

## Research Article

# Design of Corrugated Plates for Optimal Fundamental Frequency

**Nabeel Alshabatat**

*Department of Mechanical Engineering, Tafila Technical University, Tafila 66110, Jordan*

Correspondence should be addressed to Nabeel Alshabatat; nabeel963030@yahoo.com

Received 28 March 2016; Revised 16 June 2016; Accepted 27 June 2016

Academic Editor: Marc Thomas

Copyright © 2016 Nabeel Alshabatat. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates shifting the fundamental frequency of plate structures by corrugation. Creating corrugations significantly improves the flexural rigidities of plate and hence increases its natural frequencies. Two types of corrugations are investigated: sinusoidal and trapezoidal corrugations. The finite element method (FEM) is used to model the corrugated plates and extract the natural frequencies and mode shapes. The effects of corrugation geometrical parameters on simply supported plate fundamental frequency are studied. To reduce the computation time, the corrugated plates are modeled as orthotropic flat plates with equivalent rigidities. To demonstrate the validity of modeling the corrugated plates as orthotropic flat plates in studying the free vibration characteristics, a comparison between the results of finite element model and equivalent orthotropic models is made. A correspondence between the results of orthotropic models and the FE models is observed. The optimal designs of sinusoidal and trapezoidal corrugated plates are obtained based on a genetic algorithm. The optimization results show that plate corrugations can efficiently maximize plate fundamental frequency. It is found that the trapezoidal corrugation can more efficiently enhance the fundamental frequency of simply supported plate than the sinusoidal corrugation.

## 1. Introduction

Plate structures have been widely used in structural, naval, automobile, and aerospace engineering. However, plates exhibit poor vibration performance due to their lateral flexibility (i.e., plates can be deformed easily in the out-of-plane direction under static or dynamic loads). The vibration properties of plates can be modified by different methods. One objective of these methods is to shift the plate natural frequencies away from the frequency of the excitation force to avoid resonance. One cost effective method to improve the vibration characteristics of plates is to modify their shapes [1]. In this study, shape modification is used to optimize the vibration characteristics of plates. In particular, corrugation is used in plate construction to maximize the first plate natural frequency.

Corrugation of plate-like structures is commonly used to increase their strength for static loading conditions such as roofing in engineering structures, container walls, and bridge bulkheads. In the late 1960s, NASA developed corrugated

panel structures to use them in many aerospace applications such as the space shuttle and hypersonic aircraft. The corrugated nature of these panels makes them suitable for high temperature applications because the curved sections permit panel thermal expansion without inducing a significant thermal stress [2]. In 1970, Plank et al. [3] studied the best primary structure of a Mach 8 hypersonic transport wing; and they examined the ultimate load, wing flutter, panel flutter, fatigue, and creep for different wing structures. Recently, corrugated plates are used in constructing morphing wings [4].

A precise analysis of corrugated plates can be achieved by using the finite element method. However, the finite element method does not efficiently model the corrugated plates because it requires significant computing times. A more practical method to design and analyze corrugated plates is to treat them as orthotropic flat plates with equivalent rigidities (i.e., the corrugated plate is flexible in the corrugation direction and rigid in the crosswise direction). After that, an approximate solution can be obtained by solving

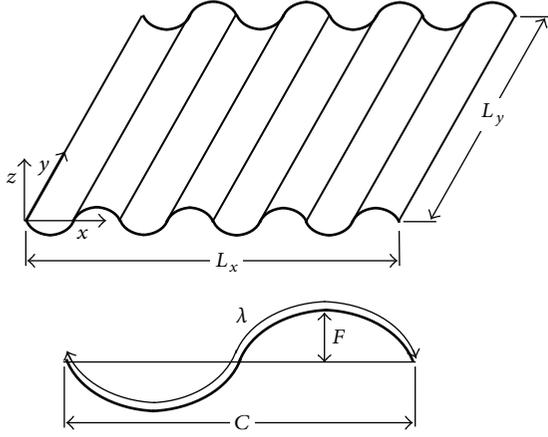


FIGURE 1: Sinusoidal corrugated plate.

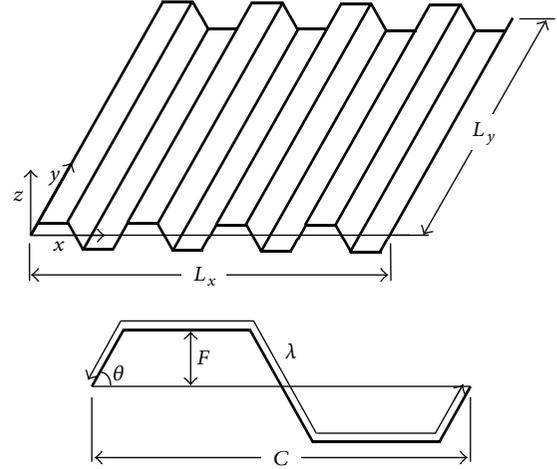


FIGURE 2: Trapezoidal corrugated plate.

the equivalent orthotropic plate problem. This approach of approximation is computationally more efficient with a little loss of precision.

The literature is rich in studies related to the behavior of corrugated plates under static loading. However, there is a limited number of papers which study the vibration of such plates. Samanta and Mukhopadhyay [5] presented formulas for the equivalent rigidities of trapezoidal corrugated plates based on energy principles; and they carried out free vibration analysis using finite element method. In addition, they carried out a nonlinear geometric analysis. Based on the nonlinear bending theory of thin shallow shells, Wang et al. [6] investigated the large amplitude vibration of corrugated circular plates with shallow sinusoidal corrugations under temperature changes. Hamilton's principle was used to derive the governing equations. Liew et al. [7] studied the free vibration of stiffened and unstiffened corrugated plates by using a mesh-free Galerkin method based on the first order shear deformation theory. Yucel and Arpacı [8] used the finite element method to study the free vibration of different types of trapezoidal and sinusoidal corrugated plates.

The objective of this study is to design corrugated plates and to optimize their fundamental frequencies. A free vibration analysis of sinusoidal and trapezoidal corrugated plates based on both the finite element method and equivalent orthotropic models is made in this paper. To demonstrate the validity of modeling the corrugated plates as orthotropic flat plates in studying the free vibration characteristics, a comparison between the results of the finite element model and equivalent orthotropic models is made. Parametric analysis and optimization examples are studied to provide a guide for design engineers. The optimal designs of the corrugated plates are found by using a genetic algorithm (GA).

## 2. Theoretical Background

Corrugated plates are assumed to have sinusoidal or trapezoidal corrugation in one direction as shown in Figures 1 and 2. The corrugated plate can be described by the number of corrugation  $N_c$ , the pitch  $C$ , the height  $F$ , and the trough

angle  $\theta$  (for trapezoidal corrugation). In this work, FEM is used for the modal analysis of corrugated plates. In addition to FEM, the corrugated plates are studied analytically based on modeling the corrugated plates as orthotropic flat plates with the same thickness and equivalent rigidities.

*2.1. Free Vibration Analysis of Orthotropic Plate.* Using Love-Kirchhoff's hypotheses, the equation of the free vibration of orthotropic rectangular thin plate of length  $L_x$  and width  $L_y$  can be written as

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad (1)$$

where  $w$  is the lateral displacement of the plate at point  $(x, y)$ ;  $D_{11}$  and  $D_{22}$  are the flexural rigidities in the  $x$  and  $y$  directions, respectively;  $H$  is the equivalent rigidity;  $h$  is the thickness; and  $\rho$  is the mass density of the plate. The solution of the harmonic motion of the plate has the form

$$w(x, y, t) = W(x, y) \cos(\omega t), \quad (2)$$

where  $W(x, y)$  is the natural mode. Substituting (2) into (1) results in the following:

$$D_{11} \frac{\partial^4 W}{\partial x^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 W}{\partial y^4} - \rho h \omega^2 W = 0. \quad (3)$$

The solution of (3) depends on the boundary conditions of the plate. For a rectangular plate simply supported on all the four sides, the solution of (3), which satisfies the boundary conditions, can be written as

$$W(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right), \quad (4)$$

where  $A_{mn}$  is a scale factor of  $(m, n)$  mode shape. Substitution of (4) into (3) gives the circular frequency  $\omega_{mn}$  which is associated to the  $W_{mn}$  mode shape as

$$\omega_{mn} = \frac{\pi^2}{\sqrt{\rho h}} \cdot \sqrt{D_{11} \left(\frac{m}{L_x}\right)^4 + 2H \left(\frac{m}{L_x}\right)^2 \left(\frac{n}{L_y}\right)^2 + D_{22} \left(\frac{n}{L_y}\right)^4} \quad (5)$$

After calculating the equivalent rigidities of the corrugated plate, the free vibration characteristics can be estimated by solving (1) either theoretically for some boundary conditions or numerically for others.

**2.2. Equivalent Orthotropic Properties of Corrugated Plates.** As mentioned previously, corrugated plates can be approximated as orthotropic flat plates. The suitable selection of the equivalent rigidities plays a significant role in the accuracy of the model. There are different equivalent rigidities available in the literature. The most complete and the most recent equivalent rigidities are adopted in this study to calculate the natural frequencies. The equivalent models are summarized here. The derivation of these models is not considered to be within the scope of this study.

The earliest complete estimation of the equivalent rigidities for sinusoidal corrugation known to the author is found in the works of Huber [9] and Seydel [10]. For many years, researchers followed the classical formulas of Huber and Seydel. Briassoulis [11] reviewed the classical formulas for the equivalent rigidities and developed more accurate expressions for the rigidities of corrugated plates:

$$\begin{aligned} D_{11} &= \frac{C}{\lambda} \frac{Eh^3}{12(1-\nu^2)}, \\ D_{22} &= \frac{EhF^2}{2} + \frac{Eh^3}{12(1-\nu^2)}, \\ D_{66} &= \frac{Eh^3}{24(1+\nu)}, \\ D_{12} &= \nu D_{11}, \end{aligned} \quad (6)$$

where  $C$  is the corrugation pitch;  $F$  is the height of corrugation (see Figure 1);  $\lambda$  is the developed length of unit corrugation;  $I$  is the moment of inertia of the plate along the corrugation direction;  $E$  and  $\nu$  are the modulus of elasticity and Poisson's ratio of plate material, respectively. For trapezoidal corrugated plates, Samanta and Mukhopadhyay [5] presented the following bending rigidities:

$$\begin{aligned} D_{11} &= \frac{C}{\lambda} \frac{Eh^3}{12}, \\ D_{22} &= \frac{EI}{C}, \end{aligned}$$

$$D_{66} = \frac{\lambda}{C} \frac{Eh^3}{24(1+\nu)},$$

$$D_{12} = 0.$$

(7)

Liew et al. [7] used the following formulas for flexural rigidities of trapezoidal corrugated plates:

$$D_{11} = \frac{C}{\lambda} \frac{Eh^3}{12(1-\nu^2)},$$

$$D_{22} = \frac{Eh}{C} \alpha + \frac{Eh^3}{12(1-\nu^2)}, \quad (8)$$

$$D_{66} = \frac{Eh^3}{24(1+\nu)},$$

$$D_{12} = \nu D_{11},$$

where  $\alpha$  is a parameter that depends on the geometry of corrugation as given in [12].

Xia et al. [13] investigated the equivalent flexural rigidities for the geometry of arbitrary corrugations. The suggested bending rigidities are

$$D_{11} = \frac{C}{\lambda} \frac{Eh^3}{12(1-\nu^2)},$$

$$D_{22} = \frac{EI}{1-\nu^2} + A \frac{Eh^3}{12(1-\nu^2)}, \quad (9)$$

$$D_{66} = \frac{\lambda}{C} \frac{Eh^3}{24(1+\nu)},$$

$$D_{12} = \nu D_{11},$$

where  $A$  depends on the geometry of corrugation. For sinusoidal corrugation,  $A$  is given by

$$A = \int_{-0.5}^{0.5} \frac{1}{\sqrt{1 + ((2\pi T/C) \cos(2\pi X))^2}} dX. \quad (10)$$

For trapezoidal corrugation  $A$  is given by

$$A = C - \frac{8}{3} \frac{F}{\tan(\theta)}. \quad (11)$$

Very recently, Ye et al. [14] derived an equivalent plate model for general corrugation. For shallow sinusoidal corrugation, the bending rigidities are

$$D_{11} = \frac{C}{\lambda} \frac{Eh^3}{12(1-\nu^2)},$$

$$D_{22} = EI + A \frac{Eh^3}{12} + \nu^2 D_{11}, \quad (12)$$

$$D_{66} = \frac{\lambda}{C} \frac{Eh^3}{12},$$

$$D_{12} = \nu D_{11}.$$

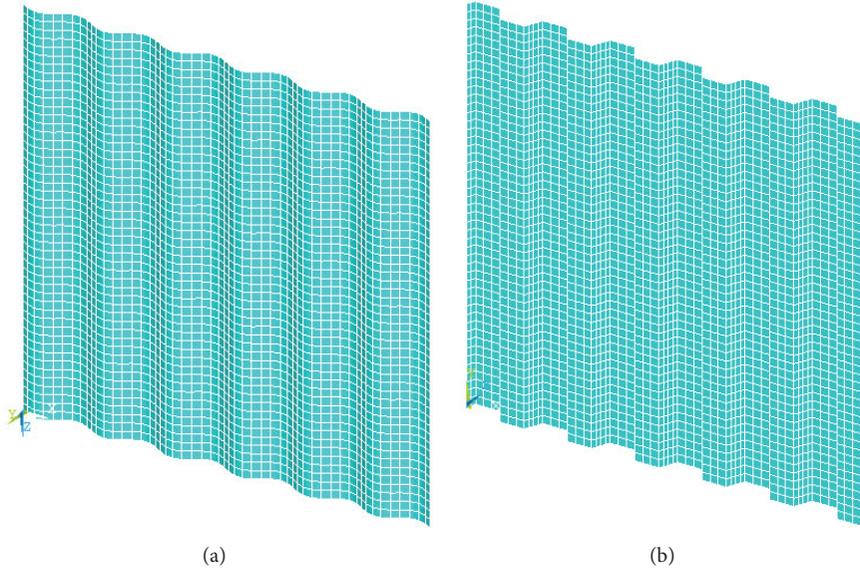


FIGURE 3: Typical finite element models for (a) sinusoidal and (b) trapezoidal corrugated plates.

Approximate values for the moment of inertia  $I$  and the developed length of a unit corrugation  $\lambda$  are available in various references [15, 16]. For more accuracy, the values of  $I$  and  $\lambda$ , in this study, are evaluated numerically. The mass density used in the equivalent orthotropic models is estimated by  $(\lambda/C)$  multiplied by the actual mass density of the corrugated plate.

### 3. Parametric Analysis

To design a corrugated plate, three independent geometrical parameters need to be determined such as the thickness of the plate  $h$ , the number of corrugation  $N$ , and the corrugation height  $F$ , or the corrugation pitch  $C$  instead of number of corrugation  $N$ , in addition to the trough angle  $\theta$  for trapezoidal corrugation. The plate under consideration is a simply-supported square plate with length  $L_x = L_y = 1$  m and thickness  $h = 0.001$  m that is made of steel with modulus of elasticity  $E = 210$  GPa, Poisson's ratio  $\nu = 0.3$ , and mass density  $\rho = 7800$  kg/m<sup>3</sup>.

The finite element method (FEM) is used for the modal analysis of corrugated plates. In particular, the plates are modelled by using ANSYS Parametric Design Language (APDL) in which the model can be built in terms of the geometrical parameters of corrugation. The shell element "shell63" with four nodes and six degrees of freedom per node is used to mesh the solid models. Convergence studies are performed and the finite element of 10 mm size is used throughout the parametric analysis. Typical finite element meshes of sinusoidal and trapezoidal corrugated plates are shown in Figure 3.

To gain some knowledge about the fundamental frequency of corrugated plates, it is important to study the effect of changing the corrugation geometrical parameters on the plate fundamental frequency. To study the effect of the corrugation height of sinusoidally corrugated plate on

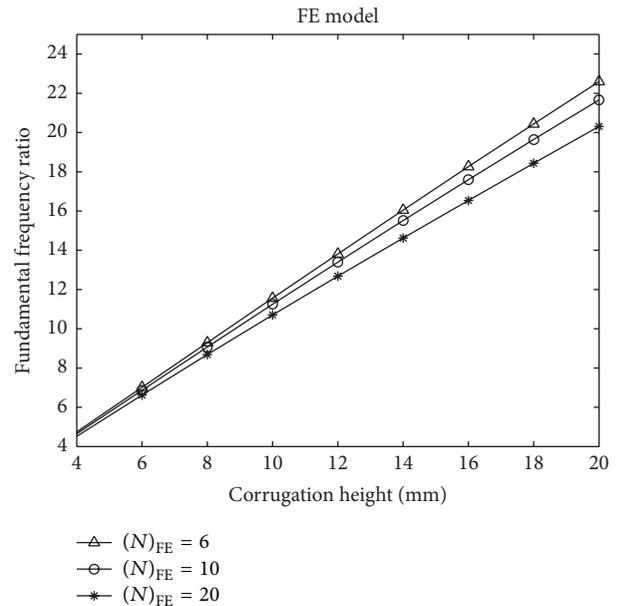


FIGURE 4: Effect of corrugation height on the fundamental frequency ratio for different number of corrugations (sinusoidal corrugation).

the fundamental frequency ratio, the number of corrugation is held constant (e.g.,  $N = 6$  or 10 or 20 corrugations). The corrugation height varies from 5 mm to 20 mm. The results are shown in Figure 4. The fundamental frequency ratio represents the ratio between the fundamental frequency of the corrugated plate and the fundamental frequency of the flat plate with the same lengths and thickness. Increasing the corrugation height would significantly increase the fundamental frequency ratio of the sinusoidally corrugated plate. This increase in the fundamental frequency resulted

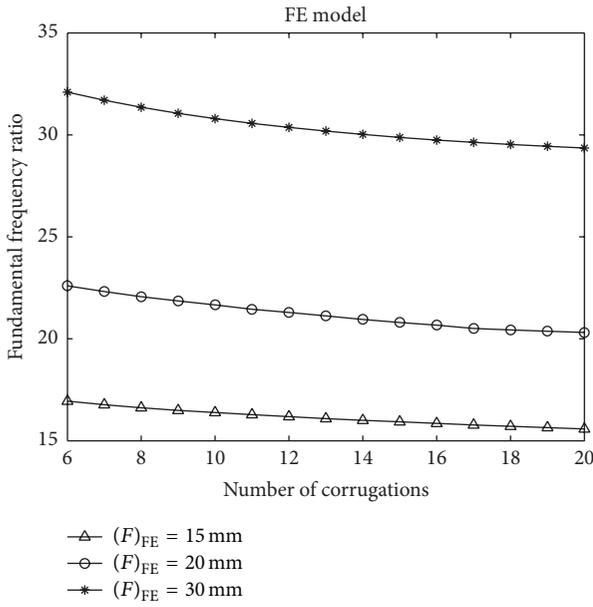


FIGURE 5: Effect of number of corrugations on the fundamental frequency ratio for different corrugation heights (sinusoidal corrugation).

from the obvious increase in the moment of inertia due to the corrugation. Figure 5 shows the change in the fundamental frequency ratio of sinusoidally corrugated plates with varying the number of corrugations from 6 to 20 and the height of corrugation is held constant (e.g.,  $F = 15$  or 20 or 30 mm). It is shown that, for small corrugation height, the frequency ratio decreases as  $N$  increases though the frequencies are still higher than those of the uncorrugated plate. For a higher corrugation number, increasing the number of corrugations ( $6 \leq N \leq 14$ ) decreases the fundamental frequency ratio. In all cases, the significant increase in the number of corrugations has a minor effect on the fundamental frequency ratio.

The effects of changing the corrugation height, number, and the trough angle on the fundamental frequency ratio of trapezoidally corrugated plate are shown in Figures 6 and 7. Figure 6 shows that the fundamental frequency ratio of trapezoidally corrugated plate increases significantly by increasing the corrugation height and the trough angle. For small corrugation height, the trough angles have almost the same effect on the fundamental frequency ratio. As shown in Figure 7, the fundamental frequency ratio of trapezoidally corrugated plate is sensitive to the increase in the number of corrugations, especially for plates with small trough angles. By increasing the number of corrugations, the fundamental frequency ratio decreases significantly when  $\theta = 45^\circ$ .

In addition to changing the natural frequencies, creating corrugations changes the plate mode shapes. For example, the first four mode shapes of a simply supported plate before and after creating 10 sinusoidal corrugations with 20 mm heights are shown in Figure 8. It is expected that this change in mode shapes results in a change in the sound radiation when the plate is subjected to an excitation force.

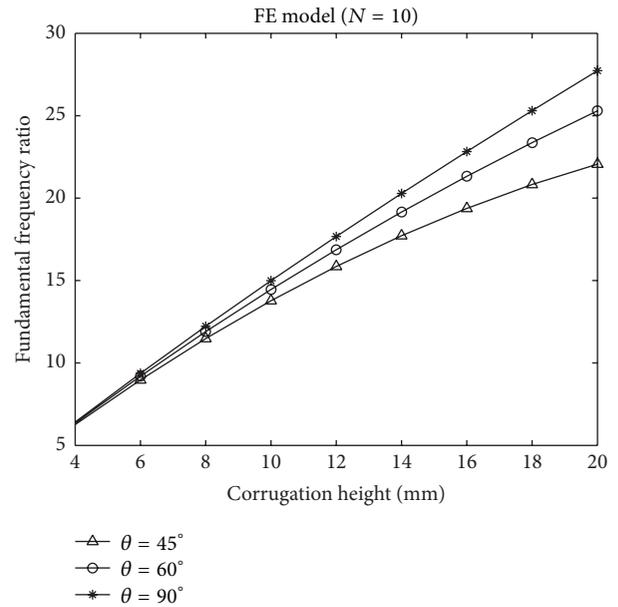


FIGURE 6: Effect of corrugation height on the fundamental frequency ratio for different trough angles (trapezoidal corrugation).

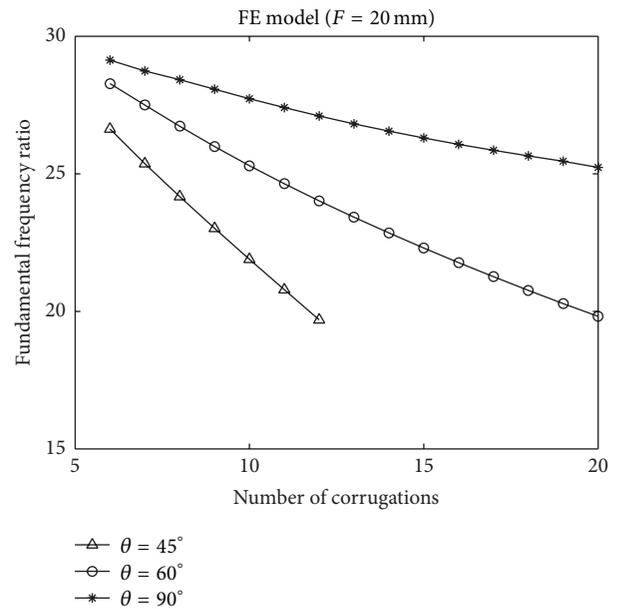


FIGURE 7: Effect of number of corrugations on the fundamental frequency ratio for different trough angles (trapezoidal corrugation).

Modeling and vibration analysis of corrugated plates using FEM can be very computational expensive and time consuming especially for the cases of high number of corrugations (i.e., high number of corrugations per unit length requires finer mesh). Thus, using FEM in design optimization or in any rigorous analysis of corrugated plates is inefficient. Treating the corrugated plate as a flat orthotropic plate is an economic alternative for using FEM. In the optimization

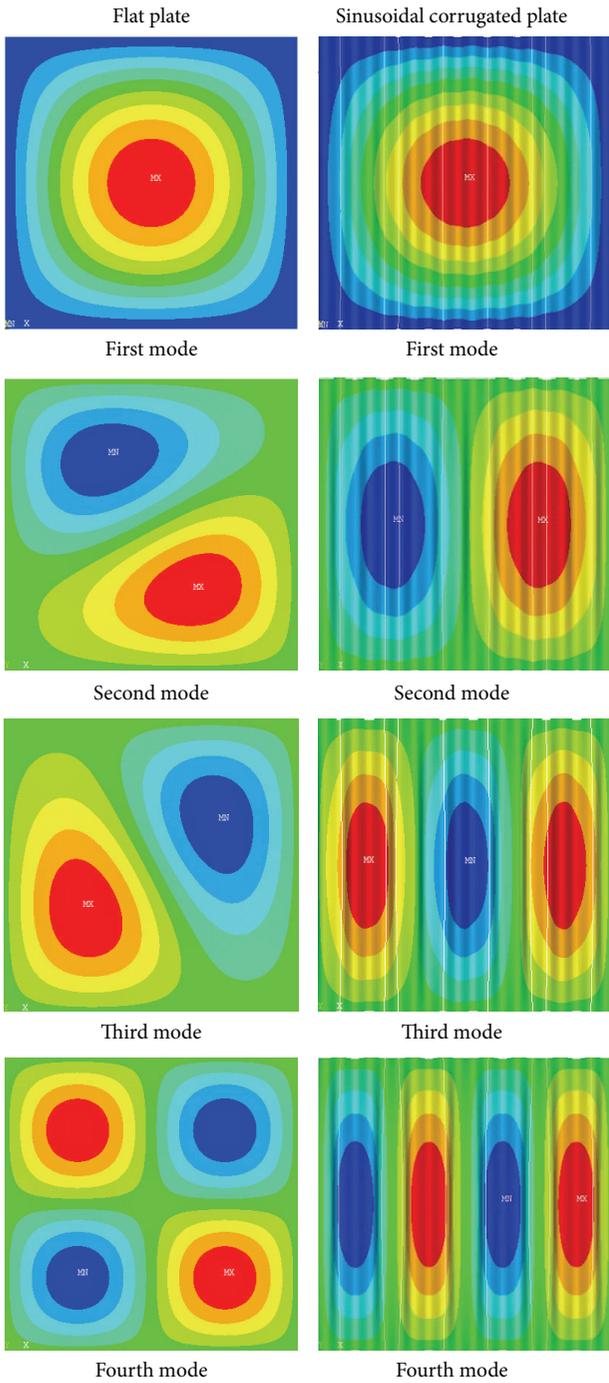


FIGURE 8: First four mode shapes of flat and sinusoidal corrugated simply supported plate.

section, the corrugated plates will be treated as orthotropic plates with equivalent rigidities. To select the most appropriate equivalent rigidities for the current study, the effect of geometric parameters of corrugation on plate fundamental frequency is investigated using equivalent orthotropic flat plate models. The rigidities in these models are based on the most complete and most recent equivalent rigidities ((6)–(12)).

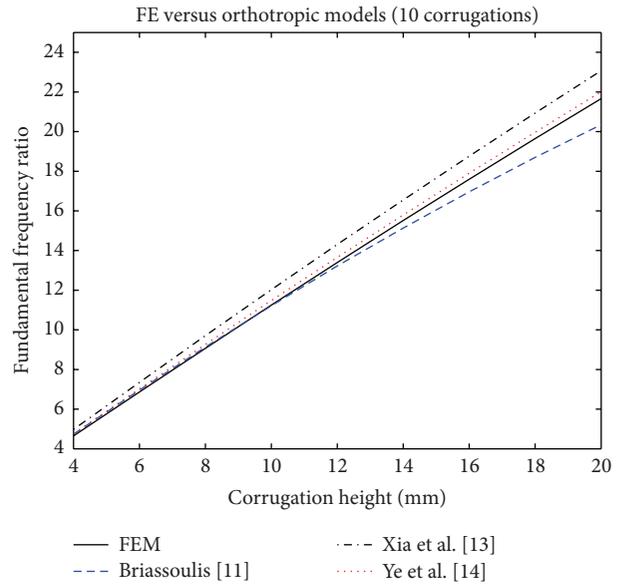


FIGURE 9: Effect of corrugation height on the fundamental frequency ratio based on finite element and equivalent orthotropic models (sinusoidal corrugation).

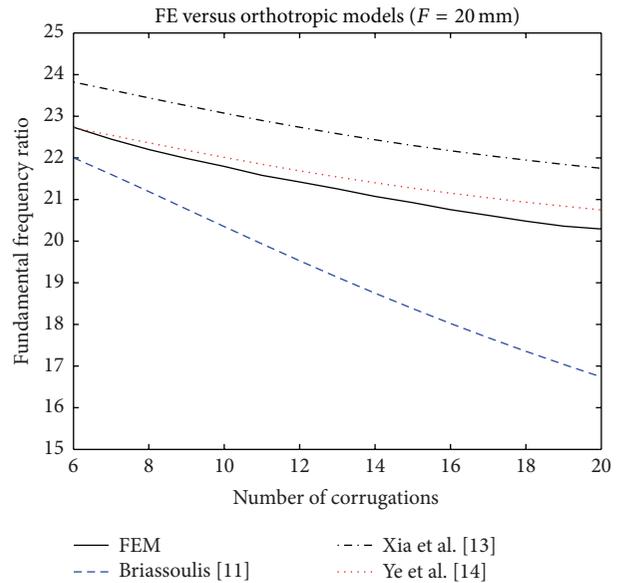


FIGURE 10: Effect of number of corrugations on the fundamental frequency ratio based on finite element and equivalent orthotropic models (sinusoidal corrugation).

The accuracy of the equivalent orthotropic models for corrugated plates is examined by comparing their results with those of the FE models. Comparison of the fundamental frequency ratio of the sinusoidal corrugated plates, computed by FEM and different orthotropic models based on the equivalent rigidities of Briassoulis [11], Xia et al. [13], and Ye et al. [14], is shown in Figures 9 and 10. It is clear that the orthotropic model that is based on the equivalent rigidities of Ye et al. [14] yields the most best agreement with the fundamental frequency values obtained using FEM. In

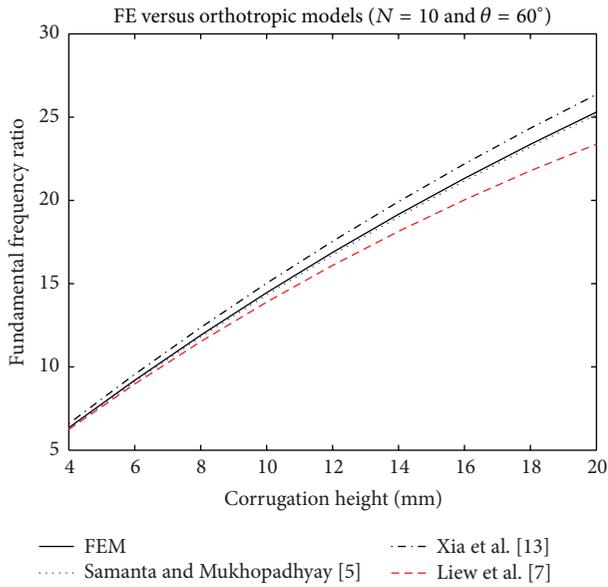


FIGURE 11: Effect of corrugation height on the fundamental frequency ratio based on finite element and equivalent orthotropic models (trapezoidal corrugation).

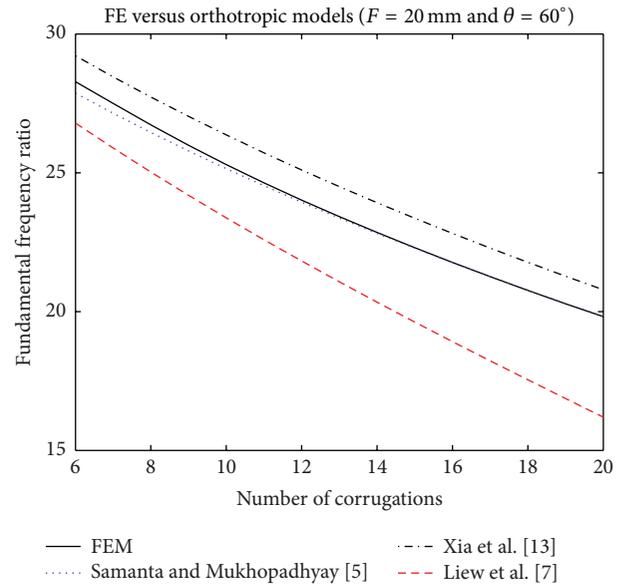


FIGURE 12: Effect of number of corrugations on the fundamental frequency ratio based on finite element and equivalent orthotropic models (trapezoidal corrugation).

general, the differences between the results of FEM and those of the equivalent orthotropic models get larger by increasing the corrugation height and number of corrugations.

For trapezoidal corrugated plates, orthotropic equivalent plates that are based on the equivalent rigidities of Samanta and Mukhopadhyay [5], Xia et al. [13], and Liew et al. [7] are compared with FE models of the actual corrugated plates in Figures 11 and 12. Like in the sinusoidal corrugated plates, a better correlation between the orthotropic models and the FE models can be obtained when the corrugation height decreases because the derivations of the equivalent rigidities are based on the assumption of shallow corrugations. Figure 12 shows that the correlation in the results of the orthotropic model that is based on equivalent rigidities of Samanta and Mukhopadhyay [5] improves with the increase in the number of corrugations.

In the optimization examples presented in the next section, the equivalent rigidities of Ye et al. [14] and Samanta and Mukhopadhyay [5] are used for sinusoidal corrugated plates and trapezoidal corrugated plates, respectively.

#### 4. Optimization Examples

In the previous section, an insight is gained on the effect of corrugation geometric parameters on a plate fundamental frequency. In this section, the focus will be on maximizing the plate fundamental frequency by using sinusoidal or trapezoidal corrugations. We consider two optimization problems. The first is to maximize the fundamental frequency of a square corrugated plate. The second is to maximize the fundamental frequency-to-mass ratio of the same plate. Three dimensions of sinusoidal corrugated plates, including plate thickness  $h$ , height of corrugation  $F$ , and number of

corrugations  $N$ , are selected as design variables. In addition to these design variables, the trough angle  $\theta$  is considered as a design variable for designing trapezoidal corrugated plates. During optimization, the number of corrugation is restricted to integer numbers. The equivalent orthotropic plate model is used to model the corrugated plates in the optimization problems.

Here, a genetic algorithm (GA) is used for optimization examples. The GAs are stochastic optimization methods that are based on the concept of biological evolution. It was introduced by Holland in 1975 [17]. In comparison with traditional optimization methods, the GAs work over a set of candidate points, which cover the entire search space instead of a single candidate point at each iteration. It, therefore, increases the probability to reach to the global optimum. GAs are not gradient-based methods. Thus, they can work with discrete design variables and do not require the objective functions to be differentiable. Also, the GAs can work with nonconvex problems. In general, GA uses a set of candidate points (called population) which cover the entire search space. Then, the objective function (fitness function) is evaluated for each candidate point. The points with relatively good objective functions are used to generate a new set of candidate points through three mechanisms known as reproduction, crossover, and mutation. In the reproduction method, the candidate point in the current generation with the best objective function is automatically passed to the next generation. The crossover method creates a new point by combining the vectors of two candidate points in the current generation. The mutation method creates a new point by randomly changing some components of a candidate point in the current generation. The algorithm continues iteration (generation) until it reaches the stopping criteria. The population size for each generation is assumed to be 70



technique on increasing the fundamental frequency of simply supported plates. Two optimization problems were considered: the maximization of corrugated plate fundamental frequency and maximization of the corrugated plate fundamental frequency-to-mass ratio. A future extension of this work will focus on the experimental verification of the results. Future work will consider the design of corrugated plates subjected to a dynamic load with respect to the minimal sound radiation.

## Competing Interests

The author declares that they have no competing interests.

## References

- [1] N. T. Alshabat and K. Naghshineh, "Optimization of the natural frequencies of plates via dimpling and beading techniques," *International Journal of Modelling and Simulation*, vol. 32, no. 4, pp. 244–254, 2012.
- [2] M. D. Musgrove and B. E. Greene, "Advanced beaded and tubular structural panels," *Journal of Aircraft*, vol. 11, no. 2, pp. 68–75, 1974.
- [3] P. P. Plank, I. F. Sakata, G. W. Davis, and C. C. Richie, "Hypersonic cruise vehicle wing structure evaluation," Tech. Rep., NASA, Washington, DC, USA, 1970.
- [4] C. Gentilinia, L. Nobilea, and K. A. Seffen, "Numerical analysis of morphing corrugated plates," *Procedia Engineering*, vol. 1, no. 1, pp. 79–82, 2009.
- [5] A. Samanta and M. Mukhopadhyay, "Finite element static and dynamic analyses of folded plates," *Engineering Structures*, vol. 21, no. 3, pp. 277–287, 1999.
- [6] Y. Wang, D. Gao, and X. Wang, "On the nonlinear vibration of heated corrugated circular plates with shallow sinusoidal corrugations," *International Journal of Mechanical Sciences*, vol. 50, no. 6, pp. 1082–1089, 2008.
- [7] K. M. Liew, L. X. Peng, and S. Kitipornchai, "Vibration analysis of corrugated Reissner-Mindlin plates using a mesh-free Galerkin method," *International Journal of Mechanical Sciences*, vol. 51, no. 9-10, pp. 642–652, 2009.
- [8] A. Yucel and A. Arpacı, "Theoretical and experimental vibration analyses of trapezoidal and sinusoidal corrugated plates," *Journal of Vibration and Control*, vol. 21, no. 10, pp. 2006–2026, 2015.
- [9] M. T. Huber, "Die Theorie der kreuzweise bewehrten Eisenbeton-platten nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten," *Bauingenieur*, vol. 4, pp. 354–360, 1923.
- [10] E. Seydel, "Shear buckling of corrugated plates," *Jahrbuch die Deutschen Versuchsanstalt für Luftfahrt*, vol. 9, pp. 233–245, 1931.
- [11] D. Briassoulis, "Equivalent orthotropic properties of corrugated sheets," *Computers and Structures*, vol. 23, no. 2, pp. 129–138, 1986.
- [12] K. Liew, L. Peng, and S. Kitipornchai, "Buckling analysis of corrugated plates using a mesh-free Galerkin method based on the first-order shear deformation theory," *Computational Mechanics*, vol. 38, no. 1, pp. 61–75, 2006.
- [13] Y. Xia, M. I. Friswell, and E. I. Saavedra Flores, "Equivalent models of corrugated panels," *International Journal of Solids and Structures*, vol. 49, no. 13, pp. 1453–1462, 2012.
- [14] Z. Ye, V. L. Berdichevsky, and W. Yu, "An equivalent classical plate model of corrugated structures," *International Journal of Solids and Structures*, vol. 51, no. 11-12, pp. 2073–2083, 2014.
- [15] S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, McGraw-Hill, New York, NY, USA, 1959.
- [16] S. Wolford, "Sectional properties of corrugated sheets determined by formula," *Civil Engineering*, vol. 103, pp. 59–60, 1954.
- [17] J. H. Holland, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, Mich, USA, 1975.



**Hindawi**

Submit your manuscripts at  
<http://www.hindawi.com>

