad \overline{\Pi}^{44} = -\eta Y_3^T - \eta Y_3, \\
\overline{\Pi}^{18} &= \left(W_2^l\right)^T, \qquad \overline{\Pi}^{28} = \eta\left(W_2^l\right)^T, \qquad \overline{\Pi}^{88} = -R^{-1}.
\end{aligned} \tag{41}$$

4. Illustrative Examples

Example 1. Consider the two-machine interconnected systems which are composed of two subsystems as follows [13]:

$$\begin{aligned}
\dot{x}_{i1}(t) &= x_{i2}(t), \\
\dot{x}_{i2}(t) &= -\frac{Dd_i}{M_{1i}} x_{i2}(t) + \frac{1}{M_{1i}} u_i(t) \\
&\quad + \sum_{j=1, j\neq i}^{2} \frac{E_{1i} E_{1j} Y_{ij}}{M_i} \\
&\quad \times \left(\cos\left(\delta_{ij}^0 - \theta_{ij}\right)\right. \\
&\quad - \cos\left(x_{i1}(t) - x_{j1}\left(t - \tau_{ij}(t)\right)\right. \\
&\quad \left.\left. + \delta_{ij}^0 - \theta_{ij}\right)\right) \\
&\quad + B_{1i} w_{i2}, \quad i = 1, 2.
\end{aligned} \tag{42}$$

We assume the two-machine interconnected systems' parameters as follows:

$$\begin{aligned}
&E_{11} = 1.017, \qquad E_{12} = 1.005, \qquad M_{11} = 1.03, \\
&M_{12} = 1.25, \qquad Dd_1 = 0.8, \qquad Dd_2 = 1.2, \\
&Y_{12} = Y_{21} = 1.98, \qquad \theta_{12} = -\theta_{21} = 1.5, \\
&\delta_{12}^0 = -\delta_{21}^0 = 1.2, \qquad B_{11} = B_{12} = \operatorname{diag}[1, 1], \\
&\tau_{12}(t) = \tau_{21}(t) = 0.6 \sin^2(t) + 1.2.
\end{aligned} \tag{43}$$

The systems are approximated by the following nine-rule fuzzy model which is the same as [13], and the details of the rules are omitted here.

Other parameters are shown as follows:

$$\begin{aligned}
A_{ri} &= \begin{bmatrix} 0 & 1 \\ -100 & -101 \end{bmatrix}, \\
B_{ri} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
v_1(t) &= \begin{bmatrix} 0 \\ 100\cos(0.5t) \end{bmatrix}, \\
v_2(t) &= \begin{bmatrix} 0 \\ 100\sin(0.5t) \end{bmatrix}.
\end{aligned} \tag{44}$$

Here, we assume that there exists the actuator failure only in the actuator of the first subsystem. Let $\underline{\alpha}_1 = 0.2$, $\overline{\alpha}_1 = 1$, $\underline{\alpha}_2 = \overline{\alpha}_2 = \alpha_2 = 1$; that is $\lambda_{11} = 0.2$, $\lambda_{21} = 1$, $\lambda_{12} = 1$. The actuator failure parameter α_1 is shown in Figure 1.

Let $\delta_1 = \delta_2 = 1$, $\eta_1 = \eta_2 = 1$, $Q_1 = Q_2 = \operatorname{diag}(1, 1)$, $R_1 = R_2 = 0.02$, according to the conditions proposed in Theorem 7; we have $K_1^l = [-42.6186 \quad -18.7306]$, $K_2^l = [-13.7741 \quad -8.6137]$, $l = 1, \ldots, 9$. For the initial condition $x_1(0) = [-1 \quad 3]^T$, $x_2(0) = [1 \quad -2]^T$ and the disturbances $w_{12}(t) = \sin(2\pi t)e^{-0.1t}$, $w_{22}(t) = \cos(2\pi t)e^{-0.1t}$, the tracking trajectories are shown in Figures 2 and 3, in which x_{rij}, $i =$

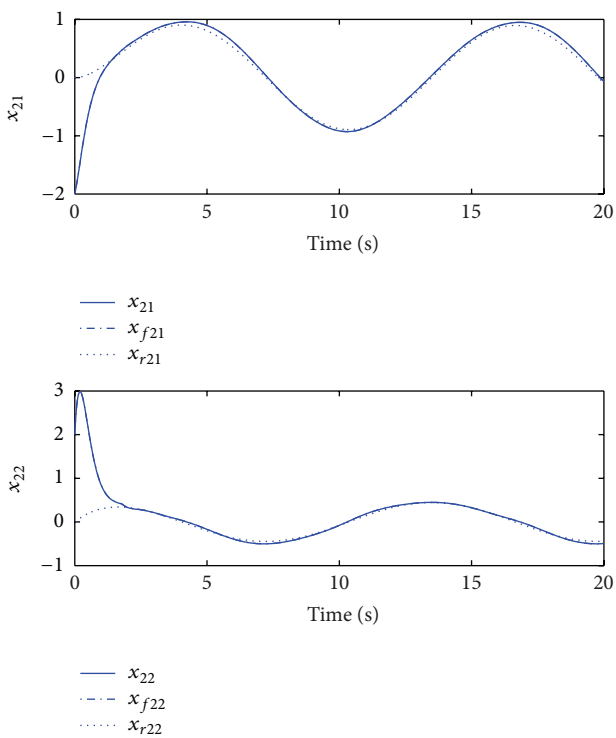

FIGURE 3: The states of subsystem 2.

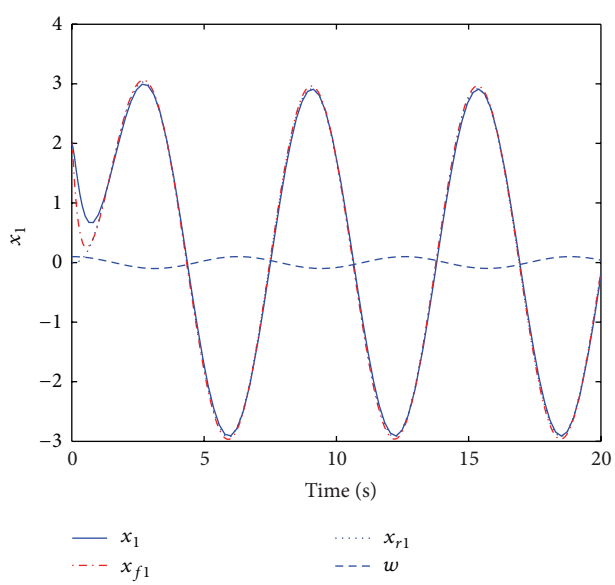

FIGURE 4: The trajectories of the state x_1, reference state x_{r1}, the state x_{f1} using [16], and the disturbance w.

$1, 2$, $j = 1, 2$, denote the states of the reference model, x_{ij}, $i = 1, 2$, $j = 1, 2$, denote the states when there are no failure actuators, and $x_{fij}, i = 1, 2$, $j = 1, 2$, denote the states when there exist partial failure actuators for $t \in [0.2, 5]$. It can be seen that there are nearly no differences between x_{fij} and x_{ij}. Therefore, when there exist the actuator failures, the performances of the system are not influenced. Compared with the result $K_1^l \approx [-140 \quad -67]$, $K_2^l \approx [-183 \quad -85]$,

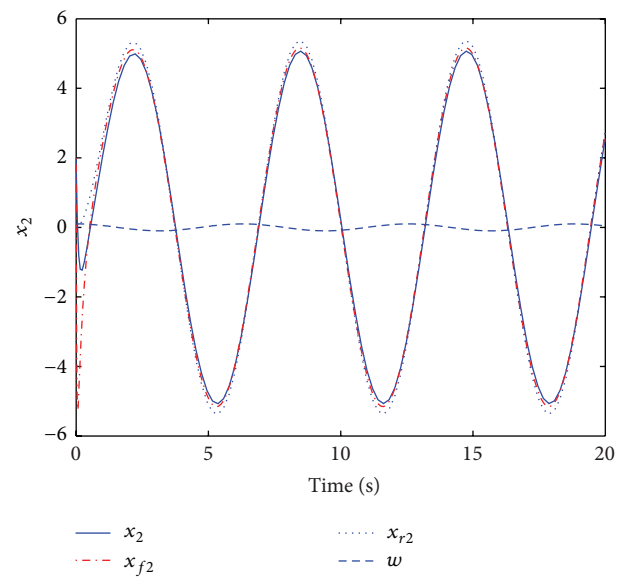

FIGURE 5: The trajectories of the state x_2, reference state x_{r2}, the state x_{f2} using [16], and the disturbance w.

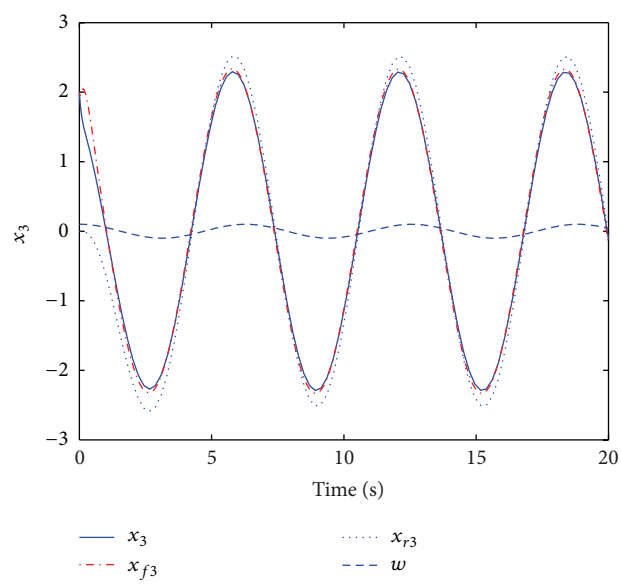

FIGURE 6: The trajectories of the state x_3, reference state x_{r3}, the state x_{f3} using [16], and the disturbance w.

$l = 1, \ldots, 9$ in [13], the gains of controllers are much smaller, and the control performances are nearly the same.

Example 2. Consider the following tracking system [16]:

$$\dot{x}(t) = \begin{bmatrix} -1.4 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w(t),$$

$$\dot{x}_r(t) = \begin{bmatrix} -1 & 1 & 0.6 \\ -0.5 & -1 & 0 \\ -1 & -1 & -3 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 8\sin(t) \\ 0 \end{bmatrix}. \tag{45}$$

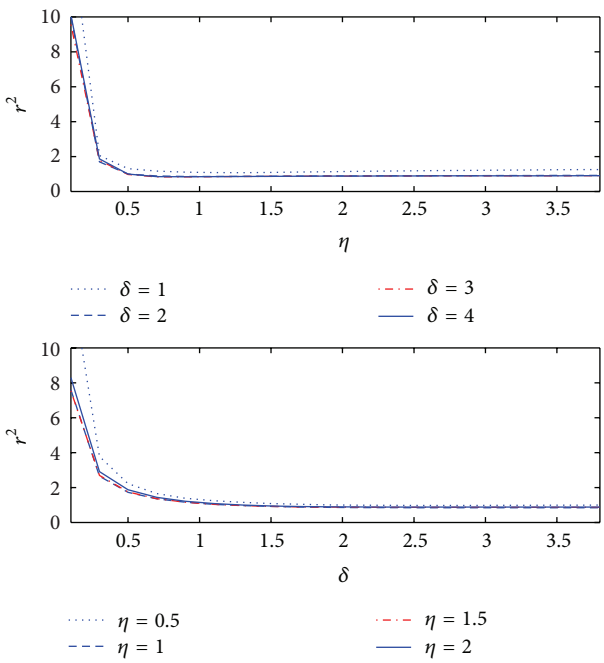

FIGURE 7: The trajectories of γ^2 about δ and η.

Let $\underline{\alpha} = 0.2$, $\overline{\alpha} = 1$, the actuator failure parameter α is the same as Example 1, $\delta = 1$, $\eta = 0.8$, $Q = \mathrm{diag}(1, 1, 1)$, $R = 0.01$, according to the conditions proposed in Corollary 9; we have $\gamma^2 = 1.1291$, $K = [-2.9191 \quad -12.6548 \quad -3.3239]$. For the initial condition $x(0) = [2 \quad 2]^T$, $x_r(0) = [0 \quad 0]^T$, and the disturbance $w(t) = 0.1\cos(t)$, the tracking trajectories are shown in Figures 4, 5, and 6. Compared with the result $\gamma^2 = 1.0751$, $K = 10^3 * [-4.1688 \quad -9.1600 \quad -1.3698]$ in [16], the gain of controller is much smaller, and the control performances are nearly the same. For the different δ, η, the trajectories of γ^2 are shown in Figure 7.

5. Conclusion

This paper investigates the robust and reliable decentralized H_∞ fuzzy tracking control issue for the fuzzy interconnected systems with time-varying delay, which consist of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then the new LMIs-based conditions with some free variables are derived to guarantee the H_∞ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

Conflict of Interests

The authors declared that they have no conflicts of interest to this work.

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