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Research Article

Modulation Instability of Ion-Acoustic Waves in Plasma with Nonthermal Electrons

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Modulational instability of ion-acoustic waves has been theoretically investigated in an unmagnetized collisionless plasma with nonthermal electrons, Boltzmann positrons, and warm positive ions. To describe the nonlinear evolution of the wave amplitude a nonlinear Schrödinger (NLS) equation has been derived by using multiple scale perturbation technique. The nonthermal parameter, positron concentration, and ion temperature are shown to play significant role in the modulational instability of ion-acoustic waves and the formation of envelope solitons.

1. Introduction

Electron-positron-ion (e-p-i) plasmas occur in many astrophysical environments such as active galactic nuclei [1], pulsar magnetospheres [2], polar regions of neutron stars [3], centres of our galaxy [4], the early universe [5, 6], and the solar atmosphere [7]. For this, over the last two decades there has been a great deal of interest in the study of nonlinear wave phenomena in e-p-i plasmas [8–12]. Positrons are produced by pair production in high energy processes occurring in many astrophysical environments. Popel et al. [9] have reported decrease in soliton amplitude in the presence of positrons. Jehan et al. [13] have shown that solitons become narrower as the concentration of positron increases. The presence of non-Maxwellian electron is common in space and astrophysical plasmas including the magnetosphere [12] and auroral zones [14]. The presence of such non-Maxwellian electrons gives rise to many interesting characteristics in the nonlinear propagation of waves including the ion-acoustic solitons [15, 16]. The solitary structures with density depression in the magnetosphere observed by the Freja satellites [17, 18] have been explained by Cairns et al. [19] by assuming electron distribution to be nonthermal. Nonlinear ion-acoustic solitary waves in e-p-i plasma have been considered by some authors [9, 20, 21] assuming ions to be cold. In practice ions have finite temperature and the ionic temperature can significantly affect the characteristics of nonlinear ion-acoustic structures [10, 22, 23]. Chawla et al. [24] have considered ion-acoustic waves in e-p-i plasma with warm adiabatic ions and isothermal electrons. Baluku and Hellberg [25] have considered ion-acoustic solitary waves in e-p-i plasma with cold ions and nonthermal electrons. Hence it is interesting to study the nonlinear ion-acoustic waves in e-p-i plasma assuming simultaneous presence of nonthermal electrons, warm negative ions, and the positrons. Recently Pakzad [11] has shown that the presence of warm ions and nonthermal electrons can modify parametric regions of existence of ion-acoustic solitary waves. A nonlinear theory of ion-acoustic waves in e-p-i plasma has been developed by Dubinov and Sazonkin [26] considering polytropic laws of compression and rarefraction for all plasma components. Survey of the past literatures shows that a large number of works on KdV type and large amplitude solitary structure formation in e-p-i plasmas have been reported. Nonlinear propagation of waves in a dispersive medium is generically subject to amplitude modulation due to carrier wave self-interaction or intrinsic nonlinearity of the medium. Modulational instability is an important phenomenon in connection with stable wave propagation. However, only a few works have been reported in recent years on the modulational instability and formation of envelope soliton in e-p-i plasmas [20, 21, 24]. It has been shown that the presence of positrons shifts the critical wave number separating the stability and instability regions to higher values and for fixed amplitude, width of envelope solitons decreases with the increase of positron concentration. Mahmood et al. [27] have studied modulational instability of ion-acoustic waves in e-p-i plasma with warm ions and isothermal electrons and positrons at the same temperature. Chawla et al. [24] have studied the effects of ion temperature, positron concentration, and positron temperature on the modulational instability of ion-acoustic waves in e-p-i plasma with isothermal electrons and positrons at different temperatures. Bains et al. [28] have considered modulational instability of ion-acoustic waves in e-p-i plasma with dust particles. Eslami et al. [29] have considered modulational instability of ion-acoustic waves in e-p-i plasma with electrons and positrons following q-nonextensive distribution. Gill et al. [21] have studied modulational instability of ion-acoustic waves in e-p-i plasma with superthermal electrons and isothermal positrons. Zhang et al. [30] have investigated modulational instability of ion-acoustic waves in e-p-i plasma with nonthermally distributed electrons and cold ions. Modulational instability and excitation of ion-acoustic envelope solitons in e-p-i plasma with nonthermal electrons have been investigated by Gill et al. [31] including ion temperature. The purpose of the present paper is to make a detailed study of modulational instability of ion-acoustic waves in e-p-i plasma including simultaneously both the effects of nonthermality of electrons and ion-temperature.

2. Basic Formulation

We consider an unmagnetized collisionless plasma consisting of warm positive ions, Boltzmann positrons, and nonthermal electrons. The normalized basic equations governing ion dynamics for one-dimensional propagation in such plasma in dimensionless form are as follows [28]:

$$\begin{gathered}
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0, \\
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{3\sigma_i}{\left(1-\chi\right)^2} n_i \frac{\partial n_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \\
\frac{\partial^2 \phi}{\partial x^2} = n_e - n_p - n_i.
\end{gathered} \tag{1}$$

In aforementioned equations, the parameters n_i, v_i are, respectively, the concentration and velocity of the positive ions; n_e and n_p are, respectively, the concentration of electrons and positrons; ϕ denotes the electrostatic potential; other parameters have their usual meaning. Different quantities are normalized as follows: the velocities by ion-acoustic speed $C_s = \sqrt{k_B T_e/m_i}$, the densities by equilibrium electron density n_{e0}, all the length x by the electron Debye length $\lambda_{\mathrm{De}} = \sqrt{k_B T_e/4e^2 n_{e0}}$, time by $\lambda_{\mathrm{De}}/C_s$, ion temperature T_i by T_e ($\sigma_i = T_i/T_e$) and the potential ϕ by $k_B T_e/e$, where k_B is the Boltzmann's constant. The nonthermal electron density is given by [19]

$$n_e = \left(1 - \beta\phi + \beta\phi^2\right)\exp\left(\phi\right), \tag{2}$$

where $\beta = 4\delta/(1+3\delta)$ measures the deviation from the thermalized state and δ determines the presence of nonthermal electrons inside the plasma. The density of Boltzmann positrons is given by

$$n_p = \chi \exp\left(-\sigma_p \phi\right), \tag{3}$$

where $\chi = n_{p0}/n_{e0}$ is the ratio between the unperturbed positron and electron number densities and $\sigma_p = T_e/T_p$ is the ratio between electron and positron temperatures. The equilibrium charge neutrality condition in normalized form is given by

$$\chi + n_{i0} = 1, \tag{4}$$

in which n_{i0} is the equilibrium ion density normalized by the equilibrium electron density.

Using (2) and (3), Poisson's equation in (1) is rewritten as

$$\frac{\partial^2 \phi}{\partial x^2} = \left(1 - \beta + \beta\phi^2\right)\exp\left(\phi\right) - \chi \exp\left(-\sigma_p \phi\right) - n_i. \tag{5}$$

3. Derivation of the Evolution Equation

Following the usual procedure we make the following Fourier expansions for the field quantities [28, 32–34]:

$$F = \varepsilon^2 F_0' + \sum_{s=1}^{\infty} \varepsilon_s \left\{F_s \exp\left(is\psi\right) + F_s^* \exp\left(-is\psi\right)\right\}, \tag{6}$$

where F stands for the field quantities n_i, v_i, and ϕ; F_0' and F_s are assumed to vary slowly with space and time; that is, they are supposed to be functions of $\xi = \varepsilon(x - C_g t)$ and $\tau = \varepsilon^2 \tau$, with ε being a small parameter and C_g the group velocity; $\psi = kx - \omega t$ (ω, k being two constants satisfying linear dispersion relation). Substituting the expansion (6) in (1) and (5) and then equating from both sides the coefficients of $\exp(i\psi)$, $\exp(2i\psi)$, and terms independent of ψ we obtain three sets of equations which we call, respectively, I, II, and III. To solve these equations we make the following perturbation expansion for the field quantities, F_0' and F_s, which we denote by X:

$$X = X^{(1)} + \varepsilon X^{(2)} + \varepsilon X^{(3)} + \cdots. \tag{7}$$

Solving the lowest order equations obtained from the set of equations I after substituting the expansion (7) we get the following solutions for the first harmonic quantities in the lowest order:

$$\begin{aligned}
n_{i1}^{(1)} &= \left(1 - \beta + \chi\sigma_p + k^2\right)\cdot\alpha, \\
v_{i1}^{(1)} &= \frac{\omega\cdot\left(1 - \beta + \chi\sigma_p + k^2\right)}{\left(1-\chi\right)}\cdot\alpha,
\end{aligned} \tag{8}$$

where

$$\alpha = \phi_1^{(1)}. \quad (9)$$

The linear dispersion relation is obtained as

$$\omega^2 = k^2 \left[\frac{(1-\chi)}{\left(1-\beta+\chi\sigma_p+k^2\right)} + \frac{3\sigma_i}{(1-\chi)} \right]. \quad (10)$$

The wave frequency is found to increase with the increase in the nonthermal parameter β and the ion temperature. On the other hand, increase in positron concentration decreases the wave frequency. In this connection it is pertinent to mention that Pakzad [35] reported an incorrect result and it was pointed out and corrected by Baluku and Hellberg [25]. If we put $\beta = 0$, $\chi = 0$, and $\sigma_i = 0$, we get the linear dispersion relation for ion-acoustic waves in *e-i* plasma as obtained by Kakutani and Sugimoto [36]. In the limit $k \to 0$ (10) leads to the normalized ion-acoustic speed (V_s) modified by the presence of positrons, ion-temperature, and non-Maxwellian electron distribution:

$$V_s^2 = \frac{(1-\chi)}{\left(1-\beta+\chi\sigma_p\right)} + \frac{3\sigma_i}{(1-\chi)}. \quad (11)$$

It agrees with the results obtained by Baluku and Hellberg [25] for the case of cold ions ($\sigma_i = 0$). Equation (11) shows that, for the case of cold ions, increase in positron concentration decreases the phase speed [15], increase in the nonthermal parameter (β) leads to an increase in phase speed, and also increase in ion temperature increases the phase speed.

First harmonic quantities in the second order are obtained from the solutions (8) by replacing $-i\omega$ by $-i\omega - \varepsilon C_g(\partial/\partial\xi) + \varepsilon^2(\partial/\partial\tau)$ and ik by $ik + \varepsilon(\partial/\partial\xi)$ and then picking out order ε terms. These are as follows:

$$\begin{aligned} \phi_1^{(2)} &= 0, \\ n_{i_1}^{(2)} &= -i2k\frac{\partial\alpha}{\partial\xi}, \\ v_{i_1}^{(2)} &= \left[\left\{ \left(\frac{\omega}{k^2} - \frac{C_g}{k} \right) \left(\frac{1-\beta+\chi\sigma_p}{1-\chi} \right) \right\} - \frac{2\omega k}{1-\chi} - \frac{k^2 C_g}{1-\chi} \right] \frac{\partial\alpha}{\partial\xi}. \end{aligned} \quad (12)$$

The second harmonic quantities in the lowest order obtained from the set of equations II after substituting the expansion (7) are as follows:

$$\begin{aligned} \phi_2^{(1)} &= A_1 \cdot \alpha^2, \\ n_{i_2}^{(1)} &= \left[\left\{ A_1 \left(1-\beta+\chi\sigma_p+4k^2\right)^2 \right\} + \left\{ \frac{\chi\sigma_p^2}{2} - \beta \right\} \right] \cdot \alpha^2, \\ v_{i_2}^{(1)} &= \frac{\omega}{k(1-\chi)} \left[\left\{ A_1\left(1-\beta+\chi\sigma_p+4k^2\right) \right\} - \frac{\left(1-\beta+\chi\sigma_p+k^2\right)^2}{(1-\chi)} + \left(\frac{\chi\sigma_p^2}{2} - \beta \right) \right] \cdot \alpha^2, \end{aligned} \quad (13)$$

where

$$\begin{aligned} A_1 = {} & \left[\left(\frac{2\omega^2}{k(1-\chi)} - \frac{6\sigma_i}{(1-\chi)} \right) \left(\frac{\chi\sigma_p^2}{2} - \beta \right) \right] - \left[\left(\frac{3\omega^2}{k(1-\chi)^2} - \frac{3\sigma_i}{(1-\chi)^2} \right) \left(1-\beta+\chi\sigma_p+k^2\right)^2 \right] \\ & \times \left(\left[\frac{6\sigma_i k}{(1-\chi)} \left(1-\beta+\chi\sigma_p+4k^2\right) \right] + 2k - \frac{\left[2\omega^2\left(1-\beta+\chi\sigma_p+4k^2\right)\right]}{k(1-\chi)} \right)^{-1}. \end{aligned} \quad (14)$$

The zeroth harmonic components generated through nonlinear self-interaction of the finite amplitude wave are obtained from the set of equations III after substituting the expansion (7):

$$\begin{aligned} \phi_0^{(1)} &= B_1 \cdot \alpha\alpha^*, \\ n_{i_0}^{(1)} &= \left[\left\{ B_1 \left(1+\chi\sigma_p\right) \right\} - 2\beta \right] \cdot \alpha\alpha^*, \\ v_{i_0}^{(1)} &= \left[\left\{ B_1 \frac{C_g\left(1+\chi\sigma_p\right)}{1-\chi} \right\} - \frac{2\beta C_g}{1-\chi} - \frac{\left\{2\omega\left(1-\beta+\chi\sigma_p+k^2\right)^2\right\}}{k(1-\chi)^2} \right] \alpha\alpha^*, \end{aligned} \quad (15)$$

where

$$\begin{aligned} B_1 = {} & \frac{6\beta\sigma_i}{1-\chi} - \frac{2\beta C_g^2}{1-\chi} - \left[\frac{\left(1-\beta+\chi\sigma_p+k^2\right)^2}{(1-\chi)^2} \left(3\sigma_i + 2C_g\frac{\omega}{k} + \frac{\omega^2}{k^2} \right) \right] \\ & \times \left(1 - \frac{C_g^2\left(1+\chi\sigma_p\right)}{1-\chi} + \left[\frac{3\sigma_i}{1-\chi}\left(1+\chi\sigma_p\right) \right] \right)^{-1}. \end{aligned} \quad (16)$$

Now in order to derive the NLS equation, we need to consider first harmonic quantities in the third order. Collecting coefficients of ε^3 from both sides of the set of equations I after substituting perturbation expansion (7), we get a set of equations for first harmonic quantities in the third order from which after proper elimination we obtain the following desired NLS equation:

$$i\frac{\partial\alpha}{\partial\tau}+P\cdot\frac{\partial^2\alpha}{\partial\xi^2}=Q\cdot\alpha\alpha^*, \tag{17}$$

$$\begin{aligned}P={}&\frac{k(1-\chi)}{2\omega\left(1-\beta+\chi\sigma_p+k^2\right)}\\&\times\left[\frac{2\omega C_g}{1-\chi}-\frac{\omega^2}{k(1-\chi)}-\frac{2\omega^2}{(1-\chi)^2}-\frac{kC_g\omega}{(1-\chi)^2}-C_g\right.\\&\times\left[\left\{\left(\frac{\omega}{k^2}-\frac{C_g}{k}\right)\left(\frac{1-\beta+\chi\sigma_p}{1-\chi}\right)\right\}-\frac{2\omega k}{1-\chi}-\frac{k^2C_g}{1-\chi}\right]\\&\left.-\frac{3\sigma_i k}{(1-\chi)^2}+\frac{\omega}{k}\left\{\left(\frac{\omega}{k^2}-\frac{C_g}{k}\right)\left(\frac{1-\beta+\chi\sigma_p}{1-\chi}\right)\right\}\right],\end{aligned} \tag{18}$$

$$Q=\frac{k(1-\chi)}{2\omega\left(1-\beta+\chi\sigma_p+k^2\right)}\left[F_2k-\frac{\omega^2F_3}{k(1-\chi)}+\frac{\omega F_1}{(1-\chi)}\right], \tag{19}$$

where

$$\begin{aligned}F_1={}&\left[\left(1+\chi\sigma_p\right)B_1-2\beta\right]\frac{\omega\left(1-\beta+\chi\sigma_p+k^2\right)}{k(1-\chi)}\\&+\left(1-\beta+\chi\sigma_p+k^2\right)\\&\times\left[\frac{C_g\left(1+\chi\sigma_p\right)}{1-\chi}-\frac{2\beta C_g}{1-\chi}-\frac{2\omega\left(1-\beta+\chi\sigma_p+k^2\right)^2}{k(1-\chi)^2}\right]\\&+\frac{\omega\left(1-\beta+\chi\sigma_p+k^2\right)}{k(1-\chi)}\\&\times\left[\frac{C_g\left(1+\chi\sigma_p\right)}{1-\chi}-\frac{2\beta C_g}{1-\chi}-\frac{\left(1-\beta+\chi\sigma_p+k^2\right)^2}{1-\chi}\right]\\&+\frac{\omega\left(1-\beta+\chi\sigma_p+k^2\right)}{k(1-\chi)}\\&\times\left[\left(1-\beta+\chi\sigma_p+4k^2\right)A_1+\left(\frac{\chi\sigma_p^2}{2}-\beta\right)\right],\\F_2={}&\left[\frac{C_g\left(1+\chi\sigma_p\right)}{1-\chi}B_1-\frac{2\beta C_g}{1-\chi}-\frac{2\omega\left(1-\beta+\chi\sigma_p+k^2\right)^2}{k(1-\chi)^2}\right]\\&\times\frac{\omega\left(1-\beta+\chi\sigma_p+k^2\right)}{k(1-\chi)}\frac{\omega^2\left(1-\beta+\chi\sigma_p+k^2\right)}{k^2(1-\chi)^2}\\&\times\left[\left(1-\beta+\chi\sigma_p+4k^2\right)+\left(\frac{\chi\sigma_p^2}{2}-\beta\right)\right.\\&\qquad\left.-\frac{\left(1-\beta+\chi\sigma_p+k^2\right)^2}{(1-\chi)}\right]\\&+\frac{3\sigma_i}{(1-\chi)^2}\left(1-\beta+\chi\sigma_p+k^2\right)\\&\times\left[\left(1+\chi\sigma_p\right)B_1-2\beta+\left(1-\beta+\chi\sigma_p+4k^2\right)A_1\right.\\&\qquad\left.+\left(\frac{\chi\sigma_p^2}{2}-\beta\right)\right],\\F_3={}&B_1\left(2\beta+\chi\sigma_p^2\right)+\beta+\beta A_1.\end{aligned} \tag{20}$$

4. Modulational Instability and Envelope Solitons

NLS equation (17) describes the nonlinear evolution of the amplitude of IAWs in e-p-i plasma with warm ions, nonthermal electrons, and Boltzmann positrons. NLS equation (17) has been studied extensively in connection with the nonlinear propagation of different wave modes. It is well known that a uniform wave train may be modulationally stable or unstable depending on the sign of the product of the group dispersive and the nonlinearity coefficient, that is, PQ. As the coefficients depend on the plasma parameters such as nonthermal parameter β, ion temperature σ_i, and positron concentration χ, the product of PQ can have both positive and negative values over different parametric regions. The wave is modulationally unstable if $PQ<0$ and the growth rate of instability has a maximum value g_m given by

$$g_m=|Q|\,\alpha_0^2, \tag{21}$$

where α_0 is the constant real amplitude of the carrier wave. For $PQ>0$, the IAW is modulationally stable. As the product can have both positive and negative signs for different values of β, σ_i, and χ, there are accordingly two types of localized solitary wave solutions of the NLS equation (17). To obtain the soliton profile we let

$$\alpha=\rho\exp(i\theta), \tag{22}$$

where ρ and θ are two real variables. Solving the resulting equations for ρ and θ with $PQ < 0$ we get the following bright envelope soliton solution:

$$\rho = \frac{\sqrt{2\,|P/Q|}}{L}\operatorname{sech}\left(\frac{\xi - U\tau}{L}\right), \tag{23}$$

where U is the envelope speed and L is the spatial width of the pulse. It encloses high frequency carrier oscillations and vanishes at infinity. On the other hand, if $PQ > 0$, a stable gray or dark soliton (a potential hole or a localized region of deceased amplitude) is obtained:

$$\rho = \frac{\sqrt{2P/Q}}{Ld}\sqrt{1 - d^2\operatorname{sech}^2\left(\frac{\xi - U\tau}{L}\right)}, \tag{24}$$

where the parameter d determines the depth of the modulation. For $d = 1$ we get a dark soliton:

$$\rho = \frac{\sqrt{2P/Q}}{Ld}\tanh\left(\frac{\xi - U\tau}{L}\right). \tag{25}$$

Thus the sign of the product PQ determines the stability/instability profile of IAWs as well as the type of soliton structure. The soliton width is determined by the ratio $|P/Q|$.

We have numerically examined different parametric regions where some of the above excitations may occur. As the coefficients P and Q depend on nonthermal parameter β, ion-to-electron temperature ratio σ_i, and positron-to-electron concentration ratio χ, these parameters would definitely determine the modulational instability and the formation of envelope solitons. Numerical plots in Figures 1–3 show P/Q as a function of k for different values of β, σ_i, and χ. It shows that the IAWs remain modulationally stable for k less than certain critical value k_c and for $k > k_c$ the wave is modulationally unstable.

In Figure 1 the variation of P/Q with wave number has been plotted for different values of nonthermal parameter (β), keeping positron concentration (χ) and ion temperature (σ_i) fixed. It shows that as β increases the value of critical wave number separating stable and unstable regions decreases. It is also noticed that as β increases the width of the dark solitons increases, but that of the bright solitons decreases.

In Figure 2 P/Q is plotted as function of k for different values of ion temperature (σ_i) taking other plasma parameters such as positron concentration (χ) and nonthermal parameter (β) as constant. It is seen that as σ_i increases critical wave number decreases; the width of dark solitons increases but that of bright solitons decreases.

Figure 3 is a P/Q versus wave number plot for different values of positron concentration (χ), keeping the values of nonthermal parameter (β) and ion temperature (σ_i) constant. It shows that as the value of χ increases the critical wave number increases. The width of dark solitons decreases and that of bright solitons increases as χ increases.

Qualitatively these results agree with those obtained by Gill et al. [31] but quantitatively there are differences. We find that the critical wave number is more sensitive to the variation in β, σ_i, and χ than that predicted by Gill et al. [31].

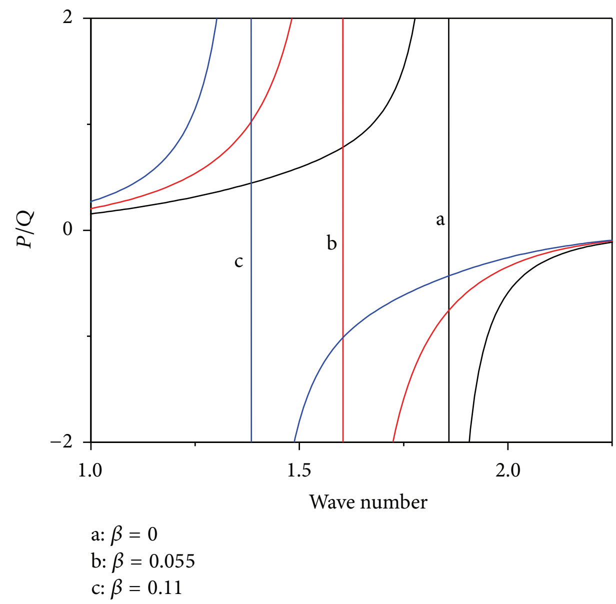

Figure 1: Plot of P/Q versus wave number k for different values of nonthermal parameter (β). Curves labelled a, b, and c correspond to $\beta = 0$, 0.055, and 0.11, respectively. $\chi = 0.22$, $\sigma_p = 0.01$, and $\sigma_i = 0.02$.

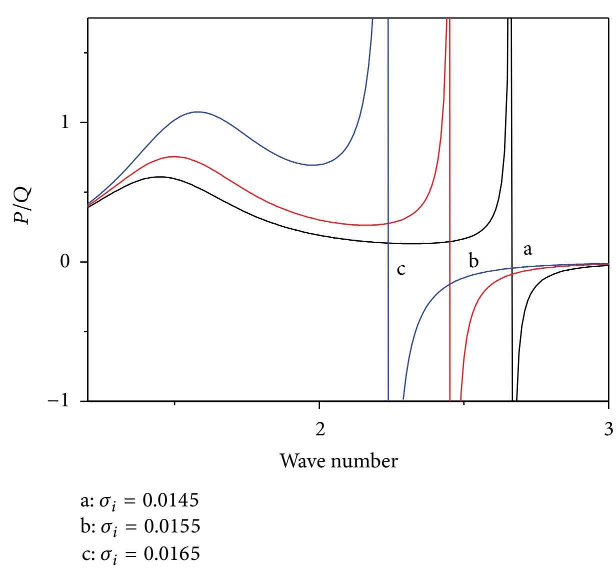

Figure 2: Plot of P/Q versus wave number k for different values of ion temperature (σ_i). Curves labelled a, b, and c correspond to $\sigma_i =$ 0.0145, 0.0155, and 0.0165, respectively. $\chi = 0.2$, $\sigma_p = 0.015$, and $\beta = 0.022$.

In addition, we have numerically studied the dependence of growth rate of instability on all the plasma parameters β, σ_i, and χ. The results are shown in Figures 4, 5, and 6. It is shown that the growth rate of instability increases with increase in the nonthermality of electrons and ion temperature but the increase of positron concentration reduces instability growth rate.

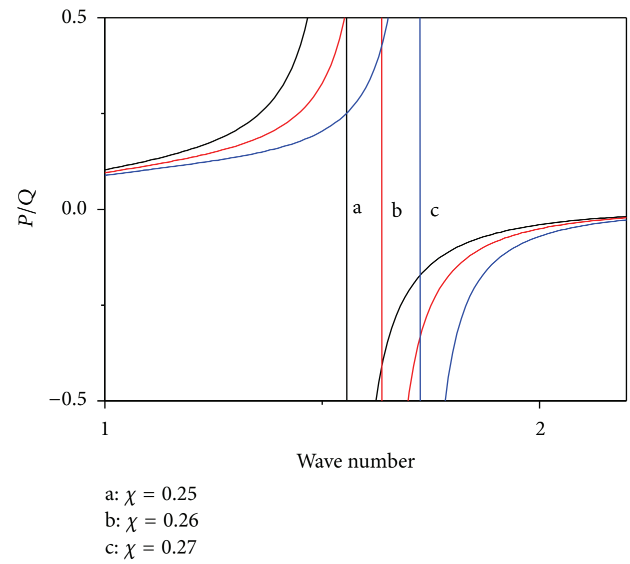

FIGURE 3: Plot of P/Q versus wave number k for different values of positron concentration (χ). Curves labelled a, b, and c correspond to χ = 0.25, 0.26, and 0.27, respectively. β = 0.022, σ_p = 0.01, and σ_i = 0.052.

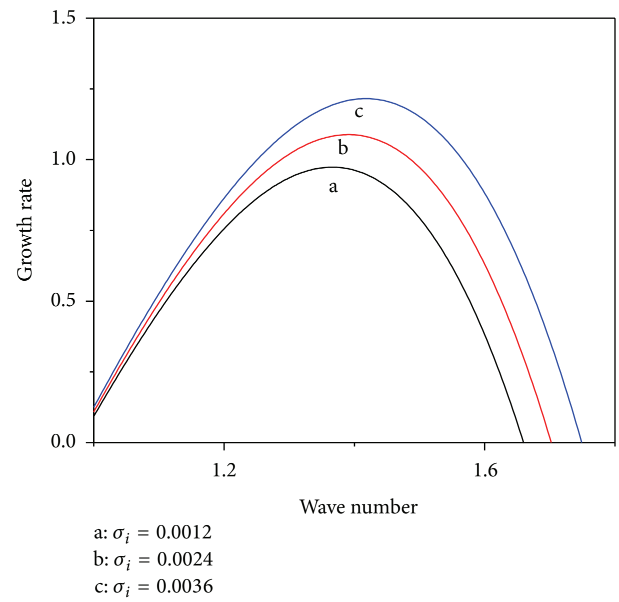

FIGURE 5: Plot of growth rate versus wave number k for different values of ion temperature (σ_i). Curves labelled a, b, and c correspond to σ_i = 0.0012, 0.0024, and 0.0036, respectively. χ = 0.001, σ_p = 0.01, and β = 0.001.

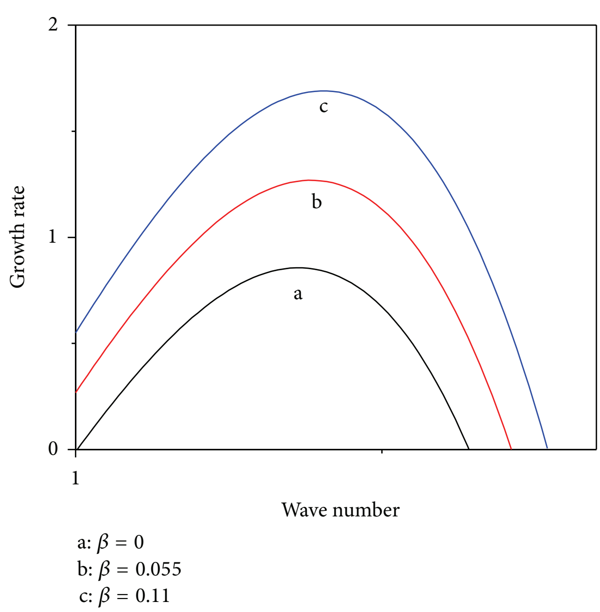

FIGURE 4: Plot of growth rate versus wave number k for different values of nonthermal parameter (β). Curves labelled a, b, and c correspond to β = 0, 0.055, and 0.11, respectively. χ = 0.02, σ_p = 0.01, and σ_i = 0.002.

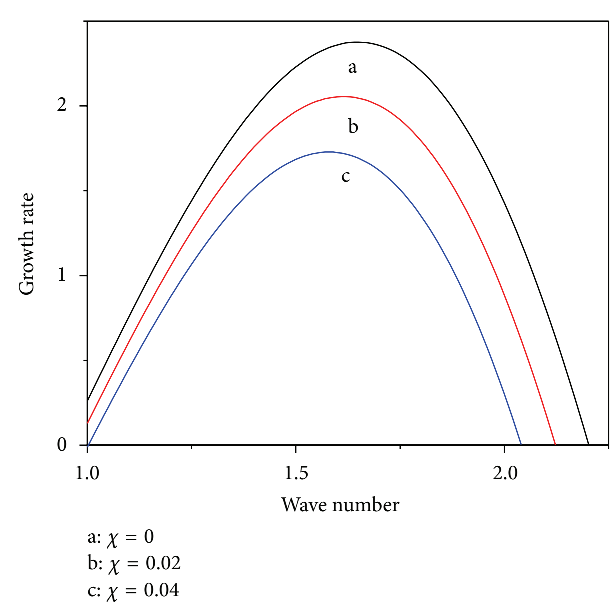

FIGURE 6: Plot of growth rate versus wave number k for different values of positron concentration (χ). Curves labelled a, b, and c correspond to χ = 0, 0.02, and 0.04, respectively. β = 0.01, σ_p = 0.01, and σ_i = 0.01.

5. Conclusions

In the present work, we have investigated modulational instability and envelope excitations of IAWs in the e-p-i plasma in detail including simultaneously the effects of nonthermality of electrons and temperatures of ions. Our main findings are summarized below.

(i) The wave frequency increases with increase in nonthermality of electrons and the temperature of ions whereas the increase in positron concentration decreases the wave frequency.

(ii) There exists a critical wave number k_c below which the wave is modulationally stable and above which the wave is modulationally unstable.

(iii) The value of the critical wave number and the characteristics of bright/dark envelope solitons depend significantly on the nonthermal parameter (β), ion temperature (σ_i), and positron concentration (χ).

Finally we would like to mention that the results presented in this paper may be useful to explain modulational instability and envelope soliton excitations of IAWs in some astrophysical and space environments where e-p-i plasmas with nonthermal electrons are present.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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