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Research Article

An Effective Math Model for Eliminating Interior Resonance Problems of EM Scattering

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It is well-known that if an E-field integral equation or an H-field integral equation is applied alone in analysis of EM scattering from a conducting body, the solution to the equation will be either nonunique or unstable at the vicinity of a certain interior frequency. An effective math model is presented here, providing an easy way to deal with this situation. At the interior resonant frequencies, the surface current density is divided into two parts: an induced surface current caused by the incident field and a resonance surface current associated with the interior resonance mode. In this paper, the presented model, based on electric field integral equation and orthogonal modal theory, is used here to filter out resonant mode; therefore, unique and stable solution will be obtained. The proposed method possesses the merits of clarity in concept and simplicity in computation. A good agreement is achieved between the calculated results and those obtained by other methods in both 2D and 3D EM scattering.

1. Introduction

Electric field integral equation (EFIE) and magnetic field integral equation (MFIE) have been widely employed to analyze electromagnetic scattering of conducting bodies [1–3]. However, interior resonance phenomena exist in solving electromagnetic scattering problems with surface integral equations. When working frequencies of conductors are near (or exactly at) to the frequencies associated with interior resonances, the single equation will become highly ill-conditioned (or singular) which makes the solution unstable or nonunique. Also, the interior resonance behavior has significant influence on the late time stability associated with time domain EFIE and MFIE [4, 5].

Several ways of dealing with this numerical problem have been proposed. Nowadays, the popular combined field integral equation (CFIE) technique to overcome this problem is a proper combination of the electric field integral equation and the magnetic field equation [6, 7]. The CFIE technique requires the calculation of both E and H impedance matrices and it is not suitable for aperture problems. The combined source integral equation (CSIE) [8, 9] technique makes up for aperture structures. Also a technique has been proposed by Mittra and Klein [10], involving application of the generalized boundary condition [11], and consists of additional points in the interior of the conductor and forces the field to be zero at those points. The problem with this technique is the fact that the chosen interior points must be carefully selected so as not to lie on nodal lines, which is not too practical for large bodies of simple shape, for which a slight change in frequency can take us from one resonance to the next, of different modal distribution. There are also some iterative methods reported in the works of Sarkar and Ergül [12, 13] to deal with this situationand they are used to compute the minimum norm solution (which produces the correct scattered fields but not the true tangential fields) and to calculate the LQSR. Another work related to the use of extended integral equations has been presented by Mautz and Harrington [8], which

involves application of the boundary element method, with observation points lying on an internal closed surface near the boundary of the scatterer. Unfortunately, the resulting matrix equations are also ill-conditioned since the internal contour can resonate by itself. The authors' proposed scheme of allowing the internal contour to vary with the wave number seems impractical and in reality does not solve the problem except in some isolated cases of electrically small simple shapes whose resonances are known. More recently, Canning [14] illustrated a matrix algebra technique known as the singular value decomposition (SVD) has been proposed for moment method calculations involving perfect conductors. Such a technique diagonalizes the matrix equation, isolating the resonant contribution, which is then omitted in the calculation. We also refer to the interesting work of Yaghjian, who originally presented his augmented electric or magnetic field equation [15], and more recently Tobin et al. [16] pointed out their drawbacks for an arbitrarily shaped, multiwavelength body and, most importantly, introduced a modification, the so-called dual-surface integral equation, which is applicable to the perfectly conducting bodies and supposedly eliminates all the spurious solutions. Finally, the modal orthogonal characteristics [17, 18] were applied to solve the same problem in two-dimensional EM scattering.

Here, we present an effective method to solve the scattering problem of conductor bodies at or in the neighborhood of the resonant frequencies in both 2D and 3D EM scattering. At the resonant frequencies, Inagaki modes, firstly applied to analyze the antenna array [19], are employed here to be validated away from resonance with a unique and stable solution to the equation. It can both stabilize the numerical calculation and yield to reliable results of the surface current density and exterior field for conductors at interior resonances.

2. Theory

The principle is elaborated as follows. The solution of anill-conditioned system of the equation will consist of (1) the correct physical solution to the problem and (2) the resonant solution, which is in conjunction with Green's Theorem which produces the nonzero complementary resonant modes. When the orthogonal modes are used to solve the electric field integral equation, the solutions will be divided into the induced modes and the resonant modes corresponding to eigenvalues. We can easily obtain the current density and exterior field from the induced modes.

2.1. Why Is the Solution Unstable? Consider a PEC scatterer excited by an incident wave and the scatterer defined by the surface S, in an impressed electric field $\overline{E}^s$ (see Figure 1).

According to the boundary condition, we get

$$\hat{n} \times \left(\overline{E}^i + \overline{E}^s\right) = 0, \quad \text{on } S. \tag{1}$$

An operator equation for the current J on S is

$$T\left(\overline{J}\right) = \hat{n} \times \overline{E}^i, \quad \text{on } S. \tag{2}$$

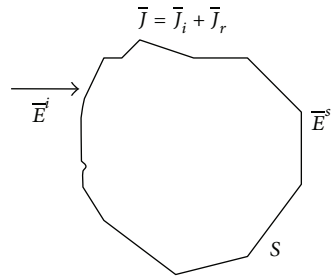

Figure 1: PEC scatterer impressed by electromagnetic field.

The operator T is defined by [2]

$$T\left(\overline{J}\right) = j\omega\mu\hat{n} \times \int_S \left[\overline{J}\left(\overline{r}'\right) + \frac{1}{k^2}\nabla' \cdot \overline{J}\left(\overline{r}'\right)\nabla\right] G\left(\overline{r}, \overline{r}'\right) ds', \tag{3}$$

where $\overline{r}$ denotes a field point, $\overline{r}'$ denotes a source point, and ω, μ, and k denote angular frequency, permeability, and wave number, respectively, of free space; $G(\overline{r}, \overline{r}')$ is the free space Green's function,

$$G\left(\overline{r}, \overline{r}'\right) = \frac{\exp\left(-jk\left|\overline{r} - \overline{r}'\right|\right)}{4\pi\left|\overline{r} - \overline{r}'\right|}. \tag{4}$$

Equation (2) is discretized into a set of simultaneous linear algebraic equations. This set of equations can be compactly rewritten in a matrix form as

$$[S]\,[J] = [Y], \tag{5}$$

where $[S]$ is the general impedance matrix, $[Y]$ denotes the general voltage vector, and $[J]$ stands for the coefficient of the current density to be determined. Usually (noninterior resonant condition) the current density of the conductor can be obtained from (5) with a unique and stable solution, but, at the frequencies associated with interior resonances, the situation will be on the contrary.

As a result of interior resonance, the current density $\overline{J}$ on S at discrete resonant frequencies will be composed of two parts: an induced surface caused by the incident field and a resonance surface current associated with the interior resonance mode:

$$\overline{J} = \overline{J}_i + \overline{J}_r, \tag{6}$$

where $\overline{J}_r$ is the resonant current density which is determined by the conducting body alone and it is independent of the incident wave; $\overline{J}_i$ is the induced current density which is jointly determined by the incident field and the conducting body itself.

If the incident $\overline{E}^i$ is removed, the electric field $\overline{E}^s$ produced by the resonant current $\overline{J}_r$ on S satisfies the following homogeneous E-field integral equation:

$$T\left(\overline{J}_r\right) = 0, \quad \text{on } S. \tag{7}$$

In other words, the solution to the inhomogeneous EFIE is not unique since the non-zero- solution to its corresponding homogeneous equation exits at some discrete frequencies. Namely, in addition to the induced current $\overline{J}_i$ determined jointly by the incident field $\overline{E}^i$ and the conducting body, there exists also the resonant current $\overline{J}_r$ at some discrete frequencies determined alone by the conducting body itself. It follows from the nonuniqueness of the solution to EFIE $T(\overline{J}) = \hat{n} \times \overline{E}^i$ that the solution to its corresponding moment matrix equation (5) will also not be unique at the interior resonant condition. The nonuniqueness of the moment matrix equation infers that the moment matrix $[S]$ is singular.

Similar to the behavior of a conducting cavity, the resonant current density $\overline{J}_r$ on a conducting body is also not able to produce scattered field $\overline{E}^s$ in space external to S. Therefore, the scattered field and the radar cross section (RCS) external to S determined by the electric field integral equation should be unique theoretically. Although resonant current theoretically does not change the exterior scattered field, it does make the method of moments analysis unstable and unreliable since the homogeneous operator equation $T(\overline{J}_r) = 0$ has non-zero-solution $\overline{J}_r$.

2.2. How to Stabilize the Solution? For the sake of simplicity, only the electric field integral equation is involved here. To solve the EFIE by method of moments, first of all, we should choose a set of expansion functions and a set of weighting functions. Here, we choose basis functions aiming at the diagonalization of the moment matrix which was proposed by Inagaki and Garbacz [19], in which the expansion functions $\{\overline{u}_n\}$ are chosen to be the eigenfunctions of a composite Hermitian operator T^*T, where T^* is the adjoint operator of T,

$$T^* T\, \overline{u}_n = \lambda_n \overline{u}_n, \tag{8}$$

the weighting functions to be the response of them

$$\overline{v}_n = T\, \overline{u}_n. \tag{9}$$

The orthogonal property of Inagaki modes [13] leads to the satisfaction of the moment matrix diagonalization condition; that is,

$$s_{mn} = \langle \overline{v}_m, T\, \overline{u}_n \rangle = \lambda_n \delta_{mn}; \tag{10}$$

namely,

$$[S] = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_N \end{bmatrix}. \tag{11}$$

So the modal solution to the operator equation becomes

$$\overline{J} = \overline{U}^T [S]^{-1} \overline{Y}. \tag{12}$$

Here $\overline{U}$ is the expansion function vector,

$$\overline{U} = [\overline{u}_1, \overline{u}_2, \ldots, \overline{u}_N], \tag{13}$$

where $\overline{Y}$ is a column excitation vector given by

$$y_n = \left\langle T\, \overline{u}_n, \hat{n} \times \overline{E}^i \right\rangle. \tag{14}$$

Then method of moments is employed here for computation of the modes. To do so, functions $f_1, f_2, \ldots, f_N$ are taken as both expansion functions and weighting functions for a moment method analysis to the operator eigenvalue equation, giving

$$\begin{gathered} u_n = \sum_{j=1}^{N} b_j^{(n)} f_j, \quad n = 1, 2, \ldots, N, \\ \sum_{j=1}^{N} b_j^{(n)} T^* T f_j = \lambda_n \sum_{j=1}^{N} b_j^{(n)} f_j, \\ \sum_{j=1}^{N} b_j^{(n)} \left\langle f_i, T^* T f_j \right\rangle = \lambda_n \sum_{j=1}^{N} b_j^{(n)} \langle f_i, f_j \rangle, \end{gathered} \tag{15}$$

whose matrix form is

$$[Q]\, \vec{B}_n = \lambda_n\, [P]\, \vec{B}_n, \quad n = 1, 2, \ldots, N. \tag{16}$$

In this matrix eigenvalue equation, both $[P]$ and $[Q]$ are $N \times N$ matrices with their elements, respectively, as

$$\begin{gathered} p_{ij} = \left\langle f_i, f_j \right\rangle, \\ q_{ij} = \left\langle f_i, T^* T f_j \right\rangle = \left\langle T f_i, T f_j \right\rangle. \end{gathered} \tag{17}$$

And $\vec{B}_n$ is a column vector composed of the unknown coefficients $\{b_j^{(n)}\}$ as

$$\vec{B}_n = \left[b_1^{(n)}, b_2^{(n)}, \ldots, b_N^{(n)} \right]^T. \tag{18}$$

The matrix $[Q]$ is simply the moment method matrix $[S]$ only if a least-squares method of moments is applied to the general operator equation in which $f_1, f_2, \ldots, f_N$ are taken to be the expansion functions and their responses $Tf_1, Tf_2, \ldots, Tf_N$ the weighting functions.

At the interior resonances, the surface current is composed of the resonant current and the induced current. The resonant current is the solution to the corresponding homogeneous E-field integral equation. It follows from the nonuniqueness of the solution to E-field integral equation that the solution to its corresponding moment matrix equation will also not be unique at the interior resonance condition. The nonuniqueness of the moment matrix equation infers that the moment matrix $[S]$ is singular.

Actually, the moment matrix will not be exactly singular, but rather, it will be highly ill-conditioned since the round-off and the truncated error exist during the computation of the matrix elements. According to the orthogonal property of Inagaki modes [13], the surface current of conductors is composed of serials of modal currents corresponding to the eigenvalue $\{\lambda_n\}$,

$$\overline{J} = \sum_{n=1}^{N} \overline{J}_n = \sum_{n=1}^{N} \frac{\left\langle T\, \overline{u}_n, \hat{n} \times \overline{E}^i \right\rangle \overline{u}_n}{\lambda_n}. \tag{19}$$

At the interior resonances, when eigenvalue λ_m is much smaller than the other eigenvalue, the corresponding modal current is resonant current and the other is the induced current. These modal currents are mutually orthogonal, so to eliminate the interior resonant mode in the Inagaki mode solution, what we need to do is just to replace the number $1/\lambda_m$ in $[S]^{-1}$ by zero. The incident current $\overline{J}_i$ can be acquired from the following equation:

$$\overline{J}_i = \sum_{n=1}^{N} \frac{\left\langle T\,\overline{u}_n, \hat{n}\times\overline{E}^i \right\rangle \overline{u}_n}{\lambda_n} - \frac{\left\langle T\,\overline{u}_m, \hat{n}\times\overline{E}^i \right\rangle \overline{u}_m}{\lambda_m}. \quad (20)$$

Theoretically, the scattered field and the radar cross section external to S determined by the electric field integral equation should be unique. However, due to the ill-condition of the moment matrix, the exterior fields from conductors will be unstable and unreliable. After filtering out the resonant current, the exterior field, obtained from the induced current, is stable and reliable.

In fact, the resonance current does not really exist on the surface of the conductors. The interior resonance problem is just caused by the deficiency of the selected mathematical model. After getting rid of the virtual resonance current, we can obtain the stable and reliable current density and exterior field of conductors at interior resonances.

3. Numerical Results

Here we confine our attention to not only two-dimensional structures but also three-dimensional structures. The presented method is applied here to get the surface current density and exterior field of conductors when they are at (or near) the interior resonant frequencies.

3.1. An Infinitely Long Circular Cylinder. As the first case for testing the approach described above, scattering from a circular conducting cylinder was examined here. It was found that when the TM plane wave normally illuminates an infinite circular cylinder, the interior resonance takesplace if $ka = 3.8214$, where k is the wave number and a is the radius, which is the numerical resonant frequency point of E_{11} mode of the same surface circular cylinder cavity.

From Figures 2 and 3, we can see that the magnitude of induced current is much smaller than that of resonant current, so the interior resonant current has completely masked the true current responsible for the scattered field. The true surface current, computed by the presented method, agrees well with the exact analytical solution. The bistatic RCS of the circular cylinder computed from the induced current is compared with the exact solution. It is clear that there are excellent agreements between the calculated results and the analytic results, as it is shown in Figure 4.

3.2. An Infinitely Long Square Cylinder. Also, an infinitely long square conducting cylinder was considered here with a TM wave incident along the axis of the cylinder. The first and the second resonant frequency of the square cylinder are, respectively, at $d/\lambda = \sqrt{2}/2$ and at $d/\lambda = \sqrt{5}/2$ theoretically.

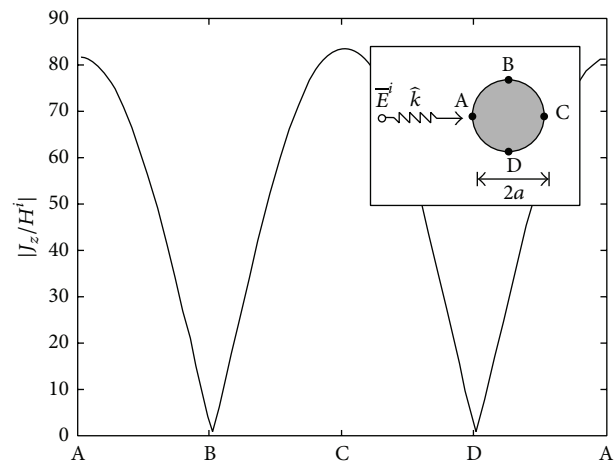

Figure 2: Resonant current on a circular cylinder, $ka = 3.8214$.

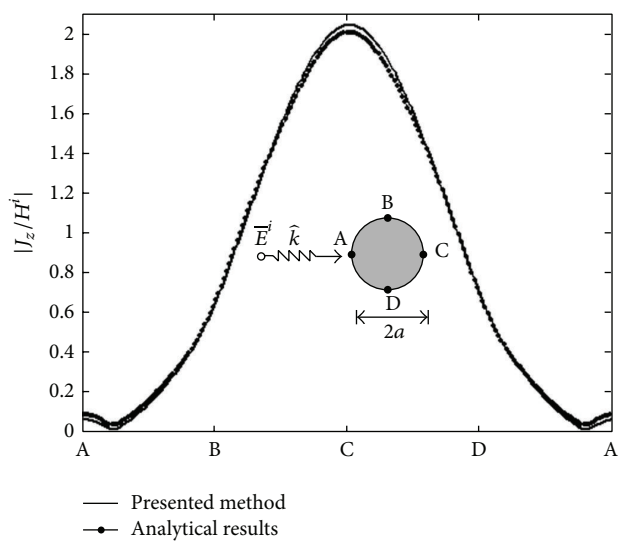

Figure 3: Induced current on a circular cylinder, $ka = 3.8214$.

We can get the first numerical resonant point at $d/\lambda = 0.70775$, using the presented method, as shown in Figure 5. Here, only the first resonant frequency is considered. At the first resonant frequency, the resonant current and induced current are shown, respectively, in Figures 6 and 7. From Figure 7, we can see that the induced current is compared well with that obtained by CFIE technique.

In Figure 8, the bistatic RCS of the square cylinder obtained through three different methods is depicted. Due to the ill-conditioned equation, the RCS of square cylinder computed by single EFIE (dash line) obviates the true result. After we filtered out the resonant modal current, the RCS (dot-dash line) produced by the induced current coincides with that obtained by CFIE method (solid line).

3.3. Two Conducting Spheres. In the end, two conducting spheres, respectively, at the $ka = 2.768$ and $ka = 4.518$, are analyzed by the presented method, where a denotes the radius

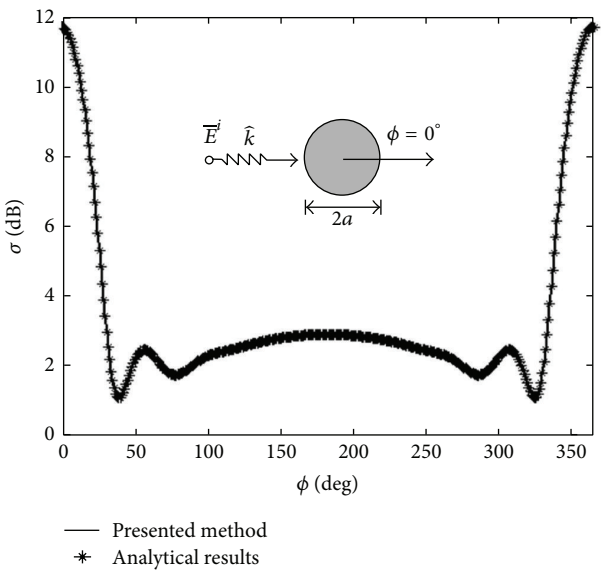

Figure 4: Bistatic RCS of a circular cylinder, $ka = 3.8214$.

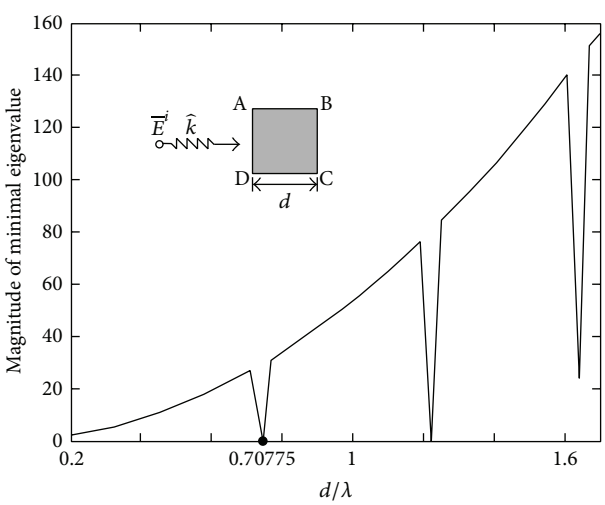

Figure 5: Minimal eigenvalue of square cylinder.

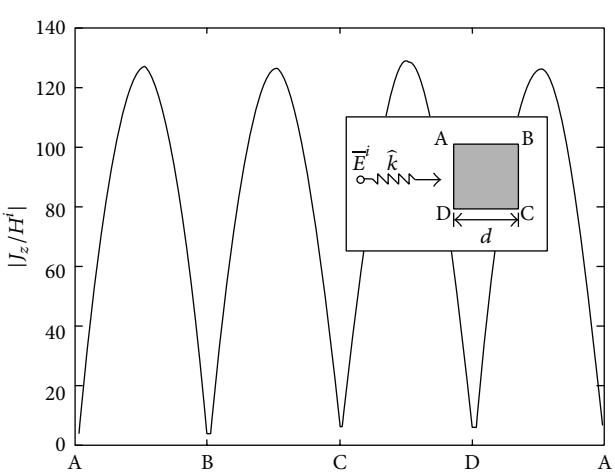

Figure 6: Resonant current on a square cylinder, $d/\lambda = 0.70775$.

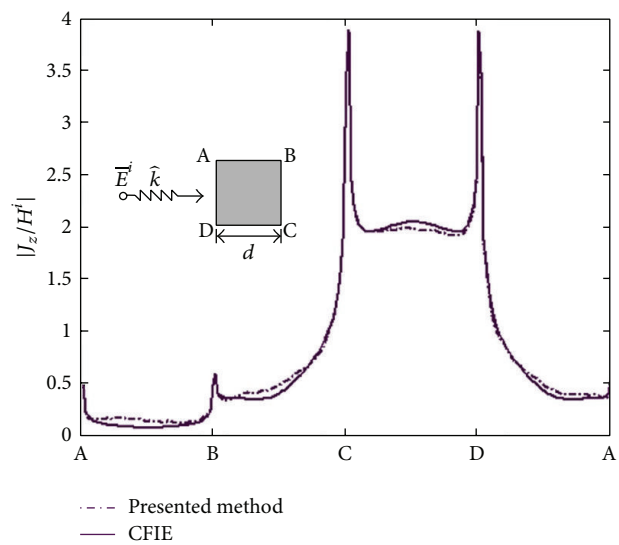

Figure 7: Induced current on a square cylinder, $d/\lambda = 0.70775$.

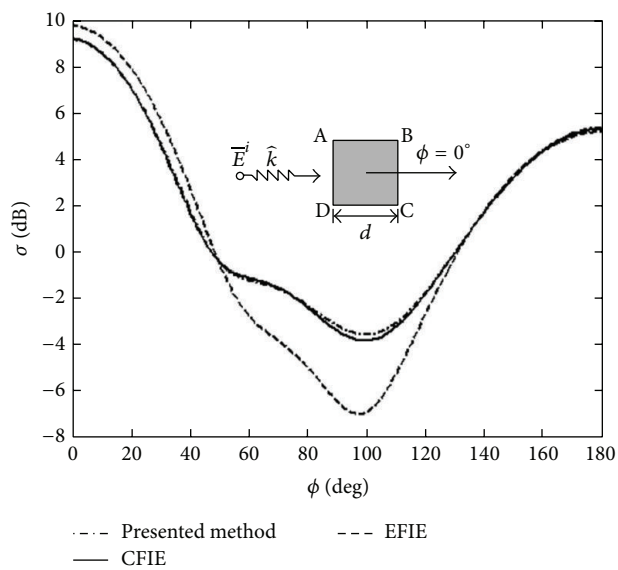

Figure 8: Bistatic RCS on square cylinder, $d/\lambda = 0.70775$ obtained by presented method (labeled IM), CFIE method, and EFIE.

of the sphere. At the interior resonant frequency, the bistatic RCS, obtained by the single EFIE method (dash line), is away from the right value. After being corrected by the presented method (dot-dash line), the bistatic RCS coincides with that calculated by CFIE technique (solid line), as shown in Figures 9 and 10.

4. Conclusion

A new scheme for eliminating interior resonance problems associated with surface integral equation is presented. The orthogonal property of Inagaki modes is used here to isolate the resonant mode, which is then omitted in the computation to obtain the right property of the conductors at the interior

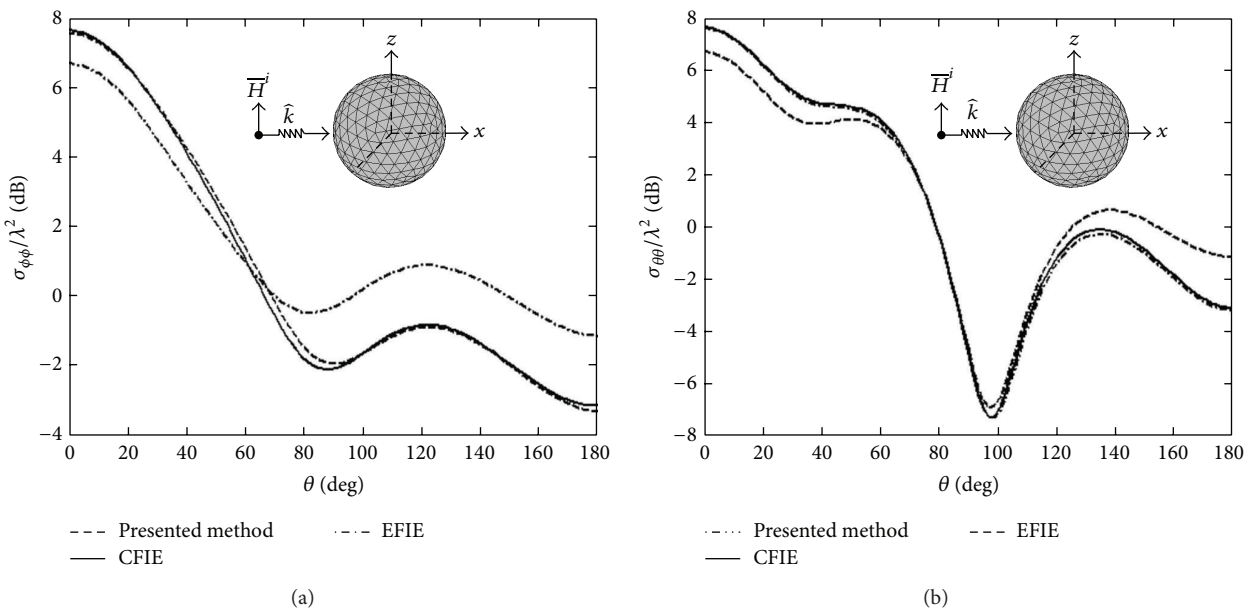

Figure 9: Bistatic RCS of a resonant sphere, $ka = 2.768$ (a) $\phi\phi$-polarization, (b) $\theta\theta$-polarization obtained by presented method (labeled IM), CFIE method, and EFIE.

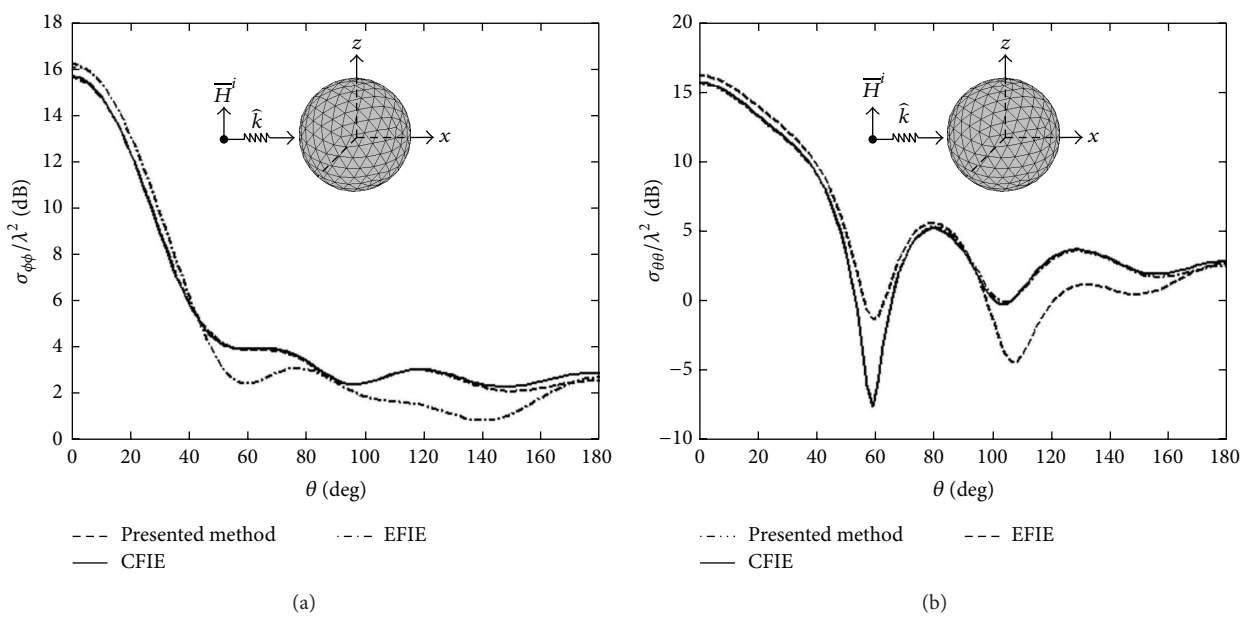

Figure 10: Bistatic RCS of a resonant sphere, $ka = 4.518$ (a) $\phi\phi$-polarization, (b) $\theta\theta$-polarization obtained by presented method (labeled IM), CFIE method, and EFIE.

resonances. This simple technique has been proven to be effective in attenuating the resonant modes and getting the unique and stable solution to EFIE when conductors are at (or near) the frequencies associated with interior resonances. Excellent numerical results, away from resonance problems, have been obtained for some shapes not only in two dimensions but also in three dimensions. Compared with other techniques, the advantages of this approach are only involved EFIE equation and easy to get the right results, but, due to the determination of eigenvalues and eigenvectors, there is a bit time-consuming for property computation at or in the interior resonance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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