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Research Article

Slope Stability Analysis of Earth-Rockfill Dams Using *MGA* and *UST*

Li Nansheng, Tang Bo, and Xie Lihui

Department of Hydraulic Engineering, School of Civil Engineering, Tongji University, Shanghai 200092, China

Correspondence should be addressed to Li Nansheng; linansheng@tongji.edu.cn

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The nonlinear *Unified Strength Theory* (*UST*), which takes into account the effect of intermediate stress and nonlinear behavior on geotechnical strength, is applied in slope stability analysis of *earth-rockfill dams* (*ERD*) in this paper. The biggest drawback for general determination of slip surface is that it must presuppose the shape of slip surface and is unable to identify the critical noncircular slip surface more accurately. This paper proposes an optimal analytic model of slope stability analysis of *ERD* and employs *modified genetic algorithm* (*MGA*) to search for the slip surface on the basis of shear failure criteria of the nonlinear *UST* without prior assumption of the shape of slip surface. The application of *MGA* dependent on Matlab toolbox to the slope stability analysis of *ERD* shows that *MGA* can consequently overcome the weakness of easily falling into local optimal solutions brought by general optimal algorithms.

1. Introduction

The material model of geotechnical strength plays a very important role in the problems of slope stability of *ERD*. Many researchers [1–4] have made a large amount of research work about the effect of applied loads and groundwater on slope stability of *ERD*, but it is still open for further research on the slope stability of *ERD* because of the complexity of the soil materials. The parameters of geotechnical strength in Slices Method based on the rigid body limiting equilibrium theory are ones related to the failure criterion of linear shear strength, just like in *Mohr-Coulomb* failure criterion that is widely applied in geotechnical engineering. *Mohr-Coulomb* failure criterion is the lower limit of a linear convex function and does not consider the effect of intermediate stress on the geotechnical strength, so the numerical results trend to more safety. However, the intrinsic complexity of the mechanical property of geotechnical materials results in that there are almost few strength criteria which can accurately describe the nonlinear fractural characteristics of soils and even the same with linear failure criteria. Some researches [5–7] have shown that the failure of geotechnical materials with nonlinear destruction is just one special case of failure criteria under linear damage and the intermediate stress effect which can enhance the slope safety factors to some degree. The major determinants causing the dams to be broken include applied forces, seepage, and earthquake. As known to all, the strength parameters of the soil play an indispensable role in the slope stability analysis of *ERD*. Many experts [1–4] have done a lot of studies on the effects produced by external loads and underground water on the slope stability; however there are still many works to be done in the field of strength theory of geotechnical material. By so far, a few hundred or more of yielding and failure criteria [8–11] have been proposed, whose applications in the geotechnical engineering have to be seriously restricted because there is not a general failure formula that is appropriate for all kinds of materials and different mechanical status. In 1991 Professor *Yu* [11] proposed the *UST* on the basis of all existing strength theories, by combining the double shear strength theory with the single shear strength theory into only one formula. *UST* is thought as the high generalization of all kinds of existing strength theories, but *UST* is only asymptotic approximation for the most of the nonlinear issues.

The Slices Method, based on the rigid body limiting equilibrium theory, should suppose the shape of slip surface in advance, so the greatest weakness is that it is difficult to cast about for the slip surface quickly and accurately corresponding to the minimum safety coefficients of slip surfaces. The geometrical configuration of the slip surfaces is assumed as in circular arc for the most of the searching algorithms, but we must take an arbitrary shape of slip surfaces for heterogeneous soil materials and discrepant distribution of pore pressure in soils. In the determination of minimum safety factor and its corresponding arbitrary slip surfaces, it might be the best choice to use the optimization methods to arrive at satisfactory results when the objective function is convex and the searching domains are irregular. But for the objective function with multipeak in the complicated searching domains, the general optimization methods often get into local optimal solutions for complex geotechnical structures and heterogeneous soil layers. On the basis of biology immune principles, a novel optimization algorithm [12–14] is proposed for solving many optimal solutions to multimodal functions. Since the late 1980s, the slope stability analysis begins to enter the period when the numerical theory develops flourishingly and finite element method becomes one method of spatial discretization which is the most widely applied; most popular methods of slope stability analysis include Shear Strength Reduction [15, 16] and Gravity Increase Method. When the Shear Strength Reduction Method is used for slope stability analysis and the mechanical states of soils are very close to limiting equilibrium, it is difficult to converge and to solve governing equations in numerical calculation. *Genetic Algorithm* (*GA*) is a global optimization algorithm, so it can overcome the shortcomings of the ordinary optimization methods that fall easily into local optimum.

How to ascertain the most critical slip surfaces faster and more accurately can be regarded as a difficult affair in the slope stability analysis of *ERD*, upon which this paper tries to apply the multivariable nonlinear optimal theory in slope stability analysis which is built on *Matlab* platform and then simulate the critical slip surfaces of *ERD* slope to find its minimum safety coefficient. For *GA* is a sort of global optimization method and can avoid simply getting into local minima, the paper will analyze slope stability by way of *GA*. How *UST* can be applied in the stability analysis of slops still makes few progresses, so this paper will try to put the linear and nonlinear *UST* applied to the stability analysis of *ERD* and compare various strength criteria to the influence of the slope stability of *ERD*. One of the difficulties of the slope stability analysis of *ERD* is how to determine the most critical slip surfaces more accurately and faster, although there have been some valuable achievements made by researchers in China, but what the most appropriate methods are still has not confirmed. Based on *Matlab* platform we try to apply the nonlinear optimization theory with multivariable to the stability analysis of *ERD* and to determine the minimum safety factor of slope stability in simulation of the critical slip surfaces of *ERD*.

2. Linear and Nonlinear *UST*

Taking a unique mechanical model as the theoretic basis, Professor *Yu* puts forward an *UST* unifying all sorts of the linear and nonlinear strength theories into one. The linear and nonlinear *UST* proposed by Professor *Yu* can be summarized as [11] follows.

If

$$\sigma_2 \leq \frac{1}{2}\left[(1+\beta)\sigma_1 + (1-\beta)\sigma_3\right], \tag{1}$$

then

$$\begin{aligned}\frac{1-n}{2}&\left[(1+b)(1+\beta)\sigma_1 - b(1-\beta)\sigma_2 - (1-\beta)\sigma_3\right]\\ &+\frac{n}{3}\left[\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\right.\\ &\left.+4\beta(\sigma_1+\sigma_2+\sigma_3)\right] = A;\end{aligned} \tag{2}$$

also if

$$\sigma_2 > \frac{1}{2}\left[(1+\beta)\sigma_1 + (1-\beta)\sigma_3\right], \tag{3}$$

then we take following formula:

$$\begin{aligned}\frac{1-n}{2}&\left[(1+\beta)\sigma_1 + b(1+\beta)\sigma_2 - (1+b)(1-\beta)\sigma_3\right]\\ &+\frac{n}{3}\left[\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\right.\\ &\left.+4\beta(\sigma_1+\sigma_2+\sigma_3)\right] = A\end{aligned} \tag{4}$$

in which

$$\begin{aligned}\beta &= \frac{3(1+n)(1+b)(1-\alpha)+2\sqrt{2}n(1-\alpha)}{3(1+n)(1+b)(1+\alpha)+8n(1-\alpha)}\\ A &= \frac{(1-n)(1+b)+2/3\sqrt{2n}}{1+\alpha}\sigma_t\\ \alpha &= \frac{1-\sin\varphi}{1+\sin\varphi}\\ \sigma_t &= \frac{2c\cos\varphi}{1+\sin\varphi},\end{aligned} \tag{5}$$

where σ_1, σ_2, and σ_3 are the first, second, and third principal stresses, respectively, α is the ratio of the tension with compression strength, β is the factor concerned with the normal stress effect, b is related coefficient to the intermediate principal stress, n is nonlinear coefficient reflecting failure criteria, c is cohesion, and φ is the inner friction angle.

When the coefficients of twins shear strength n, b, α, and β take some specific values, *UST* can be converted into existing major yield and strength criteria such as *Mohr-Coulomb* criterion as shown in Table 1. So we might think that the linear and nonlinear *UST* more comprehensively reflect the strength conditions of the practical problems of geotechnical engineering.

The applications of *UST* in practical engineering have been in a series of researches but it is still rare in the

Table 1: *UST* versus several exiting major yielding and strength criterions.

Nonlinear criterion, $0 < n \le 1$			Linear criterion, $n = 0$		
$n = 1$	$n = \alpha = 1$ $\beta = 0$	$0 < n < 1$	$n = \beta = 0$ $\alpha = 1$	$n = b = \beta = 0$	$n = b = 0$ $\alpha = 1$
Drucker-Prager criterion	Huber-Vonmises criterion	New nonlinear strength criterion	Linear unified yielding criterion	Tresca yielding criterion	Mohr-Coulomb failure criterion

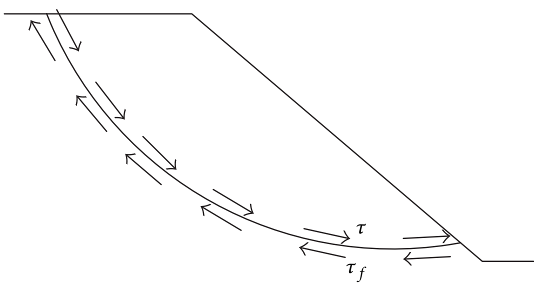

Figure 1: Slip surface.

slope stability of ERD. In order to reveal the influences of the nonlinearity and intermediate principal stresses on slope stability in the failure criteria of *UST*, we illustrate the applications of the linear and nonlinear *UST* in slope stability below.

2.1. Nonlinear UST Applied in Slope Stability. According to *Mohr-Coulomb* strength theory, the safety coefficients along the slip surface shown in Figure 1 can be defined as

$$\frac{\int \tau_f dl}{\int \tau\, dl} = \frac{\sum_{i=1}^{n} (c_i + \sigma_{ni} \tan\varphi)\,\Delta l_i}{\sum_{i=1}^{n} \tau_i \Delta l_i}, \tag{6}$$

where τ_f is the shear strength of sliding surface, τ is the factual shear stress, l means the arc length of sliding surface, n is number of soil slices, c is cohesion, φ is inner friction angle, and σ is effective stress.

To apply linear and nonlinear *UST* in the slope stability analysis, we need to rewrite formulas (2) and (4) as follows.

For $\sigma_2 \le (1/2)[(1+\beta)\sigma_1 + (1-\beta)\sigma_3]$, then

$$\begin{aligned} &\frac{1-n}{2}(\sigma_1 + b\sigma_1 - b\sigma_2 - \sigma_3) \\ &\quad + \frac{n}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \\ &= A - \frac{4}{3}n\beta(\sigma_1+\sigma_2+\sigma_3) \\ &\quad - \frac{1-n}{2}\beta(\sigma_1 + b\sigma_1 + b\sigma_2 + \sigma_3). \end{aligned} \tag{7}$$

For $\sigma_2 > (1/2)[(1+\beta)\sigma_1 + (1-\beta)\sigma_3]$, then

$$\begin{aligned} &\frac{1-n}{2}(\sigma_1 + b\sigma_2 - \sigma_3 - b\sigma_3) \\ &\quad + \frac{n}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \\ &= A - \frac{4}{3}n\beta(\sigma_1+\sigma_2+\sigma_3) \\ &\quad - \frac{1-n}{2}\beta(\sigma_1 + b\sigma_2 + \sigma_3 + b\sigma_3), \end{aligned} \tag{8}$$

where the items, related to the principal stresses at the left hand of the formulae, can be denoted by $f(\sigma_1, \sigma_2, \sigma_3)$ and the items concerned with strength parameters at the right hand are written in the form $y(c, \varphi)$. Hence, the safety coefficient along a certain potential slip surface can be depicted as

$$F_s = \frac{\int_l y(c,\varphi)\, dl}{\int_l f(\sigma_1,\sigma_2,\sigma_3)\, dl} = \frac{\sum_{i=1}^{n} y_i(c,\varphi)\,\Delta l_i}{\sum_{i=1}^{n} f_i(\sigma_1,\sigma_2,\sigma_3)\,\Delta l_i}. \tag{9}$$

So we deduce following expressions.

For $\sigma_2 \le (1/2)[(1+\beta)\sigma_1 + (1-\beta)\sigma_3]$, then

$$\begin{aligned} &y_i(c,\phi) \\ &\quad = A - \frac{4}{3}n\beta(\sigma_1+\sigma_2+\sigma_3) \\ &\quad\quad - \frac{1-n}{2}\beta(\sigma_1 + b\sigma_1 + b\sigma_2 + \sigma_3), \\ &f_i(\sigma_1,\sigma_2,\sigma_3) \\ &\quad = \frac{1-n}{2}(\sigma_1 + b\sigma_1 - b\sigma_2 - \sigma_3) \\ &\quad\quad + \frac{n}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}. \end{aligned} \tag{10}$$

For $\sigma_2 > (1/2)[(1+\beta)\sigma_1 + (1-\beta)\sigma_3]$, then

$$\begin{aligned} &y_i(c,\phi) \\ &\quad = A - \frac{4}{3}n\beta(\sigma_1+\sigma_2+\sigma_3) \\ &\quad\quad - \frac{1-n}{2}\beta(\sigma_1 + b\sigma_2 + \sigma_3 + b\sigma_3), \\ &f_i(\sigma_1,\sigma_2,\sigma_3) \\ &\quad = \frac{1-n}{2}(\sigma_1 + b\sigma_2 - \sigma_3 - b\sigma_3) \\ &\quad\quad + \frac{n}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}. \end{aligned} \tag{11}$$

By means of finite element methods to calculate the stress solutions of whole *ERD*, then extract the principal

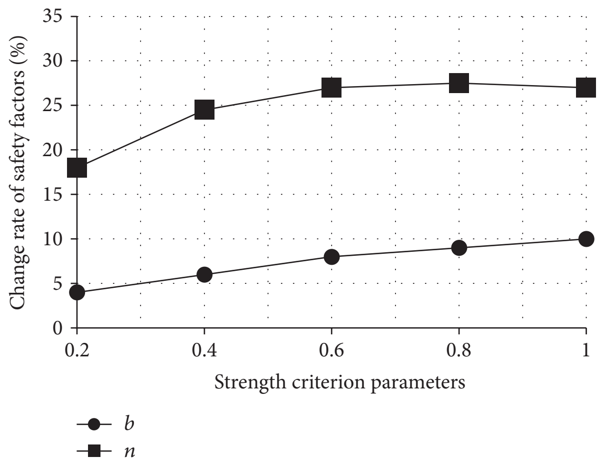

Figure 2: Strength parameters versus safety factors.

stresses σ_1, σ_2, and σ_3 which are substituted into the equations mentioned above and get the safety coefficients of presupposed slip surfaces. Consider one slip surface and substitution of the first, second, and third principal stresses into the formula above; then calculate the values of $\sum_{i=1}^{n} y_i(c,\varphi)\Delta l_i$ and $\sum_{i=1}^{n} f_i(\sigma_1,\sigma_2,\sigma_3)\Delta l_i$ according to linear and nonlinear *UST*. Eventually the safety factor can be written as

$$F_s = \frac{\sum_{i=1}^{n} y_i(c,\varphi)\Delta l_i}{\sum_{i=1}^{n} f_i(\sigma_1,\sigma_2,\sigma_3)\Delta l_i}. \tag{12}$$

For application of optimal method to decide the potential slip surface of *ERD*, we can ascertain the minimum safety factors through comparison among the outcomes obtained above.

2.2. Analysis of Parameter Sensitivity. In the analysis of slope stability of dams engineering, the applications of numerical simulation have become quite common and the accuracy of numerical simulation is closely related to the mathematical model and the values of the characteristic parameters of soil medium. Given the mathematical model of the slope stability, the material parameters are reasonable or do not have very great effect on the calculation results as shown in Figure 2. In the present paper we conclude that the linear and nonlinear *UST* associate with two main parameters, functional coefficient of intermediate principal stresses b and the nonlinear effect coefficient n, in the applications of the slope stability analysis. So we will discuss the parameter sensitivity about the influence of two parameters mentioned above on the stability of slope safety as follows. There are two kinds of sensitivity analysis, that is, the single factor and multifactorial sensitivity analysis, and we will adopt the single factor sensitivity analysis in this paper. Single factor approach, by changing the value of some factor and assuming that other factors remain immovable, compares the degree of influence of the benchmark values on the factor change.

In the following examples of the slope stability analysis, we take $n = 0$, $b = 0$, and the safety factor as the benchmark

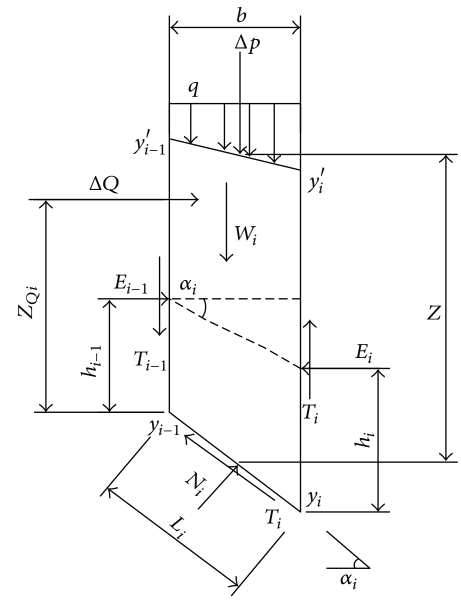

Figure 3: A general soil slice and applied forces.

values. It is illustrated in Figure 2 that the slope safety parameters change with increase of the strength parameters n and b.

As Figure 2 displays, two strength parameters n and b in *UST* have different influences on the coefficients of slope stability of *ERD*. When nonlinear coefficient n develops gradually, the safety factor increases at first and then falls back slightly, but the safety coefficients increase with accretion of the parameter b concerned with the intermediate principal stresses. From Figure 2, you can also read that the nonlinear coefficient n has greater effect on the safety coefficients of slope stability of *ERD* more than the principal stress coefficient b.

3. Stability Analysis of *ERD* by *MGA* Based on *Matlab* Platform

3.1. Formulation of Optimal Model. The Slices Method based on the rigid body limiting equilibrium method is most widely applied in the slope stability analysis. How to determine the minimum safety coefficients and corresponding slip surfaces of *ERD* is the key of algorithms adopted, this problem can fall into optimization programming in mathematical terms. If the Slices Method is adopted, the position and shape of slip surfaces are defined by two orthogonal coordinates x and y. When the width of slices is definite, the coordinate y is only optimization variable.

We choice the modified *Janbu* method [8] to calculate the safety factors of slope. Taking a general soil slice, then making force analysis is as shown in Figure 3.

The meanings of all notations can be figured out through Figure 3. If there are not external forces applied on both sides of the slices, namely, E_0, E_n, T_0, and T_n all vanish, and assuming that the number of soil slices is n, then

$$F_s = \frac{\sum_{i=1}^{n} [c_i b + (pb + \Delta T_i - ub)\tan\varphi_i]\left(\left(1+\tan^2\alpha_i\right)/\left(1+\tan\alpha_i\tan\varphi_i/F_s\right)\right)}{\sum_{i=1}^{n}[\Delta Q_i + (pb+\Delta T_i)\tan\alpha_i]} \quad (13)$$

in which

$$\tan\alpha_i = \frac{y_i - y_{i-1}}{b}, \qquad p = \frac{w_i}{b} + q + \frac{\Delta p}{b}, \quad (14)$$

and u is the pore pressure on the slip surfaces. The optimization model of slip surface of slope stability can be concluded as the following:

$$\begin{aligned} \min \quad & [abs\,(F_{s0} - F_{s1})] \\ \text{subject to:} \quad & y_0 \le y_0 \le y_0', \\ & y_1' \le y_1 \le h, \\ & \vdots \\ & y_{n-1}' \le y_{n-1} \le h, \\ & y_0 \le y_n \le y_n', \\ & -b \le y_{i+1} - y_i \le b\cdot\tan\alpha_i, \end{aligned} \quad (15)$$

where F_{s0} is initial estimated value of safety factor for iteration and F_{s1} evaluation value by substitution of F_{s0} and the shear forces between each of the soil slices into (13).

3.2. Slope Stability Analysis Based on MGA. The *Dichotomy*, *Pattern Search Method*, and *Simplex Method* are commonly adopted in slope stability analysis of *ERD* and to determine the noncircular critical slip surfaces. For the slopes with complex geometrical figuration and uneven soil layers, as well as nonconvex nonlinear constrained optimization problems, these optimization algorithms might get local optimum. However, Standard *MGA* is a global optimal algorithm in searching domain, which simulates the proceeding of natural evolution and can get the global best, and can easily overcome the shortcomings of general optimization algorithms to be the local minimum or maximum. So it is especially suitable to search for global minimum of the nonlinear problems of slope stability of *ERD*. Below we will detail the algorithm implementation of *MGA* to slope stability of *ERD*.

3.3. Procedures of Slope Stability Analysis Based on MGA. The procedures corresponding slope stability of *ERD* based on *MGA* and *UST* can be summarized as follows [14].

Step 1. Define the optimal variables $y_0, y_1, y_2, \ldots, y_n$ and constraint conditions in (15). In order to improve the efficiency of the optimal procedure of *MGA*, the geometrical configuration of the slip surfaces should be rationalized in the optimization analysis of *ERD*. According to the practical features of slope stability of *ERD*, the slope angle of the slip slices must be subject to the following condition:

$$-45^\circ \le \alpha_i \le 80^\circ, \quad (16)$$

where α_i is the angle of ith soil slice between the bottom of soil slices and horizontal coordinates. So, the last constrained condition in (15) should be modified as

$$-b \le y_{i+1} - y_i \le b\cdot\tan 80^\circ. \quad (17)$$

Step 2. Build up the objective function in (13).

Step 3. Generate original population.

Producing randomly S slip surfaces (namely S individuals) which consist of $n+1$ y-coordinates.

Step 4 (ascertaining coding method). The encoding process means transforming the variables into binary digits strings whose number of equivalent binary digits is determined by required precision. For instance, the value range of variable y_i is y_i' to h and precision of its value is for three decimal places, which means that the value range of each variable will be divided into $(h - y_i') \times 10^3$ sections. Binary string of digits (denoted by m_i) of one variable can be obtained from the following formula:

$$2^{m_i - 1} < \left(h - y_i'\right) \le 2^{m_i}. \quad (18)$$

So, individual string of digit is $\sum m_i$, which can be illustrated as shown in Figure 15.

Step 5 (ascertaining the decoding method). A factual value returned from binary digits string can be achieved by the following formula:

$$y_i = y_i' + \text{decimal}\left(\text{substring}_i\right) \times \frac{h - y_i'}{2^{m_i} - 1}, \quad (19)$$

where decimal(substring_i) is the decimal value of the variables y_i.

Step 6 (determining the fitness value of each individual). Each individual is substituted into the objective function to calculate the safety factors; if the safety factors are small, then put larger fitness value in the calculation of the safety factors and *vice versa*. Consequently, the general fitness value of colony can be presented as $f = \sum \text{eval}(u_i)$, which can deduce the probability of each selected individual $p_i = \text{eval}(u_i)/\sum \text{eval}(u_i)$. Then calculate the accumulating probability of each individual $Q_i = \sum_{j=1}^{i} p_j$, $j = 1, 2, 3, \ldots$.

Table 2: Slope safety factors evaluated by different failure criteria.

	b					
	0	0.2	0.4	0.6	0.8	1
Nonlinear						
$n = \alpha = 1, \beta = 0$	0.3007	0.3007	0.3007	0.3007	0.3007	0.3007
$n = 0.2$	1.1110	1.1170	1.1216	1.1251	1.1279	1.1302
$n = 0.4$	1.1649	1.1668	1.1681	1.1688	1.1692	1.1694
$n = 0.6$	1.1895	1.1910	1.1920	1.1926	1.1931	1.1932
$n = 0.8$	1.1976	1.1989	1.2000	1.2011	1.2019	1.2026
$n = 1$	1.1944	1.1944	1.1944	1.1944	1.1944	1.1944
Linear						
$n = 0$	0.9390	0.9700	0.9930	1.0109	1.0252	1.0368
$n = \beta = 0, \alpha = 1$	0.2814	0.2885	0.2938	0.2979	0.3012	0.3039
$n = 0, b = 1.5$			1.0583			

Step 7. Determine the operator based on the Roulette Wheel Selection. Selecting new population of a chromosome can be conducted by the next two steps:

(1) Generate a stochastic number r $(0 < r < 1)$.

(2) If $Q_{i-1} \leq r \leq Q_i$, then choose ith individual.

Step 8 (crossover operation). Select one individual node randomly, and then produce children generation by exchanging the right sections of the node between two parent generations, as in Figure 16.

If the probability of crossover is 0.3, it indicates that only 30% of individuals on the average level have been crossed. The procedure of crossing operation is as shown below.

Continue the cross process when $k \leq S$; then generate random number r_k which is a number between 0 and 1. If $r_k < 0.3$, then select the kth individual as parent generation of the crossover. Assign $k = k + 1$. End current step.

Step 9 (mutation operations). Suppose that the jth gene of ith individual is chosen for the variation; if the digit code corresponding with the gene is 1, it will change to 0 after the variation. If probability of mutation is 0.02, namely, the 2% of the individuals above the average level must get mutated. After the completion of the variation of the population we arrive at the ultimate next generation of population.

So far, we have completed the first generation calculation of slope stability based on *MGA* and *UST*. Then repeat the steps from 6 to 9 until we obtain the optimal individuals.

We can make use of *Matlab MGA* toolbox to implement the above analysis steps. Main program of *MGA* and direct searching toolbox in *Matlab* provides an interface between *Matlab* toolbox and the external. The invoking format of *Matlab* function is as follows:

$$[x \quad fval] = ga\left(@fitnessfun, n \text{ var } s, options\right), \qquad (20)$$

in which x is the optimal coordinate and $fval$ is optimal solution of objective function and both are the optimal results given by invoking *Matlab* function, $fitnessfun$ stands for objective function, n var s denotes the number of optimization variables in objective function, and *options* is a parameter option of the *MGA*. Note that default value should be invoked without passing parameter option.

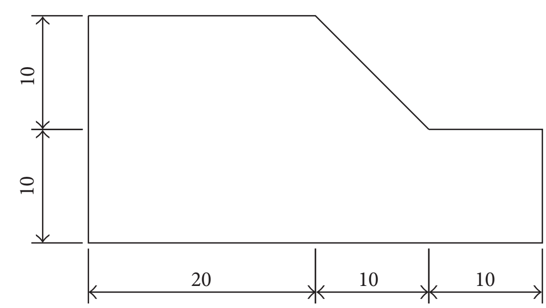

Figure 4: Single stage *ERD*.

4. Numerical Examples

4.1. Example 1. For the comparison of the general strength theory with *UST*, we choose a single stage *ERD* as an example shown in Figure 4. The intensity parameters are presumed as $c = 13\,\text{kP}$, $\varphi = 18^{\circ}$, $\rho = 19.8\,\text{kN/m}^3$, $E = 2.5\,\text{mP}$, and $\mu = 0.3$. We adopt different strength criteria, which are provided with different strength parameters n and b in *UST*, to evaluate the slope safety factors and the final results shown in Table 2.

We can evaluate that the safety factor is equal to 0.9496 by *UST* and 0.939 by *Mohr-Coulomb* failure criterion in this example using the formula below:

$$F_s = \frac{\int \tau_f dl}{\int \tau\, dl} = \frac{\sum_{i=1}^{n} \left(c_i + \sigma_{ni} \tan \phi\right) \Delta l_i}{\sum_{i=1}^{n} \tau_i \Delta l_i}. \qquad (21)$$

The relationship of slope safety coefficients F_s via b and n is shown in Figures 5 and 6.

By the calculation and analysis above the example, when $n = \alpha = 1$, $\beta = 0$ in nonlinear criterion and $n = \beta = 0$, $\alpha = 1$ in linear criterion, for the reason of no consideration of normal stress effect, so the results are not reasonable and not applicable to the materials with large tensional-compressional strength. The nonlinear coefficient n and effect

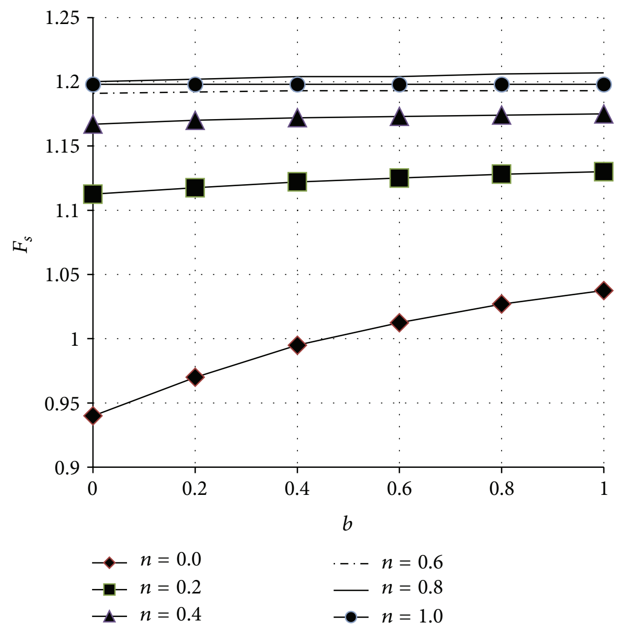

Figure 5: Slope safety factors versus strength coefficient b.

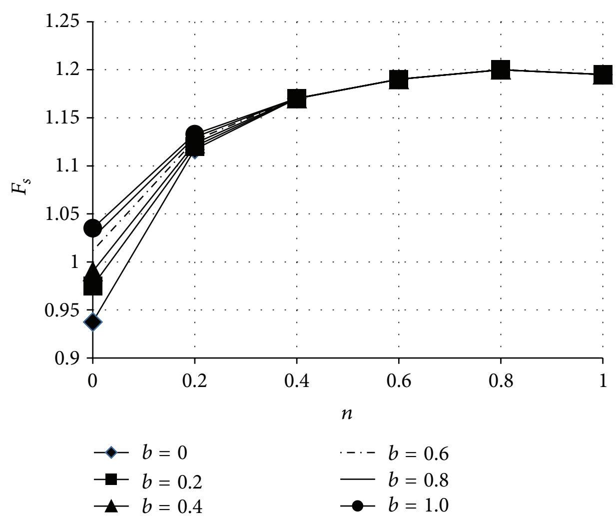

Figure 6: Slope safety factors versus strength coefficient n.

coefficient b relative to failure criteria and intermediate principal stresses, respectively, may improve the slope stability safety at some degrees, but the former to the improvement of safety factors is more obvious than the latter. From Figures 5 and 6 we can see that the increase of safety factor gets faster with the increase of parameter n at first, and when n is greater than 0.4 the effect of change of n to the safety factors becomes significantly less, and when n is greater than 0.8 the safety factors will reduce with increase of n instead. When n is less than 0.2, the increase of the parameter b on improvement of the safety factors is obvious, but if n is greater than 0.2, the change of the parameter b has little impact on the improvement of the safety factors.

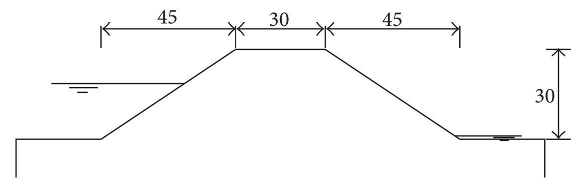

Figure 7: *ERD* (m).

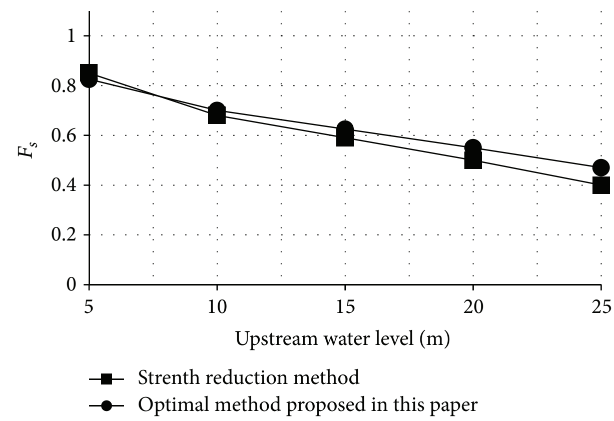

Figure 8: Safety factor versus upstream water level.

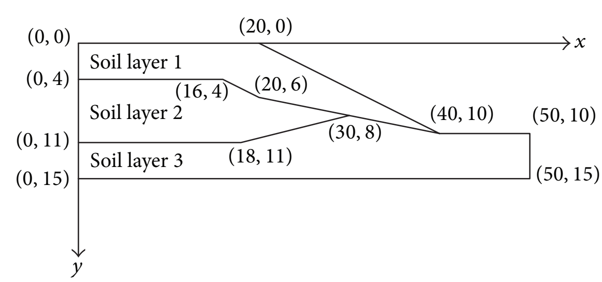

Figure 9: The structure of a slope (m).

4.2. Example 2. The practical working conditions of a typical *ERD* are demonstrated in Figure 7; The downstream water level is 1 m. Soil cohesion of *ERD* is 13 kP, inner friction angle 25°, effective cohesion 6 kP, effective inner friction angle 13°, natural weight-specific density 20 kN/m^3, and saturated unit weight of soil 23 kN/m^3. When *ERD* has the different upstream water level, the stability safety factors of *ERD*, calculated by the Strength Reduction Method and method presented in this paper, are shown in Figure 8. The results indicate that two strength theories almost give the same factors of slope stability of *ERD* under the conditions of different upstream water level.

4.3. Example 3. The geotechnical structure of an inhomogeneous slope in layers is shown in Figure 9. The material properties of the slope are listed in Table 3. The width of soil slices in the determination of the most dangerous slip surfaces by the *MGA* and *UST* is taken as 2.5 m. The slope is enforced lateral acceleration 0.15 g and the recommended safety factor

Table 3: Material properties of the slope.

Number of soil layers	Cohesion C (kP)	Inner friction angle φ	Weight-specific density γ(kN/m^3)	Modulus of elasticity E (mP)	Poisson's ratio ν	K_o
1	0	38	19.5	1.0E4	0.25	0.65
2	5.3	23	19.5	1.0E4	0.25	0.65
3	7.2	20	19.5	1.0E4	0.25	0.65

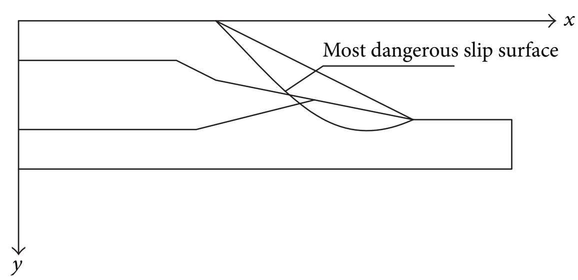

Figure 10: Most dangerous slip surface.

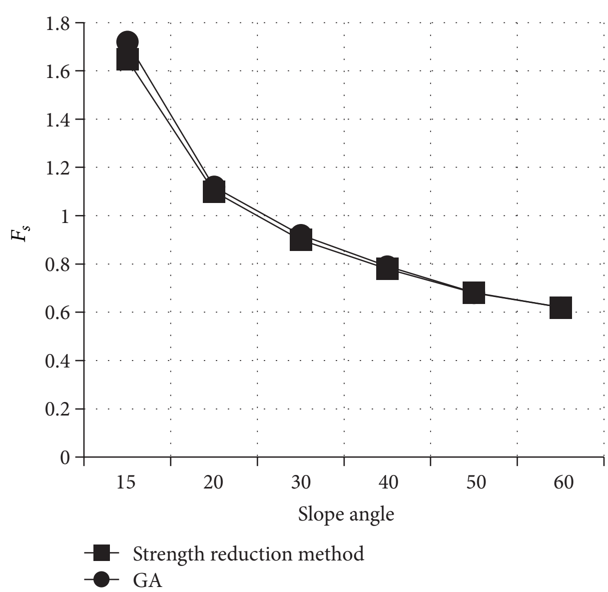

Figure 11: Safety factors versus different slope angles.

is 1.00. The initial population is specified manually and least safety factor is 1.346 at 30th generation in *MGA*. The safety factors calculated by the method proposed in this paper are 1.013; the relative error is 1.3%. The coordinates and geometrical shape of slip surface are displayed in Figure 10, respectively.

4.4. Example 4. The minimum safety factors of the slop obtained from *MGA* and Strength Reduction Method can be compared in detail as follows. Figures 11–14 depict the safety factors change with slope angles, cohesions, inner friction angles, and lateral acceleration, respectively, by Strength Reduction Method and *MGA*; all of the results show that two

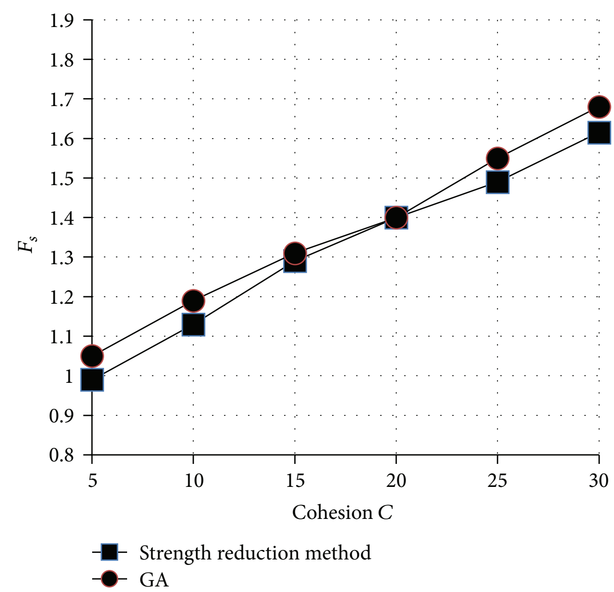

Figure 12: Safety factors versus different cohesions.

strength theories have very high consistency. Note that g is gravity acceleration in Figure 14.

5. Conclusions

The numerical results indicate that the nonlinear coefficient n and intermediate principal stress effect coefficient b in *UST* have effect on the slope stability factors at some degree. In general, nonlinear coefficient n has greater effect on the slope stability factors more than the principal stress effect coefficient b. In the geotechnical engineering, the determination of linear and nonlinear parameters in *UST* must rely on the unique characteristics of soil-rock; then geotechnical experimental research plays more effective role in the quantification of strength parameters of geotechnical material, so as to save the construction cost of the dams project.

By the above examples we can easily know that although the minimum safety factors are almost the same values deduced by *MGA* as that by Strength Reduction Method, but *MGA* is a highly efficient, simple, and good performance of fault-tolerance algorithm. Compared with the general optimization methods, *MGA* and *UST* used in the stability analysis of *ERD* possess the following specialties:

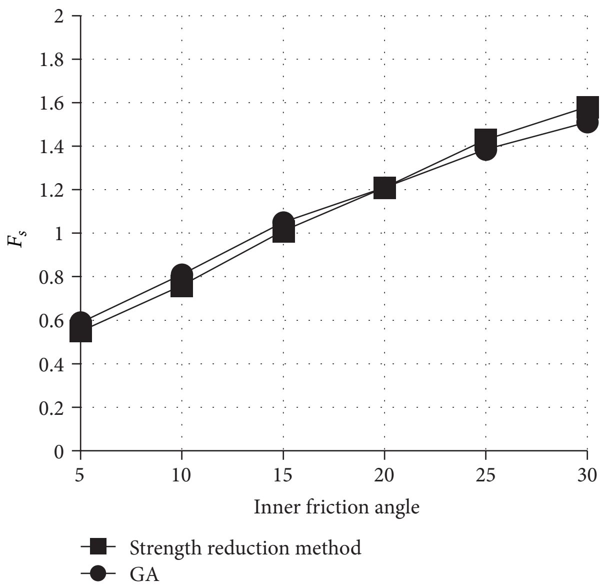

Figure 13: Safety factors versus different inner friction angles.

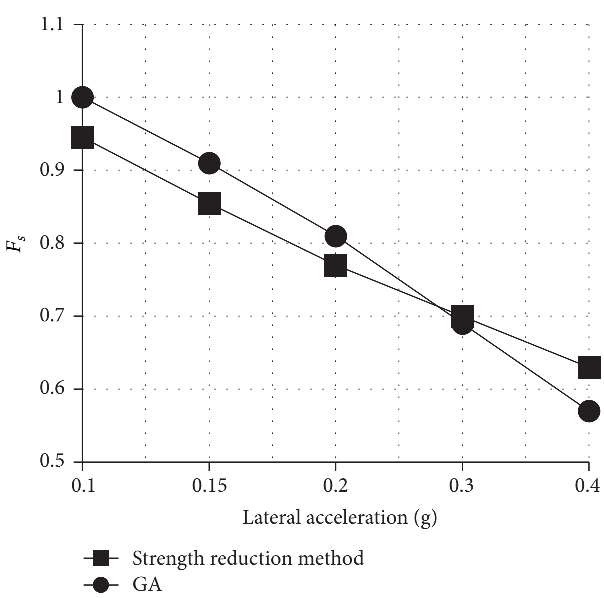

Figure 14: Safety factors versus different lateral acceleration.

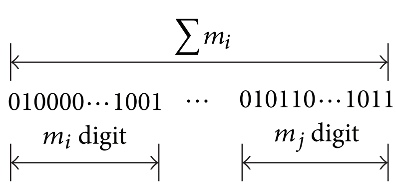

Figure 15

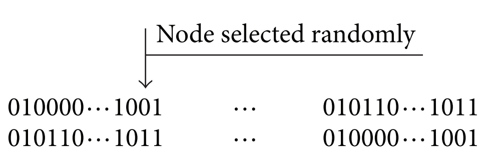

Figure 16

(1) The searching process does not operate on objective functions and optimal variables, instead of encoding the individuals on parameter set, of which *MGA* enables performing operation on structural objects (sets, series, matrix, trees, figures, chains, and charts).

(2) Searching processes from a set of optimal iterative solutions to another, use the way of handling more individuals in one group at the same time, thus reducing the possibility of falling into local optimal solution and being easy for parallel processing.

(3) The adoption of probabilistic transition rules confirms the searching direction, not the certainty searching rules.

(4) There is no specific restriction to the search space such as connectivity and convexity, only using adaptive information, and there is no need of derivatives and other auxiliary information.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] O. C. Zienkiewicz and T. Shiomi, "Dynamic behaviour of saturated porous media; the generalized Biot formulation and its numerical solution," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 8, no. 1, pp. 71–96, 1984.

[2] T.-K. Huang, "Stability analysis of an earth dam under steady state seepage," *Computers and Structures*, vol. 58, no. 6, pp. 1075–1082, 1996.

[3] M. A. B. Noori and K. S. Ismaeel, "Evaluation of seepage and stability of Duhok dam," *Al-Rafidain Engineering Journal*, vol. 19, pp. 1019–1035, 2011.

[4] G. J. Pauls, E. K. Sauer, E. A. Christiansen, and R. A. Widger, "A transient analysis of slope stability following drawdown after flooding of a highly plastic clay," *Canadian Geotechnical Journal*, vol. 36, no. 6, pp. 1151–1171, 1999.

[5] P. V. Lade, "Elasto-plastic stress-strain theory for cohesionless soil with curved yield surfaces," *International Journal of Solids and Structures*, vol. 13, no. 11, pp. 1019–1035, 1977.

[6] F. Santarelli, *Theoretical and experimental investigation of the stability of the axisymmetric borehole [Ph.D. thesis]*, University of London, London, UK, 1987.

[7] J. G. Agar, N. R. Morgenstern, and J. Scott, "Shear strength and stress-strain behavior of Athabasca oil sand at elevated temperatures and pressure," *Canadian Geotechnical Journal*, vol. 24, pp. 1–10, 1985.

[8] N. Janbu, *Slope Stability Computation in Embankment-Dam Engineering*, John Wiley & Sons, New York, NY, USA, 1973.

[9] A. W. Bishop, "The use of the slip circle in the stability analysis of slopes," *Géotechnique*, vol. 5, no. 1, pp. 7–17, 1955.

[10] S. K. Sarma, "Stability analysis of embankments and slopes," *Géotechnique*, vol. 23, no. 3, pp. 423–433, 1973.

[11] M.-H. Yu, *Unified Strength Theory and Its Applications*, Springer, 1st edition, 2004.

[12] K. S. Kahatadeniya, P. Nanakorn, and K. M. Neaupane, "Determination of the critical failure surface for slope stability analysis using ant colony optimization," *Engineering Geology*, vol. 108, no. 1-2, pp. 133–141, 2009.

[13] A. Sengupta and A. Upadhyay, "Locating the critical failure surface in a slope stability analysis by genetic algorithm," *Applied Soft Computing*, vol. 9, no. 1, pp. 387–392, 2009.

[14] A. T. C. Goh, "Genetic algorithm search for critical slip surface in multiple-wedge stability analysis," *Canadian Geotechnical Journal*, vol. 36, no. 2, pp. 382–391, 1999.

[15] T. Matsui and K. C. San, "Finite element slope stability analysis by shear strength reduction technique," *Soils and Foundations*, vol. 32, no. 1, pp. 59–70, 1992.

[16] S. Yu, L. H. Chen, Z. P. Xu, and N. Chen, "Analysis of earth-rockfill dam slope stability by strength reduction method based on nonlinear strength," in *Advances in Civil Engineering and Architecture: Advanced Material Research*, C. H. Chen, Y. Huang, and G. F. Li, Eds., vol. 243–249, pp. 2271–2275, 2011.