

Review Article

Dwarf Galaxies, MOND, and Relativistic Gravitation

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Received 5 June 2009; Revised 12 August 2009; Accepted 2 October 2009

Academic Editor: Elias Brinks

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MOND is a phenomenological modification of Newton's law of gravitation which reproduces the dynamics of galaxies, without the need for additional dark matter. This paper reviews the basics of MOND and its application to dwarf galaxies. MOND is generally successful at reproducing stellar velocity dispersions in the Milky Way's classical dwarf ellipticals, for reasonable values of the stellar mass-to-light ratio of the galaxies; two discrepantly high mass-to-light ratios may be explained by tidal effects. Recent observations also show MOND describes tidal dwarf galaxies, which form in complex dynamical environments. The application of MOND to galaxy clusters, where it fails to reproduce observed gas temperatures, is also reviewed. Relativistic theories containing MOND in the non-relativistic limit have now been formulated; they all contain new dynamical fields, which may serve as additional sources of gravitation that could reconcile cluster observations with MOND. Certain limits of these theories can also give the accelerating expansion of the Universe. The standard dark matter cosmology boasts numerous manifest triumphs; however, alternatives should also be pursued as long as outstanding observational issues remain unresolved, including the empirical successes of MOND on galaxy scales and the phenomenology of dark energy.

1. Why MOND?

In 1983, Mordehai Milgrom published a set of three papers in the *Astrophysical Journal* postulating a fundamental modification of either Newton's Law of Gravitation or Newton's Second Law [1–3]. This modification was designed to explain the Tully-Fisher relation for spiral galaxies, which relates luminosity to rotational velocity, without using dark matter. Unlike many other proposed modifications of gravity, Milgrom's is not at a particular length or energy scale, but rather at an *acceleration* scale $a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$. This is a very small acceleration, which makes any laboratory-based probes nearly impossible; the consequences of Milgrom's modification manifest themselves only on astronomical scales. In particular, Milgrom aimed to address the phenomenology of dark matter: are the departures from Newtonian gravitation we see in galaxies actually due to some breakdown of Newton's Law of Gravitation, rather than due to the action of otherwise undetected dark matter obeying the standard gravitational law?

For spiral galaxies, redshift measurements reveal how rapidly stars and neutral hydrogen gas in the disk are rotating around the center of the galaxy. It has been known since

the early 1970's that the overwhelming majority of spiral galaxies have flat rotation curves: the rotational velocity $v(R)$ at a radius R from the center of the galaxy becomes independent of R past a certain radius. (A small fraction of galaxies have rotation curves which are still increasing with R at the largest-radius data point. Only a few exhibit any significant decrease; see, [4] for an example.) This behavior is impossible to reconcile with the Newtonian gravitational field produced by the visible matter in the galaxy. At large distances, beyond the majority of the galaxy's visible mass, the Newtonian gravitational field from the visible matter begins to approximate the field from a point mass, so that rotational velocities due to the visible matter alone would drop like $v(R) \propto R^{-1/2}$, as with planetary orbital velocities in the solar system. No known spiral galaxy exhibits this behavior in its rotation curve, even though many have rotation curves probed well beyond the bulk of the visible matter via neutral hydrogen clouds. We have only two possible logical explanations: either a large amount of unseen dark matter is present to increase the gravitational field over the Newtonian value, or Newton's Law of Gravitation does not hold on the scales of spiral galaxies.

From the beginning, dark matter was considered as almost certainly the explanation for galaxy observations, buttressed by candidate dark matter particles which arise naturally in many extensions of the standard model of particle physics, particularly in supersymmetry. The dark matter hypothesis has been extremely successful at explaining a wide range of cosmological observations, including the growth of large-scale structure and gravitational lensing. Many experimental groups are in hot pursuit of the direct detection of dark matter particles, under the assumption that they interact via the weak nuclear force. Large cosmological N-body simulations have converged on detailed predictions for the distribution of dark matter on scales ranging from superclusters down to dwarf galaxies, and a large subfield of cosmology is devoted to figuring out the relationship between the theoretical dark matter distribution and the observed visible matter distribution, using a range of observations, simulations, and models. Certainly dark matter cosmology has been highly successful in explaining a wide range of observations and in providing a compelling theoretical framework for the standard cosmological model.

Is there a viable alternative? Milgrom considered the following ad hoc modification to the Newtonian law for gravitational force on a mass m due to another mass M at a distance r :

$$F = \begin{cases} ma_N, & a_N \gg a_0, \\ m(a_N a_0)^{1/2}, & a_N \ll a_0, \end{cases} \quad (1)$$

where $a_N = GM/r^2$ is the usual Newtonian acceleration due to gravity, and some smooth interpolating function between the two limits is assumed. If the Newtonian gravitational force on an object results in an acceleration greater than a_0 , the Milgrom gravitational force is proportional to r^{-2} , the same as the Newtonian force, but if the Newtonian force gives an acceleration smaller than a_0 , then the Milgrom gravitational force is larger than the corresponding Newtonian force, dropping off like r^{-1} . Remarkably, this simple modification is highly successful at reproducing the rotation curves of spiral galaxies from the visible matter distribution alone, that is, without dark matter, under the assumption that stars in disk galaxies follow circular orbits around the center of the galaxy. Many rotation curves have been fit this way; see, for example, [5–9]. Some galaxy rotation curves are not fit by this formula, but these are generally galaxies which visually appear to be out of dynamical equilibrium or which possess strong bars or other features departing from circular symmetry, and thus likely violate the assumption that stars follow circular orbits. Milgrom dubbed this force law modification MOND, for MODified Newtonian Dynamics. A large literature has developed about MOND. This paper aims to give a brief introduction to the current state of the topic with useful points of entry to the literature, but is not a complete review of the literature or the science.

A cursory analysis of the force law in (1) reveals a fundamental shortcoming: it does not conserve momentum [10]. It is simple to construct a situation with two different masses where the force of the small mass on the large mass is not the same as the force of the large mass on the small

mass. Shortly after Milgrom's original papers, Bekenstein and Milgrom proposed a generalization of the Poisson equation, instead of the Newtonian force law, which overcomes this difficulty [11]:

$$\nabla \cdot [\mu(|\nabla\Phi|/a_0)\nabla\Phi] = 4\pi G\rho, \quad (2)$$

where Φ is the physical gravitational potential, ρ is the baryonic mass density, and $\mu(x)$ is some smooth function with the asymptotic forms $\mu \sim x$, $x \ll 1$ and $\mu \sim 1$, $x \gg 1$. (The precise form of $\mu(x)$ makes little difference in the resulting phenomenology; commonly $\mu(x) = x/(1+x^2)^{1/2}$ is used. But see also [5], which claims that $\mu(x) = x/(1+x)$ gives a more realistic estimate for the visible disk mass in high surface-brightness spirals.) Note that in the regime where $|\nabla\Phi| \gg a_0$ this equation becomes the usual Poisson equation. One particular feature of this equation which differs from the usual Newtonian equation is that it is not linear: the gravitational force on an object cannot be obtained simply by adding the gravitational forces of all objects acting on it. One implication is the external field effect: a galaxy will behave according to the MOND force law with the mass of the galaxy providing the source term only as long as the internal accelerations in the galaxy are larger than the local acceleration coming from the gravitational field of external objects. This behavior was postulated in the original Milgrom papers to explain why open star clusters within our galaxy do not display MOND-like behavior, and to give a consistent explanation of how to reconcile the MOND-like behavior of different objects. The modified Poisson Equation in (2) automatically incorporates this behavior, given an arbitrary mass distribution.

Milgrom's modified force law not only explains spiral galaxy rotation curves, but also can explain a number of other phenomenological successes. It predicts that the amount of inferred dark matter in a bound system is not a function of the size of the system, but rather of the characteristic acceleration. This matches what we observe: the solar system, globular clusters, and some elliptical galaxies appear to have little or no dark matter, while dwarf galaxies and large spirals have large amounts of dark matter. But the characteristic acceleration scale of these systems successfully sorts them by dark matter content. The Milgrom force law also contains the observed Tully-Fisher Relation (luminosity scaling like the fourth power of the asymptotic rotation velocity) in spiral galaxies as a special case, implies Freeman's Law for spirals (upper limit on surface brightness), and implies the Fish Law (constant surface brightness) and the Faber-Jackson relation (luminosity scaling with the fourth power of the velocity dispersion) in elliptical galaxies. Milgrom also predicted that low surface brightness galaxies should be dark matter dominated at all radii, which was later confirmed. See [12] for an excellent review discussing the range of MOND phenomenology.

What is the effectiveness of the Milgrom force law telling us? In his original papers, Milgrom advocated that the force law is the actual fundamental force law of nature, suggesting that it either shows a modification of the law of gravity in the weak gravity regime, or a modification

of Newton's second law connecting force and acceleration. Despite observational successes, most physicists dismiss this possibility. But regardless of its fundamental status, the Milgrom force law can be viewed as an effective force law for gravitation in galaxies: if dark matter is causing the excess forces we observe, it must arrange itself in a way so that (1) represents the total gravitational force due to the combined visible and dark matter. It is not at all clear in the standard cosmological model why this should be. (Unfortunately, the immediate assertion in the original Milgrom papers that (1) represents a fundamental modification of physics caused many cosmologists to ignore the whole business. All cold dark matter advocates should still have the Milgrom Relation as part of their suite of known facts about galaxies, like the Tully-Fisher Relation; requiring dark matter models to reproduce this phenomenology might give interesting clues about the nature of the dark matter or galaxy dynamics.)

2. MOND and Dwarf Galaxies

As dwarf galaxies (particularly dwarf spheroidals) exhibit the largest discrepancies between visible matter and gravitational field of any known bound systems, they are important laboratories for testing both the dark matter and the modified gravity hypotheses. On the other hand, local dwarfs are usually irregular or spheroidal, and thus do not present an easily-measured rotation curve. Generally, redshift measurements of individual stars in dwarf spheroidals give an estimate of velocity dispersion as a function of the angular distance from the dwarf center, along with the dwarf's overall line-of-sight velocity. These measurements provide two possible probes of MOND: are internal dynamics of dwarf galaxies consistent with the MOND force law, and are the motions of dwarf galaxies around their parent galaxies consistent with the MOND force law? We briefly consider both questions here.

2.1. Internal Dynamics of Dwarfs. MOND was originally formulated to fit measured rotation curves of spiral galaxies. Extrapolating from the force law in (1) led to the prediction that low surface-brightness galaxies should have rotation curves which implied dark matter domination at all radii from the center; this prediction was eventually born out by later observations. Extending this analysis to dwarf spheroidal galaxies, which also have low surface brightness, Milgrom also predicted "... when velocity-dispersion data is available for the (spheroidal) dwarfs, a large mass discrepancy will result when the conventional dynamics is used to determine the masses." The dynamically determined mass is predicted to be larger by a factor of order 10 or more than that which is accounted for by stars [2]. This prediction was of course ultimately born out, with dwarf galaxies having the largest mass-light ratios of any known systems.

The lack of linearity in (1) implies that the gravitational force on an object is not determined solely by the mass that it orbits, but also depends on the external gravitational field. As Milgrom explained in detail [2, 11], for an object like a dwarf galaxy orbiting the Milky Way, orbits of stars

within the dwarf are governed by (1), with the mass M equal to the dwarf baryonic mass within the star's orbital radius, as long as the internal gravitational acceleration a_{in} of the star due to the dwarf mass is larger than the external gravitational acceleration a_{ex} of the dwarf as it orbits the Milky Way. Otherwise, the dwarf is in the so-called "quasi-Newtonian regime," in which case the dwarf dynamics are nearly Newtonian with an effective gravitational constant. If the internal accelerations in the dwarf are small compared to a_0 , then $G \rightarrow G/\mu(a_{\text{ex}}/a_0)$, and the MOND value for the baryonic M/L ratio is simply the Newtonian value times $\mu(a_{\text{ex}}/a_0)$. When analyzing dwarf galaxy dynamics in the context of MOND, care must be taken to determine whether the gravitational acceleration is dominated by the internal or external field.

For the quasi-Newtonian regime, estimators for the dwarf mass based on the stellar velocity dispersion are textbook formulas. In the opposite limit, where the external gravitational field is negligible (the "isolated" regime), (2) leads to the relation $M = 9\sigma_3^4/(4G a_0)$ for the mass M of a stationary, isolated system with $a_{\text{in}} \ll a_0$ [13], where σ_3 is the three-dimensional velocity dispersion of the system. We cannot directly measure σ_3 , but if the system is statistically isotropic (having the same line-of-sight velocity dispersion σ in all directions), then $\sigma_3 = \sqrt{3}\sigma$. Effects which can give significant departures from this simple mass estimator include anisotropy in the velocity dispersion for systems which are not spherically symmetric and coherent velocities due to tidal effects. These are potentially visible with sufficiently sensitive photometry and spectroscopy. Brada and Milgrom (2000) analyzed these effects in more detail in the context of MOND, showing, for example, that dwarfs are more susceptible to tidal disruption than for Newtonian gravity with dark matter haloes [14] (see also [15] for a more detailed analysis).

Stellar population studies strongly suggest that the baryonic mass-light ratio Y for dwarf satellites of the Milky Way should be in the range $1 < Y < 3$ (see, e.g., [16, 17]); these satellites generally have negligible gas contribution to their baryonic mass. A total dwarf mass derived from MOND should give a value of Y in this range, by comparing with the measured luminosity. Gerhard and Spergel [18] and Gerhard [19] analyzed seven dwarf spheroidals using MOND and claimed values for Y ranging from around 0.2 for Fornax to around 10 for Ursa Minor and Leo II. However, in a paper whimsically entitled "MOND and the Seven Dwarfs" [20], Milgrom persuasively argued that published uncertainties in the measurements of velocity dispersion used by Gerhard and Spergel were sufficient to explain the anomalous baryonic M/L ratios. He also noted how difficult it is to determine the photometric structural parameters of dwarfs, including central surface brightness, core radius, tidal radius, and total luminosity. Using measurements of dwarf parameters from Mateo (1994) [21] plus several subsequent measurements, Milgrom found that MOND was completely consistent with reasonable values for Y between 1 and 3 for all seven dwarf spheroidals considered (Sculptor, Sextans, Carina, Draco, Leo II, Ursa Minor, and Fornax). This analysis estimates the mass using only the central

velocity dispersion σ for each dwarf, and computes M in either the quasi-Newtonian or the isolated limit.

The most detailed analysis of the classical dwarf galaxies has been done by Angus (2008), incorporating many improvements in observed dwarf properties over the previous decade, and also using measured variations of the velocity dispersion with radius in each dwarf for which it is measured [22]. This analysis thus provides a substantially more stringent test of MOND compared to the Seven Dwarfs paper. Angus finds best-fit values and errors for Y , plus two parameters describing possible velocity dispersion anisotropy. Of the eight dwarfs considered, six (Sculptor, Carina, Leo I, Leo II, Ursa Minor, and Fornax) are consistent with a reasonable value of Y for the stellar content. Sextans and Draco both give discrepantly high values for Y , but within 2σ of the accepted range. Draco has been previously known to present a problem for MOND [23, 24], and the problem is worse for different measurements of velocity dispersions [25] than Angus uses. Angus notes that distance and velocity dispersion uncertainties for these two dwarfs may have a significant impact, and that possible tidal effects introduce uncertainties which are large enough to explain Angus' marginally high values of Y for these two dwarfs. However, Draco in particular shows no morphological evidence of tidal disruption. Angus speculates that it could be in the early stages of tidal heating peculiar to MOND [14] as it approaches the Milky Way. Numerical simulations would be required to decide whether this explanation is plausible. An alternate explanation is that the stellar samples are contaminated with unbound stars which are not part of the dwarf galaxy. These can inflate the inferred line-of-sight velocities in the stellar sample, leading to systematically high values for Y . (An anonymous referee for this paper claims that this is a particular issue for Sextans and Carina, while data for Draco and Ursa Minor are not yet public.)

A provocative comparison between dark matter and MOND has recently been performed using tidal dwarfs. Bournaud and collaborators [26] have measured rotational velocities for three gas-rich dwarfs forming from collisional debris orbiting NGC5291. Using the VLA, they obtained rotation curves from the 21 cm gas emission. The stellar mass in these systems is a small fraction of the gas mass, so the total mass can be determined with little uncertainty. The rotation curves are symmetric, implying that the systems are rotationally supported and in equilibrium. It is straightforward to derive mass estimates based on Newtonian gravity: the total mass in the systems is about three times the observed baryonic mass. The authors claim that this is a problem based on numerical dark matter simulations showing that gaseous dwarfs condensing out of tidal debris should contain very little dark matter [27, 28]. Tidal dwarfs have complex dynamical histories and a range of baryonic effects are important, so it is hard to know whether the comparison with simulations really is a problem. On the other hand, Milgrom [29] and Gentile et al. [30] have both shown that the tidal dwarfs are well accounted for with MOND, with *no free parameters*. This consistency is very difficult to explain in dark matter models, as tidal dwarfs form from tumultuous dynamical processes with

strong dependence on initial conditions while being affected by numerous baryonic processes. Why should their dark matter content be precisely that needed to make them land right on the MOND force law? Clearly a larger sample of these very interesting dynamical systems is an observational priority.

A recent interesting set of observations has been performed on the distant and diffuse globular cluster Palomar 14 [31], among other applications of MOND to distant globular clusters (e.g., [32–34]). The radial velocities of 17 red giant stars in the cluster have been measured with the VLT and Keck telescopes, giving an estimate of the line-of-sight velocity dispersion. This dispersion is consistent with Newtonian gravity, even though the globular cluster should be well within the MOND regime, and the authors claim the observations taken at face value rule out MOND. However, the paper also constructs a scenario where the stellar velocities could still be consistent with MOND if it were on a highly elliptical orbit in the Milky Way potential, passing in and out of the MOND regime as the external gravitational field changes. This picture is related to tidal shocking [35, 36] which can decrease the stellar velocity dispersion for dwarfs with Newtonian gravity. The cluster is very distant and faint, so prospects for obtaining a proper velocity on the sky, and thus a clearer characterization of the cluster's recent orbital history around the Milky Way, is likely remote. Mass segregation of the red giants in the cluster may also be an issue. But these systems put some significant pressure on MOND.

The other smaller Milky Way dwarfs which have recently been discovered in Sloan Digital Sky Survey data (see, e.g., [37, 38]) have even higher ratios of visible mass to inferred dark matter mass than the classical dwarf spheroidals, presenting another stringent test of MOND in galactic systems down to still smaller scales than have been so far analyzed. Sanchez-Salcedo and Hernandez [15] performed a preliminary analysis concluding that the required MOND value for Y increases at low luminosities and is too high to be realistic. This is a consequence of these dwarf spheroidals being consistent with a universal central velocity dispersion. This provocative conclusion clearly warrants further study. Systematic effects related to a host of assumptions about the Milky Way gravitational potential, tidal effects in the dwarfs, stellar interlopers in the data, and other considerations can dominate the uncertainties in these calculations.

2.2. Can a Dwarf Galaxy Rule Out MOND? The results described in the previous section raise a tricky question for MOND. On one hand, the theory is in principle more predictive than dark matter: the gravitational field of a distribution of visible matter is completely determined given the baryonic mass-light ratio Y and the distance, which sets the length scale in (1). Both of these numbers can be determined or constrained by other observations. (In practice, Y is usually left as a free parameter, and then its MOND-derived value is compared to the stellar

population to determine whether it is reasonable.) On the other hand, the gravitational field is probed via the dynamics of test objects, generally the motions of stars in the galaxy. We generally do not have available the full velocity vector or velocity dispersion, but only the line-of-sight components. The transverse components contain crucial information about the full dynamical state of the dwarf: did its past trajectory create tidal heating or disruption? To what extent is its internal velocity dispersion anisotropic? How rapidly is the external gravitational field varying? Without this information, a range of gravitational potentials are consistent with the current dynamical state of any galaxy. In the absence of other difficult observations (like proper motions, which allow reconstructing the actual galaxy trajectory), we must settle for determining whether the observed galaxy dynamics requires only believable additional assumptions about unobservable effects.

This is the same issue facing MOND fits of large galaxies as well. Large galaxies tend to have much less significant external gravitational field effects than dwarfs, leaving less freedom to explaining marginal MOND fits. The remarkable point about MOND is that, despite the limited freedom the model provides in fitting galaxy dynamics, very few galaxies, ranging over several orders of magnitude in mass and more than an order of magnitude in size, provide a true challenge for MOND.

In contrast, the dark matter situation is more forgiving. The standard cosmology in principle is completely deterministic, once the properties of dark matter are specified. Small initial density perturbations, specified by measurements of the cosmic microwave background temperature anisotropies, evolve according to completely specified equations of cosmological evolution. However, we are not yet able to make detailed *ab initio* predictions for the dark matter halo associated with a particular class of galaxies. Since the baryonic mass is coupled gravitationally to the dark matter, a complete understanding of the dark matter distribution today requires detailed knowledge of the history of the baryon distribution, and this in turn is shaped by complex baryonic physics, including heating, cooling, star formation and evolution, and energy feedback from stars and quasars. This is not simply an academic point: most of the outstanding issues with dark matter involve small scales, where the baryon densities can be comparable to the dark matter densities. Eventually, our modeling and computations may reach the point where we can simulate the evolution of realistic galaxies starting only from cosmological initial conditions, but this prospect is many years away. Until then, visible galaxies' rotation curves or velocity dispersions are generally fit using a parameterized model for the dark matter halo, and we have few constraints on the halo parameters for an individual galaxy. So for an individual galaxy, fitting a dark matter halo provides more freedom than fitting a MOND value for Υ .

2.3. Dwarfs as Dynamical Tracers. The motions of dwarf galaxies themselves as they orbit around their parent galaxy can also be used as a test of MOND. Typically, for large

elliptical galaxies, satellite galaxies are observable to much larger radii than the stars in the central galaxy, so the satellites probe the gravitational field over a larger region. However, any individual galaxy has a relatively small number of observable satellites, making gravitational potential constraints inconclusive. To get around this, Klypin and Prada [39] have used a sample of 215,000 red galaxies with redshifts between $z = 0.010$ and $z = 0.083$ from the Sloan Digital Sky Survey Data Release 4. From this catalog, they select a set of satellites according to the criteria that the satellite is at least a factor of four fainter than its host, has a projected distance from its host of less than 1 Mpc, and has a velocity difference from its host of less than 1500 km/s. This results in about 9500 satellites around 6000 host galaxies. They then form a composite satellite velocity dispersion as a function of radius for a number of bins in host galaxy luminosity by stacking all of the satellites and using a maximum-likelihood estimator. A homogeneous background level, reflecting random alignments of unrelated galaxies, is subtracted to give a final velocity dispersion for each luminosity bin.

Klypin and Prada compare this velocity dispersion with circular dark matter velocities around large isolated haloes drawn from N-body simulations, and find very good agreement. They make the assumption that the density of dwarf galaxies traces the dark matter density, which holds in dark matter simulations [40]. They further claim that velocity dispersions derived by solving the Jeans equation with the MOND gravitational potential in (2) can only fit the data using highly contrived forms for the velocity dispersion anisotropy, corresponding to highly elliptical stellar orbits. This conclusion has been disputed by Angus and McGaugh [41], who claim that the same anisotropy is actually very reasonable, based on numerical simulations of elliptical galaxy formation in MOND [42]. The two papers also disagree on a number of other points, including whether gravitational fields external to the SDSS galaxies should be relevant and what constitutes reasonable mass-light ratios for the host galaxies. Further independent analysis of this interesting data set is clearly warranted.

3. Galaxy Clusters and MOND

While the Milgrom force law, (1), and its generalization to the modified Poisson equation in (2), is a highly successful phenomenological description of galaxies ranging from the smallest dwarfs to the largest spirals, and over the full range of observed surface brightness, its efficacy does not extend to the larger scales of galaxy clusters. A typical galaxy cluster must contain a total mass several times its visible gas and stellar mass for its gas temperatures and galaxy velocities to be explained by the Milgrom force law [43]. Simply postulating an increased cluster value for the baryonic mass-light ratio Υ is not realistic, since the baryonic mass is dominated by gas and does not have the intrinsic uncertainty in Υ that stellar populations do. In the absence of large systematic errors in the cluster data, we have several choices for explaining galaxy clusters.

- (1) MOND is simply not a fundamental theory, but only a phenomenological relationship for the distribution of dark matter in galaxies, inapplicable to larger bound systems.
- (2) MOND does represent a fundamental force law, but clusters contain additional dark matter while galaxies do not.
- (3) MOND represents a fundamental force law for galaxies, but this force law gets modified on larger scales.

The first possibility is widely accepted: galaxy clusters simply rule out MOND as a modification of gravity. The second possibility is not as outlandish as it appears at first glance. If dark matter were in the form of light neutrinos with a mass of around 2 eV, for example, which had relativistic velocities at the time of their decoupling, they would not be gravitationally bound in galaxies, but could be bound in the much larger and more massive clusters [44, 45]. Current best limits on neutrino masses come from cosmology, but these assume the standard cosmological model, which may be modified if MOND actually represents a fundamental force law. Angus [46] postulates a slightly heavier (11 eV) sterile neutrino, which a numerical calculation shows can reproduce the acoustic peaks in the microwave background assuming that early-universe physics is not affected by MOND effects. Producing the forced acoustic oscillations revealed by precision measurements of the microwave background power spectrum is an important challenge for any cosmology with different dark matter or gravitation from the standard cosmology. Dynamical analysis shows that such a neutrino can reproduce observed galaxy cluster phenomenology while not affecting galaxy rotation curves [47]. Pressure is put on these models from analyses of gravitational lensing [48–50], although these all make specific assumptions about lensing in MOND, which is not uniquely determined without the context of a relativistic theory.

The most relevant observations for MOND are the gravitational lensing observations of the famous “bullet” cluster 1E 0657-558 [51] and the cluster MACS J0025.4-1222 [52]. In these systems of two merging galaxy clusters, the peak of the gravitational potential, revealed by weak lensing, is offset from the peak of the mass distribution, visible in hot gas. This offset is consistent with the clusters containing substantial dark matter; when the clusters merge, the dark matter, which is assumed to be collisionless, continues to move unimpeded, while the gas composing the bulk of the visible matter is slowed via shock heating. The offset is NOT consistent with MOND, or with *any* modified gravity theory where the gravitational field is generated solely by the visible matter: the geometry of the gravitational field is different from the geometry of the visible matter [53]. The merging-cluster observations decisively demonstrate that (2) cannot by itself describe gravitation. This is consistent with the earlier observations showing the inferred cluster dark matter content is discrepant with the MOND force law prediction, but further rules out the discrepancy being due to additional modifications of the gravitational force law on cluster scales

(case 3 above). Unlike individual galaxies, galaxy clusters *must* possess some source of gravitation in addition to that provided by the visible matter (i.e., dark matter!).

4. Relativistic MOND and Dark Matter

Many people felt relieved that the cluster lensing results meant they could finally stop hearing about all of this pesky MOND business. However, option 2 above is still viable, although most cosmologists view it as an unnaturally complicated theoretical possibility. As mentioned above, clusters could contain hot dark matter, like light neutrinos, which would be sufficient to explain the observed gravitational and gas morphology of the merging clusters. While this solution invokes two separate pieces of new physics (sufficiently massive neutrinos and a MOND force law), it is not immediately obvious how much more speculative this should be considered than standard dark matter models, given that neutrinos are already known to have non-zero masses from neutrino oscillation observations. But the most interesting option for MOND may come from a different direction.

Two years before the bullet cluster observations showed that the gravitational field must be generated by more than just the visible matter, Jacob Bekenstein published a relativistic gravitation theory which reduced to MOND in the point-source limit [54]. For many years, the fact that MOND had no relativistic basis was often cited as a compelling argument against the idea. (I sometimes joked that people claimed a relativistic MOND extension was ruled out by the Bekenstein Theorem. This theorem stated that Bekenstein had tried to do it and failed, so therefore it was not possible. Later, Bekenstein told me that he had actually never put much effort into the problem.) With a full relativistic theory of gravitation, diverse observations ranging from solar system constraints on post-Newtonian parameters to gravitational lensing, cosmological structure formation, and the expansion history of the universe, can be addressed. All of these areas were speculated about using the MOND force law or modified Poisson equation, and reasoning by analogy with general relativity. But this is clearly an unsatisfactory approach: any results contradicting observations can simply be explained away as the result of an insufficiently sophisticated theory.

Several variations on the Bekenstein proposal are now on the table [55, 56]. These theories all share the common element of additional dynamical fields which act as gravitational sources. The most elegant scheme so far has been dubbed Einstein Aether gravity [57]. A dynamical time-like vector field A_μ with $A_\mu A^\mu = -1$ is incorporated into gravity by adding a term $\mathcal{F}(\mathcal{K})$ to the Einstein-Hilbert gravitational action, where \mathcal{F} is a function to be determined and

$$\mathcal{K} \propto c_1 \nabla_\mu A_\nu \nabla^\mu A^\nu + c_2 (\nabla_\mu A^\mu)^2 + c_3 \nabla_\mu A^\nu \nabla_\nu A^\mu \quad (3)$$

with c_1 , c_2 , and c_3 undetermined constants. This is an unusual theory because the time-like vector field picks out a preferred frame at each point in spacetime (in which the time coordinate direction aligns with A) and thus violates local

Lorentz invariance. But if this symmetry breaking is small enough it would be below the level of current experimental bounds. This class of theories has a line of theoretical precedents (see, e.g., [58–60]).

Remarkably, the constants in \mathcal{K} and the function \mathcal{F} can be chosen so that the nonrelativistic limit of the theory reduces to a modified Poisson equation of the form (2): the MOND limit can be reproduced in this class of relativistic models. Much analysis has yet to be performed, but it is at least plausible that the excess gravitational forces induced by the vector field A^μ could match the departures from MOND seen in galaxy clusters. Relativistic MOND theories involving extra dynamical fields are, in some sense, an abdication of the original MOND philosophy that the visible matter creates all of the observed gravitational force in the universe; but we are forced into this by observations. The physical content of these theories is quite different from traditional models; the “dark sector” is governed by physics different from particle dark matter.

5. Challenges

Relativistic extensions of MOND have not been widely explored, in contrast to the overwhelming amount of work on the standard cosmological model. So it is simply not yet known whether particular relativistic MOND theories can reproduce all known gravitational phenomena, including gravitational lensing, post-Newtonian constraints from the solar system and binary pulsars, tidal streams, galaxy cluster gravitational potentials, and the observed dynamics of merging galaxy clusters. All of these phenomena are successfully explained by the standard dark matter cosmology, and represent a tall order for any new gravitational theory. Relativistic MOND theories also must be nonlinear, since MOND itself is, making these analyses challenging. However, relativistic MOND theories hold the promise of explaining two observations that the standard cosmology cannot: (1) the effectiveness of the Milgrom force law, equation (1), and related phenomenology of galaxy dynamics, and remarkably, (2) the accelerating expansion of the Universe.

Milgrom immediately realized that the MOND acceleration scale a_0 is the same order as cH_0 and thus also the same order as $c\Lambda^{1/2}$, where H_0 is the Hubble parameter and Λ is the cosmological constant which would explain the accelerating expansion of the universe. It is difficult to see how a_0 , which manifests itself on scales of dwarf galaxies, can be related to either H_0 or Λ in the standard cosmological model, but a relativistic basis for MOND may provide insight. Reference [57] discusses conditions under which the additional vector field in Einstein Aether gravity will influence the expansion rate of the Universe. In fact, they display a choice of \mathcal{F} which gives a late-time expansion history identical to that of general relativity with a cosmological constant. This is highly provocative: could a conceptually simple modification of relativistic gravitation explain the phenomenology of both dark matter and dark energy? The particular theory considered in [57] has substantial freedom in it, and is also challenging to analyze because it is nonlinear. But substantial

effort in this direction is clearly warranted, as the theory easily reproduces MOND-like behavior and accelerating cosmological expansion with different choices of the free parameters and the free function.

Most cosmologists pay little attention to the highly successful galaxy phenomenology of MOND, even though they should, because they do not believe in MOND as a fundamental theory. The merging galaxy cluster observations have now ruled out the possibility that (2) applied to visible matter represents the entire fundamental theory of gravitation, while dark matter cosmology has had remarkable success in explaining the growth of structure in the Universe and providing a simple standard cosmological model which explains most observations. On the other hand, dark matter cosmology still faces some serious issues, mostly related to reproducing this MOND phenomenology (with the obvious exception of dark energy). The recent observation implying tidal dwarfs obey MOND [30] is one intriguing example. We should not allow the successes of our leading theories to blind us to other possibilities.

The discovery of the accelerating expansion of the Universe changed the playing field for cosmology. Prior to the late 1990’s, many established cosmologists would greet any suggestion of a modification of gravity as the explanation for observed gravitational force excesses with contempt. Today, some of these same scientists author papers advocating a modification of gravity as a possible explanation for the observed accelerating expansion. Until the predominant picture of dark matter cosmology can explain all of the observations, other competing ideas should be pursued, either as a way of sharpening the case for dark matter cosmology, or, perhaps, uncovering an eventual replacement.

Acknowledgments

The author thank Andrew Zentner for insightful discussions about galaxy formation. Two anonymous referees helpfully pointed out recent relevant literature, and one emphasized how unbound stellar interlopers can increase inferred mass-light ratios in dwarf spheroidals. This work was partially supported by NSF Grant AST-0807790.

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