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Research Article

An Extension of the Hawkins and Simon Condition Characterizing Viable Techniques

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This paper discusses an extended version of the Hawkins and Simon condition which constitutes a synthetic formulation of the mathematical properties that viable economies must satisfy in single production models. The new version is implicit in the economic interpretations offered by them of the Hawkins and Simon condition, once a correction is introduced in one of those interpretations. Moreover, the paper details the meaning of the extended version following the interpretation of the original version proposed by Dorfman, Samuelson, and Solow. It also introduces a characteristic property of indecomposable matrices that has not previously been published.

1. Introduction

When studying the economies capable of reproducing themselves, it is useful to distinguish between first- and second-class economies. The former ones do not produce a surplus while the latter ones do produce a surplus. Furthermore, both classes may be viable or not, depending on whether they comply or not with certain requirements which, in turn, are not the same for different authors (e.g., Benítez Sánchez and Benítez Sánchez [1], p. 22; Bidard [2], pp. 13, 32; Kurz and Salvadori [3], pp. 60, 96; Pasinetti [4], pp. 63, 78). This paper presents a synthetic formulation of the mathematical properties that viable economies must satisfy in the models considered by Leontief [5] and Sraffa [6] and discusses its economic meaning. According to Benítez Sánchez and Benítez Sánchez [1], in first-class economies, the matrix of technical coefficients must be indecomposable and its Frobenius root must equal one, while, in second-class economies, the matrix must satisfy the condition established by Hawkins and Simon [7], which will be referred to with the notation HS. The synthetic formulation of these two sets of conditions constitutes an extended version (EHS) of HS that simultaneously expresses the conditions required by viable economies of the two classes. Furthermore, this paper shows that EHS is implicit in the economic interpretations of HS offered by Hawkins and Simon [7] once a correction is introduced.

Including this introduction, the paper is divided into seven sections. Section 2 presents a brief survey of the relevant literature. Section 3 presents the reference model and a mathematical characterization of the viable techniques. Section 4 introduces EHS and proves its equivalence with the two sets of conditions characterizing viable economies. It also presents a characteristic property of indecomposable matrices that, as far as the author is aware, has not been published previously. Section 5 proposes an economic interpretation of EHS based on the interpretation of HS by Dorfman et al. [8]. Section 6 points out and corrects an error in one of the economic interpretations of HS originally offered by Hawkins and Simon [7]. It also formulates an alternative definition of viable economies which helps readers to visualize these economic interpretations as well as an interpretation of EHS. The last section adds some comments of a general character.

2. A Brief Survey of the Literature

Benítez Sánchez and Benítez Sánchez [1] showed that the closed model studied by Leontief [5] and the subsistence economies introduced by Sraffa [6] may be understood as particular representations of second-class economies. This implies that if the second-class economies satisfy HS, in both cases, the corresponding coefficient matrix is indecomposable and its Frobenius root is equal to one. On this basis, they introduced the definition of viable economies reproduced in the following section, further study of which is the central theme of this paper.

Regarding the economic interpretation of HS, it is important to distinguish the two approaches introduced, respectively, by Dorfman et al. [8] and by Hawkins and Simon [7]. In the first work, the interpretation refers to the quantity of each good required to produce a unit of the same good, while in the second one, it refers to the capacity of each set of industries to produce the quantity of each good consumed by the set. Some aspects of the first interpretation have been discussed by Benítez Sánchez [9, 10], Dasgupta [11], Fujita [12], Jeong [13, 14], and Takayama [15]. In contrast, the second interpretation has been studied much less.

3. Viable Techniques

The first part of this section presents the model of a single production economy and the second one presents the definition of viable techniques.

3.1. The Model. The reference economy is integrated by n ($n \geq 1$) industries, each one producing a particular type of good labeled i or j so that $i, j = 1, 2, \ldots, n$. We will refer to a set of indexes $\{j1, j2, \ldots, jD\}$ as D-set if it contains D different indexes and, for any particular D-set, we have $d = 1, 2, \ldots, D$. We will also refer to indexes as goods. For each pair (i, j) and for each j, a_{ij} and l_j are, respectively, the quantity of i and the quantity of labor consumed directly in the production of one unit of j. Regarding these technical coefficients, we assume that $a_{ij} \geq 0$ for every (i, j) and $l_j > 0$ for every j. Furthermore, for each i, p_i is the price of good i, z_i is the sum of wages and profits corresponding to branch i per unit of good, x_i is the quantity of i produced in the corresponding industry, and c_i is the difference between this quantity and the amount of the same good that is consumed in the industrial system during the period. It is useful to write these quantities in matrix notation defining the column vectors $p = (p_1, p_2, \ldots, p_n)^T$, $z = (z_1, z_2, \ldots, z_n)^T$, $x = (x_1, x_2, \ldots, x_n)^T$, and $c = (c_1, c_2, \ldots, c_n)^T$, together with the input matrix $A = [a_{ij}]$. This enables us to represent the relations between the inputs and outputs of the different goods and the relation between each price and its production cost, respectively, by means of the first and the second of the following equation systems:

$$Ax + c = x, \tag{1}$$

$$A^T p + z = p. \tag{2}$$

The Frobenius roots of matrices A and A^T, which are equal, are represented with λ_A. Furthermore, given two matrices (A, B) or two vectors (x, y), the relations $A > B$ and $x > y$ means respectively that $a_{ij} > b_{ij}$ for every couple (i, j) and $x_j > y_j$ for every j. We define each one of the relations "<", "≥" and "≤" in a similar manner. A vector x is positive if $x > 0$ and semi-positive if $x \geq 0$ and $x \neq 0$, similar definitions are valid for matrices and positive scalars. If all the entries of a matrix or a vector are equal to zero we may represent it with 0.

A square matrix $A \geq 0$ may be interpreted as an input matrix corresponding to an economy that produces one unit of each good. Assuming this interpretation, and to simplify, we will refer to any such matrix as a technique even if the labor amounts are not indicated.

3.2. Viable Techniques. In first-class economies, $c = z = 0$, and in second-class economies, $c \geq 0, c \neq 0$, and $z \geq 0, z \neq 0$. The following definition, in which I_n is $(n \times n)$ identity matrix, is relevant to both classes.

Definition 1. Let A be a square matrix such that $A \geq 0$. A is a viable technique if it satisfies either the first two or either of the last two of the following conditions:

$$A \text{ is indecomposable,} \tag{3}$$

$$\lambda_A = 1, \tag{4}$$

$$\text{HS: all the principal minors of } [I_n - A] \text{ are greater than zero,} \tag{5}$$

$$0 \leq \lambda_A < 1. \tag{6}$$

In first-class economies, (3) and (4) are a pair of necessary and sufficient conditions for a technique to sustain an economy producing the means of production consumed and a price system to exist, allowing each branch to recover the investment made. In second-class economies, each of the conditions (5) and (6) are necessary and sufficient for a technique to sustain an economy producing any given physical surplus and a price system to exist, allowing each branch to obtain any given share of the national income. It must be underscored that, in the first case, relative prices are determined and are positive, although the unit in which they are measured has yet to be defined. Similarly, the proportions between the quantities of goods to be produced, which are positive, are fixed but not the quantities. In the second case, both quantities and prices are semipositive and are determined univocally.

In accordance with this, Definition 1 establishes the necessary and sufficient conditions in order for (1) and (2) to have a solution for each type of economy where, as shown in Benítez Sánchez and Benítez Sánchez [1], such a solution is characteristic of viable economies in both the Leontief [5] and Sraffa [6] models. Certain interesting cases are not covered by this definition in an immediate manner and do require special attention, as in the next example:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{7}$$

This matrix does not represent a viable technique because the corresponding equations (1) and (2) do not have a unique solution. The interpretation compatible with Definition 1 is that (7) represents an economy of the first class composed of two viable economies of the first class. This and other cases are studied in Benítez Sánchez and Benítez Sánchez [1] where there is also a comparison of Definition 1 with three other definitions of viable economies proposed by contemporary authors.

4. The Extended Hawkins and Simon Condition

This section presents two properties of indecomposable matrices and uses one of them to expose EHS.

4.1. Two Characteristic Properties of Indecomposable Matrices. Let A be $n \times n$ matrix such that $A \geq 0$. For each D-set, A_{D} is $D \times D$ matrix obtained from A by deleting each row j and each column j such that $j \notin \mathrm{D}$, $\lambda_{A(\mathrm{D})}$ is the Frobenius root of A_{D}, and I_D is $D \times D$ identity matrix. Therefore, the matrix $[I_D - A_{\mathrm{D}}]$ is a principal submatrix of $[I_n - A]$ and $|I_D - A_{\mathrm{D}}|$ is the corresponding principal minor. Using this notation, we can formulate HS and EHS as follows:

HS: $|I_D - A_{\mathrm{D}}| > 0$ for every D-set.

EHS: $|I_D - A_{\mathrm{D}}| > 0$ for every D-set such that $D < n$ and $|I_n - A| \geq 0$.

Therefore, EHS states that all the principal minors of $[I_n - A]$ are greater than zero, except for $|I_n - A|$ which is greater than or equal to zero. The next theorem presents two characteristic properties of indecomposable matrices required to study this condition in the following sections.

Theorem 2. *Let A be a square matrix such that $A \geq 0$. If $n > 1$, the following propositions are equivalent.*

(i) *A is indecomposable.*

(ii) *$\lambda_{A(\mathrm{D})} < \lambda_A$ for every* D*-set such that $D < n$.*

(iii) *In every* D*-set for which $1 \leq D < n$, $\sum_d a_{i,jd} < \sum_j a_{ij}$ for at least one $i \in \mathrm{D}$.*

The equivalences (i) ⇔ (ii) and (i) ⇔ (iii) follow immediately from the fact that a matrix is indecomposable if and only if it can be written as block triangular. The proposition (i) ⇒ (ii) appeared in Marcus and Minc ([16], p. 125), under heading 5.5.6 while (i) ⇔ (iii) had not been published prior to this paper, as far as I know.

4.2. Synthetic Formulation of the Viability Conditions. The following theorem relates viable economies and EHS.

Theorem 3. *Let A be a square matrix such that $A \geq 0$. The following propositions are equivalent:*

(i) *A is a viable technique.*

(ii) *EHS.*

Proof. (i) ⇒ (ii). Two cases are possible with respect to whether (3) and (4) or (5) and (6) are true. In the first case, for each D-set such that $D < n$, (ii) of Theorem 2 together with (3) and (4) imply that $\lambda_{A(\mathrm{D})} < 1$. This result together with the equivalence between (5) and (6) permit us to establish that $|I_D - A_{\mathrm{D}}| > 0$, while (4) implies that $|I_n - A| = 0$. In the second case, it is obvious that (ii) is true and that $|I_n - A| > 0$.

(ii) ⇒ (i). We have two possible cases: either $|I_n - A| > 0$ or $|I_n - A| = 0$. In the first case, (5) is true and, for this reason, A is viable. In the second case, the equation $|I_n - A| = 0$ implies that an eigenvalue of A is equal to one and, for this reason,

$$\lambda_A \geq 1. \tag{8}$$

On the other hand, given that for each D-set such that $D < n$, $|I_D - A_{\mathrm{D}}| > 0$, the equivalence between (5) and (6) implies that $\lambda_{A(\mathrm{D})} < 1$. This conclusion together with (8), and (ii) of Theorem 2 permit us to conclude that A satisfies (3). Furthermore, the equation $|I_n - A| = 0$ implies that there is a solution x, such that $x \geq 0$ and $x \neq 0$ for the equation system $Ax = x$. This result and the fact that A is indecomposable imply (4), according to (iv) of Theorem 4.B.1 by Takayama ([15], p. 372). □

It follows from this theorem that a square matrix $A \geq 0$ represents a viable production technique if and only if it satisfies EHS.

5. Economic Interpretation of HS by Dorfman, Samuelson, and Solow

This section exposes the economic interpretation of HS proposed by Dorfman et al. [8] in a form that facilitates the corresponding interpretation of EHS.

5.1. Self-Sustaining Economies. According to Dorfman et al. ([8], p. 215), the economies satisfying HS are self-sustaining, a concept that may be defined as follows.

Definition 4. An economy is self-sustaining if, for each j, the quantity of j consumed directly and indirectly in the production of one unit of j is less than one unit.

To visualize the relation between HS and this definition, it is useful to consider the next equation system that, for any given D-set, can be formulated for each k such that $1 \leq k \leq D$:

$$[I_D - A_{\mathrm{D}}]\, x^{(jk)} = c^{(jk)}, \tag{9}$$

where $c^{(jk)} = (c_{j1}, c_{j2}, \ldots, c_{jD})^T$ with $c_{jd} = 0\ \forall jd \neq jk$ and $c_{jk} = 1$ while $x^{(jk)} = (x_{j1}, x_{j2}, \ldots, x_{jD})^T$. If $[I_n - A]$ satisfies HS there is a unique vector $x^{(jk)}$ satisfying both (9) and the first of the following propositions:

$$x^{(jk)} \geq 0, \tag{10}$$

$$x_{jk} \geq 1. \tag{11}$$

In turn, (10) and the equation k in (9) imply (11).

System (9) produces each good in the quantity required to obtain a surplus of one unit of jk without considering the consumption of the goods not belonging to D. For this reason, $x_{jk} - 1$ and $(x_{jk} - 1)/x_{jk} = 1 - 1/x_{jk}$ are the total quantities of jk consumed directly and indirectly in the industries producing the D-set without considering the goods not belonging to this set, to obtain a surplus, respectively, of one unit and $1/x_{jk}$ units of jk. In the last case, the fact that one unit of jk is produced, together with (11), imply that the quantity of jk consumed is less than one unit. This result suggests the following version of Definition 4.

Definition 5. An economy is self-sustaining if, given any D-set, the quantity of any good $j \in$ D consumed directly and indirectly in the production of one unit of j in the industries producing the D-set, without considering the indirect consumption through the goods not belonging to this set, is less than one unit.

Therefore, an economy is self-sustaining if any set of industries producing a unit of a good consumes in this process, directly and indirectly through the goods produced by the set, less than one unit of the same good. Moreover, it is possible to observe that Definitions 4 and 5 are equivalent. On the one hand, the first definition implies the second one due to the fact that the quantities of a good consumed either directly or indirectly in the production of another good (not necessarily different) are nonnegative (see Jeong [13]). For this reason, in an economy satisfying Definition 4, the quantity of each good consumed in the production of the same good is less than one unit, even when, instead of the entire industrial system, only a part is considered. On the other hand, the conclusion that the second definition implies the first one follows from the remark that Definition 5 is valid for the set of all goods.

5.2. Economic Interpretation of EHS. The preceding analyses help us to visualize the following interpretation of EHS.

Proposition 6. *A technology is viable if any set of industries producing a unit of a good consumes in this process, directly and indirectly through the goods produced by the set, less than one unit of the same good. The only possible exception is the set of all goods, in which case that quantity is equal to one unit for every good.*

In a viable economy of the second class, the corresponding coefficient matrix satisfies HS. As this situation has already been treated in the preceding subsection, I will discuss here only the case of a viable economy of the first class. On the one hand, it is important to observe that the analysis developed in the previous subsection is valid for any D-set such that $D < n$. Indeed, (ii) of Theorem 2 together with (3) and (4) imply that $\lambda_{A(\mathrm{D})} < 1$ and, given the equivalence between (5) and (6), matrix $[I_D - A_\mathrm{D}]$ satisfies HS. Thus, in any set of industries not producing all the goods, the total consumption of each good produced by the set in the production of one unit of the same good is less than one unit. On the other hand, considering the set of all goods, we must remember that in any x satisfying (1) the different goods are always produced in the same proportions and, for each good, the quantity produced is equal to the quantity consumed. Therefore, to produce one unit of each good, a total consumption of one unit of the same good is required directly and indirectly.

6. Economic Interpretations of HS by Hawkins and Simon

The first part of this section discusses two economic interpretations of HS offered by Hawkins and Simon ([7], p. 248). The last two parts present some results that help us to understand the content of EHS along the lines of these interpretations.

6.1. Economic Interpretations by Hawkins and Simon. The following economic interpretation refers to second-class economies.

Proposition 7. *The condition that all principal minors must be positive means, in economic terms, that the group of industries corresponding to each minor must be capable of supplying more than its own needs for the group of products produced by this group of industries.*

The next economic interpretation refers to first-class economies. We have made some minor changes, adapting the original text to the notation followed here.

Proposition 8. *Let* $\mathrm{D} = \{1, 2, \ldots, (n-1)\}$. *If the matrix* $[I_D - A_\mathrm{D}]$ *satisfies HS and if* $|I_n - A| = 0$, *then one can say that each group of industries must be just capable of supplying its own demands upon itself and the demands of the other industries in the economy.*

Regarding this proposition, it is necessary to consider first the situation when matrix A satisfies the following condition:

$$a_{ij} > 0 \quad \forall (i, j) \text{ such that } i \neq j. \tag{12}$$

Hawkins and Simon ([7], p. 245) assume (12), which implies that A is indecomposable. As shown in the proof of Theorem 3, this result together with the assumption that $|I_n - A| = 0$ imply that matrix A satisfies (4). Thus, A is a viable technique of the first class and, for this reason, Proposition 8 is true, a conclusion explained in the next section.

However, apparently Hawkins and Simon ([7], p. 254, n. 3) inadvertently suppose that Proposition 8 is true also when $a_{ij} \geq 0 \; \forall (i, j)$ but (12) is not satisfied. This is not the case as can be discerned in the following example:

$$\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{13}$$

This technique satisfies the two conditions indicated in Proposition 8 but the second industry does not supply the demands of Good 2 from the whole industry. A corrected version of Proposition 8 can be formulated as follows.

Proposition 9. *If* $|I_D - A_D| > 0$ *for every* D*-set such that* $D < n$ *and if* $|I_n - A| = 0$*, then one can say that each group of industries must be just capable of supplying its own demands upon itself and the demands of the other industries in the economy.*

Matrix (13) does not comply with the first condition of Proposition 9 because for $\mathrm{D} = \{2,3\}$ we have $D < n$ but $|I_D - A_\mathrm{D}| = 0$. It is important to remark that, according to Theorem 3, the two conditions indicated in this proposition imply (3) and (4) and thus we can establish the following conclusion.

Proposition 10. *The techniques satisfying the conditions indicated in Propositions 7 and 9 are the same as those covered by Definition 1.*

This fact has not been previously pointed out, as far as I know. It follows from Proposition 9 that in Leontief's closed model, matrix A can be considered to be indecomposable as a general rule and not only in particular cases, contrary to what has usually been assumed (e.g., Berman and Plemmons [17], p. 262; Dorfman et al. [8], p. 255; Wurtele [18], p. 25). Benítez Sánchez and Benítez Sánchez [1] provide some additional arguments for this conclusion.

In this regard, it is important to mention that Georgescu-Roegen ([19] p. 336, n. 6) points out the error in Hawkins and Simon ([7], p. 254, n. 3) which, as already indicated, is at the base of the false assumption highlighted above. Nevertheless, he does not go any further, hence the need for the clarifications and conclusions presented here.

6.2. Convertible and Viable Techniques. Given a technique A and a vector $x > 0$, it is possible to obtain a second technique ${}^*A$ that, for each j, measures the quantities of good j with the corresponding coordinate x_j. The matrix ${}^*A$ results from multiplying each column j of A by, and dividing each row j of A by, the corresponding coordinate x_j. To this end, we define matrix $X = [x_{ij}]$, where $x_{ij} = x_j$ if $i = j$ and $x_{ij} = 0$ if $i \neq j$. Then, we have

$$ {}^*A = X^{-1} A X. \tag{14}$$

The following definition indicates the relation between A and ${}^*A$.

Definition 11. A technique $A = [a_{ij}]$ is convertible to a technique ${}^*A = [{}^*a_{ij}]$ if there is a vector $x > 0$ such that ${}^*A = [a_{ij} x_j / x_i]$.

Convertibility, thus defined, is reflexive when, for a technique A, the vector x containing a unit of each good produced by A satisfies this definition. In addition, if A is convertible to ${}^*A$, then $[a_{ij}] = [{}^*a_{ij} x_j^{-1} / x_i^{-1}]$ for $x > 0$, which implies that ${}^*A$ is convertible to A. It is important to point out that the coefficient matrices of two mutually convertible techniques have the same Frobenius root and that both are either decomposable or indecomposable.

Theorem 12. *Let A be a square matrix such that $A \geq 0$. The following propositions are equivalent.*

(i) *A is a viable technique.*

(ii) *A is convertible to a technique ${}^*A$ satisfying the following conditions:*

$$ \textit{In every } \mathrm{D}\textit{-set } \sum_d {}^*a_{i,jd} \leq 1 \ \textit{ for every } i \in \mathrm{D}. \tag{15}$$

$$ \textit{If } \ 1 \leq D < n, \ \sum_d {}^*a_{i,jd} < 1, \ \textit{ for at least one } i \in \mathrm{D}. \tag{16}$$

Proof. (i) ⇒ (ii). If A is viable, there is a vector $x > 0$ such that $[I_n - A]x = c$ in the two possible cases: in the first case, $c = 0$ and A satisfies conditions (3) and (4) whereas in the second case, $c > 0$ and A satisfies conditions (5) and (6). Using each coordinate of x as the unit of measure of the corresponding good results in a technique ${}^*A$ satisfying (ii). Indeed, in the first case, each row adds up to one in ${}^*A$ and, for this reason, (15) is true. This result and the fact that ${}^*A$ is indecomposable (due to (3)) imply (16) according to (iii) of Theorem 2. In the second case, the technique ${}^*A$ produces a surplus of each good which implies (ii).

(ii) ⇒ (i). Two cases are possible with respect to whether the inequality is satisfied or not in (15) for at least one index i in D-set of all goods. In the first case, the row sum is less than one for at least one row, which implies that, if A is indecomposable, $\lambda_A < 1$ according to the Corollary to Theorem 4.C.11 by Takayama ([15], pp. 388-389) and, for this reason, $[I_n - A]$ satisfies HS. If A is decomposable, λ_A is equal either to zero or to the Frobenius root of at least one indecomposable matrix on the main diagonal of the canonical form of A. For that matrix, condition (16) implies that the row sum is less than one for at least one row so that $\lambda_A < 1$, as was just shown. In the second case, according to (iii) of Theorem 2, (16) implies that ${}^*A$ satisfies (3). Moreover, given that each and every row sum equals one, ${}^*A$ satisfies (4) according to the theorem by Takayama just mentioned. □

Some closely related results may be found in Fisher [20] and Solow [21].

6.3. Economic Interpretation of EHS. Proposition 7, together with Proposition 9 and Theorem 12, allow for the following economic interpretation of Definition 1.

Proposition 13. *A technique is viable if and only if every group of industries is capable of supplying at least its own needs for the group of products produced by this group of industries and, with the possible exception of the group of all the industries, it can supply more of at least one of these goods.*

7. Final Remarks

As indicated in Section 6, the corrected economic interpretations of HS by Hawkins and Simon [7] imply EHS, a fact that, as far as I know, has not been discussed previously. For this reason, the necessary and sufficient conditions characterizing viable economies of the first and the second class have been considered up to now as two independent sets. The synthetic formulation of these two sets presented here highlights the existence of a common property in the mathematical structures of viable economies of the two classes. That is, if $n > 1$, matrix $[I_D - A_D]$ satisfies HS for every D-set such that $D < n$. It is worth adding that this property corresponds to the fact that the models of the first and the second class considered by Leontief [5] and Sraffa [6] are different forms to represent economies of the second class, as argued in Benítez Sánchez and Benítez Sánchez [1].

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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References

[1] A. Benítez Sánchez and A. Benítez Sánchez, "Viable techniques, Leontief's closed model, and Sraffa's subsistence economies," *International Journal of Business and Social Research*, vol. 4, no. 11, pp. 18–30, 2014.

[2] C. Bidard, *Prices, Reproduction, Scarcity*, Cambridge University Press, Cambridge, UK, 2004.

[3] H. Kurz and N. Salvadori, *Theory of Production*, Cambridge University Press, Cambridge, UK, 1995.

[4] L. Pasinetti, *Lectures on the Theory of Production*, Columbia University Press, New York, NY, USA, 1977.

[5] W. Leontief, *The structure of the American economy 1919–1939*, Oxford University Press, New York, NY, USA, 2nd edition, 1960.

[6] P. Sraffa, *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge, UK, 1960.

[7] D. Hawkins and H. A. Simon, "Note: some conditions of macroeconomic stability," *Econometrica*, vol. 17, no. 3-4, pp. 245–248, 1949.

[8] R. Dorfman, P. Samuelson, and R. Solow, *Linear Programing and Economic Analysis*, Dover, New York, NY, USA, 1986.

[9] A. Benítez Sánchez, "La forma triangular de la matriz de Leontief," *Economía: Teoría y Practica*, no. 30, pp. 115–131, 2009.

[10] A. Benítez Sánchez, "The triangular form of Leontief's matrix," *Denarius*, vol. 27, no. 2, pp. 87–103, 2013, http://ssrn.com/abstract=2389868.

[11] D. Dasgupta, "Using the correct economic interpretation to prove the Hawkins-Simon-Nikaido theorem: one more note," *Journal of Macroeconomics*, vol. 14, no. 4, pp. 755–761, 1992.

[12] Y. Fujita, "A further note on a correct economic interpretation of the Hawkins-Simon conditions," *Journal of Macroeconomics*, vol. 13, no. 2, pp. 381–384, 1991.

[13] K.-J. Jeong, "Direct and indirect requirements: a correct economic interpretation of the Hawkins-Simon conditions," *Journal of Macroeconomics*, vol. 4, no. 3, pp. 349–356, 1982.

[14] K.-J. Jeong, "The relation between two different notions of direct and indirect input requirements," *Journal of Macroeconomics*, vol. 6, no. 4, pp. 473–476, 1984.

[15] A. Takayama, *Mathematical Economics*, Cambridge University Press, New York, NY, USA, 2nd edition, 1985.

[16] M. Marcus and H. Minc, *A Survey of Matrix Theory and Matrix Inequalities*, Dover, New York, NY, USA, 1992.

[17] A. Berman and R. J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, SCIAM, Philadelphia, Pa, USA, 1994.

[18] Z. S. Wurtele, "Equilibrium in a uniformly expanding closed Leontief-type economy," *The Review of Economic Studies*, vol. 28, no. 1, pp. 23–28, 1960.

[19] N. Georgescu-Roegen, *Analytical Economics*, Harvard University Press, Cambridge, Mass, USA, 1966.

[20] F. M. Fisher, "Choice of units, column sums, and stability in linear dynamic systems with nonnegative square matrices," *Econometrica*, vol. 33, no. 2, pp. 445–450, 1965.

[21] R. Solow, "On the structure of linear models," *Econometrica*, vol. 20, no. 1, pp. 29–46, 1952.