

Corrigendum

Corrigendum to “Clusters of Galaxies in a Weyl Geometric Approach to Gravity”

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In the article titled “Clusters of Galaxies in a Weyl Geometric Approach to Gravity” [1], there was an error in equation (47) that led to a wrong relation between the acceleration due to the scale connection (a_ϕ) and the acceleration arising from the scalar field energy density (a_{sf}). The correction of (47) has consequences for the model. It makes a new run of the data evaluation based on the corrected dynamical equations necessary. The new results are given in updated Tables 3 and 4 and Figures 1–6. The overall picture of the empirical test does not change, although now three rather than two galaxy clusters agree with the model only in the 2σ range. To facilitate controlling the correction, a detailed derivation of the corrected equation (47) is given in Appendix.

Minor Corrections. “15” should be changed to “14” in the following two sections. In the abstract, the number should be corrected in the phrase “the total mass for 15 of the outlier reduced ensemble of 17 clusters seems to be predicted correctly (in the sense of overlapping 1σ error intervals).” In paragraph 6 of Introduction, it should be corrected in the sentence “For 15 of the 17 main reference clusters the empirical and the theoretical values for the total mass agree in the sense of overlapping 1σ error intervals.” Equations (28), (57), and (58) should read as follows.

$$\begin{aligned} L_{\phi^3} &= \frac{2}{3}\xi^2\eta\phi^{-2}(D_\nu\phi D^\nu\phi)^{3/2} \\ &= (\xi\phi)^2(\eta^{-1}\phi)^{-1}|\nabla\omega|^3, \end{aligned} \quad (28)$$

$$\begin{aligned} \rho_{ph} &= \tilde{\nu}\left(\frac{|a_N|}{a_o}\right)\rho_m - (4\pi G a_o)^{-1}\tilde{\nu}'\left(\frac{|a_N|}{a_o}\right)(\nabla|a_N|) \\ &\quad \cdot a_N. \end{aligned} \quad (57)$$

$$\begin{aligned} \rho_t &= \left(\frac{a_o}{|a_N|}\right)^{1/2}\rho_m + (8\pi G)^{-1}\left(\frac{a_o}{|a_N|}\right)^{1/2}\nabla(|a_N|) \\ &\quad \cdot \frac{a_N}{|a_N|}. \end{aligned} \quad (58)$$

Major Corrections, Section 2. Equation (47) should read as follows:

$$\rho_{sf} \approx (4\pi G)^{-1}\nabla^2\omega. \quad (47)$$

The paragraph following equation (47), including the old equation (48), has to be cancelled. The correction of (47) entails the following modifications in the equations of Section 2:

$$a_{add} = a_\phi + a_{sf} = 2a_\phi. \quad (49)$$

$$|a_{add}| = 2\frac{\sqrt{GM(r)}\tilde{a}_o}{r}. \quad (50)$$

$$\tilde{a}_o = \frac{a_o}{4} \approx \frac{1}{24}H[c] \approx 2.4 \cdot 10^{-9} \text{ cm s}^{-2}. \quad (51)$$

$$\rho_{sf} = \rho_{ph} = \frac{1}{2}\rho_t \quad (59)$$

$$\nabla \cdot (|\nabla\omega|\nabla\omega) = \pi G a_o \rho_m \quad (63)$$

$$\rho_{sf} = \frac{1}{2}\left(\frac{a_o}{|a_m|}\right)^{1/2}\left(\rho_m + (8\pi G)^{-1}\nabla(|a_m|) \cdot \frac{a_m}{|a_m|}\right) \quad (65)$$

$$\rho_{ph} = \rho_{sf}. \quad (66)$$

In the comment below equation (59) “three quarters” and “one quarter” should be replaced with “one-half” and the inline formula below equation (62) should be replaced with $a_{add} = a_\phi + a_{sf} = 2a_\phi$.

TABLE 3: Empirical values (M_{500} , M_{200}) and model values (M_{tot}) for total mass at r_{500} , r_{200} .

Cluster	r_{500}	$M_{\text{tot}}(r_{500})$	M_{500}	r_{200}	$M_{\text{tot}}(r_{200})$	M_{200}
Coma	1278	5.32 $^{+0.83}_{-0.58}$	6.55 $^{+0.79}_{-0.79}$	2300	13.19 $^{+2.36}_{-1.65}$	13.84 $^{+1.49}_{-1.41}$
A85	1216	4.67 $^{+0.51}_{-0.36}$	6.37 $^{+1.00}_{-1.00}$	1900	9.70 $^{+1.09}_{-0.77}$	7.71 $^{+0.8}_{-0.74}$
A400	712	1.26 $^{+0.25}_{-0.16}$	1.83 $^{+0.39}_{-0.39}$	1093	2.62 $^{+0.55}_{-0.35}$	1.48 $^{+0.21}_{-0.18}$
IIIIZw54	731	1.33 $^{+0.34}_{-0.27}$	1.91 $^{+0.58}_{-0.58}$	1350	3.09 $^{+1.30}_{-1.02}$	2.81 $^{+2.74}_{-1.10}$
A1367	893	1.82 $^{+0.29}_{-0.19}$	1.76 $^{+0.27}_{-0.54}$	1529	4.28 $^{+0.81}_{-0.62}$	4.06 $^{+0.45}_{-0.40}$
MKW4	580	0.54 $^{+0.09}_{-0.06}$	0.50 $^{+0.14}_{-0.14}$	857	1.10 $^{+0.19}_{-0.13}$	0.71 $^{+0.07}_{-0.06}$
ZwCl215	1098	3.74 $^{+0.63}_{-0.32}$	4.93 $^{+0.98}_{-0.98}$	2093	9.32 $^{+1.49}_{-1.07}$	10.37 $^{+3.51}_{-2.62}$
A1650	1087	3.42 $^{+0.40}_{-0.50}$	3.44 $^{+0.66}_{-0.66}$	2150	9.40 $^{+2.87}_{-2.31}$	11.14 $^{+5.77}_{-3.46}$
A1795	1118	3.40 $^{+0.18}_{-0.26}$	3.41 $^{+0.63}_{-0.63}$	2136	9.32 $^{+1.11}_{-0.74}$	10.99 $^{+2.26}_{-2.09}$
MKW8	715	0.86 $^{+0.18}_{-0.14}$	0.62 $^{+0.12}_{-0.12}$	1279	2.35 $^{+0.67}_{-0.54}$	2.38 $^{+1.04}_{-0.59}$
A2029	1275	6.39 $^{+0.61}_{-0.43}$	14.7 $^{+2.61}_{-2.61}$	2286	15.83 $^{+1.57}_{-1.12}$	13.42 $^{+2.43}_{-2.26}$
A2052	875	1.63 $^{+0.26}_{-0.17}$	1.39 $^{+0.28}_{-0.28}$	1250	2.94 $^{+0.47}_{-0.32}$	2.21 $^{+0.06}_{-0.08}$
MKW3S	905	1.90 $^{+0.27}_{-0.18}$	1.45 $^{+0.34}_{-0.34}$	1450	3.81 $^{+0.60}_{-0.40}$	3.46 $^{+0.36}_{-0.34}$
A2065	1008	3.92 $^{+0.72}_{-0.66}$	11.18 $^{+1.78}_{-1.78}$	2450	11.90 $^{+8.20}_{-4.61}$	16.69 $^{+21.34}_{-6.73}$
A2142	1449	7.15 $^{+0.68}_{-0.50}$	7.36 $^{+1.25}_{-1.25}$	2364	15.29 $^{+1.53}_{-1.14}$	15.03 $^{+3.9}_{-2.64}$
A2147	1064	3.32 $^{+0.53}_{-0.40}$	4.44 $^{+0.67}_{-0.67}$	1450	5.81 $^{+1.14}_{-0.90}$	3.46 $^{+1.17}_{-0.74}$
A2199	957	2.26 $^{+0.38}_{-0.28}$	2.69 $^{+0.42}_{-0.42}$	1621	4.98 $^{+0.93}_{-0.67}$	4.81 $^{+0.37}_{-0.36}$
A2255	1072	3.94 $^{+0.44}_{-0.31}$	7.13 $^{+1.38}_{-1.37}$	2271	12.27 $^{+1.87}_{-1.37}$	13.32 $^{+1.44}_{-1.19}$
A2589	848	1.92 $^{+0.33}_{-0.22}$	3.03 $^{+0.75}_{-0.75}$	1471	4.60 $^{+0.84}_{-0.58}$	3.58 $^{+3.86}_{-1.54}$

Model values $M_{\text{tot}}(r_{N00})$ and empirical values M_{N00} in $10^{14} M_{\odot}$, r_{N00} (empirical) in kpc ($N = 1, 2$).

TABLE 4: Model values for halo and baryonic masses at r_{200} .

Cluster	M_t	M_{sf}	M_{sf2}	M_{ph1}	M_{gas}	M_*	f_*	f_t
Coma	11.15	6.44	1.73	4.71	1.77	0.276	0.16	6.3
A85	8.03	4.52	1.01	3.51	1.54	0.140	0.09	5.2
A400	2.27	1.36	0.45	0.91	0.26	0.084	0.32	8.7
IIIIZw54	2.77	1.65	0.54	1.11	0.25	0.080	0.32	10.9
A1367	3.77	2.21	0.65	1.56	0.42	0.089	0.21	8.9
MKW4	0.99	0.58	0.18	0.40	0.09	0.022	0.25	10.9
ZwCl215	8.00	4.55	1.11	3.45	1.19	0.137	0.12	6.7
A1650	8.15	4.69	1.24	3.45	1.10	0.162	0.15	7.4
A1795	8.04	4.59	1.14	3.45	1.15	0.14	0.12	7.0
MKW8	2.12	1.24	0.36	0.88	0.20	0.039	0.20	10.8
A2029	12.83	7.15	1.47	5.68	2.83	0.203	0.07	4.5
A2052	2.58	1.50	0.43	1.07	0.31	0.059	0.19	8.3
MKW3S	3.35	1.95	0.55	1.40	0.39	0.072	0.18	8.5
A2065	10.28	5.80	1.32	4.48	1.48	0.142	0.10	6.9
A2142	12.55	6.95	1.35	5.60	2.59	0.159	0.06	4.8
A2147	4.83	2.77	0.71	2.06	0.87	0.118	0.14	5.5
A2199	4.36	2.52	0.68	1.84	0.55	0.088	0.16	8.0
A2255	10.35	5.84	1.33	4.51	1.76	0.166	0.09	5.9
A2589	3.99	2.33	0.68	1.65	0.52	0.105	0.20	7.7

Mass values in $10^{14} M_{\odot}$, $f_* = M_*/M_{\text{gas}}$, $f_t = (M_t/M_{\text{gas}})(r_{200})$; for r_{200} see Table 2.

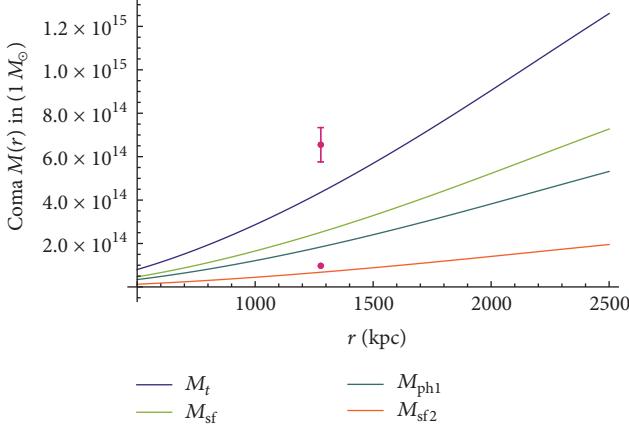


FIGURE 1: Halo components of Coma cluster: transparent matter halo $M_t = M_{sf} + M_{ph1}$, total scalar field (SF) halo M_{sf} , halo of freely falling galaxies M_{sf2} , and net phantom energy M_{ph1} (in barycentric rest system). Empirical data (violet dot and bar): baryonic masses $M_{*500} + M_{gas500}$ (dot) and M_{500} (with error intervals) at $r_{500} = 1280$ kpc.

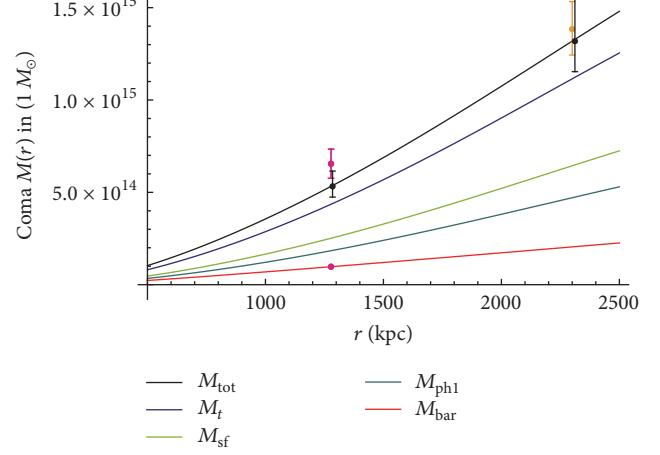


FIGURE 3: Contribution of the baryonic mass M_{bar} , of the scalar field, and the phantom energies M_{sf} , M_{ph1} to the transparent mass M_t , and to the total mass $M_{tot} = M_t + M_{bar}$ of the Coma cluster in the WST model. Model errors indicated at r_{500} , r_{200} (black). Empirical data for M_{bar} (purple dot) and for the empirically determined total mass M_{500} with error bars (purple) from [2]. Additional empirical data at r_{200} (yellowish) from [3].

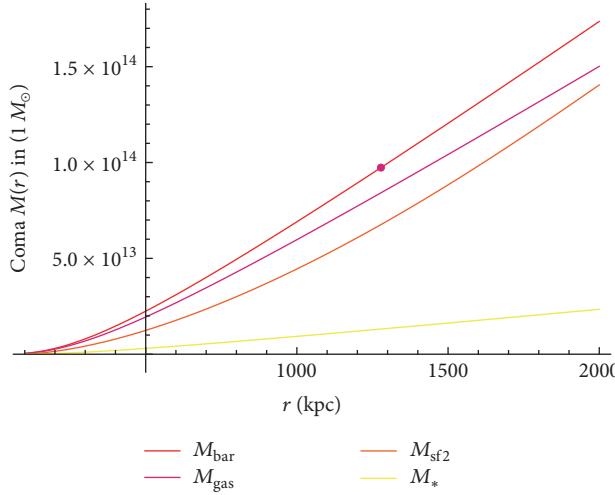


FIGURE 2: Comparison of the contribution of the scalar field halo of the galaxies M_{sf2} with the baryonic mass for the Coma cluster (empirical data for M_{bar500} violet dot).

Major Corrections, Sections 3–5. The modifications of the equations in Section 2 lead to the following changes in Section 3:

$$\rho_{sf1} = \frac{1}{2} \left(\frac{a_o}{|a_{bar}|} \right)^{1/2} \quad (69)$$

$$\cdot \left(\rho_m + (8\pi G)^{-1} \nabla (|a_{bar}|) \cdot \frac{a_{bar}}{|a_{bar}|} \right),$$

$$\rho_{ph1} = \rho_{sf1}. \quad (70)$$

$$\rho_{sf2} = \frac{1}{2} \left(\frac{a_o}{|a_{star}|} \right)^{1/2} \quad (71)$$

$$\cdot \left(\rho_{star} + (8\pi G)^{-1} \nabla (|a_{star}|) \cdot \frac{a_{star}}{|a_{star}|} \right),$$

$$\rho_{ph1} = \rho_{sf1}. \quad (74)$$

This makes a new run of the data evaluation necessary, now based on the corrected dynamical equations. The new results are given in updated Tables 3 and 4 and Figures 2–6. In the evaluation on p. 16, right column, first two sentences of the first paragraph should be replaced with the following:

For 5 clusters A85, A2255, and A2589 and the outliers A2029 and A2065, the error intervals of empirical data and model data do not overlap. For the first three of them (A85, A2255, and A2589) the model predictions are consistent with the empirical data within doubled error intervals (2σ range).

In page 20, right column (Section 5), 20, at the middle of the second paragraph “15 clusters” should be replaced with “14 clusters,” and “Two clusters …” should be substituted by “Three clusters …” at the beginning of the last phrase.

Appendix

Energy Component of the Scalar Field

In scalar field (Einstein) gauge $D_\nu \phi = \partial_\nu \phi - \varphi_\nu \phi \doteq -\phi \varphi_\nu \doteq \phi \partial_\nu \omega \doteq \phi D_\nu \omega$, and $D_\nu \phi^2 = 2\phi D_\nu \phi \doteq 2\phi^2 \partial_\nu \omega$. Similarly

$$D_\mu D^\nu \phi^2 \doteq 2D_\mu (\phi^2 \partial^\nu \omega) \doteq 2(D_\mu \phi^2 \partial^\nu \omega + \phi^2 D_\mu \partial^\nu \omega). \quad (A.1)$$

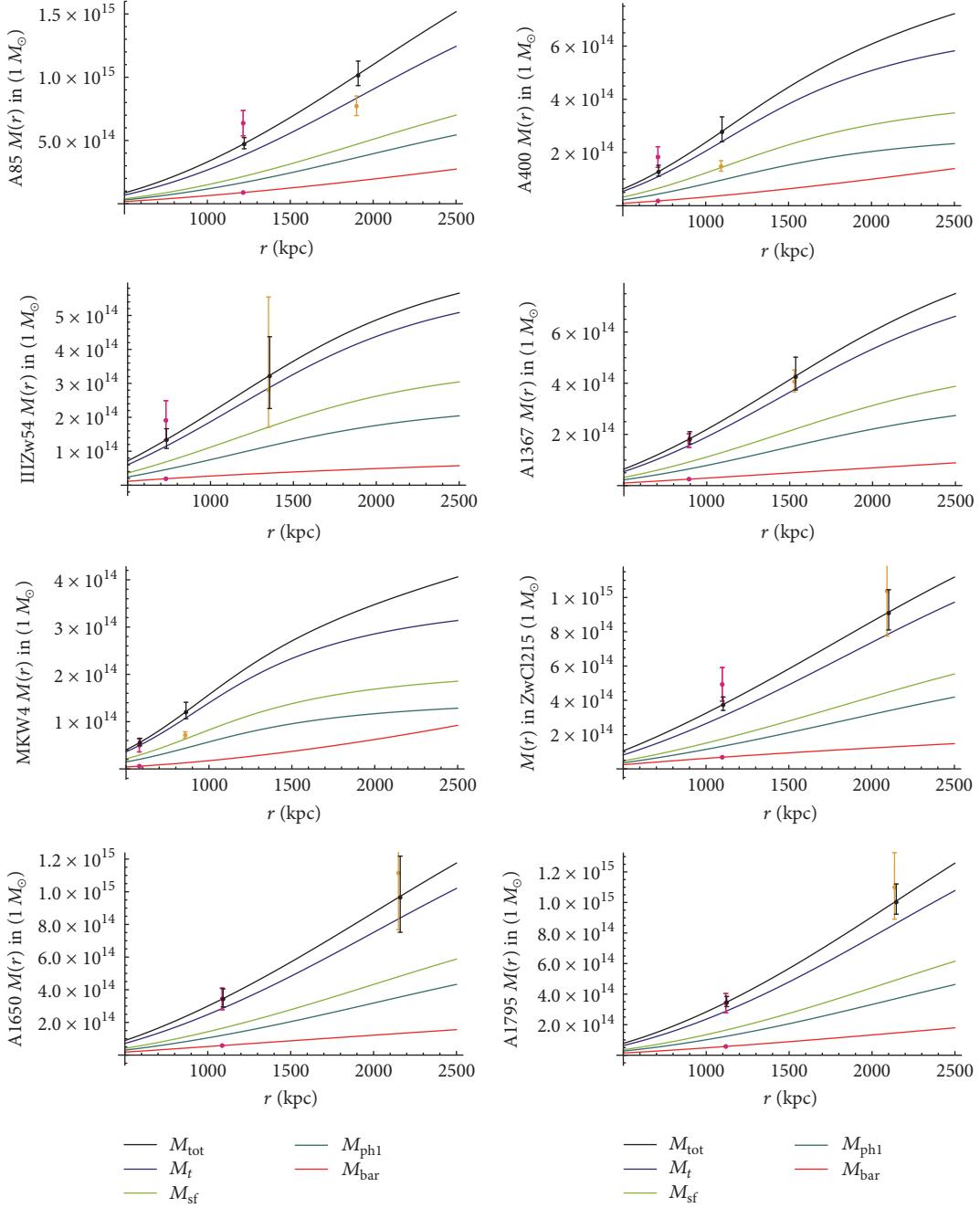


FIGURE 4: Halo models for clusters 2–9 in Table 1: total mass M_{tot} (black line) with model error bars at r_{500} , r_{200} , transparent matter halo M_t constituted by scalar field halo M_{sf} and net phantom halo (in barycentric rest system of cluster) M_{ph1} , and baryonic mass (gas and stars) M_{bar} . Empirical data for the total mass with error intervals at r_{500} (violet) from [2]. Additional empirical data at r_{200} (yellow) from [3].

Moreover,

$$\begin{aligned}
 D_\mu \partial^\nu \omega &= \nabla_\mu \partial^\nu \omega - 2\varphi_\mu \partial^\nu \omega \\
 &= \nabla_\mu \partial^\nu \omega + \delta_\mu^\nu \varphi_\lambda \partial^\lambda \omega + \delta_\lambda^\nu \varphi_\mu \partial^\lambda \omega \\
 &\quad - g_{\mu\lambda} \varphi^\nu \partial^\lambda \omega + 2\partial_\mu \omega \partial^\nu \omega
 \end{aligned} \tag{A.2}$$

leads to

$$\begin{aligned}
 D_\mu D^\nu \phi^2 &\doteq 2\phi^2 \left(2\partial_\mu \omega \partial^\nu \omega + {}_g \nabla_\mu \partial^\nu \omega + 2\partial_\mu \omega \partial^\nu \omega \right. \\
 &\quad \left. - \delta_\mu^\nu \partial_\lambda \omega \partial^\lambda \omega \right) \doteq 2\phi^2 \left({}_g \nabla_\mu \partial^\nu \omega + 4\partial_\mu \omega \partial^\nu \omega \right. \\
 &\quad \left. - \delta_\mu^\nu \partial_\lambda \omega \partial^\lambda \omega \right).
 \end{aligned} \tag{A.3}$$

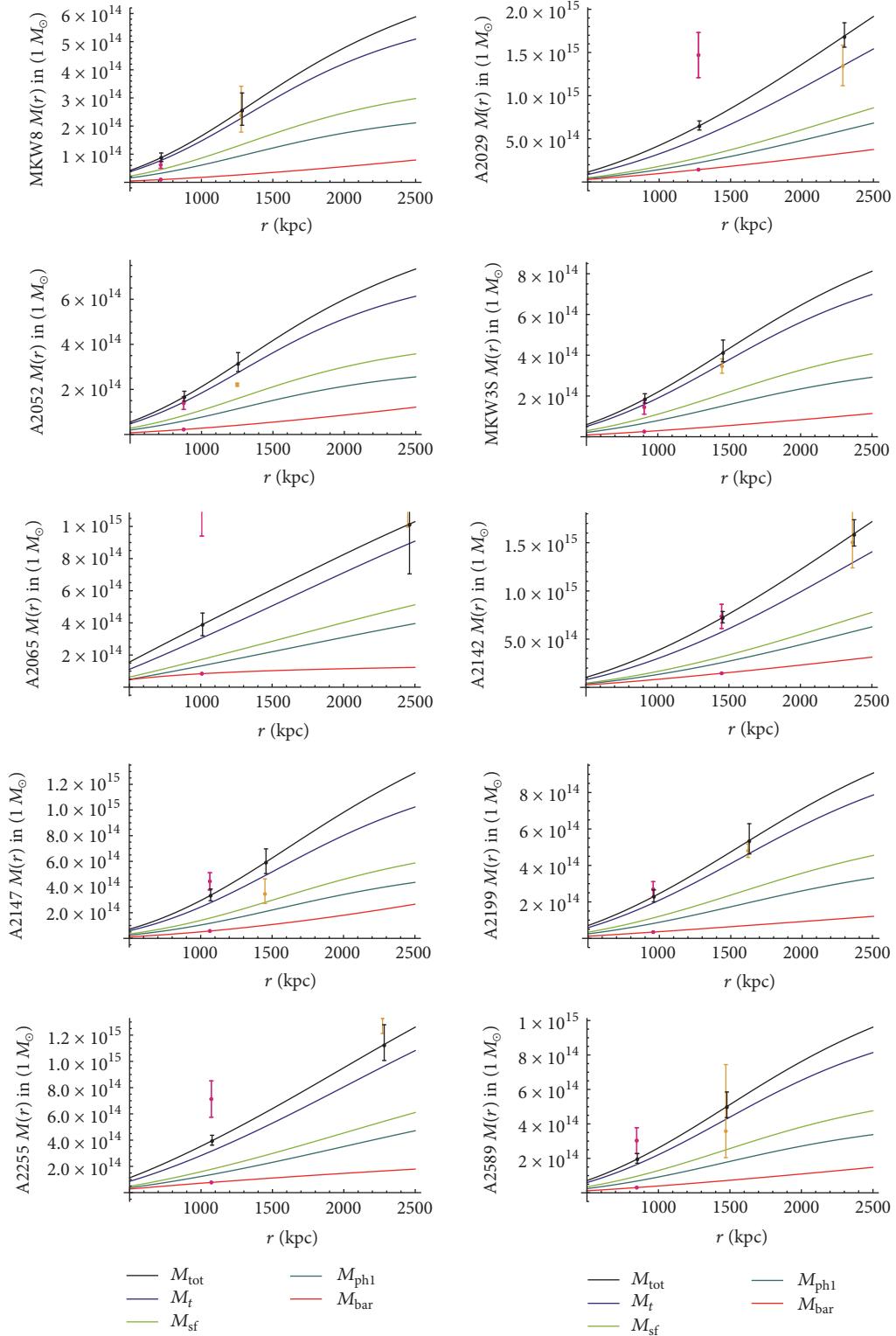


FIGURE 5: Halo models for clusters 10–19 in Table 1. For description, see Figure 4.

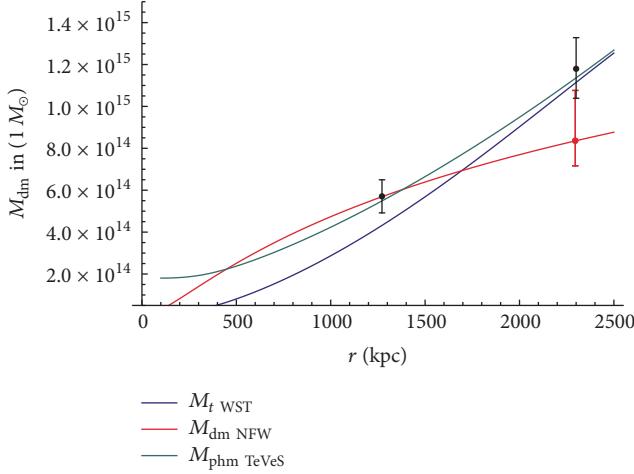


FIGURE 6: Comparison of dark/transparent/phantom mass halos for Coma in NFW, WST, and TeVeS models and free parameters of halos for NFW and TeVeS (μ_2 with neutrino core) adapted to mass data (black error bars) at $r_{500} = 1280$ kpc and $r_{200} = 2300$ kpc. Red error bar at r_{200} expresses variability of the NFW halo at this distance due to adapting it to the given error interval of mass data at r_{500} .

Thus

$$D_\lambda D^\lambda \phi^2 \doteq 2\phi^2 {}_g \nabla_\lambda \partial^\lambda \omega. \quad (\text{A.4})$$

Following (35) and (36) of the main article and using $\xi^{-1} \phi_o \doteq H$ ($H = 6a_o$, the Hubble parameter at present), the energy momentum of the scalar field,

$$\Theta = \theta^{(\text{I})} + \theta^{(\text{II})} = (\xi\phi)^2 T^{(\phi)} = (\xi\phi)^2 (T^{(\text{I})} + T^{(\text{II})}), \quad (\text{A.5})$$

becomes

$$\begin{aligned} T^{(\text{I})} &\doteq - (8\pi G)^{-1} \left(2 {}_g \nabla_\lambda \partial^\lambda \omega - 3 \partial_\lambda \omega \partial^\lambda \omega \right. \\ &\quad \left. - \frac{2}{3} \tilde{a}_o |\nabla \omega|^3 + \frac{\lambda}{4} H^2 \right) g \\ T_{\mu\nu}^{(\text{II})} &\doteq \xi^2 D_\mu D_\nu \phi^2 - 2 \frac{\partial L_\phi}{\partial g^{\mu\nu}} \doteq (8\pi G)^{-1} \left({}_g \nabla_\mu \partial_\nu \omega \right. \\ &\quad \left. + (1 - \tilde{a}_o |\nabla \omega|) \partial_\mu \omega \partial_\nu \omega - \partial_\lambda \omega \partial^\lambda \omega g_{\mu\nu} \right). \end{aligned} \quad (\text{A.6})$$

In the static case the first two terms of $T_{\mu\nu}^{(\text{II})}$ vanish and the total energy component of the scalar field turns into

$$\begin{aligned} T_{oo}^\phi &\doteq (4\pi G)^{-1} \\ &\cdot \left({}_g \nabla_\lambda \partial^\lambda \omega - \partial_\lambda \omega \partial^\lambda \omega - \frac{1}{3} \tilde{a}_o |\nabla \omega|^3 + \frac{\lambda}{8} H^2 \right). \end{aligned} \quad (\text{A.7})$$

Assuming conditions under which the terms of cosmological orders of magnitude and those of order $|\nabla \omega|^2$ can be neglected, we arrive at

$$\rho_{\text{sf}} = T_{oo}^\phi \doteq (4\pi G)^{-1} {}_g \nabla_\lambda \partial^\lambda \omega, \quad (\text{A.8})$$

like in the corrected equation (47). In the central symmetric case (44) implies $\omega(r) = \sqrt{GM\tilde{a}_o} \ln r$. In spherical coordinates $\nabla \omega = (0, \sqrt{GM\tilde{a}_o}/r, 0, 0)$ and

$$\nabla^2 \omega = {}_g \nabla_\lambda \partial^\lambda \omega \doteq \frac{\sqrt{GM\tilde{a}_o}}{r^2}. \quad (\text{A.9})$$

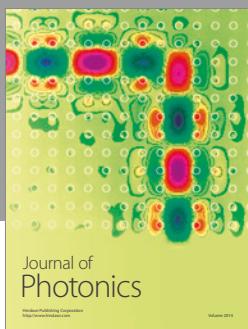
Because of $\partial_1 \omega \partial_1 \omega = GM\tilde{a}_o/r^2 \ll \sqrt{GM\tilde{a}_o}/r^2$ the approximation (47) is justified.

References

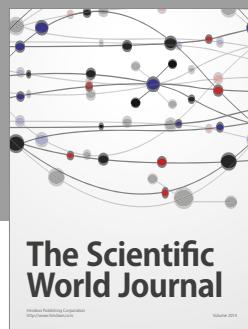
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- [2] Y.-Y. Zhang, “Corrigendum to star-formation efficiency and metal enrichment of the intracluster medium in local massive clusters of galaxies,” *Astronomy & Astrophysics*, vol. 544, no. C3, 1 page, 2012.
- [3] Reiprich and Thomas, *Cosmological Implications and Physical Properties of an X-Ray Flux-Limited Sample of Galaxy Clusters [Dissertation, thesis]*, Dissertation University Munich, Munich, Germany, 2001.



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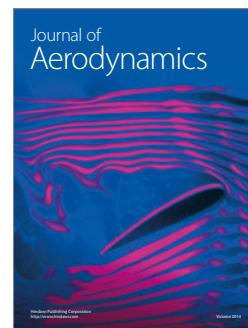
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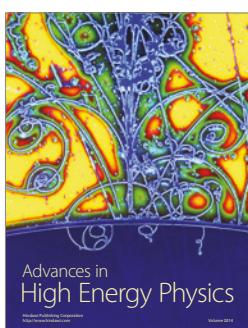
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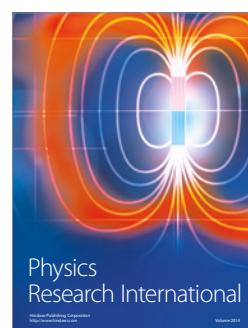
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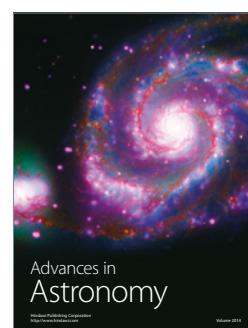
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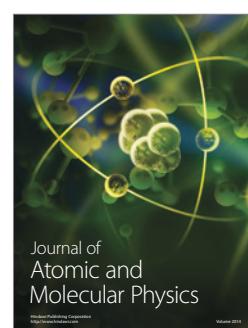
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