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Research Article

Work Criteria Function of Irreversible Heat Engines

Mahmoud Huleihil

Academic Institute for Arab Teacher Training, Beit-Berl College, 44905 Doar Beit-Berl, Israel

Correspondence should be addressed to Mahmoud Huleihil; mahmud.ana@gmail.com

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The irreversible heat engine is reconsidered with a general heat transfer law. Three criteria known in the literature—power, power density, and efficient power—are redefined in terms of the work criteria function (WCF), a concept introduced in this study. The formulation enabled the suggestion and analysis of a unique criterion—the efficient power density (which accounts for the efficiency and power density). Practically speaking, the efficient power and the efficient power density could be defined on any order based on the WCF. The applicability of the WCF is illustrated for the Newtonian heat transfer law ($n = 1$) and for the radiative law ($n = 4$). The importance of WCF is twofold: it gives an explicit design and educational tool to analyze and to display graphically the different criteria side by side and thus helps in design process. Finally, the criteria were compared and some conclusions were drawn.

1. Introduction

Finite-time thermodynamics [1, 2] has been used extensively in different fields of research [3–24]. A recent review [25] reported several key advances in theoretical investigations of efficiency at maximum power operation of heat engines. In the review, a presentation of the analytical results of efficiency at maximum power was given for the Curzon-Ahlborn heat engine, for the stochastic heat engine constructed from a Brownian particle, and for Feynman's ratchet as a heat engine [25]. The endoreversible heat engine as presented by Curzon and Ahlborn [1] considered an internally reversible heat engine working between two heat reservoirs. A Newtonian-type heat transfer was assumed. The objective was to maximize power output. Different heat transfer laws and different types of irreversibility were also considered [4–24]. In order to find the maximum power and efficiency at maximum power output, relation between the design parameters of internally and externally radiative heat engines was presented [26]. Another objective function in finite-time thermodynamics with an ecological criterion was applied to an irreversible Carnot heat engine interacting with finite thermal capacitance rates of the heat reservoirs and finite total conductance of the heat exchangers [27]. The reverse heat engine (refrigerator) was considered also for economic optimization of endoreversible operation and was carried out in [28]. The results obtained involve the following common heat transfer laws: Newton's law ($n = 1$), the linear phenomenological law in irreversible thermodynamics ($n = -1$), and the radiative heat transfer law ($n = 4$). In a different study [29], the thermoeconomic optimization of an irreversible solar-driven heat engine model was carried out using finite-time/finite-size thermodynamic theory. In this study losses were taken into account due to heat transfer across finite time temperature differences, heat leakage between thermal reservoirs and internal irreversibilities in terms of a parameter that comes from the Clausius inequality [29]. A more generalized radiative heat transfer law was introduced into a model external combustion engine with a movable piston, and effects of heat transfer laws on the optimizations of the engine for maximum work output were investigated [30]. Numerical examples for the optimizations with linear phenomenological ($n = -1$), Newton's ($n = 1$), square ($n = 2$), cubic ($n = 3$), and radiative ($n = 4$) heat transfer laws were provided, respectively, and the obtained results were compared with each other. A power analysis was conducted on a reversible Joule-Brayton cycle [31]. Although many studies have been carried out using different performance criteria, resulting in famous efficiencies (Carnot, Curzon-Ahlborn), most do not consider the sizes of the engines [31], but Gordon [24] was the first to observe that finite size of a system shows up exactly like finite time (duration). In the studies of Curzon and Ahlborn and others, researchers utilized the thermal efficiency at maximum power as a performance measure that dictates an efficiency standard for practical heat engines. As described in [31], instead of maximizing power for certain cycle parameters, the power density defined as the ratio of power to the maximum specific volume in the cycle was maximized, taking into account the effects of the engine sizes. The obtained results showed an efficiency value at the maximum power density that was always greater than that at the maximum power as given by Curzon-Ahlborn efficiency. Evaluations showed that design parameters at the maximum power density resulted in smaller and more efficient Joule-Brayton engines [31]. The maximum power density criteria and the efficient power of an engine (defined as the product of power output and efficiency) were taken as the objectives for performance analysis and optimization of an internally and externally irreversible radiative Carnot heat engine model. These objectives were approached from the standpoint of finite-time thermodynamics or entropy generation minimization. The consequences of a maximum efficient power (MEP) design were analyzed. The obtained results showed that engines designed at MEP conditions had an advantage of smaller size and were more efficient than those designed at MP and MPD conditions [32]. These findings were in agreement with those given by [31]. Other considerations were studied for a class of irreversible Carnot engines that resulted from combining the characteristics of two models found in the literature—the model in finite time and the model in finite size [33]. In a different study [34], the efficient power criterion was reconsidered analytically and finite-time thermodynamic optimization was carried out for an irreversible Carnot heat engine. The obtained results were compared with those obtained by using the maximum power and maximum power density criteria similar to the analysis done in [32]. The optimal design parameters have been derived analytically, and the effect of the irreversibilities on general and optimal performances was investigated. Maximizing the efficient power gave a compromise between power and efficiency. The derived results showed that the design parameter at the maximum efficient power condition resulted in more efficient engines than at the MP conditions, and that the MEP criterion may have a significant power advantage with respect to the MPD criterion [34], a result supporting what had been reported in [32]. In summary, different performance criteria were employed to optimize the performance of heat engines and heat pumps, using the methods of finite-time thermodynamics and taking into account endoreversible and internally irreversible conditions. The maximization of the power density is defined as the ratio of the maximum power to the maximum specific volume in the cycle. It takes into consideration the engine size instead of just maximizing its power output. The inclusion of the engine size in the calculation of its performance is a very important factor from an economical point of view. In this study, known criteria are reconsidered and the work criterion function is introduced. The formulation of this function enabled the suggestion of a performance criterion to be considered—the efficient power density (EPD), defined as the efficient power divided by the maximum volume of the working fluid, following the definitions given in [32], and its maximum value (MEPD). Details of the criteria function are given in the next section.

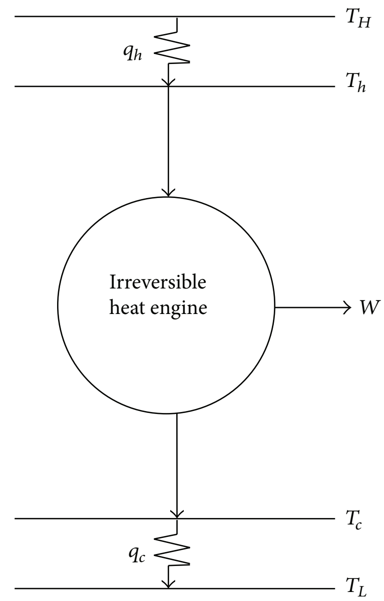


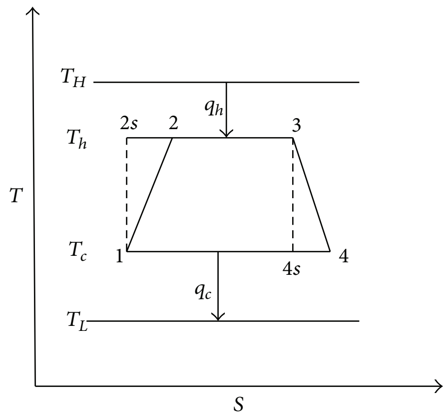
Figure 1: Schematics of irreversible heat engines showing the components and variables involved.

2. The Work Criteria Function

The irreversible heat engine, as considered earlier, in [32], was assumed to follow law of radiative heat transfer. It was analyzed to determine the maximum efficient power. In this section, the irreversible heat engine is reconsidered based on laws of general heat transfer, and criteria known in the literature (maximum power, maximum power density, and maximum efficient power) are reviewed and recast in terms of the work criteria function (WCF). The WCF allows the definition of a new criterion, the maximum efficient power density, as suggested from the result obtained. The analysis relies upon applying the first and second laws of thermodynamics written as equality, which is a common exercise that is well documented in the literature available in finite-time thermodynamics research. The schematics of the engine are given in Figure 1, and the schematics of its temperature entropy T-S diagram are depicted by Figure 2. The irreversible heat engine under consideration works between two heat reservoirs at high and low temperatures. The rate of heat input to the engine, q_h, is given by

$$q_h = k_h \left(T_H^n - T_h^n \right). \tag{1}$$

In this equation, exponent n represents well-known heat transfer rates: the Newtonian heat transfer rate ($n = 1$), the radiative heat transfer rate ($n = 4$), and so on; k_h is the thermal conductance at the hot side of the engine; T_H is the temperature of the hot reservoir; and T_h is the temperature of the working fluid at the hot side of the engine. Similarly, the



Processes

1-2s: isentropic compression

1-2: irreversible compression

2s-3, 2-3: constant temperature heat addition

3-4s: isentropic expansion

3-4: irreversible expansion

4s-1, 4-1: constant temperature heat rejection

Figure 2: Schematics of T-S diagram of the irreversible heat engine.

cooling rate or the rate of heat rejection from the heat engine, q_c, is given by

$$q_c = k_c \left(T_c^n - T_L^n\right). \tag{2}$$

In this equation, k_c is the thermal conductance at the cold side of the heat engine, T_c is the temperature of the working fluid at the cold side of the engine, and T_L is the temperature of the cold reservoir. The net power rate, w, extracted by the heat engine, follows the first law of thermodynamics for a cyclic process and is given by

$$w = q_h - q_c. \tag{3}$$

The second law of thermodynamics, accounting for internal irreversibilities, is given by

$$\frac{q_h}{T_h} - R\frac{q_c}{T_c} = 0. \tag{4}$$

In (4) R is the irreversibility factor; its value falls in the $0 < R < 1$ range.

The efficiency of the heat engine is defined as the ratio between net power extracted from the heat engine and the rate of heat absorbed by it. In mathematical symbols the efficiency is given by

$$\eta = 1 - \frac{q_c}{q_h} = 1 - \frac{1-\eta_0}{R}, \tag{5}$$

where η_0 is the efficiency of the endoreversible heat engine ($R = 1$).

Equations (1) to (4) are manipulated and rearranged in the following calculations in order to explicitly progress to the work criteria function formulation. Equation (1) is rearranged to give the temperature at the hot side of the engine by

$$T_h^n = T_H^n - \frac{q_h}{k_h}. \tag{6}$$

Similarly, based on (2), the temperature at the cold side of the engine is given by

$$T_c^n = T_L^n + \frac{q_c}{k_c}. \tag{7}$$

The efficiency of the endoreversible heat engine η_0 ($R = 1$) is a convenient choice to relate the ratio of the heat engine temperature; thus (4) could be rearranged as given by

$$\frac{T_c}{T_h} \equiv 1 - \eta_0 = R\frac{q_c}{q_h}. \tag{8}$$

Equations (6)–(8) are used to derive the explicit expression for the rate of heat input to the engine, given by

$$q_h = k_h T_H^n \frac{\left(\left(1-\eta_0\right)^n - \tau^n\right)}{\left(1-\eta_0\right)\left(\left(1-\eta_0\right)^{n-1} + (\kappa/R)\right)}. \tag{9}$$

In this equation τ is the ratio between the temperatures of the reservoirs, T_L/T_H, and κ is the ratio between the thermal conductance ratio k_c/k_h.

As stated earlier, there are different criteria that are commonly used to describe the performance of heat engines. In this study the following criteria are considered:

(i) the net power extracted from the heat engine (given by (3));

(ii) the net power density defined as the net power extracted divided by the maximum volume (see [32] for more details);

(iii) the net efficient power defined as the efficiency multiplied by the net power;

(iv) the net efficient power density defined as the net efficient power divided by the maximum volume of the working fluid;

(v) Results being valid for ideal gas (10).

The maximum volume of the working fluid, $V_{\max}$ (expressed in SI units), is adopted from [32] and is given by

$$V_{\max} = \frac{mR_gT_c}{p_{\min}}. \tag{10}$$

In this equation m is the mass of the working fluid, R_g is the ideal gas coefficient (assuming ideal gas as the working fluid), and $p_{\min}$ is the minimum pressure in the cycle. It is interesting to note that the criteria summarized in the previous page by (i)–(iv) could be addressed by the work criteria function, which is given by

$$\mathrm{WCF} = \frac{\eta^\alpha q_h}{V_{\max}^\beta}. \tag{11}$$

In this equation α and β are integers to represent the criteria considered in the study. The study uses $\alpha = 0, \beta = 0$ to give the expression of the heat input to the heat engine (see (1) and (9)). The power output extracted by the heat engine is given when $\alpha = 1$ and $\beta = 0$. The power density criterion is defined when $\alpha = 1$ and $\beta = 1$. The efficient power criterion is defined using $\alpha = 2$ and $\beta = 0$. From the suggested formalism (see (11)), one could consider the criterion defined by $\alpha = 2, \beta = 1$ to mean the efficient power density extracted by the heat engine. The WCF, therefore, provides a general statement to define efficient power and efficient power density of any order.

The WCF could be viewed as a one-dimensional function for a specific choice of variables or parameters involved. In this study, the parameter η_0 was chosen to be variable; its value spans the range from zero up to Carnot efficiency ($0 < \eta_0 < 1 - \tau$). The other parameters are each chosen to define a specific criterion. The general criteria suggested by the work function is given by

$$\mathrm{WCF} = f\left(\alpha, \beta, n, \tau, \kappa, R; \eta_0\right). \tag{12}$$

To facilitate the generation of the plots given in the next section, the temperatures at both sides of the heat engine, as derived in explicit form, are shown for the convenience of the reader. The temperature of the working fluid at the hot side of the engine is given by

$$T_h = T_H\left(1 - \frac{q_h}{k_h T n_H}\right)^{1/n}. \tag{13}$$

The temperature of the working fluid at the cold side of the engine (using (8)-(9) and (13)) is given by

$$T_c = T_H\left(1 - \eta_0\right)\left(1 - \frac{\left(1 - \eta_0\right)^n - \tau^n}{\left(1 - \eta_0\right)\left(\left(1 - \eta_0\right)^{n-1} + \left(\kappa / R\right)\right)}\right)^{1/n}. \tag{14}$$

In the next section the WCF is considered numerically and sample plots are given for illustrating its usage in analyzing different criteria for the irreversible heat engine.

3. Numerical Considerations

In this section the work criteria function is used to illustrate the different criteria aforementioned earlier. The Newtonian heat transfer law is used in the illustration, but other cases could be presented, following the same procedure.

3.1. Newtonian Heat Transfer Law ($n = 1$). The following figures (Figures 3–9) are given for the case of the Newtonian heat transfer law, with $\tau = 0.5$ (a typical value of Rankine cycle or steam turbines, for which $T_H = 600\,\mathrm{K}$ and $T_L = 300\,\mathrm{K}$). Figure 3 shows the power output of the endoreversible heat engine relative to its maximum value. The plot shows curves of the power for different values of the irreversibility factor, R, in the 0.85–1 range.

By consulting Figure 3 two points should be explained in some detail. First, the maximum efficiency is reduced from the Carnot efficiency to values given by (5). Second, the maximum power is reduced and its efficiency is shifted to the left. If we consider the extreme case, or $R = 0.85$, one could observe that the maximum efficiency and the efficiency at maximum power were reduced by approximately 20%, while the maximum power was reduced by 42%. Similar behavior is observed when considering the other performance characteristic curves, such as power density (Figure 4), efficient power (Figure 5), and efficient power density (Figure 6). It is important to note that the criteria considered in the plots, in ascending order, show a shift to the right in the location of the maximum value of the criterion under consideration. One could conclude that, for a higher order of the efficient power

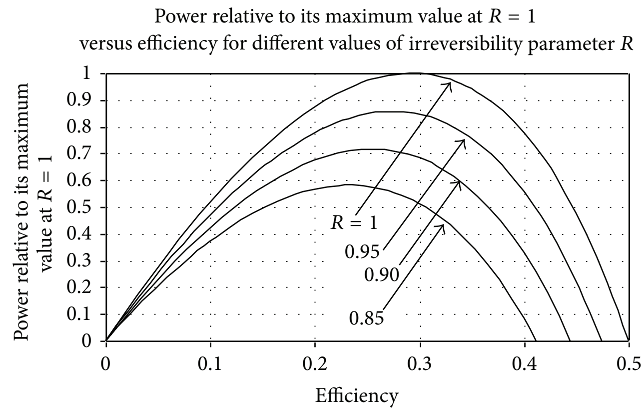


Figure 3: Power relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of R in the 0.85–1 range. The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

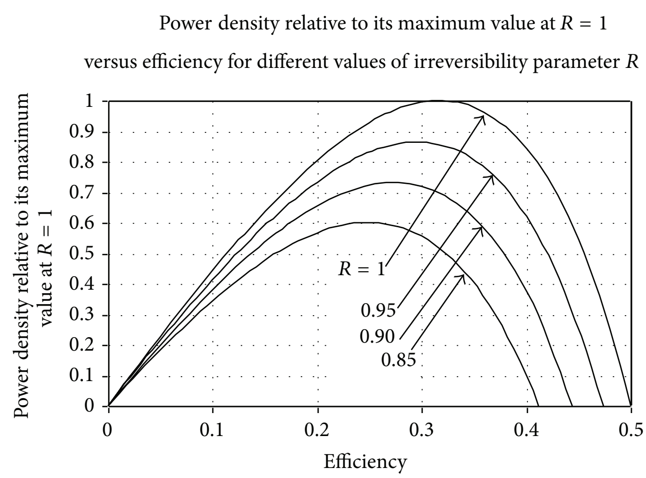


Figure 4: Power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of R in the 0.85–1 range. The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

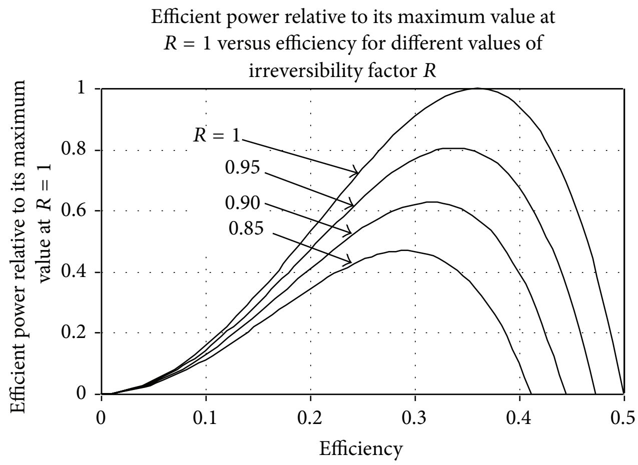


FIGURE 5: Efficient power relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of R in the 0.85–1 range. The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (or the steam turbine working between 600 K and 300 K).

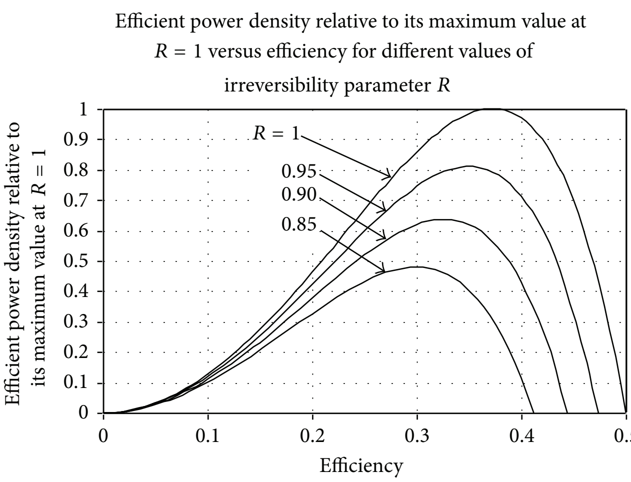


FIGURE 6: Efficient power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of R in the 0.85–1 range. The ratio between the temperature of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

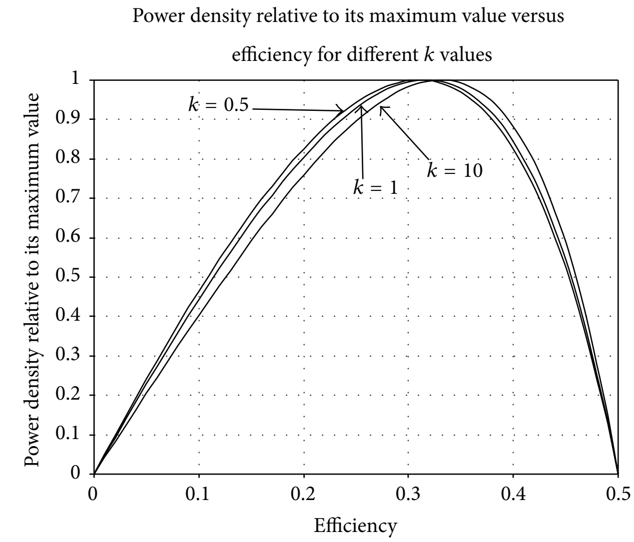


FIGURE 7: Power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of κ in the 1–10 range. The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

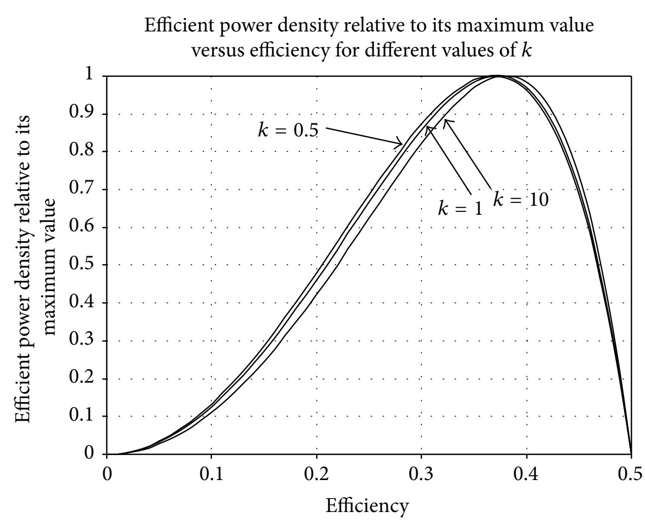


FIGURE 8: Efficient power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The plots represent values of κ in the 1–10 range. The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

or the efficient power density, a higher shift to the right in the efficiency value is produced.

The effect of thermal conductance is reported in Figures 7 and 8, for which inclusive values of κ in the 1–10 range are shown on the plots. The reasonable general conclusion is depicted, as the value of κ changes drastically, of the efficiency at maximum criteria changed approximately by 3%. For comparison purposes, power, power density, efficient power, and efficient power density criteria for the $R = 1$ case are shown on the plots given by Figure 9. The conclusions stated above are now shown explicitly, as could be noticed in this figure.

3.2. Radiative Heat Transfer Law ($n = 4$). The work criteria function could be used to easily analyze different values for different values of n (the exponent power found in the heat transfer rate expressions). For demonstrating its use, the radiative heat transfer law is considered. Figure 10 shows the criteria (similar to Figure 9) with one exception—the typical value of τ is 0.3 (a typical value of the gas turbine dictated by $T_H = 1000$ K and $T_L = 300$ K). Although the location of

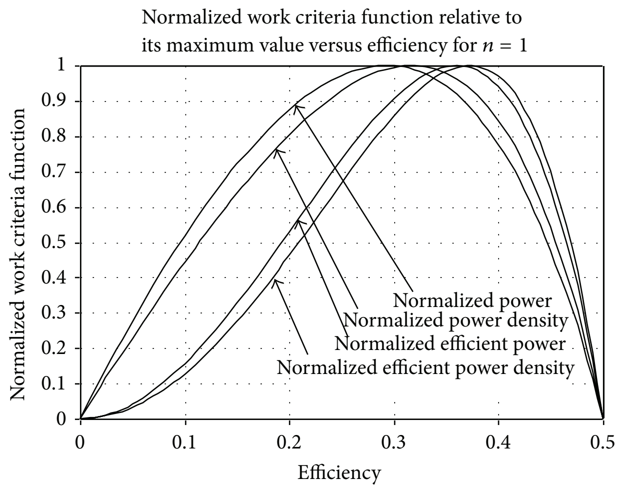


FIGURE 9: Power, power density, efficient power, and efficient power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus Newtonian heat transfer law efficiency ($n = 1$). The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.5$, a typical value for a Rankine cycle (for the steam turbine working between 600 K and 300 K).

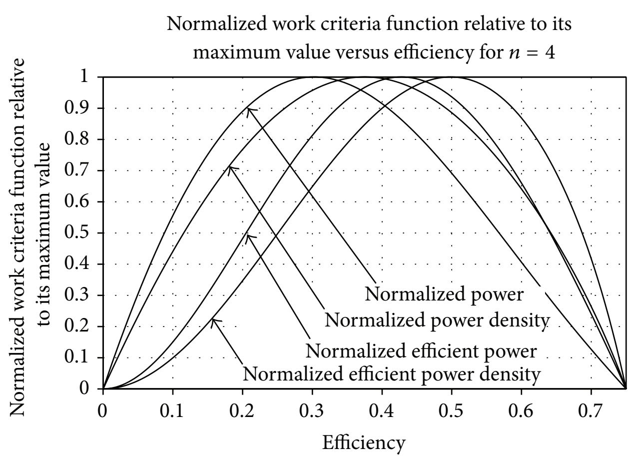


FIGURE 10: Power, power density, efficient power, and efficient power density relative to its maximum value at $R = 1$ and $\kappa = 1$ versus radiative heat transfer law efficiency ($n = 4$). The ratio between the temperatures of the reservoirs, T_L/T_H is $\tau = 0.3$, a typical value for a Brayton cycle (for the gas turbine working between 1000 K and 300 K).

the efficiency at the maximum criteria considered is changed, conclusions similar to those drawn for the case of $n = 1$ are nevertheless valid.

4. Summary and Conclusions

In the current study three criteria of the irreversible heat engine in finite time are reconsidered for a more general heat transfer law (as given by (1) and (2)). Power, power density, and efficient power were cast in a functional form called the work criteria function (WCF—see (11)-(12)). This formulation enabled the introduction of the efficient power density, a new criterion used for describing the performance characteristics of heat engines. The WCF suggests efficient power and efficient power density of different orders, represented by the exponents α and β.

Sample plots are given in Section 3 illustrating the simplicity of the WCF. The Newtonian heat transfer law ($n = 1$) served as a working example to present and compare the criteria mentioned above. The conclusions from these plots for the arbitrarily chosen value of $R = 0.85$ (ideally the value of R should approach 1) were as follows: (1) the maximal efficiency is reduced according to (5) by approximately 20%; (2) the maximum criteria were reduced by 42%; (3) the locations of the efficiency at maximum criteria were shifted to the left, similar to the shift in the maximal efficiency; (4) the smaller the value of the heat conductance ratio, the more power that could be extracted from the engine; and (5) the location of the efficiency at maximum criteria is shifted to the right while comparing the criteria considered above in their order of presentation.

The radiative heat transfer law was considered briefly in Figure 10. Similar conclusions are drawn as for the Newtonian heat transfer law with the following exceptions: the value of the efficiency at maximum criteria is different and the reservoirs temperature is 0.3, which is a typical value representing gas turbine.

Comparing the plots given in Section 3 with what was observed by previous studies, the following conclusion could be derived regarding the practical efficiency of real heat engines.

The efficiencies of any criteria of any order always fall between two extremes—the Carnot efficiency and the Curzon-Ahlborn efficiency.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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