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Research Article

A Comparative Study on Decision Making Methods with Interval Data

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Multiple Criteria Decision Making (MCDM) models are used to solve a number of decision making problems universally. Most of these methods require the use of integers as input data. However, there are problems which have indeterminate values or data intervals which need to be analysed. In order to solve problems with interval data, many methods have been reported. Through this study an attempt has been made to compare and analyse the popular decision making tools for interval data problems. Namely, I-TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), DI-TOPSIS, cross entropy, and interval VIKOR (VlseKriterijumska Optimiza-cija I Kompromisno Resenje) have been compared and a novel algorithm has been proposed. The new algorithm makes use of basic TOPSIS technique to overcome the limitations of known methods. To compare the effectiveness of the various methods, an example problem has been used where selection of best material family for the capacitor application has to be made. It was observed that the proposed algorithm is able to overcome the known limitations of the previous techniques. Thus, it can be easily and efficiently applied to various decision making problems with interval data.

1. Introduction

Engineers and managers over the world are daily faced with problems that require the selection of the best alternative from among the feasible options. Such complications are called decision making problems and encompass a wide variety of applications from design, optimisation, allotment, and screening to name a few. Often these problems present alternatives where numerous conflicting constraints are to be considered while making a decision. The attributes associated are such that maximisation of one would lead to minimisation of others. Such problems have no absolute solution and require an optimisation of all the traits to present the best possible solution. This category of problems is known as Multiple Criteria Decision Making (MCDM) problems and various MCDM techniques are used to solve such dilemmas [1–3]. MCDM techniques can be broadly classified into two main categories mainly Multiple Objective Decision Making (MODM) and Multiple Attribute Decision Making (MADM) techniques. Both techniques are based on decision making under multiple criteria consideration but they differ slightly in their method of approach.

MODM techniques require the knowledge of functional relationship that exists between various attributes associated with the alternatives. This relationship is used to formulate figure of merits (FOM) that are used to quantify the desirability or performance of a given alternative [4]. Hence, these FOMs form the basis of comparison for the alternatives. The FOMs are so formulated that their maximisation leads to a more desirable result, or towards accomplishment of one or more objectives such as low cost, longer life, and less maintenance. MADM on the other hand uses predetermined mathematical models to rank and compare the participating alternatives [5]. These mathematical models do not require the functional relationship between the attributes to be known. Often, in MADM, the participating alternatives are compared to ideal (real or hypothetical) solutions to rank their performance.

Both, MODM and MADM techniques are popular decision making tools and are freely used by the scientific and industrial community for various decision making problems [6–16]. However, most of these techniques require the availability of deterministic data and hence cannot be applied to data sets where the data is nondeterministic or is spread over a range. For such problems a deterministic value of the ranged data is either calculated or assumed for the application of decision making tools. However, doing so may not give accurate results as any modification tends to alter the nature of data. In this regard, several MADM techniques have been proposed to enable the decision maker (DM) to take decisions on problems dealing in data set with ranged or nondeterministic data [17–24]. Through this study we have made an attempt to compare the working, advantages, and limitations of such techniques with a new proposed ranking procedure to deal with nondeterministic or interval data sets.

To the best of our knowledge, this is the first attempt to utilize indeterminate data for ranking purposes using a traditional approach.

2. Techniques for Selection

Several techniques have been proposed to deal with problems having nondeterministic or interval data. Most of these techniques require the basic knowledge of TOPSIS and VIKOR which are two popular MADM selection and ranking tools. The basic underlying mechanism of the two techniques is as explained below.

2.1. TOPSIS. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a MADM technique which was first proposed by Hwang and Yoon [25]. TOPSIS implies that any given decision matrix with m alternatives and n attributes can be represented on a n-dimensional hyper-plane with m points. *The location of these points are determined by the value of their attributes.* TOPSIS compares and ranks alternatives based on two sets of ideal solutions known as the positive and negative ideal solutions. The ideal solutions are data driven; that is, the positive ideal solution contains data; that is, the most desirable from among all the alternatives and similarly negative ideal solution contains data points that are the least desirable from among all the alternatives. The ranking is determined by calculating the Euclidean distance of an alternative from these two ideal solutions. The alternative which has the largest distance from the negative solution and least distance from the positive solution is termed as the best.

TOPSIS uses vector normalisation for scaling of data. TOPSIS is a very versatile and popular MADM tool among the scientific community. Since inception, many new methods have been proposed to modify the classical TOPSIS approach to suit specific problems. These include fuzzy TOPSIS [26], R-TOPSIS [27], and M-TOPSIS [28] to name a few. TOPSIS has been used previously to make a number of multidimensional selection problems and has been shown to have good agreement with MODM approach [7, 8]. The TOPSIS method involves the following steps for a decision matrix having m alternatives and n attributes (Table 1).

Table 1: Decision matrix with m alternatives and n attributes.

		Attributes				
		C_1	C_2	C_3	$\cdots$	C_n
Alternatives	A_1	a_{11}	a_{12}	a_{13}		a_{1n}
	A_2	a_{21}	a_{22}	a_{23}		a_{2n}
	$\vdots$					
	$\vdots$					
	A_m	a_{m1}	a_{m2}	a_{m3}		a_{mn}

Step 1. Construction of normalised decision matrix:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m}\left(a_{ij}\right)^2}}; \quad \forall j. \tag{1}$$

Step 2. Construction of weighted normalised decision matrix:

$$V_{ij} = \left[r_{ij}\right]_{m*n} * \left[W_j\right]_{n*n}^{\text{diagonal}}. \tag{2}$$

Step 3. Determination of the positive ideal and negative ideal solutions: the positive ideal solution V_j^+ and the negative ideal solution V_j^- are given by

$$\begin{gathered} V_j^+ = \left\{\left(\max V_{ij}, j \in J_1\right), \left(\min V_{ij}, j \in J_2\right), i = 1, 2, 3 \ldots, m\right\}; \quad \forall j \\ V_j^- = \left\{\left(\min V_{ij}, j \in J_1\right), \left(\max V_{ij}, j \in J_2\right), i = 1, 2, 3 \ldots, m\right\}; \quad \forall j, \end{gathered} \tag{3}$$

where J_1 corresponds to benefit criteria and J_2 corresponds to cost criteria.

Step 4. Calculate the distances d_i^+ and d_i^- from the positive ideal and negative ideal solutions, respectively:

$$\begin{gathered} d_i^+ = \left\{\sum_{j=1}^{n}\left(V_{ij} - V_j^+\right)^2\right\}^{1/2}; \quad \forall i, \\ d_i^- = \left\{\sum_{j=1}^{n}\left(V_{ij} - V_j^-\right)^2\right\}^{1/2}; \quad \forall i. \end{gathered} \tag{4}$$

Step 5. Determine relative closeness of alternatives to the ideal solution:

$$\text{cl}_i^+ = \frac{d_i^-}{d_i^+ + d_i^-}; \quad \forall i, \tag{5}$$

where $0 \le \text{cl}_i^+ \le 1$. Alternatives with higher magnitude of closeness are preferred.

2.2. VIKOR. VlseKriterijumska Optimiza-cija I Kompromisno Resenje (VIKOR) is a compromise ranking and selection technique. Like TOPSIS, VIKOR is also a MADM technique and implies similar principles of application. However, the two differ slightly in that VIKOR uses linear normalisation technique [6]. It also uses a compromise (utility) weight

Table 2: Decision matrix with interval data.

		Attributes				
		C_1	C_2	C_3	$\cdots$	C_n
Alternatives	A_1	$[a_{11}^l, a_{11}^u]$	$[a_{12}^l, a_{12}^u]$	$[a_{13}^l, a_{13}^u]$		$[a_{1n}^l, a_{1n}^u]$
	A_2	$[a_{21}^l, a_{21}^u]$	$[a_{22}^l, a_{22}^u]$	$[a_{23}^l, a_{23}^u]$		$[a_{2n}^l, a_{2n}^u]$
	$\vdots$					
	$\vdots$					
	A_m	$[a_{m1}^l, a_{m1}^u]$	$[a_{m2}^l, a_{m2}^u]$	$[a_{m3}^l, a_{m3}^u]$		$[a_{mn}^l, a_{mn}^u]$

for the decision maker to choose his policy which could be optimistic, pessimistic, and neutral. In VIKOR, the best solution will have the VIKOR index closest to 0 and the worst alternative will have VIKOR index closest to 1. VIKOR is also a popular MADM technique and has been used by scholars to solve many multiple criteria problems. VIKOR has also been modified to adapt to various specific problems. The steps involved for ranking alternatives for a decision matrix having m alternatives and n attributes using VIKOR are as follows (Table 1).

Step 1. Calculation of normalised decision matrix (optional):

$$f_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m}(a_{ij})}}; \quad \forall j. \tag{6}$$

Step 2. Determination of ideal and negative ideal solutions: the ideal solution A^* and the negative ideal solution A^- are determined as

$$A^* = \left\{\left(\max f_{ij}, j \in J\right) \text{ or } \left(\min f_{ij}, j \in J'\right)\right\} = \left\{f_1^*, f_2^*, \ldots, f_n^*\right\}.$$
$$A^- = \left\{\left(\min f_{ij}, j \in J\right) \text{ or } \left(\max f_{ij}, j \in J'\right)\right\} = \left\{f_1^-, f_2^-, \ldots, f_n^-\right\}. \tag{7}$$

Here, J corresponds to benefit criteria and J' corresponds to cost criteria.

Step 3. Calculation of utility measure and regret measure:

$$S_i = \sum_{j=1}^{n} W_j \frac{\left(f_j^* - f_{ij}\right)}{\left(f_j^* - f_j^-\right)}; \quad \forall i,$$
$$R_i = \max_j \left[W_j \frac{\left(f_j^* - f_{ij}\right)}{\left(f_j^* - f_j^-\right)}\right]; \quad \forall i, \tag{8}$$

where S_i and R_i represent the utility measure and regret measure, respectively, and W_j is the relative weight assigned to the jth attribute.

Step 4. Determination of VIKOR index:

$$Q_i = \nu \left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-\nu)\left[\frac{R_i - R^*}{R^- - R^*}\right]; \quad \forall i, \tag{9}$$

where Q_i represents the VIKOR value for the ith alternative and ν is the group utility weight; it is generally taken as 0.5 (neutral):

$$S^* = \min_i (S_i);$$
$$S^- = \max_i (S_i);$$
$$R^* = \min_i (R_i);$$
$$R^- = \max_i (R_i). \tag{10}$$

The alternative with the least value of VIKOR index Q_i is preferred.

3. Techniques for Decision Making with Interval Data

Many of the real life problems are multidimensional in nature. Most of those dimensions have a degree of uncertainty associated with them. This uncertainty causes the variation in data and leads to indeterminate outcome. When the degree of uncertainty is low we can assume a crisp value to represent the average outcome. However, when the degree of uncertainty is high we can no longer assume a crisp value but report the data in a ranged format (Table 2). Most of the decision making problems require crisp data as input. However, this requires assumptions to be made which can change the nature of data and thus affect the results. In order to overcome this problem, need was felt for a technique which could use interval data as input. Several methods have been reported for solving decision making problems involving interval data. They can be mainly categorised into two categories, namely, fuzzy based approaches [17–19] and direct approaches [20–24]. The fuzzy based approaches make use of linguistic variables and membership functions to assign the ranged data to particular fuzzy variables. Though appreciable, doing so requires remodelling of the data to fuzzy variables and back to crisp values to make decision. This can affect the results as at times the membership rules implemented to classify the data may not be able to completely comprehend the overlapping intervals that may exist in the attributes. Also, fuzzy variables assigned to a particular membership are equal. It means that though the span of intervals can be different, alternatives assigned to the same fuzzy variable will be equal. This can result in loss of useful data.

The direct techniques on the other hand do not attempt to change the nature of data and thus are able to preserve information. Several methods have been reported for decision making using direct interval data. Some of the popular methods are being explained as follows.

3.1. I-TOPSIS. I-TOPSIS was proposed by Jahanshahloo et al. [20] to extend basic TOPSIS to data involving intervals. The steps for applying I-TOPSIS for a decision matrix having m alternatives and n attributes are as follows.

Step 1. Calculation of normalised decision matrix:

$$n_{ij}^{u} = \frac{a_{ij}^{u}}{\sqrt{\sum_{i=1}^{m}\left(a_{ij}^{u}\right)^{2} + \left(a_{ij}^{l}\right)^{2}}}; \quad \forall j$$
$$n_{ij}^{l} = \frac{a_{ij}^{l}}{\sqrt{\sum_{i=1}^{m}\left(a_{ij}^{u}\right)^{2} + \left(a_{ij}^{l}\right)^{2}}}; \quad \forall j. \tag{11}$$

Here, the superscript u and l denote the upper and lower limits of the attribute, respectively. This is done to preserve the interval property of the attributes and to scale down all the values in the range of $[0, 1]$.

Step 2. Construction of weighted normalised decision matrix:

$$v_{ij}^{u} = w_j * n_{ij}^{u}; \quad \forall j,$$
$$v_{ij}^{l} = w_j * n_{ij}^{l}; \quad \forall j. \tag{12}$$

Here, w_j is the weight assigned to the jth attribute.

Step 3. Determination of the ideal solutions:

$$A^{+} = \left\{\left(\max v_{ij}^{u}, j \in I\right) \text{ or } \left(\min v_{ij}^{l}, j \in J\right)\right\} = \left\{v_1^{+}, \ldots, v_n^{+}\right\},$$
$$A^{-} = \left\{\left(\min v_{ij}^{u}, j \in I\right) \text{ or } \left(\max v_{ij}^{l}, j \in J\right)\right\} = \left\{v_1^{-}, \ldots, v_n^{-}\right\}. \tag{13}$$

Here, superscript + and − are used to denote positive and negative ideal solutions, respectively; I is associated with benefit criteria and J is associated with cost criteria.

Step 4. Calculation of distances (separation) of alternatives from ideal solutions:

$$d_i^{+} = \left\{\sum_{j\in I}^{1-n}\left(v_{ij}^{l} - v_j^{+}\right)^2 + \sum_{j\in J}^{1-n}\left(v_{ij}^{u} - v_j^{+}\right)^2\right\}^{1/2}; \quad \forall i$$
$$d_i^{-} = \left\{\sum_{j\in I}^{1-n}\left(v_{ij}^{u} - v_j^{-}\right)^2 + \sum_{j\in J}^{1-n}\left(v_{ij}^{l} - v_j^{-}\right)^2\right\}^{1/2}; \quad \forall i. \tag{14}$$

Step 5. Determination of ranking index R_i:

$$R_i = \frac{d_i^{-}}{d_i^{-} + d_i^{+}}. \tag{15}$$

The alternative with the the highest value of ranking index R_i is preferred.

3.2. DI TOPSIS. DI TOPSIS or direct interval TOPSIS was proposed by Sevastjanov and Tikhonenko [22]. This method serves as an improvement to the previously proposed I-TOPSIS method. It was claimed that I-TOPSIS presented the ideal solutions in terms of real values and not intervals which can lead to wrong results in the case of intersecting or overlapping intervals. DI TOPSIS claims to eliminate this short coming by creating ideal solutions with interval data. The steps for application of DI TOPSIS are as follows.

Step 1. Calculation of normalised decision matrix:

$$n_{ij}^{u} = \frac{a_{ij}^{u}}{\sqrt{\sum_{i=1}^{m}\left(a_{ij}^{u}\right)^{2} + \left(a_{ij}^{l}\right)^{2}}}; \quad \forall j,$$
$$n_{ij}^{l} = \frac{a_{ij}^{l}}{\sqrt{\sum_{i=1}^{m}\left(a_{ij}^{u}\right)^{2} + \left(a_{ij}^{l}\right)^{2}}}; \quad \forall j. \tag{16}$$

Step 2. Construction of weighted normalised decision matrix:

$$v_{ij}^{u} = w_j * n_{ij}^{u}; \quad \forall j,$$
$$v_{ij}^{l} = w_j * n_{ij}^{l}; \quad \forall j. \tag{17}$$

Up to this point, the two methods are the same. However, the difference lies in the assumption of ideal solutions and the method used to calculate and compare the separation of the alternatives with the ideal solutions.

Step 3. Determination of ideal solutions:

$$A^{+} = \left\{\left(\max\left[v_{ij}^{u}, v_{ij}^{l}\right\}, j \in K_b\right) \text{ or } \left(\min\left[v_{ij}^{u}, v_{ij}^{l}\right], j \in K_c\right)\right\}$$
$$A^{-} = \left\{\left(\min\left[v_{ij}^{u}, v_{ij}^{l}\right], j \in K_b\right) \text{ or } \left(\max\left[v_{ij}^{u}, v_{ij}^{l}\right], j \in K_c\right)\right\}. \tag{18}$$

Step 4. Calculation of separation from ideal solutions Δ_{A-B}.

The separation Δ_{A-B} is calculated as the difference in the midpoints of the two interval ranges as follows:

$$S_i^+ = \frac{1}{2}\left[\sum_{j\in K_b}\left\{\left(v_j^{+L}+v_j^{+U}\right)-\left(v_{ij}^L+v_{ij}^U\right)\right\}\right. \\ \left.+\sum_{j\in K_c}\left\{\left(v_{ij}^L+v_{ij}^U\right)-\left(v_j^{+L}+v_j^{+U}\right)\right\}\right],$$
$$S_i^- = \frac{1}{2}\left[\sum_{j\in K_b}\left\{\left(v_{ij}^L+v_{ij}^U\right)-\left(v_j^{-L}+v_j^{-U}\right)\right\}\right. \\ \left.+\sum_{j\in K_c}\left\{\left(v_j^{-L}+v_j^{-U}\right)-\left(v_{ij}^L+v_{ij}^U\right)\right\}\right]. \tag{19}$$

Step 5. Calculation of relative closeness to the ideal solution RC_i:

$$\mathrm{RC}_i = \frac{S_i^-}{S_i^- + S_i^+}. \tag{20}$$

Alternatives with the highest value of RC_i are preferred.

3.3. Cross Entropy. Cross entropy is another method used to solve decision making problems with interval data sets [23]. It is different from the rest of the models because it uses cross entropy measurement between the ideal solution and the alternatives to evaluate the relative closeness. It is claimed that by using this method it is possible to evaluate the similarity between two systems without using an actual distance. Thus, the ranking index obtained is informative of their relative performance in a better manner. The steps for application of cross entropy are as follows.

Step 1. Calculation of the normalised decision matrix:

$$\tilde{z}_{ij} = \frac{\tilde{a}_{ij}}{\sqrt{\sum_{i=1}^m \tilde{a}_{ij}^2}}; \quad (\text{attribute is positive})$$
$$\tilde{z}_{ij} = \frac{1/\tilde{a}_{ij}}{\sqrt{\sum_{i=1}^m \left(1/\tilde{a}_{ij}^2\right)}}; \quad (\text{attribute is negative}). \tag{21}$$

Step 2. Calculate the weighted normalised decision matrix:

$$\tilde{v}_{ij} = w_j * z_{ij}. \tag{22}$$

Here, the weight is calculated using a linear programming model and $\sum w_j = 1$.

Step 3. Determination of the positive ($\tilde{v}_j^*$) and negative ($\tilde{v}_j^-$) ideal solutions:

$$\tilde{v}_j^* = \left[v_j^u, v_j^l\right] = \left[\max v_j^u, \max v_j^l\right];$$
$$\tilde{v}_j^- = \left[v_j^u, v_j^l\right] = \left[\min v_j^u, \min v_j^l\right]. \tag{23}$$

Step 4. Calculation of cross entropy:

$$d_i^* = \sum_{j=1}^n \left\{v_{ij} * \log\frac{v_{ij}}{\tilde{v}_{ij}^*} + \left(1-v_{ij}\right) * \log\frac{\left(1-v_{ij}\right)}{\left(1-\tilde{v}_{ij}^*\right)}\right\};$$
$$d_i^- = \sum_{j=1}^n \left\{v_{ij} * \log\frac{v_{ij}}{\tilde{v}_{ij}^-} + \left(1-v_{ij}\right) * \log\frac{\left(1-v_{ij}\right)}{\left(1-\tilde{v}_{ij}^-\right)}\right\}. \tag{24}$$

Step 5. Calculation of relative closeness to the idea solution:

$$C_i^* = \frac{d_i^-}{d_i^* + d_i^-}. \tag{25}$$

Here, the alternative with the highest value of C_i^* is preferred.

3.4. VIKOR (for Interval Data). VIKOR was extended for decision models with interval data by Sayadi et al. [24]. It was claimed that representation of the ranking index in the form of crisp or integer form could result in loss of data. Hence, in interval VIKOR the ranking index is calculated in the form of interval only. The following steps are used for the application of interval VIKOR.

Step 1. Determination of the positive and negative ideal solutions:

$$A^* = \left\{\left(\max f_{ij}^u, j\in I\right) \text{ or } \left(\min f_{ij}^l, j\in J\right)\right\} = \left\{f_1^*, \ldots, f_n^*\right\}$$
$$A^- = \left\{\left(\min f_{ij}^l, j\in I\right) \text{ or } \left(\max f_{ij}^u, j\in J\right)\right\} = \left\{f_1^-, \ldots, f_n^-\right\}. \tag{26}$$

Here, I is associated with benefit criteria and J is associated with cost criteria.

Step 2. Computation of $[S_i^u, S_i^l]$ and $[R_i^u, R_i^l]$ intervals:

$$S_i^l = \sum_{j\in I} w_j\left(\frac{f_j^* - f_{ij}^u}{f_j^* - f_j^-}\right) + \sum_{j\in J} w_j\left(\frac{f_{ij}^l - f_j^*}{f_j^- - f_j^*}\right),$$
$$S_i^u = \sum_{j\in I} w_j\left(\frac{f_j^* - f_{ij}^l}{f_j^* - f_j^-}\right) + \sum_{j\in J} w_j\left(\frac{f_{ij}^u - f_j^*}{f_j^- - f_j^*}\right),$$
$$R_i^l = \max\left\{w_j\left(\frac{f_j^* - f_{ij}^u}{f_j^* - f_j^-}\right) j\in I; w_j\left(\frac{f_{ij}^l - f_j^*}{f_j^- - f_j^*}\right) j\in J\right\},$$
$$R_i^u = \max\left\{w_j\left(\frac{f_j^* - f_{ij}^l}{f_j^* - f_j^-}\right) j\in I; w_j\left(\frac{f_{ij}^u - f_j^*}{f_j^- - f_j^*}\right) j\in J\right\}. \tag{27}$$

Step 3. Compute the ranking interval $Q_i = [Q_i^l, Q_i^u]$:

$$Q_i^l = \nu\left[\frac{S_i^l - S^*}{S^- - S^*}\right] + (1-\nu)\left[\frac{R_i^l - R^*}{R^- - R^*}\right]; \quad \forall i$$
$$Q_i^u = \nu\left[\frac{S_i^u - S^*}{S^- - S^*}\right] + (1-\nu)\left[\frac{R_i^u - R^*}{R^- - R^*}\right]; \quad \forall i. \tag{28}$$

Table 3: Proposed algorithm decision matrix.

		Attributes								
		C_1^l	C_1^u	C_2^l	C_2^u	C_3^l	C_3^u	$\cdots$	C_n^l	C_n^u
Alternatives	A_1	a_{11}^l	a_{11}^u	a_{12}^l	a_{12}^u	a_{13}^l	a_{13}^u		a_{1n}^l	a_{1n}^u
	A_2	a_{21}^l	a_{21}^u	a_{22}^l	a_{22}^u	a_{23}^l	a_{23}^u		a_{2n}^l	a_{2n}^u
	$\vdots$									
	$\vdots$									
	A_m	a_{m1}^l	a_{m1}^u	a_{m2}^l	a_{m2}^u	a_{m3}^l	a_{m3}^u		a_{mn}^l	a_{mn}^u

Here, Q_i represents the ith alternative VIKOR value and ν is the group utility weight; it is generally taken as 0.5 (unsupervised):

$$S^* = \min_i \left(S_i^l\right); \qquad S^- = \max_i \left(S_i^u\right);$$
$$R^* = \min_i \left(R_i\right); \qquad R^- = \max_i \left(R_i\right). \tag{29}$$

These intervals can now be compared using interval comparison methods as per the requirements of the decision maker. Under the comparison of two interval numbers, $[a^l, a^u]$ and $[b^l, b^u]$, the following four conditions can exist.

(1) If these interval numbers have no intersection, the minimum interval number is the one that has lower values. In other words, if $a^u \leq b^l$, then we choose $[a^l, a^u]$ as the minimum interval.

(2) If two interval numbers are the same, both have the same priority.

(3) In situations that $a^l \leq b^l \leq b^u \leq a^u$, we select the minimum interval as follows: if $\alpha * (b^l - a^l) \geq (1-\alpha)(a^u - b^u)$, then $[a^l, a^u]$ is selected as the lower interval or else $[b^l, b^u]$ is selected as the lower interval.

(4) In a condition where $a^l < b^l < a^u < b^u$, if $\alpha * (b^l - a^l) \geq (1-\alpha)(b^u - a^u)$, then $[a^l, a^u]$ is selected as the lower interval or else $[b^l, b^u]$ is selected as the lower interval.

Here α is used as the optimistic weight of the user. For a rational user the value of α is taken as 0.5, while a value higher than this is used to favour the larger intervals and a smaller value is used when smaller intervals are preferred.

3.5. Proposed Algorithm. The purpose of this study was to enable the use of ranged or indeterminate data to be applied to point based decision making techniques without any appreciable loss of form or information. It is a common trend with the conventional interval data techniques to employ only the upper and lower limits of the spread for evaluation purposes. This is done because the statistical nature of the data is relatively incapable to alter the ranking by large. Thus, it not considered while the implementation of the decision technique. Exceptions to this rule are observed for fuzzy decision making.

Considering the aforementioned logic and after observing the data handling associated with direct methods, it can be concluded that most of the limitations can be overcome by slight modification to the approach itself. An interval $[a, b]$ is defined by its end points "a" and "b". The intermediate points of the interval play no active role in rank determination, and hence they can be safely omitted from the calculation. Incorporation of this additional step reduces the original interval $[a, b]$ to a two-point system without any loss of information. This elimination is similar to the one used for reducing the number of attributes where the degree of correlation is high [29]. Now, if the upper and lower limits of the data are treated as individual attributes (Table 3), the problem can be solved using basic point based approach. The suggested technique reduces the original decision matrix (Table 2) to simplified decision matrix (Table 3) as shown in Figure 1.

In order to test the effectiveness of the new approach, we incorporated the additional step to the conventional TOPSIS approach. TOPSIS has been chosen because it has been shown to be in good agreement with MODM results and experimental demonstrations through our previous works [13–16]. The decision matrix can be modified to make it compatible with the conventional TOPSIS approach. This modified TOPSIS approach can be used effectively to overcome the shortcomings associated with the other interval methods. Furthermore, a data having a crisp value can also be represented as an interval with the same value for upper and lower limits. To test the effectiveness of the proposed algorithm and compare it with the existing methods we make use of an example problem.

4. Results and Discussion

4.1. Problem Formulation. To compare the performance of different methods and the proposed algorithm we make use of an example problem. The given decision matrix (Table 4) represents four different material families used for capacitor applications. The selected problem requires selection of the best family for electrical energy storage. The considered properties are breakdown voltage and dielectric constant. It is required to determine the best material family from among the given alternatives. Both the properties are associated with benefit criteria, that is, maximisation of both properties is desirable. The data is such that the intervals present in the properties are over lapping.

4.2. Comparative Analysis. A two-dimensional representation of TOPSIS, I-TOPSIS, and DI-TOPSIS has been given in

Table 4: Decision matrix for the problem.

Materials	Breakdown voltage (MV/mm)		Dielectric constant ε_r	
	Lower	Upper	Lower	Upper
Polymer-based nanodielectric	4	7	220	370
Ferroelectric glass ceramic	3	7	170	370
Metal-glass nanocomposite	4	6	220	320
Ferroelectric ceramic	3	6	170	320

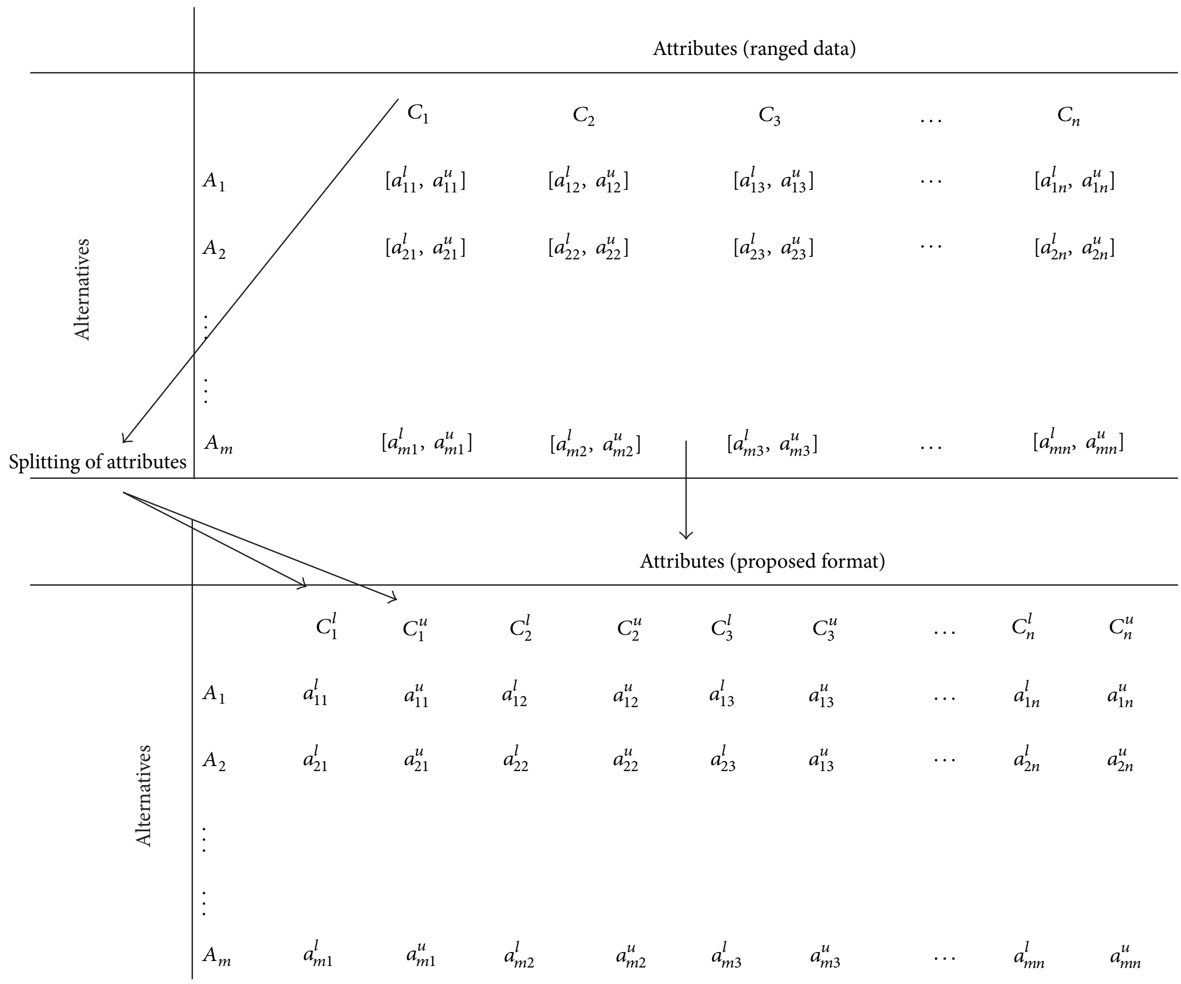

Figure 1: Figurative description of data conversion technique to be employed for proposed algorithm.

Figures 2, 3, and 4, respectively, for a two-attribute problem, where both attributes are associated with benefit. The positive ideal solution for TOPSIS lies at the intersection of the maxima of all the associated attributes (Figure 2). This is an imaginary point and represents the desirable properties of the ideal alternative. Similarly, the negative ideal solution lies at the intersection of the minima of all the associated attributes. The separation measure from the ideal solutions is done using Euclidean distance.

In I-TOPSIS the data has intervals and as such cannot be represented as points on the plane. To this effect we use rectangles where the dimensions of the rectangle represent the range of the respective property (Figure 3). Here the positive ideal solution is a point which lies at the intersection of the maxima of the upper limits of the associated attributes, representing the highest point of desirability. Similarly, the negative ideal solution lies at the intersection of the minima of the lower limit of the associated attributes. The separation measure is calculated as the Euclidean distance between the geometrical centre of the alternatives and the ideal solutions.

For DI-TOPSIS the ideal solutions are calculated as ranged data. The positive ideal solution is derived as the difference between the maxima of the upper and the lower limits of the associated attributes (Figure 4). This can be interpreted as the range between the highest value in the upper and lower limits, respectively, that are associated with

Table 5: Rankings and index for applied techniques.

Materials	I-TOPSIS		DI-TOPSIS		Cross entropy		Interval VIKOR		Proposed method	
	Index	Rank	Index	Rank	Index	Rank	Index	Rank	Index	Rank
Polymer-based nanodielectric	0.57884	1	1	1	1	1	[0, 0.75]	1	1.00000	1
Metal-glass nanocomposite	0.50000	2	0.5	2	0.76604	2	[0.25, 0.75]	2	0.64345	2
Ferroelectric glass ceramic	0.50000	2	0.5	2	0.234306	3	[0, 1]	3	0.35655	3
Ferroelectric ceramic	0.42116	4	0	4	0	4	[0.25, 1]	4	$3.16E-7$	4

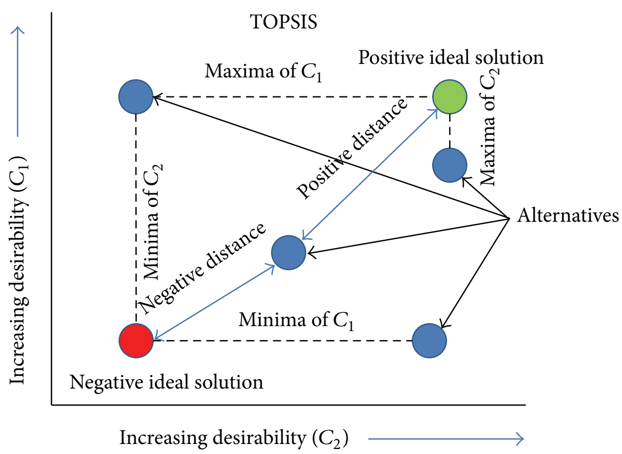

Figure 2: Graphical representation of TOPSIS with two attributes.

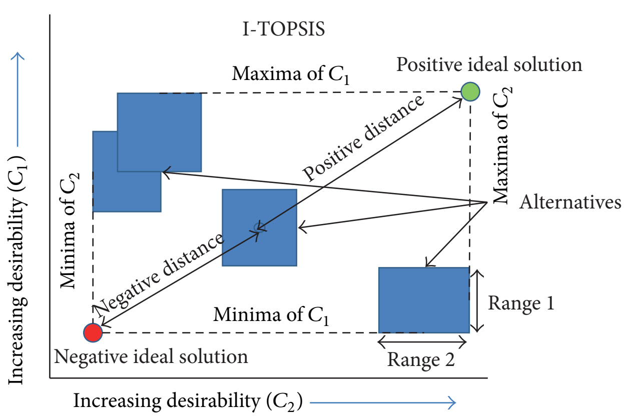

Figure 3: Graphical representation of I-TOPSIS with two attributes.

the attribute. Similarly, the negative ideal solution is taken as the range between the minima of the upper and lower limits associated with the attribute, respectively. The separation measure is calculated as the sum of the difference between the centre points of the intervals of the alternative and the ideal solution. The separation measure, distances, and index calculated for VIKOR, interval VIKOR, and cross entropy cannot be represented as physical distances so it is not possible to plot them graphically.

The results of the ranking using various methods are given in Table 5. It can be observed that I-TOPSIS and DI-TOPSIS have failed to differentiate between the ferroelectric glass ceramic and metal glass nanocomposite, evident from their ranking index. This is due to the fact that both the methods use midpoint location for separation measurement. Thus, for alternatives having coinciding midpoints (geometric centres for multidimensional problems) both I-TOPSIS and DI-TOPSIS cannot be used for differentiating them. This is the same reason why cross entropy can easily differentiate between the alternatives as it does not use a geometric distance for separation or similarity measurement. However, cross entropy is limited in the manner that it can only be used for real numbers. Integer values (negative integers) cannot be used as the logarithm used for entropy measurement can only work with positive values. It is for the same reason that oscillating data will have to be modified for use with cross entropy method. This can affect the nature of data and thus can have adverse effects on the results. Extension of VIKOR for interval data is also able to easily differentiate between the given alternatives. However, its limitation lies in the fact that the VIKOR index calculated for decision making is in the form of interval also. This gives the decision maker the flexibility to use different interval comparison methods as per the requirement. However, this makes the comparison a tedious task as intervals have to be compared individually. Thus, for problems having overlapping intervals for VIKOR index using high number of alternatives, the comparison and ranking become computationally taxing.

The proposed algorithm on the other hand suffers from none of the above-mentioned drawbacks. It can differentiate between alternatives having overlapping data sets as well as data sets which have coinciding midpoints. Integers can be used as data points and they return a crisp value as the final ranking index. This enables the decision maker to easily rank and compare different alternatives with little effort. For the special case of data sets in which one completely overlaps the other and also has coincidental geometric centres, the proposed algorithm favours the data set with the lower spread. This is acceptable as a wider spread is generally associated with a higher value of uncertainty. Thus, for majority of the problems where the certainty of the outcome is important, selection of alternative with lesser data spread will be beneficial. For such problems, the proposed algorithm would be an excellent way to make decisions where the input data is nondeterministic or has interval values.

5. Conclusions

Through this study an attempt has been made to compare various decision making methods for problems having interval data. These methods are I-TOPSIS, DI-TOPSIS, cross

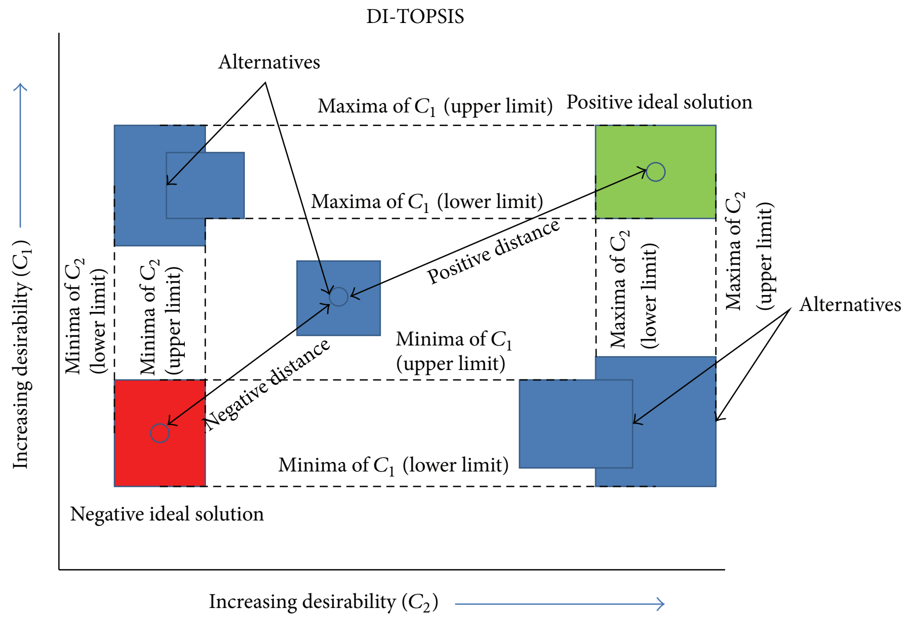

Figure 4: Graphical representation of DI-TOPSIS with two attributes.

entropy, and interval VIKOR. Finally, a new algorithm has been proposed to solve interval data problems using basic TOPSIS approach. To compare the effectiveness of the techniques an example problem has been used. It was observed that I-TOPSIS and DI-TOPSIS techniques have failed to distinguish between alternatives having coincident geometric data centre points. Cross entropy and interval VIKOR can easily distinguish the data points but have their own set of limitations. Cross entropy can only accept positive values and thus cannot be used with integer data. Interval VIKOR gives the result in the form of intervals, which then have to be compared individually making it computationally taxing for a large number of alternatives. The proposed algorithm does not have the limitations of the previous methods and thus can be easily and efficiently applied to problems with interval data.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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