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Research Article

Mathematical Modeling to Predict the Geometrical and Physical Properties of Bleached Cotton Plain Single Jersey Knitted Fabrics

A. Fouda, A. El-Hadidy, and A. El-Deeb

Textile Engineering Department, Faculty of Engineering, Mansoura University, Mansoura 35516, Egypt

Correspondence should be addressed to A. Fouda; abdo_faheem@mans.edu.eg

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This paper presents a novel mathematical model of bleached cotton plain single jersey knitted fabrics. The mathematical model is used to deduce the fabric geometrical relationships that can be useful for forecasting the properties of the fabric before production. A practical verification is carried out at different cotton yarn counts and twist factors. The obtained mathematical and practical results are deeply studied and analyzed. The results showed a good agreement between the proposed mathematical model and the practical one. The finished fabric weight is predicted at different yarn counts and loop lengths, and a forecasting weight chart is deduced. This chart will undoubtedly help the producers to enhance the fabric productions. In addition, an actual yarn diameter in the fabric measuring method is carried out and the fabric thickness is estimated consequently. The obtained results proved that the plain single jersey thickness is proportional to three times of the actual measured yarn diameter.

1. Introduction

Knitted fabrics are distinguished from the comfort point of view as compared to woven fabrics. Therefore, it is widely used in sportswear, underwear, and other comfortable fabrics. Over all types of knitted fabrics, plain single jersey knitted fabric is considered as a wide spread type. So, many research studies were focused on this type of fabrics.

Generally, the research studies depend on mathematical models to forecast the fabric properties that help to save frequent production time and efforts and in addition to achieve production of fabrics with high quality.

The knitted fabric weight per unit area is predicted using appropriate software called proKNIT [1]. But the prediction of the fabric weight is dependent on the calculated dimensional parameters of K_c, K_w, K_s, and R, which have been entered into the system, but these constants are used widely with wool knitted fabrics.

The denominated deviation rate DR [2] was found to have an excellent statistical correlation with the aspect of the knitted fabrics. For fine gauge knitted fabrics, it is recommended to have DR values low to 25%, for classified products those like of excellent quality, which is not verified experimentally.

Considering the analytical and experimental procedures for estimating elastic properties of a plain weft-knitted fabric and of polymer composite materials reinforced by it [3] and the elastic moduli of the cotton yarn and knitted fabrics, having different load span and knitting directions, based on the leaf and glaskin model, a numerical (FEM) elastic properties averaging model was elaborated.

The feasibility of assessing yarns with the wool comfort meter (WCM) to predict the comfort properties of the corresponding single jersey-knitted fabrics is examined [4]; inclusion of knitting gauge and cover factor slightly improved predictions. This indicates that evaluation at the yarn stage would be a reliable predictor of knitted fabric comfort, and thus yarn testing would avoid the time and expense of fabric construction; such correlation is only limited to wool knitted fabrics.

Considering the applicability of finite element method to analyze the bagging behavior of plain single jersey weft knitted fabrics in terms of bagging resistance [5], the numerical modeling of the bagging resistance of the plain knitted fabric using solid elements and yarn transverse isotropic properties is in good agreement with experimental values. Finally, this paper demonstrates a good practical verification of the proposed mathematical study, but in the rare field for uses of knitted fabrics.

Obviously, most of these researches did not differentiate between the mathematical and practical results, except for a few researches that differentiate the mechanical properties only. Most of these researches did not focus on bleached cotton plain single jersey knitted fabrics.

The aims of this paper are construction of a mathematical model that represents the geometrical properties of finished (bleached) plain single jersey knitted fabric, practical verification of the proposed mathematical model, and finally deducing some relationships to study the geometrical properties of the fabric.

2. Experimental Work and Tests Methodology

The specifications of the used machine are circular single jersey knitting machine, model ALBI, gauge 28, diameter 17 inch, and number of feeders 34. The fabrics are produced at loop length of 2.87 mm. The material used is cotton 100% Giza 86 and eight yarn counts are used (18, 22, 26, 30, 36, 40, 45, and 60N_e). All of the yarns are produced at three levels of yarn twist factor (2.8, 3.5, and 4.2α_e).

Although it is known that the use of yarns with fine counts with respect to machine gauge and high twist factor gives spirality in plain single jersey knitted fabrics, a wide range of twist factor and yarn count was used to study the compatibility of the practical results with the proposed mathematical model.

All fabrics are finished (full bleached) before tests on Thies Jet machine. The bleached solution contains the following agents with liquor ratio L.R (1 : 8):

(1) sequestering agent (0.9 : 1.25 g/lit),

(2) soap (1.25 g/lit),

(3) sodium hydroxide 50° Be (5 g/lit),

(4) peroxide stabilizer (1.6 : 1.9 g/lit),

(5) hydrogen peroxide 50% (10 g/lit).

The process was carried out at 100°C for 40 min. At 80°C the optical bleaching agent was used (0.375 g/lit), followed by rising the temperature to 100°C again for 20 min. Washing off was carried at 80°C for 10 min, followed by acid bath washing using acetic acid (1.25 g/lit) at 60°C for 20 min. Cold rinsing was followed by hot rinsing at 80°C for 10 min. Softening agent (5 g/lit) was used at 45°C for 30 min, followed by the final cold rinsing.

WPC, CPC, thickness, and weight tests are carried out and measured practically.

During wales density test, two needles are removed and there is a spread of 288 needles between them. The distance between the two removed needles is measured in the finished fabric and the wales are calculated b:

$$(\mathrm{WPC}) = \frac{288}{\text{distance between the removed needles (cm)}}. \tag{1}$$

The courses density test is carried out by inserting a different color cone during the production and the length of ten repeats is measured. The course of finished fabric is then calculated by

$$(\mathrm{CPC}) = \frac{\text{Number of feeders} \times 10}{\text{length of ten repeats (cm)}}. \tag{2}$$

The fabric thickness is measured according to ASTM D1777 standard.

The fabric weight is measured using a digital balance of two decimal digits accuracy.

3. Mathematical Knitting Model

In this section the mathematical model of the plain single jersey knitted fabric is deduced. The model is presented through two stages, namely, idealized general mathematical model and geometrical model at maximum set in only wales directions. Regarding the first model, a general model is assumed with general assumptions, and then mathematical equations are proved. Through the second model, new assumptions are introduced and the final equations are deduced to find out the suitable mathematical model.

3.1. Idealized General Mathematical Model of Plain Single Jersey Knitted Fabrics. In this model, the general assumptions are as follows.

(1) Yarns are circular cross section and can be touched but they cannot be compressed.

(2) Yarns are formed in straight lines and half circles.

According to these assumptions, the yarn appearance is drawn as shown in Figure 1.

To find out the mathematical equations that represent the plain single jersey knitted fabric, one repeat is taken into account and its boundaries are $UXYZ$ as shown in Figure 1. The dimensions of the loop under study are illustrated in Figure 2.

As shown in Figure 2, the loop head and the two legs can be drawn graphically by assuming an appropriate value for their radius of curvature (R). To avoid jamming, the radius of curvature should satisfy the following condition:

$$R < \left(\frac{W}{2} - \frac{d}{2}\right), \tag{3}$$

where W is the wale space and d is the yarn diameter.

According to the assumptions, the right stem can be graphically drawn by connecting the points (n m) and (p q) which can be simply determined as $n\ p = m\ q = d$. Similarly, the left stem can be drawn by the same technique.

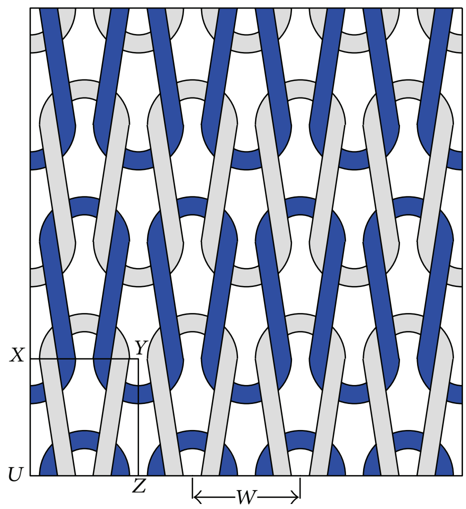


Figure 1: Simulated appearance of open plain single jersey knitted fabric.

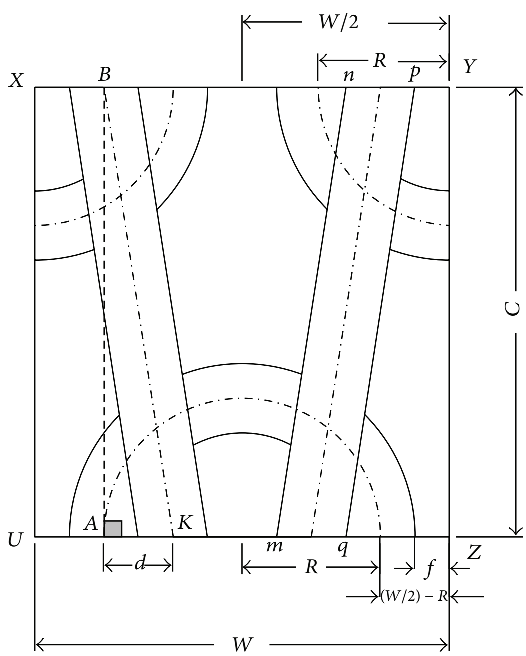


Figure 2: Graphical representation of plain single jersey general model.

Proof. According to Figure 2, the following relationships can be deduced:

$$f = \frac{W}{4} - d. \tag{4}$$

The value of R as a function of the dimensions can be determined as follows:

$$\begin{gathered} \frac{W}{2} = R + \frac{d}{2} + f, \\ \therefore R = \frac{W}{4} + \frac{d}{2}. \end{gathered} \tag{5}$$

The loop length value (l) can be determined from the following equations:

$$l = 2\pi R + 2BK, \tag{6}$$

$$\therefore l = \frac{\pi W}{2} + \pi d + 2\sqrt{C^2 + d^2}, \tag{7}$$

where $BK = \sqrt{C^2 + d^2}$ and b is the course space.

The tightness factor (T.F) can be estimated according to the following relationship:

$$\text{T.F} = \frac{l \times d}{W \times C}. \tag{8}$$

Substituting by (8),

$$\therefore \text{T.F} = \frac{\pi W d/2 + \pi d^2 + 2d\sqrt{C^2 + d^2}}{W \times C}. \tag{9}$$

□

Regarding (7), it is found that the loop length (l) is a function of the yarn diameter (d). However, practically, it is found that there is no relationship between the loop length and the yarn diameter. Therefore, new assumptions should be introduced to find out a suitable model as discussed in the following section.

3.2. Geometrical Model at Maximum Set in Only Wales Directions. Due to the nature of knitted fabrics finishing processes in the form of rope, the knitted fabrics are exposed to tension in the length direction, which tends to close any spaces in between wales on the expense of increasing spaces in course direction.

Therefore, in this model, beside the general assumption, the following assumption is introduced.

(1) The loop length is fixed.

(2) The wale space at maximum set is equal to four times of the yarn diameter.

According the overall assumption, the wale space, course space, and tightness factor can be estimated as a function of yarn diameter.

Figure 3 shows the simulated appearance of the maximum set in wales direction for plain single jersey knitted fabric.

To derive the final mathematical equations for maximum set in wales direction of plain single jersey knitted fabric, the repeat $UXYZ$ is taken into account as shown in Figure 3. The dimensions of the loop under study are illustrated in Figure 4.

Proof. According to Figure 4, the following relationships can be deduced.

The value of wale space (W) can be determined as

$$\begin{gathered} W = 4d, \\ W = \frac{10}{\text{WPC}}\,(\text{mm}), \\ \therefore \text{Max .wales} = \frac{10}{W}, \end{gathered} \tag{10}$$

$$\because d = \frac{1}{28\sqrt{N_e}} \text{ inch}. \tag{11}$$

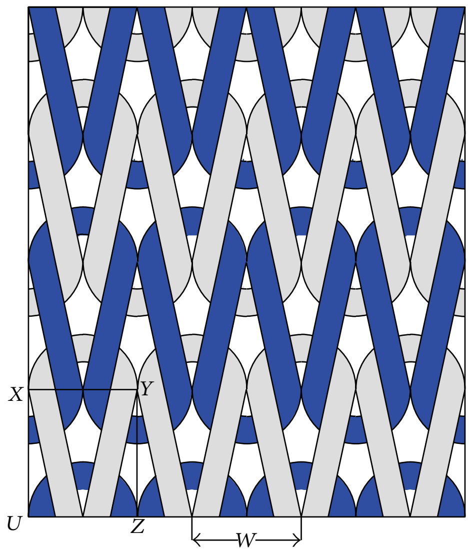


Figure 3: Simulated appearance of jersey knitted fabric touched in wales directions.

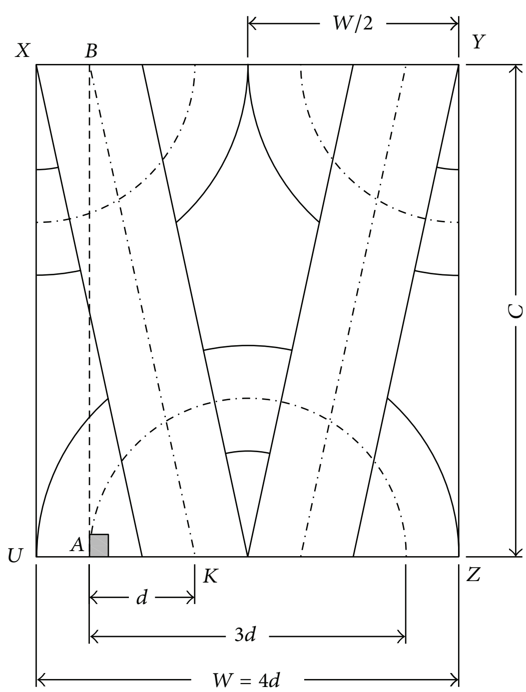


Figure 4: Graphical representation of touched in wales direction for plain single jersey model.

So, the maximum wales can be calculated as

$$\therefore \text{Max .wales} = 2.756\sqrt{N_e},$$
$$\therefore \text{Max .wales} = \frac{66.97}{\sqrt{\text{Tex}}}, \tag{12}$$

where N_e is English yarn count and Tex is Tex yarn count.

The value of loop length can be determined from the following equation:

$$l = 3\pi d + 2BK. \tag{13}$$

From the triangular ABK, the distance BK can be easily determined as

$$BK = \sqrt{d^2 + C^2}. \tag{14}$$

Substituted in (13) by the value of BK, the relationship of the course space (C) can be deduced as follows:

$$\therefore l = 3\pi d + 2\sqrt{d^2 + C^2},$$
$$\sqrt{d^2 + C^2} = \left(\frac{l - 3\pi d}{2}\right),$$
$$C^2 = \frac{(l - 3\pi d)^2}{4} - d^2,$$
$$\therefore C = \sqrt{\frac{(l - (3/4)\,\pi W)^2}{4} - \left(\frac{W}{4}\right)^2}, \tag{15}$$
$$\because C = \frac{10}{\text{CPC}}\,(\text{mm}),$$
$$\therefore \text{CPC} = \frac{10}{C}\,(\text{courses/cm}),$$
$$\therefore \text{Stitch density (SD)}$$
$$= \text{CPC} \times \text{WPC}\left(\text{S/cm}^2\right).$$

The T.F can be determined from the following equations:

$$\text{T.F} = \frac{\text{Covered area}}{\text{Total area}} = \frac{\text{loop length }(l) * d}{W * C},$$
$$\therefore \text{T.F} = \frac{\left((3/4)\,\pi W + 2\sqrt{(W/4)^2 + C^2}\right) \times d}{W \times C}, \tag{16}$$
$$\text{Fabric weight (F.W)} = \frac{l \times \text{SD} \times \text{Tex}}{100}\left(\text{g/m}^2\right),$$
$$\text{Fabric Bulkiness (F.B)} = \frac{\text{F.W}}{h \times 1000}\left(\text{g/cm}^3\right),$$

where SD is the stitch density (S/cm^2) and h is the fabric thickness (mm). □

The calculation of fabric thickness is discussed in details in Section 4.6.

Figure 5 shows the systematical steps that conclude the calculation procedures to get T.F and stitch density from the input known parameters which are yarn diameter and loop length.

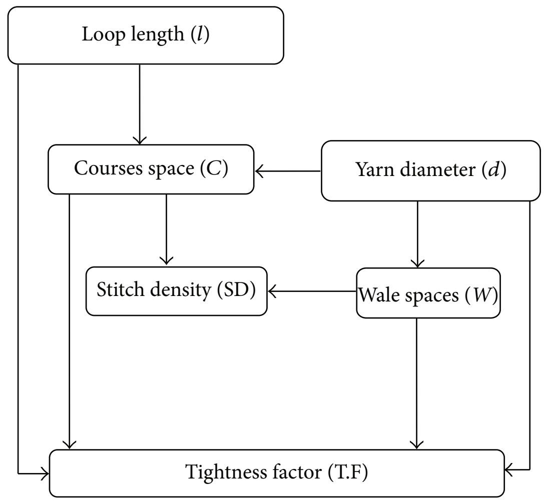


FIGURE 5: Output of the final mathematical model (touched model in wales direction).

4. Results and Discussions

In this section, a comparison between practical and theoretical results is presented. In addition, yarn count and twist factor effects on geometrical properties of plain single jersey knitted fabrics are studied and analyzed.

4.1. Wales Density (WPC). Figure 6 illustrates the actual effect of yarn count on the wales density at different levels of twist factors and a comparison with the results is obtained from the mathematical model.

Practically, it can be seen that. By increasing the yarn count from 9.8 to 32.8 Tex, the wales density decreased by 22 percent. On the other hand, the same trend is observed from the results obtained from the mathematical model, with different slope.

It can be justified that increasing the yarn diameter leads to increasing of the wale space; therefore, the wales density decreases.

Therefore, as compared to the mathematical model results, the yarn counts of 22.7 Tex can be considered as tight fabric because the yarns may be touched and compressed. On the other hand, the yarn counts less than 19.7 can be considered as open fabric.

Practically, it can be noted that the yarn twist factor has no significant effect.

4.2. Course Density (CPC). Figure 7 presents the practical and mathematical results of the yarn count effect and the yarn twist factor on the density courses.

From a practical point of view, it can be seen that, by increasing the yarn count from 9.8 to 32.8 Tex, the course space decreases and consequently the density courses increase by 33 percent. On the other hand, the same trend is observed from the results obtained from the mathematical model.

By a comparison with the mathematical model results, the fabric produced from the yarn that counts less than 19.7 Tex

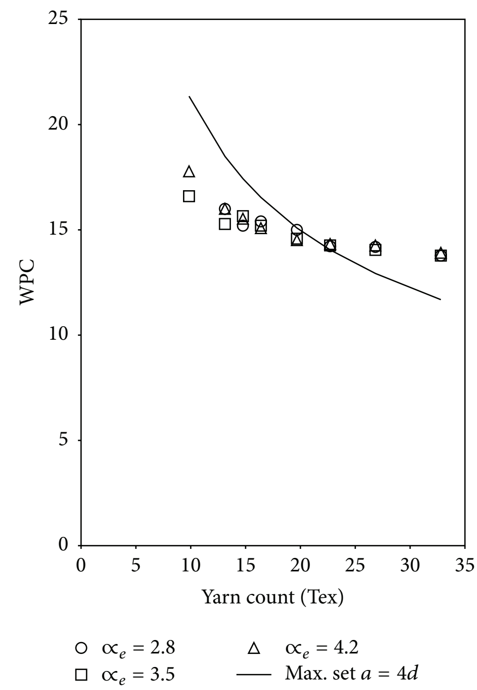


FIGURE 6: Effect of yarn count and twist factor on the wales density.

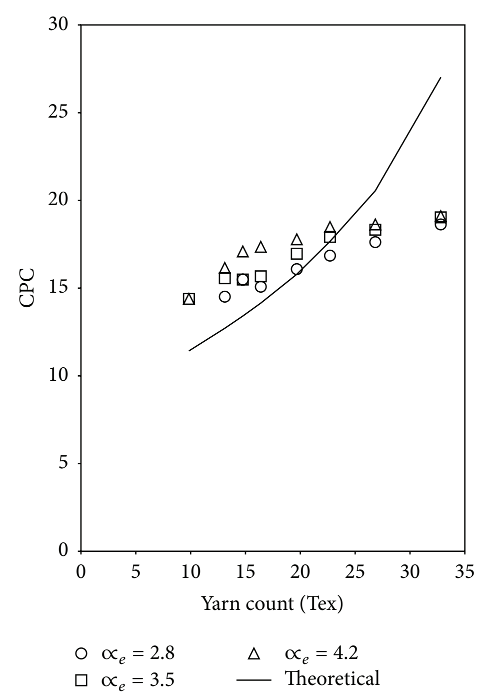


FIGURE 7: Effect of yarn count and twist factor on the course density.

could be tight in contrast with the results obtained for the wales density (see Figure 6). The nature of the knitted fabrics can be understood where the length increases as the width decreases and vice versa.

It should be noted that the yarn twist factor has a small effect at the fine counts. For instance, by increasing of the yarn

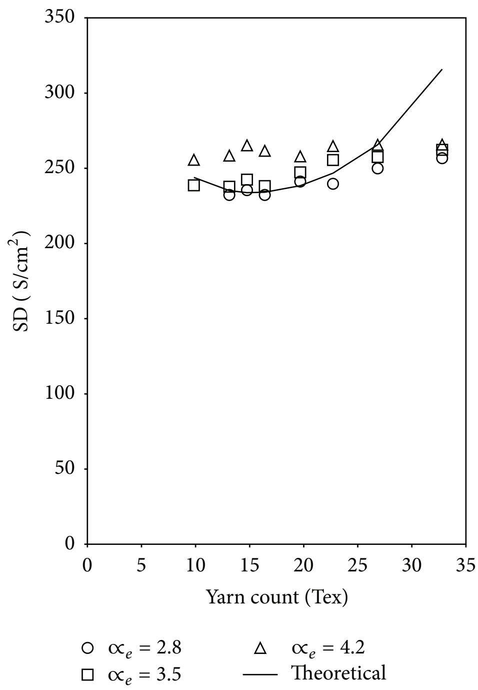


FIGURE 8: Effect of yarn count and twist factor on the stitch density.

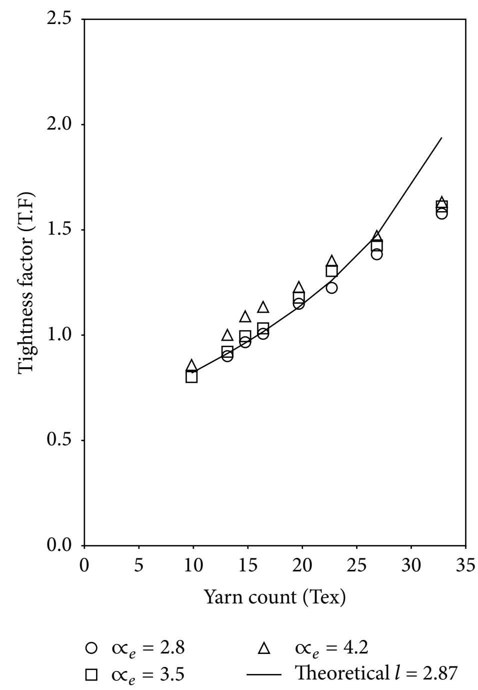


FIGURE 9: Effect of yarn count and twist factor on the fabric tightness.

twist factor from 2.8 to 4.2, the course density increases by 11%. It is due to the increasing of twist liveliness that leads to inclining of the knitting loop and in consequence the loop high decreases and as a result, the course density increases. Obviously, this is observed in the fine counts.

4.3. Stitch Density (SD). The effect of yarn count and yarn twist factor on the stitch density SD (S/cm^2) is shown in Figure 8.

It can be seen that the stitches are nearly constant over the range of yarn Tex tested, that is because the stitch density is the result of multiplying the course and wales densities, where the first factor is directly proportional and the other is inversely proportional to the yarn count.

On the other hand, the yarn twist factor has a small effect at the fine counts. For instance, by increasing of the yarn twist factor from 2.8 to 4.2, the stitch density increases by 12.6%.

It can be noted that there is an agreement between the practical and theoretical results except at the yarn count of 32.8 Tex; there is a difference of 18.9%.

4.4. Fabric Tightness. Figure 9 illustrates the effect of the yarn count and the yarn twist factor on tightness factor (T.F).

It should be noted that the tightness factor increased by 103.5% as the yarn Tex increased from 9.8 to 32.8 Tex. This happened due to increasing the yarn diameter and increasing the covered area because the loop length is constant.

Also, it can be seen that the yarn twist factor has no effect and there is a complete agreement between the practical and theoretical results.

From the obtained results, an equation is deduced to predict the tightness factor for finished plain single jersey knitted fabric. The equation is given by

$$\text{T.F} = 0.0014\ \text{Tex}^2 - 0.0113\ \text{Tex} + 0.8168, \tag{17}$$

where T.F is the tightness factor and Tex is the yarn count. The correlation factor of the given equation is estimated by 0.99.

4.5. Fabric Weight. The effect of the yarn count and yarn twist factor on the weight (g/m^2) at loop length of 2.87 mm is presented in Figure 10.

It can be seen that, by increasing the yarn count from 9.8 to 32.8 Tex, the weight is increased significantly by three times (285%). It can be accounted that by increasing the yarn diameter the stitch weight increases and as a result, the overall weight increases.

Also, it should be noted that there is a total agreement between the practical and theoretical results and the effect of yarn twist factor is insignificant.

The weight trend changing at different loop lengths is obtained and shown in Figure 11.

As it is clear, the required weight can be obtained at different loop lengths and yarn counts. For instance, weight of 180 g/m^2 can be obtained at yarn count of 24 Tex or 29 Tex with loop length of 3.1 mm or 2.3 mm, respectively.

4.6. Fabric Thickness. Figure 12 shows the effect of yarn count and yarn twist factor on fabric thickness (mm). From a practical point of view, it can be seen that the thickness increased

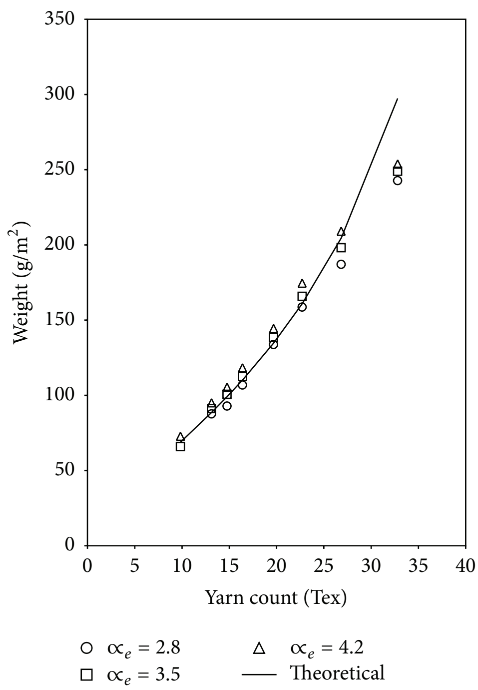


Figure 10: Effect of yarn count and twist factor on the fabric weight.

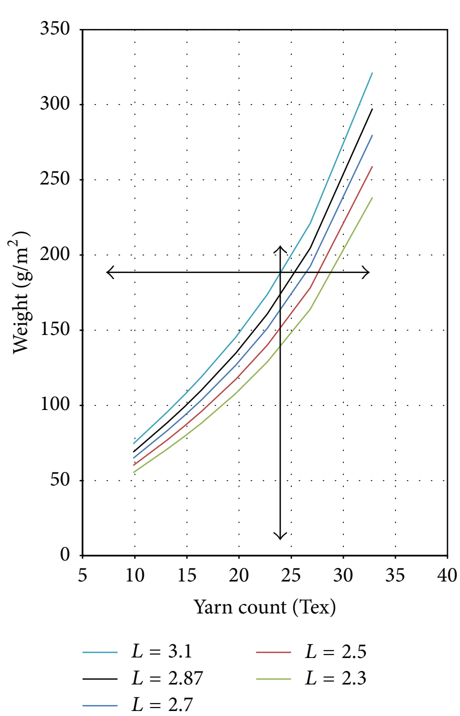


Figure 11: Empirical predicted weight of finished cotton plain single jersey knitted fabrics.

by a percentage of 48.15 as the yarn Tex increased from 9.8 to 32.8 Tex and it is logically acceptable because by increasing Tex the diameter increases.

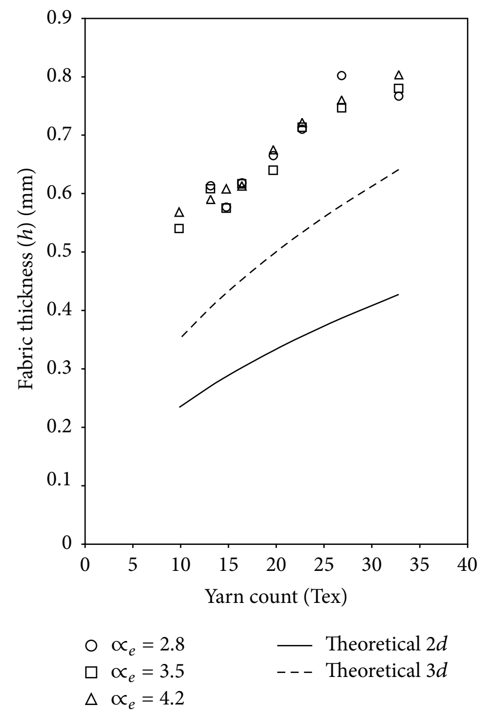


Figure 12: Effect of yarn count and twist factor on the fabric thickness.

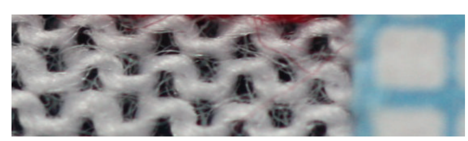


Figure 13: The optical pictures to represent the actual diameter measuring test.

Theoretically, the thickness of plain single jersey knitted fabric is estimated at two and three times of yarn diameter (d) and the results are plotted as solid and dashed curves as shown in Figure 12.

The yarn diameter can be calculated by (11).

It should be noted that neither 2d nor 3d is in agreement with the practical results and it dissents the well-known formulations published before which stated that the thickness of the plain single jersey knitted fabric is proportional to two times of the yarn diameter given by [6]

$$\text{Fabric thickness}\,(h) = 2 \times d\,(\text{mm})\,. \tag{18}$$

To find out the exact thickness at different yarn counts, an actual yarn diameter measuring test is carried out. Figure 13 presents the optical images for one plain single jersey knitted fabric sample of 26N_e yarn count with twist factor of 3.5α_e.

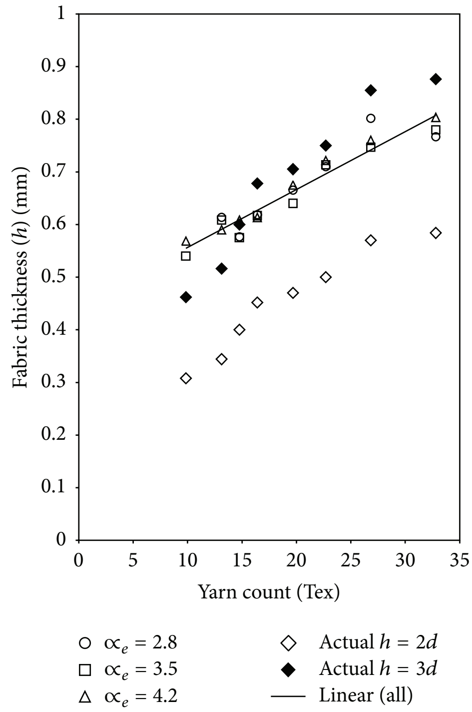


Figure 14: Verification of actual measured yarn diameter and fabric thickness.

Table 1: The actual measurements of yarn diameter.

N_e	18	22	26	30	36	40	45	60
d (mm)	0.292	0.285	0.25	0.235	0.226	0.2	0.186	0.177

For all samples, the actual yarn diameter in the fabric is measured by using scaled paper and 12.8 mega pixels camera; then the diameter of the yarn is measured at seven different positions for each sample, and then the average value is estimated. The results are recorded in Table 1.

Thickness of plain single jersey knitted fabric is estimated using the actual measured yarn diameter at different yarn counts. The thickness is estimated at two and three times of the yarn diameter and the results are plotted as shown in Figure 14.

It can be seen that there is an acceptable agreement between the practical results and the mathematical results obtained at three times of the actual measured yarn diameter especially at 14.8, 16.4, 19.7, and 22.7 yarn counts.

For clarification and justification purpose of the obtained results, a plain single jersey knitted fabric real sample, simulated appearance, and cross section are presented as shown in Figure 15.

It can be seen that, due to the incline of the loop as a result of the yarn twist liveliness existence, two legs of loops are overlapped one over the other as shown in Figure 15(a) and it is presented as shown in Figure 15(b). It should be noted that the wale space became three times of the yarn diameter instead of four times at the maximum set. Therefore, the area of interloping contains three yarn diameters as shown in Figure 15(c).

4.7. Fabric Bulkiness. The effect of the yarn count and yarn twist factor on the knitted fabric bulkiness (g/cm^3) at loop length of 2.87 mm is presented in Figure 16.

It can be seen that, by increasing the yarn count from 9.8 to 32.8 Tex, the fabric bulkiness is increased significantly by 167%. It can be accounted that the decreasing of wale space as a result of yarn diameter increasing leads to increasing of course space. Consequently, the overall fabric bulkiness increases.

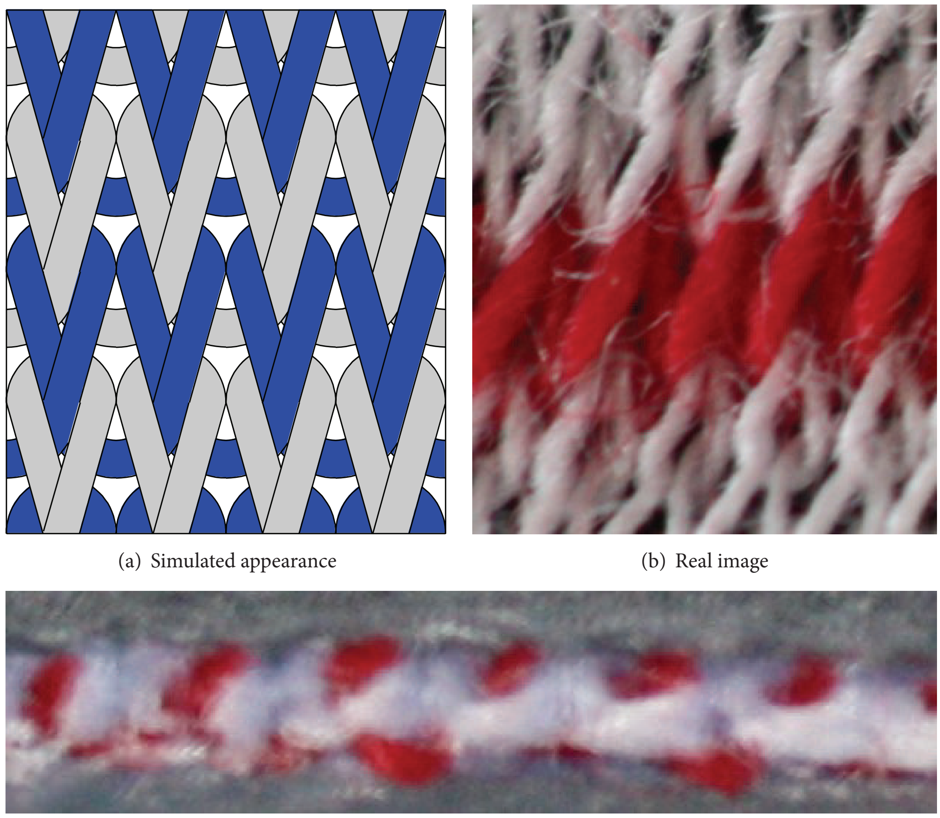
Also, it should be noted that there is a complete agreement between the practical and theoretical results and the effect of yarn twist factor is also insignificant.

5. Conclusion

In this paper, a mathematical model of plain single jersey knitted fabric was presented and the results were verified and compared to practical results. Obviously, the results have shown a significant agreement between the mathematical and practical results.

In addition, a weight forecasting relationship was deduced and verified practically. An empirical chart was also presented at different loop lengths and yarn counts.

Furthermore, the yarn diameter was measured actually and the fabric thickness as a function of yarn diameter was deduced. The obtained thickness relationship showed that



(a) Simulated appearance

(b) Real image

(c) Cross section

FIGURE 15: Plain single jersey vision by variety methods.

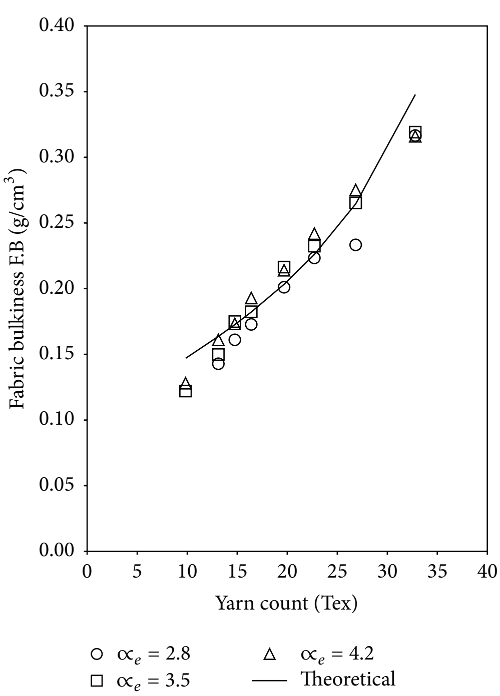


FIGURE 16: Effect of yarn count and twist factor on the fabric bulkiness.

the thickness is proportional to three times of the actual measured yarn diameter.

Geometrical Parameters

N_e: English yarn count
Tex: Direct yarn count
WPC: Wales per cm
CPC: Courses per cm
SD: Stitches density (S/cm^2)
R: Radius of loop head
W: Wale space
C: Course space
l: Loop length (mm)
T.F: Tightness factor
d: Yarn diameter
F.W: Fabric weight (g/m^2)
F.B: Fabric bulkiness (g/cm^3)
h: Fabric thickness (mm).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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