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Research Article

Optimization of Natural Frequencies and Sound Power of Beams Using Functionally Graded Material

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This paper presents a design method to optimize the material distribution of functionally graded beams with respect to some vibration and acoustic properties. The change of the material distribution through the beam length alters the stiffness and the mass of the beam. This can be used to alter a specific beam natural frequency. It can also be used to reduce the sound power radiated from the vibrating beam. Two novel volume fraction laws are used to describe the material volume distributions through the length of the FGM beam. The proposed method couples the finite element method (for the modal and harmonic analysis), Lumped Parameter Model (for calculating the power of sound radiation), and an optimization technique based on Genetic Algorithm. As a demonstration of this technique, the optimization procedure is applied to maximize the fundamental frequency of FGM cantilever and clamped beams and to minimize the sound radiation from vibrating clamped FGM beam at a specific frequency.

1. Introduction

Functionally graded materials (FGMs) are new composite materials that are fabricated to have spatially continuous variation of material properties. This variation of material properties is achieved by the gradient variation of the relative volume fraction of its material constituents within the solid. The main advantage of FGMs over the laminated composite materials is that the material properties in FGMs are changed seamlessly and smoothly throughout the length or the depth of the structure, whereas the material properties of laminated composite are discontinuous resulting in an abrupt change in mechanical properties between layers which can result in delamination and cracks [1]. The concept of FGM was first introduced in Japan in 1984 during the space-plane project where the materials used were required to survive in environments with high-temperature gradients. The FGMs are widely used in engineering applications such as spacecraft heat shields, rocking-motor casings, critical engine components, and the biomedical industry. The processing and manufacturing of FGMs is addressed by others [2–7] and is not considered to be within the scope of this paper.

Several studies have considered the dynamic behavior of FGM beams. For example, Lambros et al. [8] investigated the optimal design of uniform nonhomogeneous beams. They varied the modulus of elasticity distribution through the beam length, assuming constant density, to maximize its fundamental frequency. Sankar [9] presented an elasticity solution of an FGM beam in which Young's modulus and the density are assumed to vary through the thickness in an exponential manner. Qian and Ching [10] proposed a solution for static and dynamic behavior of FGM cantilever beam based on the meshfree local Petrov-Galerkin method. In this case, the material distribution varies along the thickness and the length of the beam according to a simple power law. Aydogdu and Taskin [11] studied the free vibration analysis of FG simply supported beams. They assumed that the material properties vary in the thickness direction according to exponential and power laws. Ying et al. [12] studied the bending and free vibration of functionally graded beams resting on an elastic foundation. They presented two-dimensional elasticity solution in which trigonometric series are adopted to transfer the partial differential state equation into an ordinary differential equation. Sina et al. [13] presented a theory to analyze the free vibration of FGM beams. They assumed that the material properties vary through the thickness according to a simple power law. Recently, Alshorbagy et al. [14] investigated the vibration of FGM beams using finite element method. The material constituents of these beams were assumed to vary through the thickness or the length of the beam according to a simple power law. They investigated the effects of boundary conditions, power law exponent index, and beam slenderness ratio on the free vibration characteristics of these beams.

Similar to other beam structures, FGM beams may be subjected to dynamic excitations resulting from unbalanced rotating or oscillating equipment. Reducing the vibration and noise which is radiated from such structures is important in engineering design. There are different approaches to improve the vibration and acoustic characteristics of structures. One approach is to shift the natural frequencies away from the frequency of the excitation force. For example, Qian and Batra [15] considered maximizing the first two natural frequencies of a cantilever FGM plate based on a higher-order shear and normal deformable plate theory and a meshless local Petrov-Galerkin method. They assumed the material distributed according to a simple power law along the thickness (z) and the length (x) directions as

$$V(x,z) = \left(\frac{1}{2} + \frac{z}{h}\right)^p \left(\frac{x}{L}\right)^q, \tag{1}$$

where V is the volume fraction of one of the constituent materials, h is the thickness of the plate, L is the length of the plate, and p and q are the indices of the power law which control the volume fraction profile of the constituent material in the xz-plane. Their material fraction distribution model has two optimization variables only which are the indices of the power law. They concluded that the materials distribution should be varied only along the length of the plate rather than along the thickness direction. Goupee and Vel [16] proposed a method to optimize the natural frequencies of FGM thick beam using element-free Galerkin method. They maximized each of the first three natural frequencies of the thick beam. The beam is analyzed as a two-dimensional body with plain strain assumption. Their design variables are the volume fraction values at a number of grid points. Due to the material property discontinuities between grids, they used third order transition functions to obtain a smooth transition in material properties.

In addition to shifting the natural frequencies, material tailoring has been used to minimize the radiation of sound from vibrating beams. Material tailoring of structures to achieve a minimum sound radiation at a single frequency was studied first by Naghshineh and Koopmann [17]. In their approach, the radiation problem was decoupled from the structural vibration problem. They found the optimal surface velocity distribution, which produced the minimum radiation condition (weak radiator), and then they tailored the modulus of elasticity and density of a beam structure in discrete segments such that it would vibrate as a weak radiator. Marburg et al. [18] investigated the minimization of sound radiation from finite beams over a frequency range of interest. They optimized the distribution of density and modulus of elasticity to minimize the sound power of vibrating beams over a wide frequency range containing about eight natural frequencies. In both [17, 18], the beams are divided into segments, and the material properties of each segment are independent; that is, the modulus of elasticity and the material density are not physically related and continuity conditions are not enforced. To the best of the authors' knowledge, no study has been conducted to date investigating the minimization of the sound radiation from vibrating FGM beams.

The objective of this work is to demonstrate that one can maximize the fundamental frequency of FGM beams and to minimize the radiated sound power from vibrating FGM beams. As mentioned above, researchers have studied FGMs by applying the classical power law or exponential law to describe the distribution of volume fractions through the structure. This paper also presents two novel laws for describing the volume fractions through an FGM beam that give engineers a powerful tool for design optimization. The first one is a more complex power law and the second one is a trigonometric law.

In this paper, the natural frequencies and the vibration response of the FGM beams are computed using the finite element method. Then, the radiated sound power from the vibrating beam is calculated based on the Lumped Parameter Model (LPM) [19]. Finally, an optimizer based on the method of Genetic Algorithm (GA) is used to determine the optimal profiles of volume fraction of beam constituent materials with the goal of maximizing the fundamental frequency or minimizing the sound radiation of FGM beams.

The paper is organized as follows. The next section will give the description of the FGM, the proposed material distribution laws, and the governing equations for the vibration and acoustic analysis. Then, optimization examples are described and their results are reported in Section 3. Finally, our conclusions are presented in Section 4.

2. Theoretical Background

2.1. Functionally Graded Material. Consider a beam made of functionally graded material which consists of a mixture of two materials V_1 and V_2, where the combination of these two materials (V_1+V_2) makes up one hundred percent of the beam material. Thus, we assume that each of these variables has a range between zero and one (i.e., $0 \leq V_1 \leq 1$ and $0 \leq V_2 \leq 1$) and $V_1 + V_2 = 1$. At each position, x, constituent 1 (V_1) is defined first and the volume fraction of constituent 2 (V_2) is then found. The beam material is graded along its length; the volume fractions of the constituents are continuous functions of the beam longitudinal axis. Using computer-aided manufacturing techniques makes it possible to produce FGMs with complex parameters and multidimensional gradients. FGMs with material properties varying in the axial direction can be

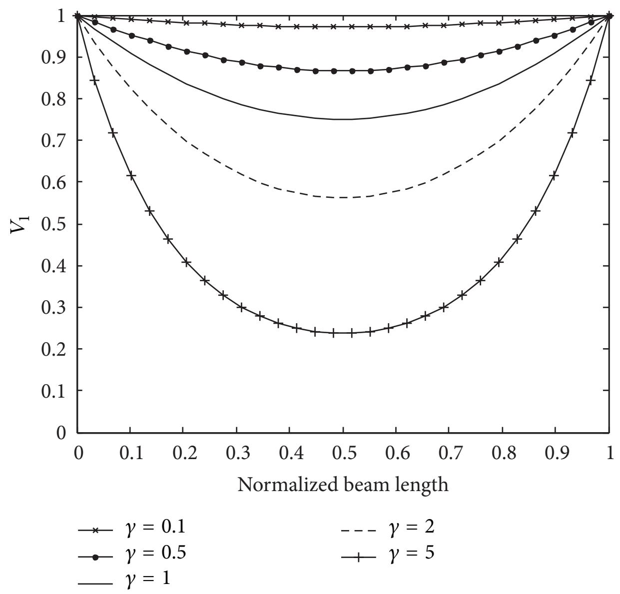

Figure 1: Variation of the volume fraction profile (power law of (2), $C = \alpha = 1$ and $\beta = 2$).

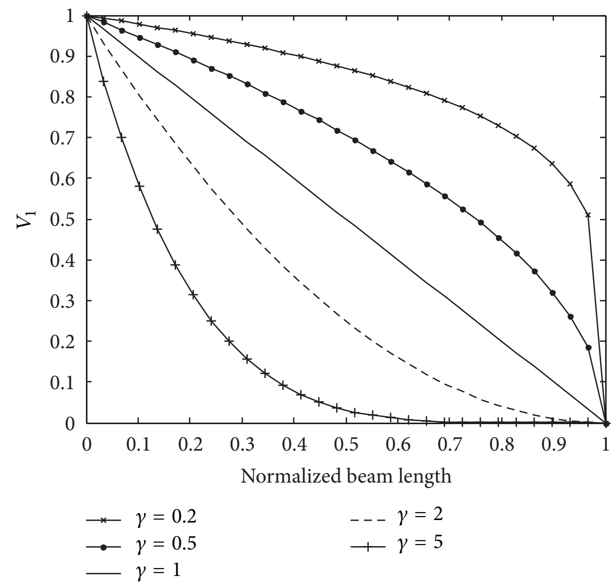

Figure 2: Variation of the volume fraction profile (power law of (2), $\alpha = 0$).

manufactured by ultraviolet irradiation [8]. In this work, it is proposed that the volume fraction of constituent 1 follows four-parameter power law distribution or four-parameter trigonometric distribution:

$$V_1 = C\left[1 - \left(\frac{x}{L}\right) + \alpha\left(\frac{x}{L}\right)^{\beta}\right]^{\gamma}, \quad (2)$$

or

$$V_1 = C\left[\frac{1}{2} - \frac{\alpha}{2}\sin\left(\frac{\eta\pi x}{L} + \phi\right)\right]^{\gamma}, \quad (3)$$

where the parameters C, α, β, γ, η, and ϕ control the volume fraction variation through the FGM beam length. The values of these parameters must be chosen so that $0 \le V_1 \le 1$. For example, assuming $C = 1$ and $\gamma = 0$ in (2), then $V_1 = 1$; that is, the whole beam is made from constituent 1. On the other hand, assuming $C = 0$ in (2), then $V_1 = 0$; that is, the whole beam is made from constituent 2. Another example is that, assuming $C = \alpha = 1$ and $\beta = 2$ in (2), the material fraction profile is shown in Figure 1. As shown in Figure 1, the volume fraction of constituent 1 decreases from 1 at the end to a minimum value at $x = L/2$. The classical power law distribution can be obtained by assuming $\alpha = 0$ in (2) as shown in Figure 2. Some volume fraction profiles which are described by (3) are shown in Figures 3, 4, and 5. As can be seen from Figures 1–5, (2) and (3) provide a flexible description of material volume fraction profile which gives engineers a powerful tool for design optimization as will be shown later in this paper.

The finite element method is employed to model the FGM beam. The length of the beam is divided into equal lengths elements. Each element has two nodes and three degrees of freedom at each node as shown in Figure 6 (i.e., the degrees of freedom are the transverse and axial displacements and the slope). Each element was modeled as a homogeneous material with properties assigned according to the FGM properties at its centroid.

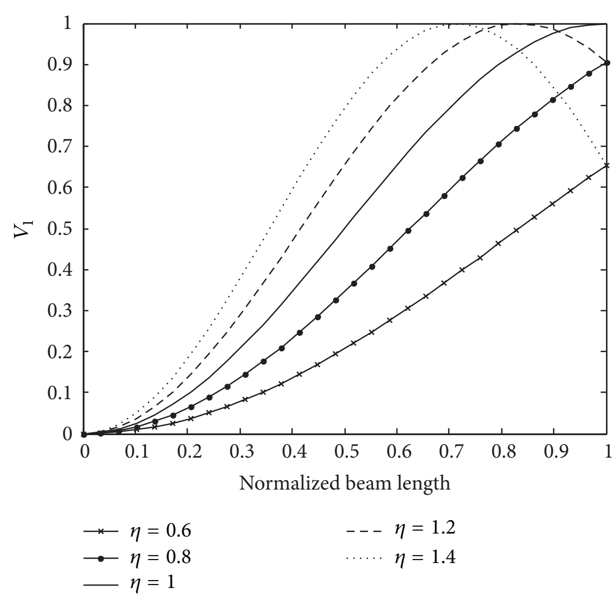

Figure 3: Variation of the volume fraction profile (trigonometric law of (3), $C = \alpha = \gamma = 1$ and $\phi = \pi/2$).

2.2. Estimation of Mechanical Properties of FGMs. Some papers [14, 20, 21] present the FGM properties in terms of modulus of elasticity and density in a specific direction. However, in this paper we have described the FGM structural materials in terms of the volume fraction of its constituents. In order to design one such FGM structure, it may be necessary to obtain an approximation of its modulus of

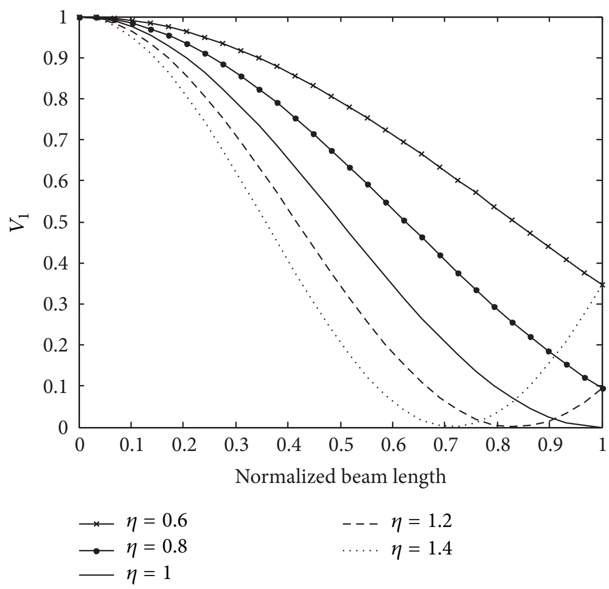

Figure 4: Variation of the volume fraction profile (trigonometric law of (3), $C = \alpha = \gamma = 1$ and $\phi = -\pi/2$).

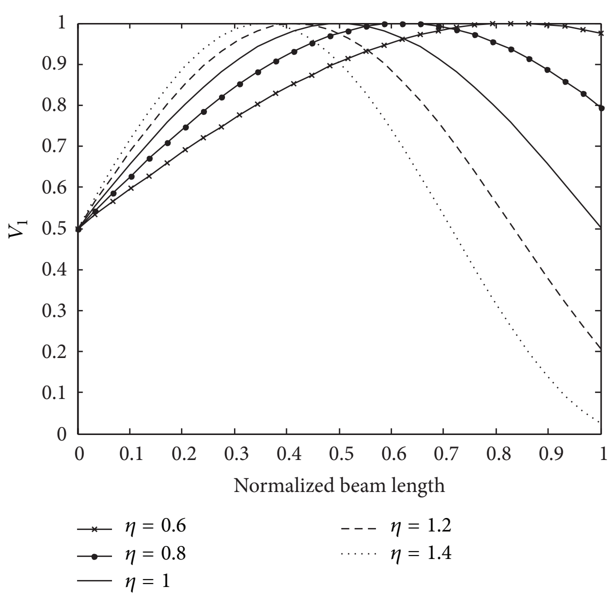

Figure 5: Variation of the volume fraction profile (trigonometric law of (3), $C = \alpha = \gamma = 1$ and $\phi = 0$).

elasticity and density as a function of the coordinate x. Thus, we have utilized a homogenization method similar to that used by Mori and Tanaka [22] in order to estimate the effective material moduli at any point in the FGM beam. The Mori-Tanaka method is based on a micromechanical model [22] where the effective local bulk modulus K and effective local shear modulus G at any point in the FGM are found from

$$\frac{K - K_1}{K_2 - K_1} = \frac{V_2}{1 + 3\left(1 - V_2\right)\left(K_2 - K_1\right) / \left(3K_1 + 4G_1\right)}, \quad (4)$$

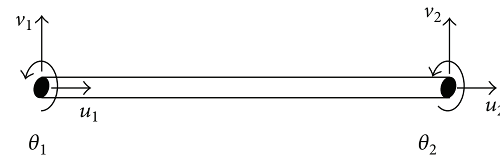

Figure 6: Beam element.

$$\frac{G - G_1}{G_2 - G_1} = \frac{V_2}{1 + \left(1 - V_2\right)\left(G_2 - G_1\right) / \left(G_1 + f_1\right)}, \quad (5)$$

where

$$f_1 = \frac{G_1\left(9K_1 + 8G_1\right)}{6\left(K_1 + 2G_1\right)}. \quad (6)$$

In (4)–(6), the subscripts 1 and 2 refer to constituent 1 and constituent 2, respectively. Having found the effective bulk modulus K and the effective shear modulus G at any point along the length of the beam, the effective modulus of elasticity at a point can then be calculated using

$$E = \frac{9KG}{3K + G}. \quad (7)$$

The mass density at a point in the FGM can be calculated by the rule of mixtures

$$\rho = \rho_1 V_1 + \rho_2 V_2, \quad (8)$$

where ρ_1 and ρ_2 are the densities of constituent 1 and constituent 2, respectively.

2.3. Structural and Acoustic Analysis. Finite element method is used here in order to perform the modal as well as harmonic analysis of the FGM beams. For the free vibration, the finite element method provides the following system of linear equations which is solved as an eigenvalue/eigenvector problem

$$\left[K - \omega_i^2\ M\right]\{\phi_i\} = \{0\}, \quad (9)$$

where $[K]$ is the global stiffness matrix of the beam, $[M]$ is the global mass matrix of the beam, $\{\phi_i\}$ is the mode shape of the ith mode, and ω_i is the natural frequency of the ith mode. To calculate the radiated sound power level of a vibrating beam, the velocity distribution throughout the length of the beam is required. The nodal velocity vector of an excitation frequency ω can be calculated as a function of the nodal excitation frequency vector as [23]

$$\{v\} = \sum_{i=1}^{N} \frac{\{\phi_i\}\{\phi_i\}^T\{F\}}{\omega_i^2 - \omega^2 + j\eta_i\omega_i}, \quad (10)$$

where $\{F\}$ is the force vector, η_i is the damping loss factor corresponding to the ith mode, ω is the frequency of the excitation force, N is the total number of modes considered in the summation, and $j = \sqrt{-1}$. In order to simplify the study of the sound radiation from a vibrating FGM beam, we assume that the beam is located within an infinite rigid baffle. With reference to Figure 7, the sound pressure can be calculated by using Rayleigh Integral [24]

$$p(\vec{r}) = \frac{j\omega\rho}{2\pi} \int_S \frac{v_n(\vec{r}_s) e^{-jkR}}{R} dS, \tag{11}$$

where k is the acoustic wave number ($k = \omega/c$), c is the speed of sound in ambient fluid, ρ is the density of the surrounding fluid (i.e., air), $\vec{r}$ is the position vector of the observation point, $\vec{r}_s$ is the position vector of the elemental surface δS, $v_n(\vec{r}_s)$ is the normal velocity of δS, and $R = |\vec{r} - \vec{r}_s|$. The acoustic intensity at any point $\vec{r}_s$ is given by [24]

$$I(\vec{r}) = \frac{1}{2} \operatorname{Re}\{p(\vec{r}) v_n^*(\vec{r}_s)\}, \tag{12}$$

where Re{ } is the real part of the number inside the bracket and the v_n^* represents the conjugate complex of v_n. The radiated sound power can be calculated by integrating the intensity over a surface surrounding the radiating beam or the surface of the beam as

$$W = \frac{1}{2} \int_S \operatorname{Re}\{p v_n^*\} dS. \tag{13}$$

The sound power for some simple cases can be calculated analytically. However, numerical methods are commonly used for calculating the sound power in more complicated or real-world problems. Different approximate methods are used to estimate the radiated sound power such as Equivalent Radiated Power (ERP) [25], volume velocity (VV) approach [26], and Lumped Parameter Model (LPM) [19]. The calculation of sound power based on ERP and VV is faster than the calculation of sound power based on LPM. However, the LPM gives most accurate results among the three methods [25]. In this study, the LPM is used to estimate the sound power of vibrating FGM beams. LPM is based on dividing the radiated surface into elements and characterizing the amplitude of the radiation from each element by its volume velocity. The sound power based on LPM is given by [19]

$$W = \frac{1}{2} \sum_{\mu=1}^{N_e} \sum_{\nu=1}^{N_e} \mathfrak{R}_{\mu\nu} u_\mu^* u_\nu, \tag{14}$$

where N_e is the number of elements, $\mathfrak{R}_{\mu\nu}$ is the acoustic resistance of element μ to element ν, and u_μ and u_ν are the volume velocities of elements μ and ν, respectively. The acoustic resistance is given as

$$\mathfrak{R}_{\mu\nu} = \frac{k\rho c}{2\pi} \frac{\sin(kr_{\mu\nu})}{r_{\mu\nu}}, \tag{15}$$

where $r_{\mu\nu}$ is the distance between element μ and element ν centers. The singularity of (15) at the point where the source and receiver coincide has vanished by finding the limit as the distance between the elements goes to zero; then (15) can be written as [27]

$$W = \frac{k\rho c}{4\pi} \sum_{\mu=1}^{N_e} \left(k u_\mu^* u_\mu + \sum_{\nu=1}^{N_e} \frac{\sin(kr_{\mu\nu})}{r_{\mu\nu}} u_\mu^* u_\nu \right). \tag{16}$$

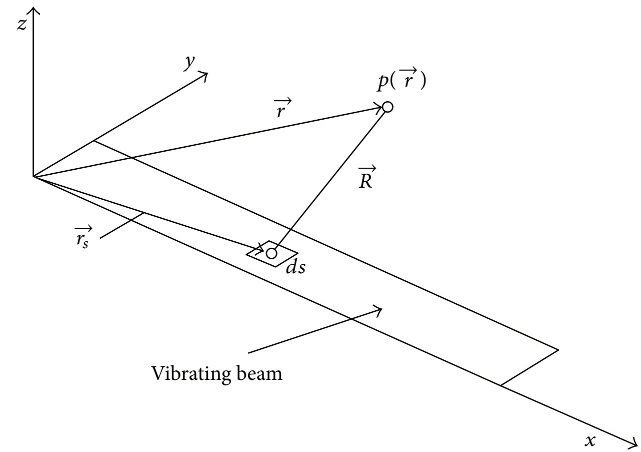

Figure 7: Coordinate system of a baffled beam radiator.

Table 1: Material properties of steel and aluminum.

Property	Steel	Aluminum
E (GPa)	210	70
ρ (kg/m^3)	7800	2700

3. Optimization Examples

Beams are widely used in many engineering applications. Similar to other structural members, beams are frequently subjected to dynamic excitations. Thus, reducing the vibration and noise radiation of such structures is a goal of many structural engineers. One approach to reduce the vibration and sound radiation is to shift the natural frequencies away from the frequency of the excitation force. In this paper, we are interested in shifting the natural frequencies of FGM beams, as well as in minimizing the radiated sound power from vibrating FGM beams. Genetic Algorithm (GA) is used as a stochastic and heuristic optimization method. The FGM beam considered in the optimization examples is composed of steel (constituent 1) and aluminum (constituent 2). The properties of steel and aluminum are listed in Table 1. It is important to note that there may be issues in actually fabricating a beam made of these two materials. These fabrication issues are considered beyond the scope of this paper and will not be addressed here. The choice of these materials is purely for the sake of the demonstration of the concept in our case.

3.1. Optimizing the Natural Frequencies of FGM Beams. In this section, examples are presented to show the efficiency of designing the FGM beams to maximize their natural frequencies. The optimal design of the FGM beam is based on the optimization of material distribution throughout beam length (i.e., the optimization of material distribution is achieved by optimizing the volume fractions of the material constituents). The first example shows the case of maximizing the fundamental frequency of an beam cantilever FGM, while, in the second example, the fundamental frequency of a clamped FGM beam is maximized. It should be noted that

Table 2: The optimum design parameters of cantilever FGM beam which maximize the fundamental frequency.

	Ω_1	C	α	γ	β	η	ϕ
Steel	0.5595	—	—	—	—	—	—
Aluminum	0.5491	—	—	—	—	—	—
FGM-1	0.7557	1.000	0.000	1.704	Any	—	—
FGM-2	0.8232	1.000	1.000	1.601	—	1.304	−2.086

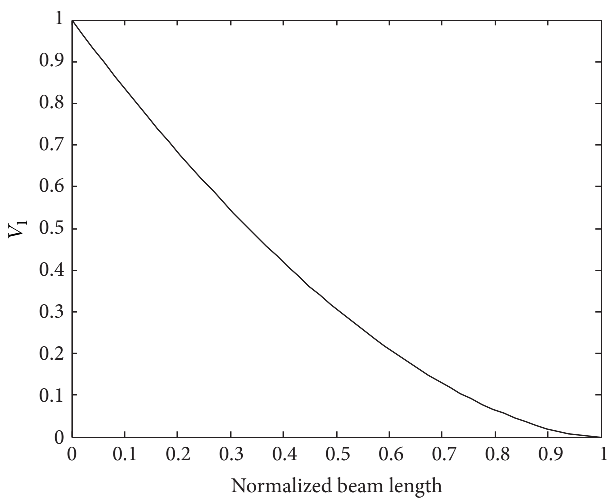

Figure 8: The steel volume fraction optimal distribution which maximizes the fundamental frequency of an FGM cantilever beam (power law of (2)).

we could have just as easily aimed to minimize these natural frequencies.

The results are presented in terms of the nondimensional frequency

$$\Omega = \omega L^2 \sqrt{\frac{\rho_1 A}{E_1 I}}, \tag{17}$$

where A is the beam cross-section area, I is the beam section-section moment of inertia, L is the beam length, ω is the natural frequency, and ρ_1 and E_1 are the density and modulus of elasticity of the steel, respectively.

In the first example, the objective is to find the optimal volume fraction of material constituents in order to maximize the fundamental frequency of a cantilever beam. In other words, we look for the best values of parameter C, α, β, and γ in the four-parameter power law distribution (see (2)) and the best values of parameters C, α, η, ϕ, and γ in the five-parameter trigonometric law distribution (see (3)) to maximize the fundamental frequency of the FGM cantilever beam. The beam under consideration has a length $L = 1$ m, width $b = 0.05$ m, and thickness $h = 0.01$ m. During optimization, the mass of the FGM beam is bounded by the mass of a uniform beam made of aluminum and steel, respectively (i.e., $m_{\text{Al. beam}} \leq m_{\text{FGM beam}} \leq m_{\text{St. beam}}$). The GA is used to find the optimal parameters in (2) and (3). The population in each generation is assumed to be 100. The stopping criterion is chosen to be a tolerance of less than 10^{-3} for the objective function. In general, the optimization problem is defined as follows:

$$\begin{aligned} &\text{Maximize} && \Omega_1 \\ &\text{Subject to} && 0 \leq V_1 \leq 1 \\ &&& C_{\min} \leq C \leq C_{\max} \\ &&& \alpha_{\min} \leq \alpha \leq \alpha_{\max} \\ &&& \gamma_{\min} \leq \gamma \leq \gamma_{\max} \\ &&& \beta_{\min} \leq \beta \leq \beta_{\max} \quad (\text{In } (2)) \\ &&& \eta_{\min} \leq \eta \leq \eta_{\max} \quad (\text{In } (3)) \\ &&& \phi_{\min} \leq \phi \leq \phi_{\max} \quad (\text{In } (3)). \end{aligned} \tag{18}$$

In this example, V_1 is the steel constitute, $C_{\min} = 0$, $C_{\max} = 1$, $\alpha_{\min} = 0$, $\alpha_{\max} = 1$, $\gamma_{\min} = 0$, $\gamma_{\max} = 15$, $\beta_{\min} = 1$, $\beta_{\max} = 15$, $\eta_{\min} = -2$, $\eta_{\max} = 2$, $\phi_{\min} = -\pi$, and $\phi_{\max} = \pi$. The optimal design parameters which maximize the fundamental frequency are shown in Table 2. The results depicted in the second column of Table 2 show that the fundamental frequency of the optimal FGM beam is greater than that of either steel or aluminum beam. Using the optimal material distribution based on the power law (2) shifts the fundamental frequency 35.1% and 37.6% higher than the fundamental frequency of a steel and an aluminum beam, respectively. The steel volume fraction optimal distribution that results in the largest fundamental frequency based on the power law (2) is shown in Figure 8. Using the optimal material distribution based on the trigonometric law (3) shifts the fundamental frequency 47.1% and 49.9% higher than the fundamental frequency of a steel and an aluminum beam, respectively. The steel volume fraction optimal distribution that results in the largest fundamental frequency based on the trigonometric law (3) is shown in Figure 9. As shown in Figures 8 and 9, the optimized FGM cantilever beam consists of steel near the fixed end and aluminum near the free end (i.e., stiffer and heavier material near the fixed end and more compliant and lighter material at the free end). The increased stiffness due to steel at the fixed end of the cantilever beam compensates the disadvantage of increased mass.

In the second example, the objective is to find the optimal volume fraction of material constituents in order to maximize the fundamental frequency of a clamped beam. In this example, a beam with the same dimensions as in the previous example is considered. The optimization problem is the same as the problem shown in (18). By using GA,

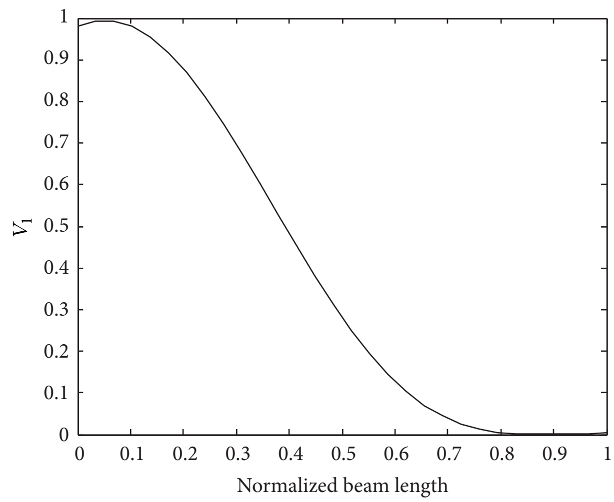

Figure 9: The steel volume fraction optimal distribution which maximizes the fundamental frequency of an FGM cantilever beam (trigonometric law of (3)).

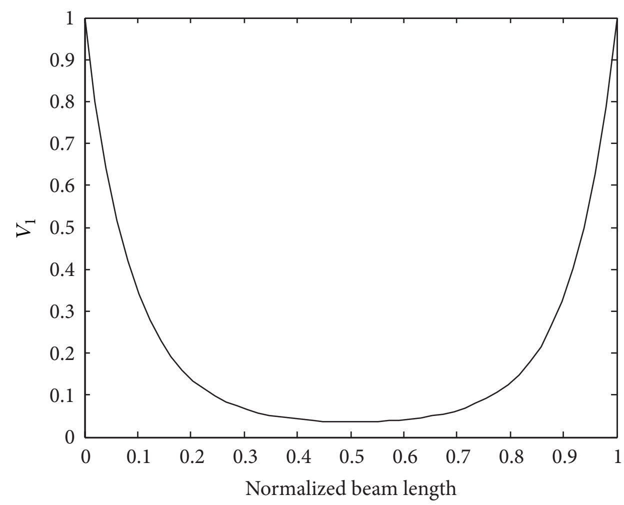

Figure 10: The steel volume fraction optimal distribution which maximizes the fundamental frequency of a beam FGM clamped (power law of (2)).

the optimal design parameters which yield the maximum fundamental frequency of the clamped beam are found and summarized in Table 3. The results depicted in the second column of Table 3 show that the fundamental frequency of the optimal FGM beam is greater than that of both steel and aluminum beam. Using the optimal material distribution based on the power law (2) shifts the fundamental frequency 14.8% and 17.8% higher than the fundamental frequency of a steel and an aluminum beam, respectively. The steel volume fraction optimal distribution that results in the largest fundamental frequency based on the power law (2) is shown in Figure 10. Using the optimal material distribution based on the trigonometric law (3) shifts the fundamental frequency 29.7% and 33.1% higher than the fundamental frequency of a steel and an aluminum beam, respectively. The steel volume fraction optimal distribution that results in the largest fundamental frequency based on the trigonometric law (3) is shown in Figure 11. As shown in Figures 10 and 11, the optimized clamped FGM mainly consists of steel near the fixed ends and aluminum near the middle of the beam. It can be concluded from the previous examples that using compliant and light material at the higher modal strain area and using stiffer and heavier material near the fixed ends have a significant effect on increasing the fundamental frequency. The areas near the fixed ends of the beams represent the areas of high bending moment. Thus, increasing the local stiffness at these areas has the greatest effect on the natural frequencies of the beam.

3.2. Minimizing the Radiated Sound Power of Vibrating FGM Beams. In this section, an example is presented to demonstrate the efficiency of designing the FGM in order to minimize the radiated sound power of vibrating clamped beams. In this example, we look for the design of a clamped FGM beam that exhibits a minimum sound radiation condition at a specific frequency. The beam under consideration has

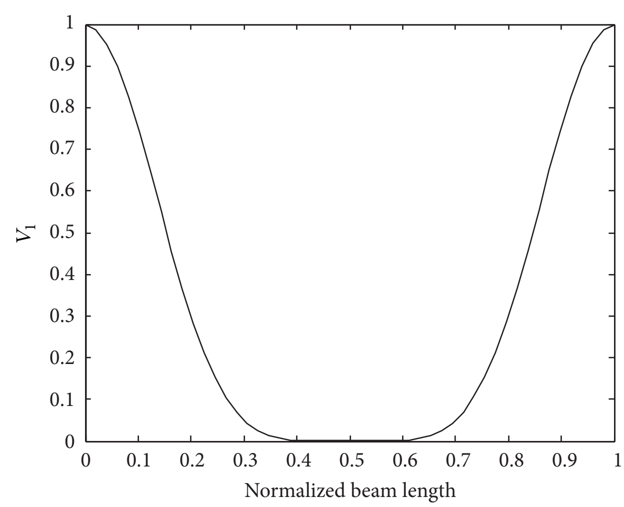

Figure 11: The steel volume fraction optimal distribution which maximizes the fundamental frequency of a beam FGM clamped (trigonometric law of (3)).

the same dimensions as the beams in the previous examples and it has a loss factor of 0.02. It is excited by a harmonic force at point $x = 0.35L$ (see Figure 12(a)), having a uniform amplitude of 5 N in the frequency range of 20–300 Hz. The location of the excitation is selected such that this force will excite all the modes within the frequency range of interest. The clamped beam is assumed to be in an infinite baffle. The surrounding medium is the air with 415 rayle acoustical impedance. The sound power is calculated numerically based on LPM by using (16). The predicted spectrum of sound power radiated from both steel and aluminum beam is shown in Figure 12(b) for the clamped boundary condition. The peaks in this plot represent the sound radiation at beam's natural frequencies. According to the finite element analysis, the first three modes of the steel beam occur at 53.3, 147.0,

Table 3: The optimum design parameters of clamped FGM beam which maximize the fundamental frequency.

	Ω_1	C	α	γ	β	η	ϕ
Steel	3.5603	—	—	—	—	—	—
Aluminum	3.4942	—	—	—	—	—	—
FGM-1	4.0869	1.000	1.000	11.010	2.066	—	—
FGM-2	4.6175	1.000	1.000	2.833	—	1.998	−1.567

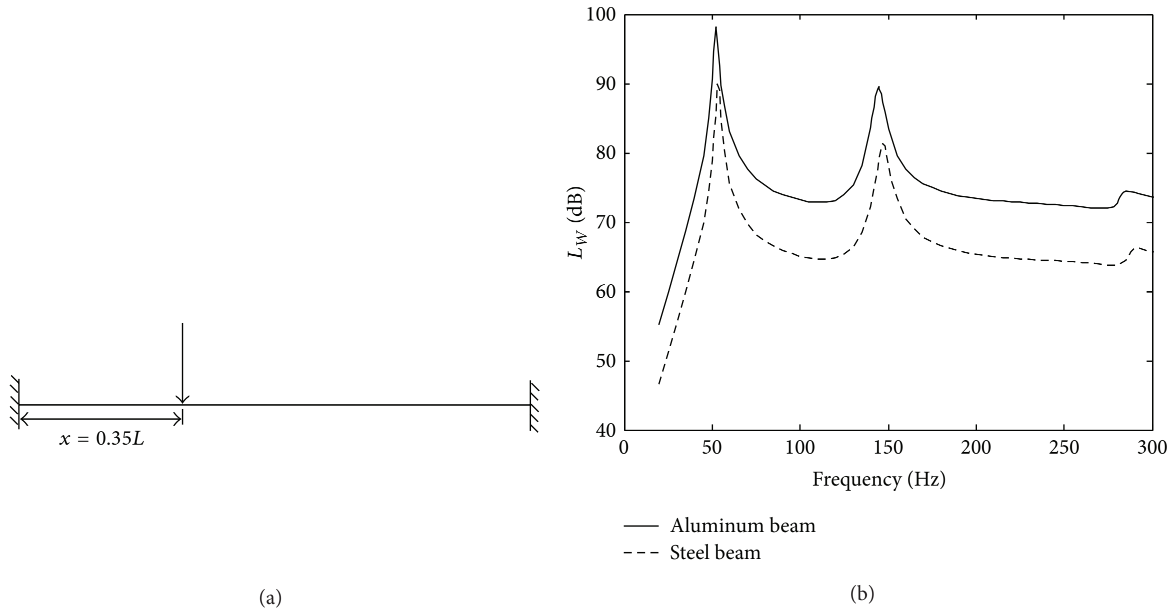

Figure 12: (a) Beam excited by a point force and (b) sound power level spectrum of the clamped steel and aluminum beams.

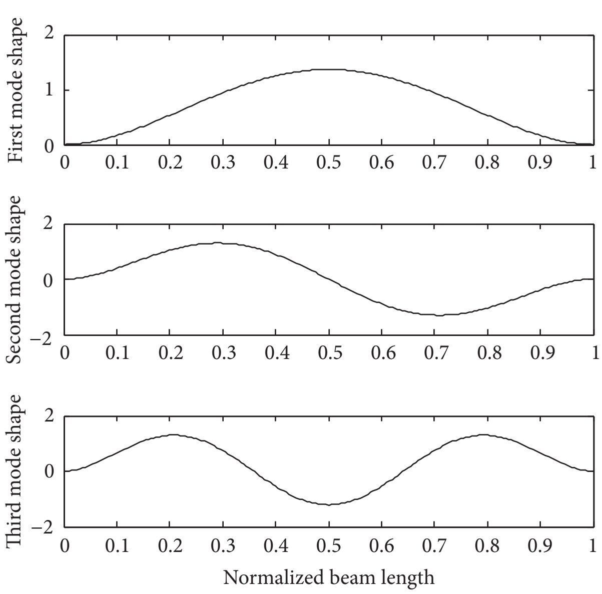

Figure 13: The first three mode shapes of clamped beams.

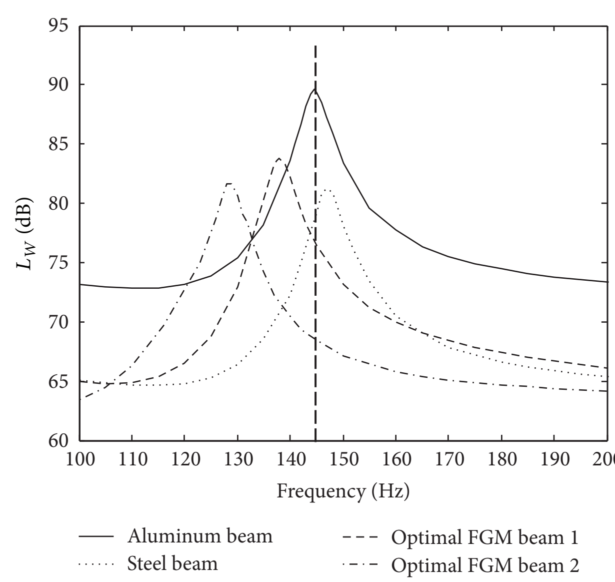

Figure 14: Sound power level spectrum of the optimized clamped beams at 145 Hz compared with steel and aluminum beams.

and 288.1 Hz, respectively, while the first three modes of the aluminum beam occur at 53.3, 144.3, and 282.7 Hz, respectively. Figure 13 shows the first three mode shapes of the steel and aluminum beams. The mode shapes will be the same for both beams since each of them is made of one material.

In this example, we aim to minimize the sound radiation from a clamped FGM beam at a frequency of 145 Hz by optimizing the volume fractions steel and aluminum fractions throughout the beam length. The optimization problem is

Table 4: The optimum design parameters of a baffled clamped FGM beam which minimize the sound radiation at 145 Hz.

	L_W (dB)	C	α	γ	β	η	ϕ
Steel	79.2	—	—	—	—	—	—
Aluminum	89.3	—	—	—	—	—	—
FGM-1	76.5	0.799	0.000	0.386	Any	—	—
FGM-2	68.4	0.999	1.000	0.512	—	1.989	1.761

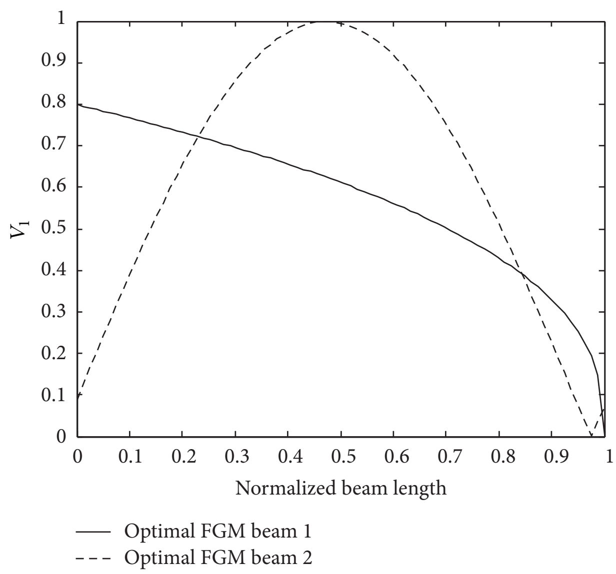

Figure 15: The steel volume fraction optimal distributions that result in the minimum sound power at 145 Hz.

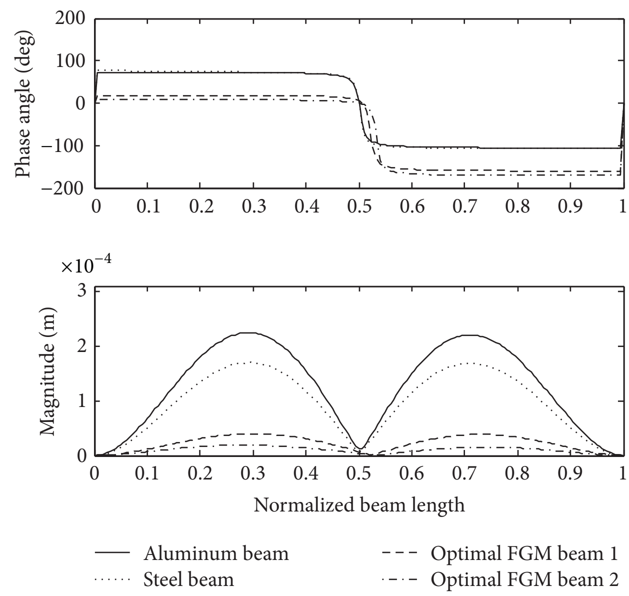

Figure 16: Comparison of the beam response in terms of lateral displacement at 145 Hz.

the same as the problem which is illustrated in (18), but with minimizing the sound power level (L_W) at 145 Hz instead of maximizing Ω_1. Figure 14 shows the sound power level of the optimized beams compared with those of steel and aluminum beams. The sound power level of steel beam is 79.2 dB at frequency of 145 Hz, while the sound power level of aluminum beam at the same frequency is 89.3 dB. The optimal design parameters which minimize the sound radiation at 145 Hz are listed in Table 4. The steel volume fraction optimal distributions that result in the minimum sound power at 145 Hz based on the power law (labeled as optimal FGM beam 1) and the trigonometric law (labeled as optimal FGM beam 2) are shown in Figure 15. By creating the optimal clamped FGM beam based on the power law, the sound power radiation at 145 Hz is 76.5 dB which is 2.7 dB and 12.8 dB less than the sound power radiation of steel and aluminum beams at 145 Hz, respectively. By creating the optimal clamped FGM beam based on the trigonometric law, the sound power radiation at 145 Hz is 68.4 dB which is 10.8 dB and 20.9 dB less than the sound power radiation of steel and aluminum beams at 145 Hz, respectively. As shown in Figure 16, the reduction of the sound power resulted from the significant drop in the beam vibration response achieved by using the optimal FGM distribution. This decrease in the vibration response results from shifting the second natural frequency far away from 145 Hz.

4. Conclusions

This paper demonstrates the use of the FGM in order to effectively maximize the fundamental frequency of beam structures or to minimize the radiated sound power from vibrating beams. The optimum design of the FGM beam is achieved by the shaping volume fraction distribution of material constituents throughout beam length. In this study, two novel laws of the volume fractional distribution of the constituents of beam structure were considered. The first law is a four-parameter power law and the second law is a five-parameter trigonometric law. Mori-Tanaka method was used for estimating the effective material properties at any point in the beam. The design approach coupled the finite element method for modal and harmonic analysis, the Lumped Parameter Model for acoustic analysis, and Genetic Algorithm as an optimization tool.

The design method is applied to maximize the fundamental frequency of a cantilever beam or to maximize the fundamental frequency of clamped beam. Also, it is applied to minimize the sound radiation from a vibrating baffled clamped beam at a single frequency. The numerical results show that the proposed material distribution laws give more flexibility than that of the classical power law in designing FGM beams. The optimization examples demonstrated that the proposed methodology can be effectively used to improve the vibration and acoustic characteristics of FGM beams. Future extensions of this work will include the optimal design of FGM plates.

Although this study focused on the idea of using FGM in order to alter the dynamic characteristics of beam structures, future studies should also take into consideration the manufacturing limitations of fabricating such materials. In addition, it would be beneficial to consider other design criteria within the optimization process in order to ensure the ability of the design to withstand operating loading conditions.

Conflict of Interests

The authors declare there is no conflict of interests regarding the publishing of this paper.

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