Research Article

Relations between the Spectral Indices and Flux Densities of Eight Blazars

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Relations between the flux densities (F) and spectral indices (α) can help us analyze the emission process. In this paper, we choose 8 blazars (0235+164, 0430+052, 1156+295, 3C345, 1308+326, 1413+135, 3C454.3, and 1749+096) from the University of Michigan Radio Observatory (UMRAO) database to study the relations between the spectral indices (α) and flux densities at 14.5GHz ($F_{14.5}$). The main results are the following. (1) There are strong anticorrelations between α and $F_{14.5}$, with the correlation coefficient r in the range from -0.33 to -0.87 . (2) The $\alpha - F_{14.5}$ distributions show elliptic appearance, which have been fitted by elliptic curves. (3) For most of the sources, the time intervals of $\alpha - F_{14.5}$ elliptic circle (T_c) are consistent with the quasi-periodicities calculated by the averaged light curves and spectral variances ($P_{\alpha F}$).

1. Introduction

Blazars are a subclass of active galactic nuclei (AGNs). They show some extreme properties, such as violent optical variability, core dominance, superluminal motion, and so on [1, 2]. Blazars can be divided into two subclasses, BL Lacertae objects (BL Lacs), and flat-spectrum radio quasars (FSRQs). BL Lacs are characterized by featureless optical spectra or weak emission lines [3]. Flat radio spectrum and broad optical emission lines are typical for FSRQs [1]. Generally, the division between the two subclasses is based on the equivalent width (EW) of the optical broad emission lines; BL Lacs show $EW < 5\text{\AA}$ [1, 4–6].

The time scale of optical variability is an important quantity which is often used to probe the physical processes in blazars. The time scales are in the range from minutes to years and can be divided into three types: intraday variability (IDV), short-term variability, and long-term variability.

Usually, the intraday variability and short-term variability are nonperiodic, but the long-term variability is quasi-periodic. The long-term optical variabilities of blazars have been discussed by many papers [3, 7–18]. The explanation about the long-term variabilities can be the binary black-hole

model, the thermal instability model and the perturbation model and so on [14].

For blazars, there are many papers discussing relationship between the spectrum and flux density [16, 19–26]. Dai et al. (2009) [27] presented long-term *BVRI* observations and discussed the correlations between color index and brightness. Beaklini & Abraham (2014) [28] used variability at 7 mm to find evidence of shocks and precession in the jet. Fan et al. (2014) [15] analyzed the relation between *V*-band flux density (F_V) and spectral index (α) for two nearby quasars and found two different relations below and above $F_V = 28$ mJy. When $F_V < 28$ mJy, F_V , and α showed anticorrelation, when $F_V > 28$ mJy, F_V , and α showed positive correlation. Yuan & Fan (2015) [29] found that, in 3C273 and 3C446, there was elliptic structure in the distributions between the flux densities and spectral indices. Wierzcholska et al. (2015) [22] analyzed the color-magnitude correlations of 30 blazars and observed the bluer-when-brighter behaviors. Carnerero et al. (2017) [23] pointed out that the flux densities and spectral variability can be compatible with jet models including at least two emitting regions that can change their orientation with respect to the line of sight. Isler et al. (2017) [24] provided an explanation of the long-term optical-infrared

TABLE 1: The spectral indices of 8 blazars.

Name (1)	z (2)	Type (3)	N (4)	α_{min} (5)	α_{max} (6)	$\bar{\alpha}$ (7)
0235 + 164	0.94	BL	129	-0.798 ± 0.001	0.463 ± 0.012	-0.082 ± 0.279
0430 + 052	0.033	F	116	-0.384 ± 0.001	0.469 ± 0.018	0.127 ± 0.200
1156 + 295	0.7247	F	66	-0.779 ± 0.023	0.264 ± 0.006	-0.221 ± 0.249
1308 + 326	0.998	BL	128	-0.713 ± 0.017	0.231 ± 0.012	-0.283 ± 0.236
1413 + 135	0.2467	BL	56	-1.098 ± 0.001	-0.183 ± 0.038	-0.583 ± 0.260
3C 345	0.593	F	187	-0.733 ± 0.013	0.296 ± 0.024	-0.136 ± 0.230
1749 + 096	0.322	BL	125	-1.213 ± 0.023	-0.008 ± 0.001	-0.503 ± 0.266
3C 454.3	0.859	F	162	-0.339 ± 0.038	0.517 ± 0.013	0.171 ± 0.217

color variabilities in some blazars and presented a general scheme which can apply to these variabilities.

The spectral indices are associated with the emission properties, and the flux densities demonstrate the optical variability properties. The study about the relations between the spectral indices and flux densities can combine the emission properties and optical variabilities. Generally, for BL Lacs, when the sources become brighter, the spectrum becomes flatter and when the sources become fainter, the spectrum becomes steeper.

This paper is arranged as follows. In Section 2, we calculate the spectral indices; in Section 3, we analyze the relations between the radio spectral index α and the 14.5 GHz radio flux density $F_{14.5}$; in Sections 4 and 5, we give the discussion and conclusions, respectively.

2. The Spectral Indices

Based on the UMRAO (University of Michigan Radio Astronomy Observatory) database (<https://dept.astro.lsa.umich.edu/datasets/umrao.php>), we collected the radio flux densities at 4.8 GHz, 8 GHz, and 14.5 GHz and used the following method to calculate their averaged flux densities and spectral indices, similarly to Yuan & Fan (2011) [30] and Yuan et al. (2014) [31]. Firstly, at each band (4.8 GHz, 8 GHz, and 14.5 GHz), we average the flux densities within the same bin and obtain N sets of data: $D_i(J_i, F_{4.8|j}, F_{8|j}, F_{14.5|j})$ ($i \in N$), which represent the flux density at each band. Secondly, based on the relation,

$$F_\nu \propto \nu^{-\alpha} \quad (1)$$

($\nu = 4.8$ GHz, 8 GHz and 14.5 GHz), we obtain

$$\log F_\nu = -\alpha \log \nu + C \quad (2)$$

and use linear fit to calculate the spectral indices (α).

There are 8 blazars (0235+164, 0430+052, 1156+295, 3C345, 1308+326, 1413+135, 3C454.3, and 1749+096) with known quasi-periodicity and enough α values ($N > 50$). We use them to build a sample, for which, the spectral indices (α) and the averaged flux densities within the respective bins are displayed in Figure 1. The upper panels show the averaged flux densities at 4.8 GHz, 8 GHz, and 14.5 GHz, and the lower panels show the spectral indices. The detailed descriptions about α have been listed in Table 1, where Col. 1: Name, Col.

2: redshift z , Col. 3: Type, ‘F’: FSRQ, ‘BL’: BL object, Col. 4: number of α values, Col. 5: α_{min} (the minimum α), Col. 6: α_{max} (the maximum α), and Col. 7: $\bar{\alpha}$ (the averaged α).

3. The Relations between the Spectral Indices and Flux Densities

3.1. Methods. For two variable data sets, x_i, y_i ($i = 1, n$), we use the linear fitting to analyze their correlations, $y = (k \pm \Delta k)x + (k_0 \pm \Delta k_0)$, where k is the slope, k_0 is the intersection, p is the Student’s t probability, and n is the number of points in the data set. The Pearson’s correlation coefficient r is expressed as [32–34]

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}, \quad (3)$$

where $\bar{x}$ is the averaged value of x_i and $\bar{y}$ is the averaged value of y_i .

When x, y show elliptical appearance, we use the following elliptic curve to make the elliptic fit:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0, \quad (4)$$

with a, b, c, d, e, f being free parameters and $|a|$ being normalized.

Considering x as $F_{14.5}$ and y as α , we make the following calculations.

3.2. The Linear Correlations. We use the linear correlations to analyze the relations between $F_{14.5}$ and α . The results are shown in Figure 2, where the solid lines indicate the fit between $F_{14.5}$ and α . The slope k , the intersection k_0 , the correlation coefficient r , and the chance probability p are listed in Table 2, where Col. 1: Name, Col. 2: the slope k , Col. 3: the slope error Δk , Col. 4: the intersection k_0 , Col. 5: the intersection error Δk_0 , Col. 6: the correlation coefficient r , and Col. 7: the chance probability p . Among the eight target sources, r values are in the range from -0.869 to -0.328 , so α and $F_{14.5}$ show strong anticorrelations.

3.3. The Elliptic Fitting. For every targeted source, based on time sequence, and the variable trend of spectral indices (α) dependent on flux densities, we can find the elliptic appearances in the $\alpha - F_{14.5}$ distributions, as seen in Figure 3, where

FIGURE 1: The averaged flux densities and spectral variances of the 8 sources, i.e., 0235+164, 0430+052, 1156+295, 1308+326, 1413+135, 1641+399 (3C 345), 1749+096, and 2251+158 (3C 454.3) as a function of time measured in JD. The averaged flux densities are shown in the upper panels and the spectral variances are shown in the lower panels, in which the filled circles stand for 4.8 GHz, the crosses stand for 8 GHz, and the open circles stand for 14.5 GHz.

FIGURE 2: The correlations between the spectral indices (α) and flux densities ($F_{14.5}$). The solid lines indicate linear fits between α and $F_{14.5}$.

the filled circles, crosses, and open circles stand for the different elliptic appearances. For the whole elliptic appearances, the variable trends are anticlockwise. We use the elliptic curve to make fit, and the results are shown in Figure 3 and listed in Table 3.

In Table 3,

- Col. 1: Name;
- Col. 2: sequence number of elliptic circle;
- Col. 3: T_B , beginning time in Julian Date (JD);

- Col. 4: T_L , end time of elliptic circle in JD;
- Col. 5: the time span in yr;
- Col. 6-11: the coefficients of elliptic fitting.

4. Discussion

Phenomenon that there are cycle structures in blazar variabilities can help us study the theoretical models in both optical and radio light curves. This method has been used in analysis of the quasi-periodic behavior in some blazars

TABLE 2: The linear correlations between the spectral indices and flux densities.

Name	k	Δk	k_0	Δk_0	r	p
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0235 + 164	-0.12	0.02	0.18	0.04	-0.56	< 0.01%
0430 + 052	-0.16	0.02	0.61	0.06	-0.63	< 0.01%
1156 + 295	-0.35	0.03	0.45	0.07	-0.78	< 0.01%
1308 + 326	-0.09	0.02	-0.01	0.07	-0.33	0.01%
1413 + 135	-0.30	0.02	0.03	0.05	-0.87	< 0.01%
3C 345	-0.05	0.01	0.38	0.04	-0.67	< 0.01%
1749 + 096	-0.13	0.01	-0.07	0.04	-0.71	< 0.01%
3C 454.3	-0.06	0.01	0.76	0.04	-0.78	< 0.01%

TABLE 3: The elliptic fitting of 8 blazars.

Name	Seq.	T_B	T_L	$Span$	a	b	c	d	e	f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0235 + 164	1	2444460.459	2446363.609	5.21	0.055	0.242	0.867	-0.226	-0.332	0.159
	2	2446406.692	2448333.603	5.28	0.006	0.238	0.873	-0.255	-0.257	0.214
	3	2448499.623	2450431.415	5.29	0.037	0.206	0.934	-0.171	-0.226	0.064
0430 + 052	1	2444452.616	2446851.019	6.57	0.042	0.207	0.532	-0.274	-0.661	0.401
	2	2447096.251	2449293.781	6.02	-0.024	-0.132	-0.664	0.182	0.629	-0.336
1156 + 295	1	2444740.566	2445899.332	3.17	-0.14	-0.404	-0.412	0.439	0.585	-0.336
	2	2445980.910	2447706.918	4.73	-0.091	-0.383	-0.501	0.328	0.633	-0.296
	3	2449194.517	2451045.216	5.07	0.141	0.428	0.623	-0.412	-0.396	0.286
1308 + 326	1	2443753.007	2448787.811	13.78	-0.046	-0.259	-0.951	0.129	0.006	-0.095
	2	2448792.829	2451203.896	6.6	-0.033	-0.223	-0.577	0.209	0.685	-0.319
1413 + 135	1	2444502.914	2447492.380	8.18	-0.091	-0.479	-0.857	0.064	-0.132	-0.084
3C 345	1	2443657.970	2447349.477	10.11	0.003	0.075	0.659	-0.068	-0.668	0.33
	2	2447419.046	2451142.448	10.19	-0.008	-0.115	-0.509	0.116	0.749	-0.39
1749 + 096	1	2444384.487	2445934.132	4.24	-0.092	-0.276	-0.782	0.352	-0.04	-0.421
	2	2445980.169	2447567.092	4.35	-0.044	-0.217	-0.847	0.224	0.034	-0.394
	3	2447655.821	2451095.560	9.42	0.033	0.285	0.932	-0.079	0.137	0.155
3C 454.3	1	2444543.911	2447805.613	8.93	-0.002	-0.057	-0.465	0.063	0.782	-0.406
	2	2448006.686	2451143.235	8.59	-0.002	-0.059	-0.599	0.055	0.736	-0.304

[17, 18, 35, 36]. Villata & Raiteri (1999) [37] pointed out that the quasi-periodic behavior of Mkn501 can be associated with the helical structures of the jet. Pyatunina et al. (2006) [38] presented that the activity of blazars can be characterized by two parameters: the time scale for the activity of the central engine, T_{act} , and the time scale for the evolution of the jet, T_{evo} . They also proposed that T_{act} determines an 'activity cycle' for the source. Pyatunina (2007) [17] defined the activity cycles as the time intervals between the successive 'core' outbursts. They obtained cycle times of more than 14 yr for 1308+326, about 12 yr for 2223-052, and 12.4 yr for 2251+158 (3C 454.3). *Our circle durations are 6.6 years and 13.78 years for 1308+326, which are consistent with Pyatunina (2007) [17]. For 2251+158, our results show that 8.93 years is shorter than Pyatunina (2007) [17]. Main reason for this should be that we averaged the light curves with the fixed time intervals and then lead to the less observations.*

So, studies about the activity cycles are very important for testing the theoretical models, the jet structure, the properties of central engine, and predicting the likely outbursts.

According to the time interval calculated by the elliptic fitting, we can divide the total light curves into different parts, as seen in Figure 4, and then compare the time intervals of elliptic circle with the quasi-periodicities calculated by the averaged light curves and spectral variances.

The most common tool for periodicity analysis of both evenly and unevenly sampled signals is the periodogram method, which is an estimator of the signal energy in the frequency domain [39]. Lomb (1976) [40] introduced a modified form of this method.

We used the Lomb's periodogram method to calculate the periodicity and used the half width at half maximum (HWHM) to calculate the corresponding error. *For every targeted sources, based on the averaged light curves and spectral indices, we calculate the two periodic signals, then place them together, and choose the common signal as the last periodicity, $P_{\alpha\bar{F}}$. The period signals are displayed in Figure 5, where the upper panels show the averaged flux density, and the lower panels show the spectral variances. The red noise [41] with the noise levels 80%, 90%, 95%, and 99% is also shown in Figure 5.*

FIGURE 3: The $\alpha - F_{14.5}$ circles of the target sources. The elliptic solid lines, dotted lines, and dashed lines indicate the elliptic fit results based on the time sequences.

FIGURE 4: For each source, based on the time interval of elliptic circles, the light curves can be divided into different parts.

For every targeted source, the time interval of elliptic circle (T_c), the periodicities calculated from the averaged light curves, and spectral variances ($P_{\alpha F}$) are listed in Table 4. In order to make comparison, we quoted the quasi-periodicities (Q_F) calculated by Fan, Liu & Yuan (2007) [14], which were calculated from the whole light curves and listed in Table 4.

We use a linear fitting to compare T_c , $P_{\alpha F}$, and Q_F . The results show that $P_{\alpha F} = (0.90 \pm 0.52)T_c - (0.88 \pm 3.63)$, with $r=0.58$, $p=13.3\%$; $Q_F = (0.85 \pm 0.27)T_c + (0.63 \pm 1.92)$, with $r=0.78$, $p=2.1\%$, as seen in Figure 6, with the black line standing for T_c vs. $P_{\alpha F}$ and the red line standing for T_c vs. Q_F . The

two fitting results T_c vs. $P_{\alpha F}$ and T_c vs. Q_F can be consistent with each other, which show strong correlations between the time interval of elliptic circle and quasi-periodicities not only calculated from the whole light curve Q_F , but also calculated from the averaged light curves and spectral variances ($P_{\alpha F}$).

Quasi-periodicity is a hot research topic in radio variability of blazars, but the reason for the periodicity is unclear. There are many models proposed to explain this phenomenon, for example, the binary black-hole model, the thermal instability model, and the perturbation model [14]. Many authors [42–45] apply the model of a moving shock in a relativistic jet with a helical magnetic field to explain the flux

FIGURE 5: The quasi-periodicities of 8 targeted sources. The upper panels are derived from the light curves and the lower panels are from spectral indices. The black lines indicate periodic signals. The red, green, blue, and cyan lines show 80%, 90%, 95%, and 99% red noise level, respectively.

TABLE 4: The time spans and long-term quasi-periodicities of 8 blazars.

Name	T_c /yr	$P_{\alpha\bar{F}}$ /yr	Q_F /yr
0235 + 164	5.21, 5.28, 5.29	1.82 ± 0.57	$5.7 \pm 0.3, 10.0 \pm 1.3$
0430 + 052	6.56, 6.02	$2.98 \pm 0.07, 6.61 \pm 0.84$	$4.3 \pm 0.2, 11.8 \pm 1.1, 13.6 \pm 1.2$
1156 + 295	3.17, 4.73, 5.07	$2.68 \pm 0.15, 3.39 \pm 0.16$	$3.3 \pm 0.1, 3.4 \pm 0.1, 10.4 \pm 1.2$
1308 + 326	6.60, 13.78	9.15 ± 1.75	$5.9 \pm 0.4, 11.8 \pm 1.1$
1413 + 135	8.18	1.79 ± 0.06	8.2 ± 0.4
3C 345	10.11, 10.19	10.13 ± 1.08	$10.9 \pm 0.7, 11.3 \pm 0.4$
1749 + 096	4.24, 4.35, 9.41	2.37 ± 0.36	$6.3 \pm 0.4, 9.0 \pm 0.9$
3C 454.3	8.93, 8.59	6.05 ± 1.06	$6.3 \pm 0.2, 11.8 \pm 1.1$

FIGURE 6: Comparison between T_c and $P_{\alpha\bar{F}}$, Q_F . The black line stands for T_c vs. $P_{\alpha\bar{F}}$, and the red line stands for T_c vs. Q_F .

density variances. The $\alpha - F_{14.5}$ circles might come from the helical jet produced by the binary black holes.

5. Conclusion

In this paper, we choose eight blazars from the UMRAO data base to calculate the spectral indices. We average the light curves with the fixed interval, calculate the averaged flux densities ($\bar{F}$) and spectral indices (α), and then compare the relations between α and $\bar{F}$.

For each source, we obtain strong anticorrelations between α and $F_{14.5}$, which can combine the emission properties and radio variabilities.

Based on time sequence, the $\alpha - F_{14.5}$ distributions showed elliptic appearances and the variable trends are anticlockwise. We use the elliptic curve to make fit, find that the variation directions of elliptic cycle are anticlockwise, and then obtain the cycle durations. Based on the averaged light curves and spectral variances, we calculate the quasi-periodicities and compare them with the cycle durations. The results show that they are consistent with each other, so the elliptic appearances should

come from the helical structures of jets or the jet models containing at least two emitting regions.

Data Availability

The text formatting data used to support the findings of this study come from the University of Michigan Radio Observatory (UMRAO), which are public. The main results have been listed in Tables 1–3, which can be used by others.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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