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Research Article

Exact Traveling Wave Solutions for Wick-Type Stochastic Schamel KdV Equation

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F-expansion method is proposed to seek exact solutions of nonlinear partial differential equations. By means of Hermite transform, inverse Hermite transform, and white noise analysis, the variable coefficients and Wick-type stochastic Schamel KdV equations are completely described. Abundant exact traveling wave solutions for variable coefficients Schamel KdV equations are given. These solutions include exact stochastic Jacobi elliptic functions, trigonometric functions, and hyperbolic functions solutions.

1. Introduction

In this paper, we investigate the variable coefficients Schamel KdV equations [1, 2]:

$$u_t + \left[g_1(t)\,u^{1/2} + g_2(t)\,u\right]u_x + g_3(t)\,u_{xxx} = 0, \qquad (t,x) \in \mathbb{R}_+ \times \mathbb{R}, \tag{1}$$

where $g_1(t)$, $g_2(t)$, and $g_3(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Random wave is an important subject of stochastic partial differential equations (SPDEs). Many authors have studied this subject. Wadati first introduced and studied the stochastic KdV equations and gave the diffusion of soliton for KdV equation under Gaussian noise in [3, 4] and others [5–9] also researched stochastic KdV equations. Xie first introduced Wick-type stochastic KdV equations on white noise space and showed the auto-Backlund transformation and the exact white noise functional solutions in [10]. Furthermore, Xie [11–14] and Ghany et al. [15–21] researched some Wick-type stochastic wave equations using white noise analysis.

In this paper we use F-expansion method for finding new periodic wave solutions of nonlinear evolution equations in mathematical physics, and we obtain some new periodic wave solutions for Schamel KdV equations. This method is more powerful and will be used in further works to establish more entirely new solutions for other kinds of nonlinear partial differential equations arising in mathematical physics. The effort in finding exact solutions to nonlinear equations is important for the understanding of most nonlinear physical phenomena, for instance, the nonlinear wave phenomena observed in the fluid dynamics, plasma, and optical fibers [1, 2]. Many effective methods have been presented such as homotopy analysis method [22], variational iteration method [23, 24], tanh-function method [25–27], homotopy perturbation method [28–30], tanh-coth method [26, 31, 32], Exp-function method [33–38], Jacobi elliptic function expansion method [39–42], and F-expansion method [43–46]. The main objective of this paper is using F-expansion method to construct the exact traveling wave solutions for Wick-type stochastic Schamel KdV equations via the Wick-type product, Hermite transform, and white noise analysis. If (1) is considered in a random environment, we can get stochastic Schamel KdV equations. In order to give the exact solutions of stochastic Schamel KdV equations, we only consider this problem in white noise environment.

We will study the following Wick-type stochastic Schamel KdV equations:

$$U_t + \left[G_1(t) \diamond U^{\diamond 1/2} + G_2(t) \diamond U\right] \diamond U_x + G_3(t) \diamond U_{xxx} = 0, \tag{2}$$

where "$\diamond$" is the Wick product on the Kondratiev distribution space $(\mathcal{S})_{-1}$ and $G_1(t), G_2(t)$, and $G_3(t)$ are $(\mathcal{S})_{-1}$ valued functions [47].

2. Description of the F-Expansion Method

In order to simultaneously obtain more periodic wave solutions expressed by various Jacobi elliptic functions to nonlinear wave equations, we introduce an F-expansion method which can be thought of as a succinctly overall generalization of Jacobi elliptic function expansion. We briefly show what F-expansion method is and how to use it to obtain various periodic wave solutions to nonlinear wave equations. Suppose a nonlinear wave equation for $u(t,x)$ is given by

$$\Psi_1(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0, \tag{3}$$

where $u = u(t,x)$ is an unknown function and Ψ_1 is a polynomial in u and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of a deformation F-expansion method.

Step 1. Look for traveling wave solution of (3) by taking

$$u(t,x) = u(\xi), \qquad \xi(t,x) = kx + \int_0^t \theta(\tau)\, d\tau + c. \tag{4}$$

Hence, under the transformation in (4), then, (3) can be transformed into ordinary differential equation (ODE) as follows:

$$\Psi_2\left(u, \theta u', ku', k^2 u'', k^3 u''', \ldots\right) = 0. \tag{5}$$

Step 2. Suppose that $u(\xi)$ can be expressed by a finite power series of $F(\xi)$ of the form

$$u(t,x) = u(\xi) = \sum_{i=0}^{N} a_i F^i(\xi), \tag{6}$$

where $a_0, a_1, \ldots, a_N$ are constants to be determined later, while $F'(\xi)$ in (6) satisfies

$$\left[F'(\xi)\right]^2 = PF^4(\xi) + QF^2(\xi) + R \tag{7}$$

and hence holds for $F(\xi)$:

$$\begin{gathered} F'F'' = 2PF^3F' + QFF', \\ F'' = 2PF^3 + QF, \\ F''' = 6PF^2F' + QF', \\ \ldots, \end{gathered} \tag{8}$$

where P, Q, and R are constants.

Step 3. The positive integer N can be determined by considering the homogeneous balance between the highest derivative term and the nonlinear terms appearing in (5). Therefore, we can get the value of N in (6).

Step 4. Substituting (6) into (5) with condition (7), we obtain polynomial in $F^i(\xi)[F'(\xi)]^j$ $(i = 0 \pm 1, \pm 2, \ldots,\ j = 0, 1)$. Setting each coefficient of this polynomial to be zero yields a set of algebraic equations for $a_0, a_1, \ldots, a_N$ and θ.

Step 5. Solving the algebraic equations with the aid of Maple we have $a_0, a_1, \ldots, a_N$ and θ can be expressed by P, Q, and R. Substituting these results into F-expansion (6), then a general form of traveling wave solution of (3) can be obtained.

Step 6. Since the general solutions of (6) have been well known for us, choose properly P, Q, and R in ODE (7) such that the corresponding solution $F(\xi)$ of it is one of Jacobi elliptic functions (see Appendices A, B, and C) [43–45].

3. Exact Traveling Wave Solutions of (2)

In this section, we apply Hermite transform, white noise theory, and F-expansion method to explore soliton and periodic wave solutions for (2). Applying Hermite transform to (2), we get the deterministic equation

$$\begin{aligned} &\widetilde{U}_t(t,x,z) + \left\{\widetilde{G}_1(t,z)\widetilde{U}^{1/2}(t,x,z) \right.\\ &\qquad \left. + \widetilde{G}_2(t,z)\widetilde{U}(t,x,z)\right\}\widetilde{U}_x(t,x,z) \\ &\qquad + \widetilde{G}_3(t,z)\widetilde{U}_{xxx}(t,x,z) = 0, \end{aligned} \tag{9}$$

where $z = (z_1, z_2, \ldots) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter. To look for the traveling wave solution of (3), we make the transformations $\widetilde{G}_1(t,z) := g_1(t,z)$, $\widetilde{G}_2(t,z) := g_2(t,z)$, $\widetilde{G}_3(t,z) := g_3(t,z)$, and $\widetilde{U}(t,x,z) =: u(t,x,z)$, $u = v^2$, $v(t,x,z) = V(\xi)$, with

$$\xi(t,x,z) = k\left[x - \int_0^t \theta(\tau,z)\, d\tau\right] + c, \tag{10}$$

where k and c are arbitrary constants which satisfy $k \neq 0$ and $\theta(t,z)$ is a nonzero function of the indicated variables to be determined later. Thus, (3) can be transformed into the following ODE:

$$-\theta VV' + \left[g_1V^2 + g_2V^3\right]V' + g_3k^2\left[VV''' + 3V'V''\right] = 0, \tag{11}$$

where $V' = dV/d\xi$. The balancing procedure implies that $N = 1$. Therefore, in view of F-expansion method the solution of (3) can be expressed in the form

$$V(t,x,z) = V(\xi) = a_0 + a_1F(\xi(t,x,z)), \tag{12}$$

where a_0, a_1 are constants to be determined later. Substitute (12) with conditions (7) and (8) into (11) and collect all terms with the same power of $F^i(\xi)[F'(\xi)]^j (i = 0, \pm 1, \pm 2, \ldots,\ j = 0, 1)$ as follows:

$$\begin{aligned}&\left[-\theta a_0 a_1 + g_1 a_0^2 a_1 + g_2 a_0^3 a_1 + g_3 k^2 a_0 a_1 Q\right] F'\\&\quad + \left[-\theta a_1^2 + 2 g_1 a_0 a_1^2 + 3 g_2 a_0^2 a_1^2 + 4 g_3 k^2 a_1^2 Q\right] F F'\\&\quad + \left[g_1 a_1^3 + 3 g_2 a_0 a_1^3 + 6 g_3 k^2 a_0 a_1 P\right] F^2 F'\\&\quad + \left[g_2 a_1^4 + 12 g_3 k^2 a_1^2 P\right] F^3 F' = 0.\end{aligned} \tag{13}$$

Setting each coefficient of $F^i(\xi)[F'(\xi)]^j$ to be zero, we get a system of algebraic equations which can be expressed by

$$\begin{aligned}&\left(-\theta + g_1 a_0 + g_2 a_0^2 + g_3 k^2 Q\right) a_0 a_1 = 0,\\&\left(-\theta + 2 g_1 a_0 + 3 g_2 a_0^2 + 4 g_3 k^2 Q\right) a_1^2 = 0,\\&\left(g_1 a_1^2 + 3 g_2 a_0 a_1^2 + 6 g_3 k^2 a_0 P\right) a_1 = 0,\\&\left(g_2 a_1^2 + 12 g_3 k^2 P\right) a_1^2 = 0,\end{aligned} \tag{14}$$

with solving the above system to get the following coefficients:

$$\begin{aligned}a_1 &= \pm\sqrt{\frac{-12 k^2 g_3(t,z) P}{g_2(t,z)}},\\a_0 &= -\frac{2 g_1(t,z)}{5 g_2(t,z)},\\\theta &= \frac{-6 g_1^2(t,z) + 25 k^2 g_2(t,z) g_3(t,z) Q}{25 g_2(t,z)}.\end{aligned} \tag{15}$$

Substituting coefficient (15) into (12) yields general form solutions to (2):

$$u(t,x,z) = \left[-\frac{2 g_1(t,z)}{5 g_2(t,z)} \pm ik\sqrt{\frac{12 g_3(t,z) P}{g_2(t,z)}} F(\xi(t,x,z))\right]^2, \tag{16}$$

with

$$\begin{aligned}&\xi(t,x,z)\\&= k\left\{x - \int_0^t \left[\frac{-6 g_1^2(\tau,z) + 25 k^2 g_2(\tau,z) g_3(\tau,z) Q}{25 g_2(\tau,z)}\right] d\tau\right\}\\&\quad + c.\end{aligned} \tag{17}$$

From Appendix A, we give the special cases as follows.

Case 1. If we take $P = 1$, $Q = (2 - m^2)$, and $R = (1 - m^2)$, then $F(\xi) \to \mathrm{cs}(\xi)$;

$$\begin{aligned}&u_1(t,x,z)\\&= \left[-\frac{2 g_1(t,z)}{5 g_2(t,z)} \pm ik\sqrt{\frac{12 g_3(t,z)}{g_2(t,z)}} \mathrm{cs}(\xi_1(t,x,z))\right]^2,\end{aligned} \tag{18}$$

with

$$\begin{aligned}&\xi_1(t,x,z)\\&= k\left\{x - \int_0^t \left[\frac{-6 g_1^2(\tau,z) + 25 k^2 g_2(\tau,z) g_3(\tau,z)\left(2 - m^2\right)}{25 g_2(\tau,z)}\right] d\tau\right\}\\&\quad + c.\end{aligned} \tag{19}$$

In the limit case when $m \to o$, we have $\mathrm{cs}(\xi) \to \cot(\xi)$; thus (18) becomes

$$\begin{aligned}&u_2(t,x,z)\\&= \left[-\frac{2 g_1(t,z)}{5 g_2(t,z)} \pm ik\sqrt{\frac{12 g_3(t,z)}{g_2(t,z)}} \cot(\xi_2(t,x,z))\right]^2,\end{aligned} \tag{20}$$

with

$$\begin{aligned}&\xi_2(t,x,z)\\&= k\left\{x - \int_0^t \left[\frac{-6 g_1^2(\tau,z) + 50 k^2 g_2(\tau,z) g_3(\tau,z)}{25 g_2(\tau,z)}\right] d\tau\right\}\\&\quad + c.\end{aligned} \tag{21}$$

In the limit case when $m \to 1$, we have $\mathrm{cs}(\xi) \to \mathrm{csch}(\xi)$; thus (18) becomes

$$\begin{aligned}&u_3(t,x,z)\\&= \left[-\frac{2 g_1(t,z)}{5 g_2(t,z)} \pm ik\sqrt{\frac{12 g_3(t,z)}{g_2(t,z)}} \mathrm{csch}(\xi_3(t,x,z))\right]^2,\end{aligned} \tag{22}$$

with

$$\begin{aligned}&\xi_3(t,x,z)\\&= k\left\{x - \int_0^t \left[\frac{-6 g_1^2(\tau,z) + 25 k^2 g_2(\tau,z) g_3(\tau,z)}{25 g_2(\tau,z)}\right] d\tau\right\}\\&\quad + c.\end{aligned} \tag{23}$$

Case 2. If we take $P = 1/4$, $Q = (m^2 + 1)/2$, and $R = (1 - m^2)^2/4$, then $F(\xi) \to \mathrm{sn}\,\xi/(\mathrm{cn}\,\xi \pm \mathrm{dn}\,\xi)$ and

$$\begin{aligned}&u_4(t,x,z)\\&= \left[-\frac{2 g_1(t,z)}{5 g_2(t,z)} \pm ik\sqrt{\frac{3 g_3(t,z)}{g_2(t,z)}}\right.\\&\qquad\left.\times\left\{\frac{\mathrm{sn}(\xi_4(t,x,z))}{\mathrm{cn}(\xi_4(t,x,z)) \pm \mathrm{dn}(\xi_4(t,x,z))}\right\}\right]^2,\end{aligned} \tag{24}$$

with

$$\xi_4(t,x,z) = k\left\{x-\int_0^t\left[\frac{-12g_1^2(\tau,z)+25k^2g_2(\tau,z)\,g_3(\tau,z)\left(m^2+1\right)}{50g_2(\tau,z)}\right]d\tau\right\} + c. \quad (25)$$

In the limit case when $m \to o$, we have $\operatorname{sn}\xi/(\operatorname{cn}\xi \pm \operatorname{dn}\xi) \to \sin\xi/(\cos\xi \pm 1)$; thus (24) becomes

$$u_5(t,x,z) = \left[-\frac{2g_1(t,z)}{5g_2(t,z)} \pm ik\sqrt{\frac{3g_3(t,z)}{g_2(t,z)}}\left\{\frac{\sin\xi_5(t,x,z)}{\cos\xi_5(t,x,z)\pm 1}\right\}\right]^2, \quad (26)$$

with

$$\xi_5(t,x,z) = k\left\{x-\int_0^t\left[\frac{-12g_1^2(\tau,z)+25k^2g_2(\tau,z)\,g_3(\tau,z)}{50g_2(\tau,z)}\right]d\tau\right\} + c. \quad (27)$$

In the limit case when $m \to 1$, we have $\operatorname{sn}\xi/(\operatorname{cn}\xi \pm \operatorname{dn}\xi) \to \tanh(\xi)/2\operatorname{sech}(\xi) = (1/2)\sinh(\xi)$; thus (24) becomes

$$u_6(t,x,z) = \left[-\frac{2g_1(t,z)}{5g_2(t,z)} \pm \frac{ik}{2}\sqrt{\frac{3g_3(t,z)}{g_2(t,z)}}\sinh\left(\xi_3(t,x,z)\right)\right]^2. \quad (28)$$

Case 3. If we take $P = 1/4, Q = (1-2m^2)/2$, and $R = 1/4$, then $F(\xi) \to \operatorname{ns}(\xi) \pm \operatorname{cs}(\xi)$ and

$$u_7(t,x,z) = \left[-\frac{2g_1(t,z)}{5g_2(t,z)} \pm ik\sqrt{\frac{3g_3(t,z)}{g_2(t,z)}} \times \left\{\operatorname{ns}\left(\xi_6(t,x,z)\right) \pm \operatorname{cs}\left(\xi_6(t,x,z)\right)\right\}\right]^2, \quad (29)$$

with

$$\xi_6(t,x,z) = k\left\{x-\int_0^t\left[\frac{-12g_1^2(\tau,z)+25k^2g_2(\tau,z)\,g_3(\tau,z)\left(1-2m^2\right)}{50g_2(\tau,z)}\right]d\tau\right\} + c. \quad (30)$$

In the limit case when $m \to o$, we have $\operatorname{ns}(\xi) \pm \operatorname{cs}(\xi) \to \csc(\xi) \pm \cot(\xi)$; thus (29) becomes

$$u_8(t,x,z) = \left[-\frac{2g_1(t,z)}{5g_2(t,z)} \pm ik\sqrt{\frac{3g_3(t,z)}{g_2(t,z)}} \times \left\{\csc\left(\xi_5(t,x,z)\right) \pm \cot\left(\xi_5(t,x,z)\right)\right\}\right]^2. \quad (31)$$

In the limit case when $m \to 1$, we have $\operatorname{ns}(\xi) \pm \operatorname{cs}(\xi) \to \coth\xi \pm \operatorname{csch}\xi$; thus (29) becomes

$$u_9(t,x,z) = \left[-\frac{2g_1(t,z)}{5g_2(t,z)} \pm \frac{ik}{2}\sqrt{\frac{3g_3(t,z)}{g_2(t,z)}} \times \left\{\coth\xi_7(t,x,z) \pm \operatorname{csch}\xi_7(t,x,z)\right\}\right]^2, \quad (32)$$

with

$$\xi_7(t,x,z) = k\left\{x+\int_0^t\left[\frac{12g_1^2(\tau,z)+25k^2g_2(\tau,z)\,g_3(\tau,z)}{50g_2(\tau,z)}\right]d\tau\right\} + c. \quad (33)$$

Remark that there are other solutions for (2). These solutions come from setting different values for the coefficients P, Q, and R (see Appendices A, B, and C). The above-mentioned cases are just to clarify how far our technique is applicable.

4. White Noise Functional Solutions of (2)

In this section, we employ the results of Section 3 by using Hermite transform to obtain exact white noise functional solutions for Wick-type stochastic Schamel KdV equations (2). The properties of exponential and trigonometric functions yield the fact that there exists a bounded open set $\mathbf{H} \subset \mathbb{R}_+ \times \mathbb{R}$, $\rho < \infty$, $\lambda > 0$ such that the solution $u(t,x,z)$ of (9) and all its partial derivatives which are involved in (9) are uniformly bounded for $(t,x,z) \in \mathbf{H}\times K_\rho(\lambda)$, continuous with respect to $(t,x) \in \mathbf{H}$ for all $z \in K_\rho(\lambda)$, and analytic with respect to $z \in K_\rho(\lambda)$, for all $(t,x) \in \mathbf{H}$. From Theorem 4.1.1 in [47], there exists $U(t,x,z) \in (\mathcal{S})_{-1}$ such that $u(t,x,z) = \widetilde{U}(t,x)(z)$ for all $(t,x,z) \in \mathbf{H}\times K_\rho(\lambda)$ and $U(t,x)$ solves (2) in $(\mathcal{S})_{-1}$. Hence, by applying the inverse Hermite transform to the results of Section 3, we get exact white noise functional solutions of (2) as follows.

(i) Exact stochastic Jacobi elliptic functions solutions:

$$U_1(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm ik\sqrt{\frac{12G_3(t)}{G_2(t)}}\diamond \mathrm{cs}^{\diamond}(\Xi_1(t,x))\right]^{\diamond 2},$$

$$U_2(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm ik\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond\left\{\frac{\mathrm{sn}^{\diamond}(\xi_2(t,x))}{\mathrm{cn}^{\diamond}(\Xi_2(t,x))\pm \mathrm{dn}^{\diamond}(\Xi_2(t,x))}\right\}\right]^{\diamond 2},$$

$$U_3(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm ik\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond\left\{\mathrm{ns}^{\diamond}(\Xi_3(t,x))\pm \mathrm{cs}^{\diamond}(\Xi_3(t,x))\right\}\right]^{\diamond 2}, \quad (34)$$

with

$$\Xi_1(t,x)=k\left\{x-\int_0^t\left[\frac{-6G_1^{\diamond 2}(\tau)+25k^2G_2(\tau)\diamond G_3(\tau)(2-m^2)}{25G_2(\tau)}\right]d\tau\right\}+c,$$

$$\Xi_2(t,x)=k\left\{x-\int_0^t\left[\frac{-12G_1^{\diamond 2}(\tau)+25k^2G_2(\tau)\diamond G_3(\tau)(m^2+1)}{50G_2(\tau)}\right]d\tau\right\}+c,$$

$$\Xi_3(t,x)=k\left\{x-\int_0^t\left[\frac{-12G_1^{\diamond 2}(\tau)+25k^2G_2(\tau)\diamond G_3(\tau)(1-2m^2)}{50G_2(\tau)}\right]d\tau\right\}+c. \quad (35)$$

(ii) Exact stochastic trigonometric solutions:

$$U_4(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm\sqrt{\frac{-12k^2G_3(t)}{G_2(t)}}\diamond \cot^{\diamond}(\Xi_4(t,x))\right]^{\diamond 2},$$

$$U_5(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm ik\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond\left\{\frac{\sin^{\diamond}\Xi_5(t,x)}{\cos^{\diamond}\Xi_5(t,x)\pm 1}\right\}\right]^{\diamond 2},$$

$$U_6(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm ik\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond\left\{\csc^{\diamond}(\Xi_5(t,x))\pm\cot^{\diamond}(\Xi_5(t,x))\right\}\right]^{\diamond 2}, \quad (36)$$

with

$$\Xi_4(t,x)=k\left\{x-\int_0^t\left[\frac{-6G_1^{\diamond 2}(\tau)+50k^2G_2(\tau)\diamond G_3(\tau)}{25G_2(\tau)}\right]d\tau\right\}+c,$$

$$\Xi_5(t,x)=k\left\{x-\int_0^t\left[\frac{-12G_1^{\diamond 2}(\tau)+25k^2G_2(\tau)\diamond G_3(\tau)}{50G_2(\tau)}\right]d\tau\right\}+c. \quad (37)$$

(iii) Exact stochastic hyperbolic solutions:

$$U_7(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm\sqrt{\frac{-12k^2G_3(t)}{G_2(t)}}\diamond \mathrm{csch}^{\diamond}(\Xi_6(t,x))\right]^{\diamond 2},$$

$$U_8(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm\frac{ik}{2}\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond \sinh^{\diamond}(\Xi_6(t,x))\right]^{\diamond 2},$$

$$U_9(t,x)=\left[-\frac{2G_1(t)}{5G_2(t)}\pm\frac{ik}{2}\sqrt{\frac{3G_3(t)}{G_2(t)}}\diamond\left\{\coth^{\diamond}\Xi_7(t,x)\pm \mathrm{csch}^{\diamond}\Xi_7(t,x)\right\}\right]^{\diamond 2}, \quad (38)$$

with

$$\Xi_6(t,x)=k\left\{x-\int_0^t\left[\frac{-6G_1^{\diamond 2}(\tau)+25k^2G_2(\tau)\diamond G_3(\tau)}{25G_2(\tau)}\right]d\tau\right\}+c,$$

$$\Xi_7(t,x) = k\left\{x + \int_0^t \left[\frac{6G_1^{\diamond 2}(\tau) + 25k^2 G_2(\tau)\diamond G_3(\tau)}{50G_2(\tau)}\right] d\tau\right\} + c. \tag{39}$$

We observe that, for different forms of G_1, G_2, and G_3, we can get different types of exact stochastic functional solutions of (2) from (34)–(38).

5. Example

It is well known that Wick version of function is usually difficult to evaluate. So, in this section, we give non-Wick version of solutions of (2). Let $W_t = \dot{B}_t$ be the Gaussian white noise, where B_t is the Brownian motion. We have the Hermite transform [47]:

$$\widetilde{W}_t(z) = \sum_{i=1}^{\infty} z_i \int_0^t \mu_i(s)\,ds. \tag{40}$$

Since

$$\begin{gathered}
\exp^{\diamond}(B_t) = \exp\left(B_t - \frac{t^2}{2}\right), \quad \text{we have}\\
\sin^{\diamond}(B_t) = \sin\left(B_t - \frac{t^2}{2}\right),\\
\cos^{\diamond}(B_t) = \cos\left(B_t - \frac{t^2}{2}\right),\\
\cot^{\diamond}(B_t) = \cot\left(B_t - \frac{t^2}{2}\right),\\
\csc^{\diamond}(B_t) = \csc\left(B_t - \frac{t^2}{2}\right),\\
\coth^{\diamond}(B_t) = \coth\left(B_t - \frac{t^2}{2}\right),\\
\operatorname{csch}^{\diamond}(B_t) = \operatorname{csch}\left(B_t - \frac{t^2}{2}\right),\\
\sinh^{\diamond}(B_t) = \sinh\left(B_t - \frac{t^2}{2}\right).
\end{gathered} \tag{41}$$

Suppose that

$$G_1(t) = \eta_1 G_3(t), \qquad G_2(t) = \eta_2 G_3(t), \qquad G_3(t) = \sigma(t) + \eta_3 W_t, \tag{42}$$

where η_1, η_2, and η_3 are arbitrary constants and $\sigma(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$. Therefore, for $G_1(t)G_2(t)G_3(t) \neq 0$, exact white noise functional solutions of (2) are as follows:

$$\begin{aligned}
U_{10}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm ik\sqrt{\frac{12}{\eta_2}}\cot\Pi_1(t,x)\right]^2,\\
U_{11}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm ik\sqrt{\frac{3}{\eta_2}}\left\{\frac{\sin\Pi_2(t,x)}{\cos\Pi_2(t,x) \pm 1}\right\}\right]^2,\\
U_{12}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm ik\sqrt{\frac{3}{\eta_2}}\left\{\csc\Pi_2(t,x) \pm \cot\Pi_2(t,x)\right\}\right]^2,
\end{aligned} \tag{43}$$

with

$$\begin{aligned}
\Pi_1(t,x) &= k\left[x - \left(\frac{-6\eta_1^2 + 50k^2\eta_2}{25\eta_2}\right) \times \left\{\int_0^t \sigma(\tau)\,d\tau + \eta_3\left[B_t - \frac{t^2}{2}\right]\right\}\right] + c,\\
\Pi_2(t,x) &= k\left[x - \left(\frac{-12\eta_1^2 + 25k^2\eta_2}{50\eta_2}\right) \times \left\{\int_0^t \sigma(\tau)\,d\tau + \eta_3\left[B_t - \frac{t^2}{2}\right]\right\}\right] + c,
\end{aligned}$$

$$\begin{aligned}
U_{13}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm ik\sqrt{\frac{12}{\eta_2}}\operatorname{csch}\Pi_3(t,x)\right]^2,\\
U_{14}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm \frac{ik}{2}\sqrt{\frac{3}{\eta_2}}\sinh\Pi_3(t,x)\right]^2,\\
U_{15}(t,x) &= \left[-\frac{2\eta_1}{5\eta_2} \pm \frac{ik}{2}\sqrt{\frac{3}{\eta_2}}\left\{\coth\Pi_4(t,x) \pm \operatorname{csch}\Pi_4(t,x)\right\}\right]^2,
\end{aligned} \tag{44}$$

with

$$\begin{aligned}
\Pi_3(t,x) &= k\left[x - \frac{-6\eta_1^2 + 25k^2\eta_2}{25\eta_2}\left\{\int_0^t \sigma(\tau)\,d\tau + \eta_3\left[B_t - \frac{t^2}{2}\right]\right\}\right] + c,\\
\Pi_4(t,x) &= k\left[x + \frac{12\eta_1^2 + 25k^2\eta_2}{50\eta_2}\left\{\int_0^t \sigma(\tau)\,d\tau + \eta_3\left[B_t - \frac{t^2}{2}\right]\right\}\right] + c.
\end{aligned} \tag{45}$$

Table 1

$[F'(\xi)]^2 = PF^4(\xi) + QF^2(\xi) + R,$			
P	Q	R	$F(\xi)$
m^2	$-1-m^2$	1	$\mathrm{sn}\xi,\ \mathrm{cd}\xi = \dfrac{\mathrm{cn}\xi}{\mathrm{dn}\xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$\mathrm{dn}\xi$
1	$-1-m^2$	m^2	$\mathrm{ns}\xi = \dfrac{1}{\mathrm{sn}\xi},\ \mathrm{dc}\xi = \dfrac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$\mathrm{nc}\xi = \dfrac{1}{\mathrm{cn}\xi}$
m^2-1	$2-m^2$	-1	$\mathrm{nd}\xi = \dfrac{1}{\mathrm{dn}\xi}$
$1-m^2$	$2-m^2$	1	$\mathrm{sc}\xi = \dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$
$-m^2(1-m^2)$	$2m^2-1$	1	$\mathrm{sd}\,\xi = \dfrac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$\mathrm{cs}\xi = \dfrac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$\mathrm{ds}\xi = \dfrac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$
$\dfrac{m^4}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{1}{4}$	$\dfrac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi},\ \dfrac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi}$
$\dfrac{m^2}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{m^2}{4}$	$\mathrm{sn}\xi \pm i\mathrm{cn}\xi,\ \dfrac{\mathrm{dn}\xi}{i\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{cn}\xi},\ \dfrac{m\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi}$
$\dfrac{1}{4}$	$\dfrac{1-2m^2}{2}$	$\dfrac{1}{4}$	$\mathrm{ns}\xi\pm\mathrm{cs}\xi,\ \dfrac{\mathrm{cn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{dn}\xi},\ \dfrac{\mathrm{sn}\xi}{1\pm\mathrm{cn}\xi},$
$\dfrac{m^2-1}{4}$	$\dfrac{m^2+1}{2}$	$\dfrac{m^2-1}{4}$	$\dfrac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi}$
$\dfrac{1-m^2}{4}$	$\dfrac{m^2+1}{2}$	$\dfrac{1-m^2}{4}$	$\mathrm{nc}\xi\pm i\mathrm{sc}\xi\dfrac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi}$
$\dfrac{-1}{4}$	$\dfrac{m^2+1}{2}$	$\dfrac{-(1-m^2)^2}{4}$	$m\mathrm{cn}\xi\pm\mathrm{dn}\xi$
$\dfrac{1}{4}$	$\dfrac{m^2+1}{2}$	$\dfrac{(1-m^2)^2}{4}$	$\dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi\pm\mathrm{dn}\xi}$
$\dfrac{1}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{m^2}{4}$	$\mathrm{ns}\xi\pm\mathrm{ds}\xi$

6. Summary and Discussion

We have discussed the solutions of SPDEs driven by Gaussian white noise. There is a unitary mapping between the Gaussian white noise space and the Poisson white noise space. This connection was given by Benth and Gjerde [48]. From [47, section 4.9] and by the aid of the connection, we can derive some stochastic exact soliton solutions, which are Poisson white noise functions in (2). In this paper, using Hermite transformation, white noise theory, and F-expansion method, we study the white noise functional solutions for Wick-type stochastic Schamel KdV equations. This paper shows that F-expansion method is sufficient to solve the stochastic nonlinear equations in mathematical physics. The method which we have proposed in this paper is standard, direct, and computerized method, which allows us to do complicated and tedious algebraic calculation. It is shown that the algorithm can be also applied to other nonlinear SPDEs in mathematical physics such as modified Hirota-Satsuma coupled KdV, KdV-Burgers, modified KdV Burgers, Sawada-Kotera, and Zhiber-Shabat equations and Benjamin-Bona-Mahony (BBM) equations. Since (2) has other solutions of Jacobi elliptic functions, trigonometric functions, and hyperbolic functions if we select other values of P, Q, and R (see Appendices A, B, and C), there are many other exact traveling wave solutions for Wick-type stochastic Schamel KdV equations.

Appendices

A.

The Jacobi elliptic functions degenerate into trigonometric functions when $m \to 0$:

$$\mathrm{sn}\,\xi \longrightarrow \sin\xi, \qquad \mathrm{cn}\,\xi \longrightarrow \cos\,\xi, \qquad \mathrm{dn}\,\xi \longrightarrow 1,$$

$$\mathrm{sc}\,\xi \longrightarrow \tan\,\xi,$$

$$\mathrm{sd}\,\xi \longrightarrow \sin\,\xi, \qquad \mathrm{cd}\,\xi \longrightarrow \cos\xi,$$

$$\mathrm{ns}\,\xi \longrightarrow \csc\xi, \qquad \mathrm{nc}\,\xi \longrightarrow \sec\xi, \qquad \mathrm{nd}\,\xi \longrightarrow 1,$$

$$\begin{gathered}\operatorname{cs}\xi \longrightarrow \cot\xi, \\ \operatorname{ds}\xi \longrightarrow \csc\xi, \qquad \operatorname{dc}\xi \longrightarrow \sec\xi.\end{gathered} \tag{A.1}$$

B.

The Jacobi elliptic functions degenerate into hyperbolic functions when $m \to 1$:

$$\begin{gathered}\operatorname{sn}\xi \longrightarrow \tan\xi, \qquad \operatorname{cn}\xi \longrightarrow \operatorname{sech}\xi, \qquad \operatorname{dn}\xi \longrightarrow \operatorname{sech}\xi, \\ \operatorname{sc}\xi \longrightarrow \sinh\xi, \\ \operatorname{sd}\xi \longrightarrow \sinh\xi, \qquad \operatorname{cd}\xi \longrightarrow 1, \\ \operatorname{ns}\xi \longrightarrow \coth\xi, \qquad \operatorname{nc}\xi \longrightarrow \cosh\xi, \qquad \operatorname{nd}\xi \longrightarrow \cosh, \\ \operatorname{cs}\xi \longrightarrow \operatorname{csch}\xi, \\ \operatorname{ds}\xi \longrightarrow \operatorname{csch}\xi, \qquad \operatorname{dc}\xi \longrightarrow 1.\end{gathered} \tag{B.1}$$

C.

The ODE and Jacobi elliptic functions: for relation between values of P, Q, and R and corresponding $F(\xi)$ in ODE, see Table 1.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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