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Research Article

Nonexistence of Harberger-Laursen-Metzler Effect with Endogenous Time Preference in an Imperfect Capital Market

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This paper investigates the spending and current-account effects of a permanent terms-of-trade change in a dynamic small open economy facing an imperfect world capital market, where the households' subjective discount rate is a function of savings. Under the assumption that the bond holdings are measured in terms of home goods, it is shown that when the discount rate is a decreasing function of savings, there does not necessarily exist a stable state; however, when the discount rate is an increasing function of savings, a saddle-path stable steady state comes into existence and the Harberger-Laursen-Metzler effect does not exist unambiguously; that is, an unanticipated permanent terms-of-trade deterioration leads to a cut in aggregate expenditure and a current-account surplus. The short-run effects obtained by the technique by Judd (1985, 1987) and Zou (1997) are consistent with the results from the long-run analysis and diagrammatic analysis.

1. Introduction

This paper aims at studying the Harberger-Laursen-Metzler (hereafter H-L-M) effect that a terms-of-trade deterioration causes a reduction in national savings and a current-account deficit, in a dynamic small open economy. In this model, the subjective discount rate is allowed to be increasing or decreasing in savings and the home country faces an imperfect international capital market, where the cost of borrowing is an increasing function of its indebtedness. We find that a terms-of-trade deterioration results in a cut in consumption and a current-account surplus, which are contrary to the H-L-M effect.

Many theoretical and empirical studies on the time preference have been carried out in Uzawa [1], Obstfeld [2], Becker and Mulligan [3], Laibson [4], Gootzeit et al. [5], Das [6], Gong [7], Hirose and Ikeda [8], and so forth. Uzawa [1] believed that the subjective discount rate is an increasing function of instantaneous utility, implying that rich people are more impatient. This is a crucial assumption for the existence of steady-state stability. However, many argue that this assumption is unappealing and should be questioned on intuitive grounds. According to Das [6], the assumption of decreasing marginal impatience is "intuitively more plausible at least for low-income groups." Marshall [9] believed that the future utility depends not only upon future consumption, but also upon the act of saving. Gootzeit et al. [5] formalize Marshall's idea by making the discount factor on future utility to be an increasing function of current savings. It means that the subjective discount rate is a decreasing function of current savings; that is, the more the people save today, the greater the utility of their consumption in the future is. They offer three alternative interpretations of the preference:

> *Firstly, a Fisherian interpretation would be that an increase in savings makes the consumer more patient. A second interpretation is a variation on the perennial theme of the "spirit of capitalism," the desire to accumulate wealth as an end in itself (Weber, 1920, 1958 [10, 11]). A recent literature (Zou, 1994, 1995, 1998 [12–14]; Bakshi and Chen, 1996 [15]; Smith, 1999, 2001 [16, 17]) models*

the spirit of capitalism by incorporating the stock of wealth as an argument in the utility function. Marshall provides another view of the spirit of capitalism. It is not the stock of wealth, but rather the accumulation of wealth, or the flow of savings that confers utility. It is not the amount accumulated, but rather the act of accumulating that matters. Thirdly, Marshallian preferences make the discount factor a function of current consumption, albeit indirectly, through saving. . . . The Marshallian taste for savings provides an underlying psychological rationale for why the discount factor should be a decreasing function of consumption . . ..

It seems that all of the three interpretations can be questioned. First of all, an increase in savings leads people to image a higher level of consumption in the future. For example, man who eats an apple every day may expect more apples per day in the future after he increases his current savings. An apple cannot satisfy him anymore. That means the future utility with the same consumption will be less after savings increase: the more the people save today, the less the utility of their consumption in the future is. That is to say, savings make people more impatient. So the subjective discount rate should be an increasing function of current savings. Secondly, according to the "spirit of capitalism," people derive utility from the accumulated wealth. A better formalization is to put this factor in the utility function, rather than put it in the subjective discount rate. If savings are always positive, wealth will increase all the time. Then utility will increase as time goes on when consumption does not change. However, according to Marshallian time preference in Gootzeit et al. [5], positive savings lead to a decrease in both discount factor and utility in course of time which is contrary to the "spirit of capitalism." Finally, according to Das [6], the subjective discount rate which is a decreasing function of consumption is intuitively more plausible than the one in Uzawa [1]. Since savings decrease with consumption, the subjective discount rate which is an increasing function of savings seems to be the normal case, other than the one presented in Marshallian time preference.

Adding to the complexity as well as the realism of our analysis, we weaken the assumption of Marshallian time preference to focus on two cases, Cases 1 and 2. In Case 1, the representative household has Marshallian time preference and, in Case 2, the subjective discount rate is an increasing function of savings.

Obstfeld [2] examines the H-L-M effect with Uzawa [1] utility function. He shows that an unanticipated permanent worsening of the terms of trade will cause a surplus in the current-account if initial steady-state bond holdings measured in units of foreign goods are nonpositive and consumption measured in units of domestic goods will fall. He studies both the case with perfect capital mobility and the case with imperfect mobility. The two cases have the same result which is contrary to those obtained in Harberger [18] and Laursen and Metzler [19].

Huang and Meng [20] base their analysis on Das [6] utility function with the discount rate being a decreasing function of instantaneous utility. Their analysis shows that if the economy is initially at steady state, the short-run response of an adverse permanent terms-of-trade shock is that spending rises sharply and then both spending and bond holding fall gradually to new, lower long-run levels. This result reverses the findings in Obstfeld [2] and is consistent with the H-L-M effect.

Angyridis and Mansoorian [21] study the H-L-M effect in a perfect capital market when the households have Marshallian time preference in Gootzeit et al. [5], where the subjective discount rate is a decreasing function of current savings. However, they have supposed that the concave utility function U is negative to satisfy the inequality $U''U-(U')^2>0$, which guarantees their system to be saddle-point stable. Angyrids and Mansoorian also show that an adverse terms-of-trade change occasions a deficit in the current-account.

The present paper revises the H-L-M effect in an imperfect international capital market. We find that, in Case 1, when households have Marshallian time preference, there does not necessarily exist a stationary state; in Case 2, when the discount rate is an increasing function of savings, a saddle-point stable steady state comes into existence. Then we investigate the long-run and short-run effects of terms-of-trade deterioration on consumption and bond holdings and find that a terms-of-trade deterioration leads to a cut in consumption and a current-account surplus, which is contrary to the H-L-M effect.

The rest of this paper is organized as follows. Section 2 describes the lifetime maximization problem of the representative agent in a world of imperfect capital mobility when the bond holdings are measured in terms of home goods. Section 3 obtains the steady state and examines the effects of a permanent deterioration in the terms of trade on steady-state consumption and bond holdings. Section 4 investigates the short-run effects of terms-of-trade deterioration on consumption and the current-account at the initial equilibrium, using a technique developed by Judd [22, 23], Zou [24], and Cui et al. [25]. Some concluding remarks are presented in Section 5.

2. The Model

We consider an infinite-horizon representative agent model with a downward-sloping bond curve; that is, the cost of borrowing faced by the home country is an increasing function of its indebtedness to the rest of the world. This small economy cannot influence the terms of trade between home and foreign goods. This price is expected to remain fixed forever and any price changes take households by surprise. The representative agent is to select the consumption level of imported goods and home goods in fixed supply and the bond holdings to maximize its discounted utility; namely,

$$U=\int_0^\infty u\left(c_t^f,c_t^h\right)e^{-\int_0^t\delta(\dot{b}_s)ds}dt, \tag{1}$$

subject to the given initial bond holdings b_0 and the budget constraint

$$\dot{b}_t = y - pc_t^f - c_t^h + r(\hat{b}_t) b_t, \tag{2}$$

where c_t^f and c_t^h denote consumption of the foreign and home goods at time t, respectively. Term $\dot{b}_t$ is the change of bond holdings, that is, the savings at time t. Function $u(\cdot,\cdot)$ is the instantaneous utility function and $\delta(\cdot)$ denotes the subjective discount rate. Term b_t denotes the household's bond holdings at time t, $\hat{b}_t$ is the aggregate bond holdings, p is the price of foreign goods in terms of home goods, y is the household's fixed endowment of the home goods, and $r(\hat{b}_t)$ denotes the world rate of interest as a function of $\hat{b}_t$. In solving the optimization problem, the representative household takes both $\hat{b}_t$ and $r(\hat{b}_t)$ as given, but in equilibrium we have $\hat{b}_t = b_t$.

The following assumption characterizes the instantaneous utility function $u(\cdot,\cdot)$ and the discount-rate function $\delta(\cdot)$.

Assumption 1. (i) Utility function $u(\cdot,\cdot)$ is nonnegative, strictly increasing in both of its arguments, strictly concave, and twice continuously differentiable. In addition,

$$\lim_{c_t^f \to 0} u_{c_t^t} = \lim_{c_t^h \to 0} u_{c_t^h} = \infty. \tag{1a}$$

(ii) Case 1:

$$\begin{aligned} &\delta(\cdot) > 0; \\ &\delta'(\cdot) < 0; \\ &\delta''(\cdot) > 0; \\ &\delta(0) = \delta_0. \end{aligned} \tag{1b}$$

Case 2:

$$\begin{aligned} &\delta(\cdot) > 0; \\ &\delta'(\cdot) > 0; \\ &\delta''(\cdot) > 0; \\ &\delta(0) = \delta_0. \end{aligned} \tag{1c}$$

In order to avoid noninterior solutions to household's lifetime consumption problem, we make the assumption in (i) which follows the assumption in Obstfeld [2].

Assumption (1b) in (ii) follows Marshallian time preference in Gootzeit et al. [5]: the accumulation of wealth makes people more patient, but at a decreasing rate. Furthermore, a person whose wealth is constant discounts the future at the same rate as a "Fisherian" consumer, $\delta(0) = \delta_0$. In Fisher [26], the discount rate is a constant δ_0. The assumption $\delta''(\cdot) > 0$ is consistent with Gootzeit et al. [5] in the discrete-time case, which means that the marginal impatience increases with the saving. The following example satisfies Assumption (1b) in (ii):

$$\delta(\dot{b}) = \delta_0 + \vartheta\left[e^{-\dot{b}} - 1\right] \quad \text{for } \vartheta > 0,\ \delta_0 > \vartheta. \tag{3}$$

Assumption (1c) in (ii) corresponds to the assumption that savings make people more impatient; the discount rate is an increasing function of savings, which is intuitively more plausible than Marshallian time preference. This assumption corresponds to the time preference in Das [6], where the author modified the Uzawa time preference [1]. One example satisfying Assumption (1c) in (ii) is

$$\delta(\dot{b}) = \delta_0 + \vartheta\left[e^{\dot{b}} - 1\right] \quad \text{for } \vartheta > 0. \tag{4}$$

It can be shown that Assumption 1 guarantees global monotonicity and quasiconcavity of U.

The following assumption characterizes the world rate of interest $r(\hat{b}_t)$.

Assumption 2. Consider

$$\begin{aligned} r(\hat{b}_t) &> 0, \\ r'(\hat{b}_t) &< 0, \\ r'(\hat{b}_t)\hat{b}_t + r(\hat{b}_t) &> 0. \end{aligned} \tag{5}$$

Here the assumption $r'(\hat{b}_t) < 0$ is in line with the general notion of imperfect asset substitutability, which is used in Obstfeld [2] and Huang and Meng [20].

In order to get the optimal consumption plan, the discounted integral of lifetime consumption must be not greater than the capitalized value of lifetime income plus initial bond holdings. The household is bound by the second constraint on its consumption and bond holdings; that is,

$$b_0 + \int_0^\infty y e^{-\rho_t} dt - \int_0^\infty \left(pc_t^f + c_t^h\right) e^{-\rho_t} dt \ge 0, \tag{6}$$

where

$$\rho_t \equiv \int_0^t r(b_s)\, ds. \tag{7}$$

After changing the variables from t to ρ, the left-hand side of (6) becomes

$$b_0 + \int_0^\infty \left(\frac{db}{d\rho} - b_\rho\right) e^{-\rho} d\rho = \lim_{\rho \to \infty} b_\rho e^{-\rho}. \tag{8}$$

Since b_ρ converges to a finite value, this limit is zero. Thus constraint (6) may be ignored in deriving necessary conditions for optimality which is the same as the one in Obstfeld [2].

The household's problem is to choose consumption paths c^f and c^h to

$$\begin{aligned} &\text{maximize} \quad \int_0^\infty u\left(c_t^f, c_t^h\right) e^{-\Delta_t} dt, \\ &\text{subject to} \quad \text{(i)}\ \Delta_t = \int_0^t \delta\left(\dot{b}_s\right) ds, \\ &\qquad\qquad\quad \text{(ii)}\ \dot{b}_t = y - pc_t^f - c_t^h + r\left(\hat{b}_t\right) b_t, \\ &\qquad\qquad\quad \text{(iii)}\ c_t^f, c_t^h \geq 0, \end{aligned} \tag{9}$$

for a given initial stock b_0 of the bond holdings.

According to Assumption (1a), the constraint (iii) above can never be binding and may be ignored in solving this problem.

We replace the utility function in (1) with the indirect utility function

$$V(p, z) \equiv \sup\left\{u\left(c^f, c^h\right) \mid pc^f + c^h = z\right\}, \tag{10}$$

where the representative household must maximize its instantaneous utility, by giving the price and its chosen level of expenditure on consumption goods in general, to maximize its lifetime welfare. According to Obstfeld [2], $V(p,z)$ is strictly concave in z, and therefore

$$\begin{aligned} V_{zz} &< 0, \\ V &> zV_z. \end{aligned} \tag{11}$$

We simplify this problem by assuming that the indirect utility function is separable in consumption z and price p; that is, $V_{zp} = 0$. Otherwise, the H-L-M effect may not exist. One of the examples is as follows.

Let the standard utility function be

$$\begin{aligned} u\left(c_t^f, c_t^h\right) &= \frac{\left[C\left(c_t^f, c_t^h\right)\right]^{1-\sigma}}{1-\sigma}, \\ C\left(c_t^f, c_t^h\right) &= \left[\omega^{1/\theta}\left(c_t^h\right)^{(\theta-1)/\theta} + (1-\omega)^{1/\theta}\left(c_t^f\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}, \end{aligned} \tag{12}$$

where $\sigma > 0$ is the CRRA coefficient, $\omega \in (0,1)$ is the weight on the home produced goods, and $\theta > 0$ is the elasticity of substitution between home and imported goods. With the CES aggregate, we have the expenditure of the agent, z_t, as

$$z_t = pc_t^f + c_t^h = P(p)\, C\left(c_t^h, c_t^f\right), \tag{13}$$

where $P(p)$ is given by

$$P(p) = \left[\omega + (1-\omega) p^{1-\theta}\right]^{1/(1-\theta)}. \tag{14}$$

Thus the agent's indirect utility function is

$$V = \int_0^\infty \left[P(p)\right]^{\sigma-1}\left(\frac{z_t^{1-\sigma}}{1-\sigma}\right) e^{-\int_0^t \delta(\dot{b}_s) ds} dt. \tag{15}$$

The agent's problem is to maximize his/her utility. Since p is a constant parameter, the maximized problem has nothing to do with p. The problem can be rewritten as

$$\begin{aligned} &\max \quad V^* = \int_0^\infty \left(\frac{z_t^{1-\sigma}}{1-\sigma}\right) e^{-\int_0^t \delta(\dot{b}_s) ds} dt \\ &\text{s.t.} \quad \dot{b}_t = y - z_t + r\left(\hat{b}_t\right) b_t, \end{aligned} \tag{16}$$

where $V^* = V[P(p)]^{1-\sigma}$. Therefore, the optimal paths $\{b_t^*, z_t^*\}_{t=0}^\infty$ are not affected by the change in price p. In other words, with this standard utility function we have

$$\frac{d\bar{b}}{dp} = \frac{d\bar{z}}{dp} = 0; \tag{17}$$

that is, there is no long term effect on the steady state $(\bar{b}, \bar{z})$. It follows that the unexpected permanent change in price p has no short-run effect on bond holdings and consumption. Thus there is no H-L-M effect with this utility function. So we assume $V_{zp} = 0$ in this paper to avoid the above situation.

The second simplification is to change variable in (9) from t to Δ. Following the work of Obstfeld [2], we use the fact

$$d\Delta = \delta\left(\dot{b}_t\right) dt, \tag{18}$$

which reduces the household's problem (1) to that of choosing a path for z to

$$\max \int_0^\infty \frac{V(p,z)}{\delta\left(\dot{b}\right)} e^{-\Delta} d\Delta \tag{19}$$

subject to

$$\frac{db}{d\Delta} = \frac{y - z + r\left(\hat{b}\right) b}{\delta\left(\dot{b}\right)}, \tag{20}$$

for the initial bond holdings b_0.

Following Arrow and Kurz [27], the Hamiltonian associated with this problem is

$$H(b, z, \lambda) = \frac{V(p,z) + \lambda\left[y - z + r\left(\hat{b}\right) b\right]}{\delta\left(\dot{b}\right)} \tag{21}$$

with $\lambda = \lambda_\Delta$ being the shadow price of savings.

Necessary conditions for the optimization are

$$\lambda = \frac{V_z \delta + V\delta'}{\delta - \left[y - z + r\left(\hat{b}\right) b\right]\delta'}, \tag{22}$$

$$\begin{aligned} \dot{\lambda} &= \lambda\left[\delta - r\left(\hat{b}\right)\right] \\ &\quad + \left\{V + \lambda\left[y - z + r\left(\hat{b}\right) b\right]\right\} \frac{\delta'}{\delta} r\left(\hat{b}\right). \end{aligned} \tag{23}$$

In addition, the optimal policy satisfies the transversality condition

$$\lim_{\Delta \to \infty} \lambda_\Delta e^{-\Delta} = 0. \tag{24}$$

In equilibrium, the flow constraint at time t is (letting $\hat{b}_t = b_t$)

$$\dot{b}_t = y - z_t + r\left(b_t\right) b_t. \tag{25}$$

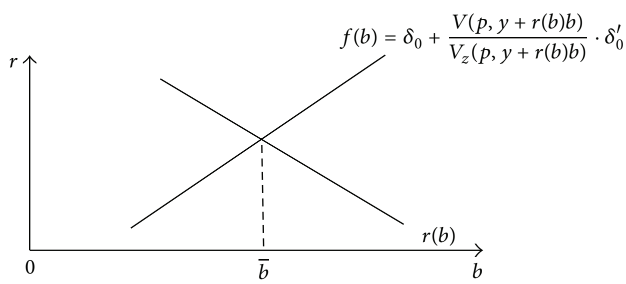

Figure 1: The unique steady state.

3. Dynamics and Long-Run Analysis

3.1. Dynamic System. We transform the differential equations (23) and (25) into a system involving only b_t and z_t. Condition (22) shows that λ_t can be written as a function of b_t and z_t

$$\lambda_t = \lambda\left(b_t, z_t\right). \tag{26}$$

Taking the time derivative of the above equation and combining with (23) yield the dynamics for consumption as

$$\dot{z}_t = \frac{\lambda_t\left[\delta - r\left(b_t\right)\right] + \left\{V\left(p, z_t\right) + \lambda_t\left[y - z_t + r\left(b_t\right)b_t\right]\right\}\left(\delta'/\delta\right)r\left(b_t\right) - \left[y - z_t + r\left(b_t\right)b_t\right]\lambda_b\left(b_t, z_t\right)}{\lambda_z\left(b_t, z_t\right)} = \Phi\left(b_t, z_t\right). \tag{27}$$

3.2. The Steady State. The full dynamics of the economy is described by two differential equations (25) and (27) with the transversality condition.

The steady state can be characterized as

$$\bar{\lambda}\left(\delta_0 - r\left(\bar{b}\right)\right) + \bar{V}\frac{\delta_0'}{\delta_0}r\left(\bar{b}\right) = 0, \tag{28}$$

$$y - \bar{z} + r\left(\bar{b}\right)\bar{b} = 0, \tag{29}$$

where the bar over the functions indicates that it is evaluated at the steady state $(\bar{b}, \bar{z})$.

Combining (22) with (29), we have

$$\bar{\lambda} = \bar{V}_z + \bar{V}\frac{\delta_0'}{\delta_0}. \tag{30}$$

According to (28) and (30), we obtain

$$\bar{V}_z\left(\delta_0 - r\left(\bar{b}\right)\right) + \bar{V}\delta_0' = 0. \tag{31}$$

In order to get the steady-state values for b and z, combining (29) with (31) yields

$$r\left(\bar{b}\right) = \delta_0 + \frac{V\left(p, y + r\left(\bar{b}\right)\bar{b}\right)}{V_z\left(p, y + r\left(\bar{b}\right)\bar{b}\right)} \cdot \delta_0'. \tag{32}$$

Let $f(b) = \delta_0 + V(p, y + r(b)b)/V_z(p, y + r(b)b) \cdot \delta_0'$; then $r(\bar{b}) = f(\bar{b})$.

The LHS of (32) is a downward-sloping curve. Differentiating the RHS of (32), we have

$$f'(b) = \delta_0' \cdot \frac{\left(V_z\right)^2 - VV_{zz}}{\left(V_z\right)^2} \cdot \left(r'(b)\,b + r(b)\right). \tag{33}$$

According to Assumption 2 and (30)

$$\begin{aligned} f'(b) &< 0 \quad \text{if } \delta'(\cdot) < 0 \text{ in Case 1,} \\ f'(b) &> 0 \quad \text{if } \delta'(\cdot) > 0 \text{ in Case 2.} \end{aligned} \tag{34}$$

It means that the RHS of (32) is a downward-sloping curve in Case 1 and upward-sloping curve in Case 2. It is obvious that there exists a unique point of intersection, which is a steady-state value for b, in Case 2. However, there does not necessarily exist a unique $\bar{b}$ in Case 1. Figure 1 illustrates Case 2.

We now study the local stability of the unique steady state in Case 2. The resulting linear approximation of z is

$$\dot{z}_t \simeq \bar{\Phi}_b\left(b_t - \bar{b}\right) + \bar{\Phi}_z\left(z_t - \bar{z}\right), \tag{35}$$

where

$$\begin{aligned} \bar{\Phi}_b = {} & \frac{\bar{\lambda}_b\left[\delta_0 - 2r\left(\bar{b}\right) - r'\left(\bar{b}\right)\bar{b}\right] - \bar{\lambda}r'\left(\bar{b}\right) + \bar{\lambda}\left[r'\left(\bar{b}\right)\bar{b} + r\left(\bar{b}\right)\right]\left(\delta_0'/\delta_0\right)\left(\delta_0 + r\left(\bar{b}\right)\right) + \bar{V}r'\left(\bar{b}\right)\left(\delta_0'/\delta_0\right)}{\bar{\lambda}_z} \\ & + \frac{\bar{V}\left(\left(\delta_0''\delta_0 - \left(\delta_0'\right)^2\right)/\left(\delta_0\right)^2\right)r\left(\bar{b}\right)\left[r'\left(\bar{b}\right)\bar{b} + r\left(\bar{b}\right)\right]}{\bar{\lambda}_z}, \end{aligned}$$

$$\overline{\Phi}_z = \frac{\overline{\lambda}_z\left[\delta_0 - r\left(\overline{b}\right)\right] - \overline{\lambda}\left(\delta_0'/\delta_0\right)\left(\delta_0 + r\left(\overline{b}\right)\right) - \overline{V}r\left(\overline{b}\right)\left(\left(\delta_0''\delta_0 - \left(\delta_0'\right)^2\right)/\left(\delta_0'\right)^2\right) + \overline{V}_z r\left(\overline{b}\right)\left(\delta_0'/\delta_0\right) + \overline{\lambda}_b}{\overline{\lambda}_z}. \tag{36}$$

Because of

$$\overline{\lambda} = \frac{\overline{V}_z r\left(\overline{b}\right)}{\delta_0},$$

$$\overline{\lambda}_b = \frac{\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\overline{V}_z\left[\delta_0' - \left(\left(\delta_0 - r\left(\overline{b}\right)\right)/\delta_0'\right)\cdot\delta_0''\right]}{\delta_0},$$

$$\overline{\lambda}_z = \frac{\overline{V}_{zz}\delta_0 + \overline{V}_z\left(\left(\delta_0 - r\left(\overline{b}\right)\right)/\delta_0'\right)\cdot\delta_0''}{\delta_0}, \tag{37}$$

functions $\overline{\Phi}_b$ and $\overline{\Phi}_z$ above can be rewritten as

$$\begin{aligned}\overline{\Phi}_b = {} & \frac{\overline{V}_z\left\{\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\delta_0'\left[\delta_0 + r\left(\overline{b}\right)\right] - \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0\delta_0''/\delta_0'\right)\right\}}{\overline{V}_{zz}\delta_0 + \overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)} \\ & + \frac{\overline{V}_z\left\{-\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]^2\delta_0' + \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]^2\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right) - \delta_0 r'\left(\overline{b}\right)\right\}}{\overline{V}_{zz}\delta_0 + \overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)},\end{aligned} \tag{38}$$

$$\begin{aligned}\overline{\Phi}_z = {} & \frac{\overline{V}_{zz}\delta_0\left[\delta_0 - r\left(\overline{b}\right)\right] + \overline{V}_z\delta_0' r'\left(\overline{b}\right)\overline{b} - \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)}{\overline{V}_{zz}\delta_0 + \overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)} \\ & + \frac{\overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''\delta_0/\delta_0'\right)}{\overline{V}_{zz}\delta_0 + \overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)}.\end{aligned} \tag{39}$$

Moreover, the linear approximation of b is

$$\dot{b}_t = \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\left(b_t - \overline{b}\right) - \left(z_t - \overline{z}\right). \tag{40}$$

Thus the linearized dynamic system can be written as

$$\begin{pmatrix}\dot{b}\\ \dot{z}\end{pmatrix} = \begin{pmatrix} r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right) & -1 \\ \overline{\Phi}_b & \overline{\Phi}_z\end{pmatrix}\begin{pmatrix} b - \overline{b} \\ z - \overline{z}\end{pmatrix}. \tag{41}$$

The determinant of the Jacobean matrix of (41) is

$$\det(J) = \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\overline{\Phi}_z + \overline{\Phi}_b = \frac{\overline{V}_{zz}\delta_0\left[\delta_0 - r\left(\overline{b}\right)\right]\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right] + \overline{V}_z\delta_0'\delta_0\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right] - \overline{V}_z\delta_0 r'\left(\overline{b}\right)}{\overline{V}_{zz}\delta_0 + \overline{V}_z\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)}. \tag{42}$$

From (31) we have

$$\begin{aligned}\left[\delta_0 - r\left(\overline{b}\right)\right] > 0 \quad & \text{if } \delta'(\cdot) < 0 \text{ in Case 1}, \\ \left[\delta_0 - r\left(\overline{b}\right)\right] < 0 \quad & \text{if } \delta'(\cdot) > 0 \text{ in Case 2}.\end{aligned} \tag{43}$$

As a result, we conclude this subsection that $\det(J) < 0$ in Case 2 and the sign of $\det(J)$ cannot be determined in Case 1. Thus the saddle point exists only in Case 2.

The following analysis is based on the assumption in Case 2 of Assumption 1.

3.3. The Optimal Path. Figure 2 displays the dynamic behavior of the economy described by (25) and (27) in the region where the feasibility constraint ((9)(iii)) is respected. Equation (25) defines the locus of points in the z-b plane for which $\dot{b} = 0$. That is to say, a rise in spending must be matched by a rise in interest income if the current-account is to remain in balance. Since

$$\frac{dz}{db} = \left[r'(b)\,b + r(b)\right] > 0, \tag{44}$$

the curve described by (25) is upward-sloping one.

From (27) $\dot{z} = 0$ if

$$\dot{b}_t = y - z_t + r(b_t)\, b_t = 0,$$
$$r(b) = \delta_0 + \frac{V}{V_z} \cdot \delta_0'. \tag{45}$$

From Assumption 2 and (11), we have

$$r'(b) < 0,$$
$$\frac{\partial (V/V_z)}{\partial z} = \frac{(V_z)^2 - VV_{zz}}{(V_z)^2} > 0, \tag{46}$$

and thus the curve described by (45) is downward-sloping one.

Note that z is not necessarily stationary on the curve described by (45). The point of intersection of the two curves in the plane is the steady state where both b and z are stationary. Bond holdings are increasing to the right of the curve $\dot{b} = 0$ and decreasing to its left. We note for future reference that the point $(0, y)$ always lies on the $\dot{b} = 0$ line.

Since point $\overline{E}$ is a saddle point, it is obvious that there exists a unique convergent path that the economy chooses. According to the movement of b, the convergent path is a little steeper than the curve $\dot{b} = 0$. To see this, if we denote the negative root of the Jacobian matrix of (41) as $\theta < 0$ and the associated eigenvector as (M, N), then $b_t - \overline{b} = Me^{\theta t}$ and $z_t - \overline{z} = Ne^{\theta t}$. From the first low of the Jacobian matrix, we have $[r'(b)b + r(b) - \theta]M - N = 0$, so that

$$\frac{z_t - \overline{z}}{b_t - \overline{b}} = \frac{N}{M} = \left[r'(b)\, b + r(b) - \theta\right] > \left[r'(b)\, b + r(b)\right]. \tag{47}$$

Therefore, the projection of the convergent path to the steady state $\overline{E}$ is steeper than the curve $\dot{b} = 0$.

3.4. The Harberger-Laursen-Metzler Effect. We first consider the effect of an increase in p on the steady-state equilibrium. Differentiating (29) and (31) with respect to p yields

$$\frac{d\overline{z}}{dp} = \left[r'(b)\, b + r(b)\right] \frac{d\overline{b}}{dp},$$
$$\overline{V}_{zz}\left[\delta_0 - r\left(\overline{b}\right)\right] \frac{d\overline{z}}{dp} - \overline{V}_z r'(b) \frac{d\overline{b}}{dp} + \overline{V}_z \delta_0' \frac{d\overline{z}}{dp} + \overline{V}_p \delta_0' = 0, \tag{48}$$

from which we obtain

$$\frac{d\overline{b}}{dp} = \frac{\overline{V}_p \delta_0'}{\overline{V}_z r'\left(\overline{b}\right) - \overline{V}_{zz}\left[\delta_0 - r\left(\overline{b}\right)\right]\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right] - \overline{V}_z \delta_0' \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]} > 0,$$
$$\frac{d\overline{z}}{dp} = \frac{\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]\overline{V}_p \delta_0'}{\overline{V}_z r'\left(\overline{b}\right) - \overline{V}_{zz}\left[\delta_0 - r\left(\overline{b}\right)\right]\left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right] - \overline{V}_z \delta_0' \left[r'\left(\overline{b}\right)\overline{b} + r\left(\overline{b}\right)\right]} > 0. \tag{49}$$

Equation (49) means that an unanticipated permanent deterioration of terms of trade results in a rise in steady-state bond holdings and consumption.

The change in bond holdings $\overline{b}$ can be seen from Figure 3. Since

$$\frac{d(V/V_z)}{dp} = \frac{V_z V_p}{(V_z)^2} < 0, \tag{50}$$

an increase in price p causes a downward-shift of the f-curve, so that steady-state bond holdings $\overline{b}$ increase. In Figure 4 we can see the effect on both bond holdings and consumption. A rise in p leads to an upward-shift of the curve $r(b) = \delta_0 + (V/V_z) \cdot \delta_0'$; at the same time, the curve $\dot{b} = 0$ does not move, so that both steady-state bond holdings and consumption increase.

We can also find out the short-run effect in Figure 4. The convergent path leading to the new steady state $\overline{E}'$ necessarily passes below the initial steady state $\overline{E}$. The bond holdings cannot change instantaneously, the consumption jumps to point E' immediately, and the current-account goes up to surplus (this will be proved in the next section). However, after the shock, bond holdings b and consumption z increase along the convergent path from point E' to the new steady state $\overline{E}'$.

The intuition is easy to grasp. The steady states of bond holdings and consumption are a higher level than before. Since income is fixed and bond holdings cannot be changed instantaneously, consumption will be cut to gain a rise in bond holdings, resulting in a current-account surplus. This is contrary to the H-L-M effect.

4. Short-Run Analysis

In the last section, we discussed the effects of an unanticipated permanent deterioration on the steady-state consumption and bond holdings. In this section, the short-run effects of this shock on consumption and the current-account at the initial equilibrium will be examined.

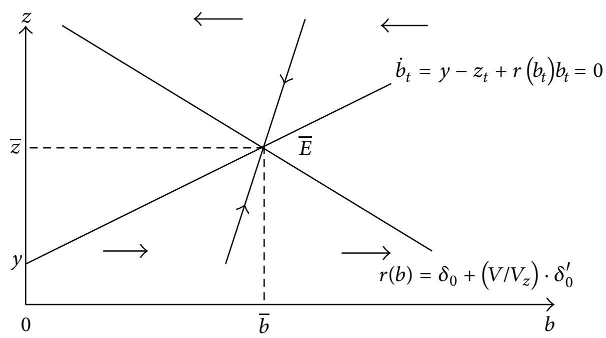

Figure 2: Saddle stability of the steady state.

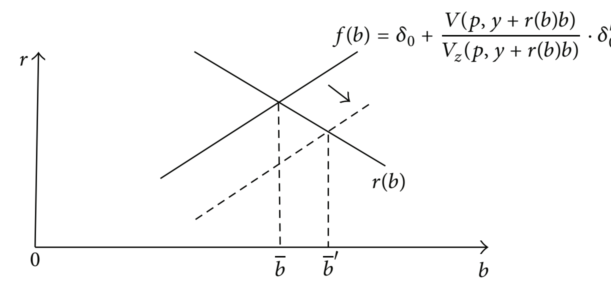

Figure 3: Change of the steady state with the increase of price p.

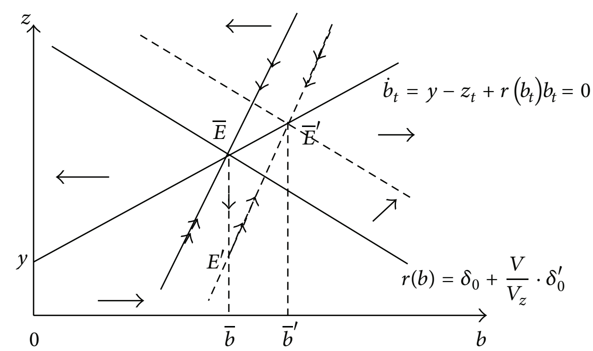

Figure 4: Nonexistence of Harberger-Laursen-Metzler effect.

Following the short-run analysis proposed by Judd [22, 23], Zou [24], and Cui et al. [25], suppose that the economy is in the steady-state $\overline{b}$ and $\overline{z}$ with the terms of trade $\overline{p}$ at time $t = 0$. Also at time $t = 0$, the terms of trade change as

$$p' = \overline{p} + \varepsilon p(t), \tag{51}$$

where ε is a parameter and function $p(t)$ represents the intertemporal change in a magnitude-free fashion.

Substituting (51) into the dynamical system (25)–(27), differentiating them with respect to ε and evaluating the derivatives at $\varepsilon = 0$ yield

$$\begin{pmatrix} \dot{b}_\varepsilon \\ \dot{z}_\varepsilon \end{pmatrix} = \begin{pmatrix} r'(\overline{b})\overline{b} + r(\overline{b}) & -1 \\ \overline{\Phi}_b & \overline{\Phi}_z \end{pmatrix} \begin{pmatrix} b_\varepsilon \\ z_\varepsilon \end{pmatrix} + \begin{pmatrix} 0 \\ g(t) \end{pmatrix}, \tag{52}$$

where

$$\begin{aligned} b_\varepsilon(t) &= \frac{\partial b(t,0)}{\partial \varepsilon}, \\ z_\varepsilon(t) &= \frac{\partial z(t,0)}{\partial \varepsilon}, \\ \dot{b}_\varepsilon(t) &= \frac{\partial[\partial b(t,0)/\partial\varepsilon]}{\partial t}, \\ \dot{z}_\varepsilon(t) &= \frac{\partial[\partial z(t,0)/\partial\varepsilon]}{\partial t}, \\ g(t) &= \frac{\overline{V}_p \delta_0' \delta_0}{\overline{V}_{zz}\delta_0 + \overline{V}_z[\delta_0 - r(\overline{b})](\delta_0''/\delta_0')}\cdot p(t). \end{aligned} \tag{53}$$

The Jacobian matrix in (52) has two eigenvalues, denoted by $\mu > 0$ and $\theta < 0$, respectively. As in Judd [22, 23], the Laplace transform can be used to solve (52). The Laplace transform of a function $f(t)$ $(t > 0)$ is another function $F(s)$, where

$$F(s) = \int_0^\infty f(t)e^{-st}dt. \tag{54}$$

Let $B_\varepsilon(s)$, $Z_\varepsilon(s)$, $P(s)$, and $G(s)$ be the Laplace transforms of $b_\varepsilon(t)$, $z_\varepsilon(t)$, $p(t)$, and $g(t)$, respectively. Then the ordinary differential equations in (52) are transformed into

$$\begin{gathered} s\begin{pmatrix} B_\varepsilon \\ Z_\varepsilon \end{pmatrix} = \begin{pmatrix} r'(\overline{b})\overline{b} + r(\overline{b}) & -1 \\ \overline{\Phi}_b & \overline{\Phi}_z \end{pmatrix}\begin{pmatrix} B_\varepsilon \\ Z_\varepsilon \end{pmatrix} \\ + \begin{pmatrix} b_\varepsilon(0) \\ G(s) + z_\varepsilon(0) \end{pmatrix} \end{gathered} \tag{55}$$

$$\text{with } G(s) = \frac{\overline{V}_p\delta_0'\delta_0}{\overline{V}_{zz}\delta_0 + \overline{V}_z[\delta_0 - r(\overline{b})](\delta_0''/\delta_0')}\cdot P(s),$$

which are inhomogeneous linear algebraic equations about B_ε and Z_ε. Solving the two equations for B_ε and Z_ε, we have

$$\begin{aligned} &\begin{pmatrix} B_\varepsilon \\ Z_\varepsilon \end{pmatrix} \\ &= \begin{pmatrix} s - r'(\overline{b})\overline{b} - r(\overline{b}) & 1 \\ -\overline{\Phi}_b & s - \overline{\Phi}_z \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ G(s) + z_\varepsilon(0) \end{pmatrix}, \end{aligned} \tag{56}$$

where we have $b_\varepsilon(0) = 0$ for the bond holdings that cannot jump initially. To determine the initial consumption change $z_\varepsilon(0)$, we note that the steady state is saddle-point stable and the bond holdings and consumption are bounded when $s = \mu$. However, when $s = \mu$, the matrix in (56) is singular and its inverse does not exist. Thus the first right-hand equation of (56) is zero. That is,

$$G(\mu) + z_\varepsilon(0) = 0. \tag{57}$$

Then we have

$$z_{\varepsilon}(0) = -\frac{\overline{V}_{p}\delta_0'\delta_0}{\overline{V}_{zz}\delta_0 + \overline{V}_{z}\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)} \cdot P(\mu), \quad (58)$$

which shows that the initial jump of the consumption is always negative related to the discounted, future terms-of-trade shock because the coefficient of $P(\mu)$ is always negative.

According to (52) and (58), we have

$$\begin{aligned} \dot{b}_{\varepsilon}(0) &= -z_{\varepsilon}(0) \\ &= \frac{\overline{V}_{p}\delta_0'\delta_0}{\overline{V}_{zz}\delta_0 + \overline{V}_{z}\left[\delta_0 - r\left(\overline{b}\right)\right]\left(\delta_0''/\delta_0'\right)} \cdot P(\mu), \end{aligned} \quad (59)$$

which means that any future deterioration in the terms of trade makes the current-account meet a surplus today since the coefficient of $P(\mu)$ is positive. The intuition is as follows: the long-run consumption and bond holdings will increase, but the bond holdings cannot be changed immediately; thus the household cuts its consumption today to react to the deterioration in the terms of trade. Then the current-account meets a surplus and both bond holdings and consumption will increase to a new steady state.

The short-run effects obtained with the above technique conform to the analysis in the last section; that is, the deterioration in the terms of trade will result in a current-account surplus, which is contrary to the H-L-M effect.

5. Conclusions

This paper has examined the terms-of-trade deterioration in a small open economy facing an imperfect world capital market. It is shown that, in the small open economy with imperfect capital mobility, there does not necessarily exist a steady state under Marshallian time preference. However, a saddle-point stable steady state comes into existence when the discount rate is an increasing function of savings.

Under the assumption that bond holdings are determined in terms of home goods, we find that the permanent terms-of-trade deterioration leads to an increase in the steady-state bond holdings and consumption. From the diagrammatic analysis and short-run analysis, we find that consumption will be cut immediately and the current-account will meet a surplus, which is contrary to the H-L-M effect.

The present paper assumes that the subjective discount rate is an increasing function of savings, which needs empirical and theoretical justification. Moreover, this paper could be extended in the following ways. Firstly, we could analyse the effects of various shocks such as fiscal policy, monetary policy, and distortionary taxation. Secondly, this small open economy model could be extended to a big-country model with both capital accumulation and foreign asset holdings. Thirdly, we could extend this model to a RBC model, incorporating labor supply and production. Fourthly, we could examine the effect of exchange rate between two countries on the current-account. Finally, we could examine the H-L-M effect in a two-country world economy with endogenous time preference.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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