

Research Article

Measuring Track-Related Pointing Errors on the Nanshan Radio Telescope with an Optical Pointing Telescope

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The need for precise radio telescope pointing has driven great attention to investigating the effect of antenna local deformation or irregularity, such as unevenness in the azimuth track. Although the track-alidade interaction model is commonly used to investigate the pointing effect of track unevenness (such as the works on DSN 34 m antennas and the Green Bank Telescope), few experiments have been done to test the effectiveness of this model independently from the overall pointing model. To address this issue, a method utilizing an optical pointing telescope (OPT) for determining the impact of track unevenness on pointing is proposed. This method uses a group of reference pointing data collected by an OPT mounted at the bottom of the antenna alidade to extract the twist effect of the alidade from the radio telescope pointing data, thus compared with the predicted twist by the track-alidade model. This method was applied to the 26 meter Nanshan Radio Telescope (NSRT 26 m), achieving good agreement with the model-predicted values.

1. Introduction

The pointing accuracy of radio telescopes is typically required to achieve 1/10th of the beam full width at half maximum (FWHM). As the frequency and diameter of radio telescopes increase, their beam FWHM decreases, necessitating higher pointing accuracy requirements. For instance, the Green Bank Telescope (GBT) has 4" blind pointing and 2.8" offset pointing requirements [1] and has achieved a local offset pointing performance of 1.2'' in recent observations [2]. ALMA requires 2" blind pointing and 0.6" offset pointing performance [3]. In addition, QTT demands a 5" pointing accuracy in the 1.3 cm band and a 1.5" pointing accuracy in the 3 mm band [4]. These pointing requirements demand an understanding of additional factors beyond the conventional considerations such as encoder errors and axis geometry errors. These factors include deformation of the alidade, backup structure, and subreflector support legs, caused by gravity,

temperature fluctuations, and wind disturbance, as well as azimuth track local unevenness [5-8]. A model which investigates the relationship between the track unevenness and radio telescope pointing through the interaction between the track and the alidade (track-alidade model for short) was introduced by Constantikes [9] and Xue et al. [10]. Although this model has been commonly used in research regarding the pointing effect of antenna track [5, 9, 11], few experiments have been performed to test the effectiveness of this model through astronomical measurements. The track unevenness profile can be obtained through various industrial methods, such as theodolites or inclinometers, and then the remaining problem of the validation test is obtaining the model-predicted pointing effect through astronomical pointing measurements. Because radio telescope pointing measurement data combines effects of pointing errors from many parts, it remains a challenge to isolate track-related pointing errors in the radio telescope pointing measurement results.

To identify the track-related pointing effects from the radio telescope pointing data, Gawronski [5] used the unique pointing features caused by the gaps between track segments on the Deep Space Network's 34 m antennas. Lew [12] also studied the effects of track segment gaps on the pointing on the Torun 32 m radio telescope. Wen et al. [13] measured the track height profile of the 26-meter Nanshan Radio Telescope (NSRT 26 m), which had no significant gaps, and predicted pointing errors through the trackalidade model, but did not identify the track-related effects from radio telescope pointing data. Previous successful measurements were predominantly based on the gaps in the track, which may not be effective for radio telescopes with less significant track gaps. As welded tracks become more common, this method becomes less viable. Our research proposes a method that uses an optical pointing telescope (OPT) to extract the pointing errors resulting from the track-induced twisting of the alidade from the radio telescope's overall pointing data. This method is not dependent on the gaps of the track and was tested on the NSRT 26 m radio telescope.

2. Methods

2.1. Radio Telescope Pointing Model. A basic pointing model for an Altitude-Azimuth (AltAz) mount telescope includes axis errors, encoder errors, and gravitational elastic

deformation [14]. Based on this model, many radio telescopes have developed their individual pointing models by including individual corrective terms [2, 3, 15, 16]. Equation (1) represents the Field System (FS) recommended pointing model (https://github.com/nvi-inc/fs/blob/main/pdplt/ pdplt), which is commonly used among many radio telescopes in the Very Long Baseline Interferometry (VLBI) stations and meets the pointing requirements well. In equation (1), x and y denote the azimuth and elevation angles of the telescope observations, and ex and ey correspond to the pointing correction values at the observation angles, while P1~P22 represent the model parameters. Specifically, P1 and P7 are the zero-point errors associated with azimuth and elevation encoders, and P9 and P12 indicate their slope errors. P2 is only relevant for equatorial mounts and is usually masked in AltAz mount telescopes. P3 represents the errors caused by nonorthogonality of mount axes, and P₄ represents errors caused by nonorthogonal between the elevation axis and the radio beam. P5 and P6 represent tilt errors of the azimuth axis. P8 denotes the elastic deformation caused by gravity. The rest of the parameters are established empirically and are typically produced by complex mechanisms, lacking a physically defined meaning attributed by the FS. However, some known encoder errors have functional forms $\cos(x)$ and $\cos(2x)$ [6], [17], which should contribute in terms P₁₃, P₁₄, P₁₇, P₁₈, P₁₀, and P₁₁.

$$ex = P_1 - P_2 \cos(phi) \tan(y) + P_3 \tan(y) - P_4 \sec(y) + P_5 \sin(x) \tan(y) - P_6 \cos(x) \tan(y) + P_{12} x + P_{13} \cos(x) + P_{14} \sin(x) + P_{17} \cos(2x) + P_{18} \sin(2x), ey = P_5 \cos(x) + P_6 \sin(x) + P_7 + P_8 \cos(y) + P_{9}y + P_{10} \cos(y) + P_{11} \sin(y) + P_{15} \cos(2x) + P_{16} \sin(2x) + P_{19} \cos(8y) + P_{20} \sin(8y) + P_{21} \cos(x) + P_{22} \sin(x).$$
(1)

2.2. The Track-Alidade Interaction Model Analysis

$$\begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = \frac{1}{2\sqrt{2}r} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -\frac{\sqrt{2}h}{r} & \frac{\sqrt{2}h}{r} & -\frac{\sqrt{2}h}{r} & \frac{\sqrt{2}h}{r} \end{bmatrix} \begin{bmatrix} Z\left(A + \frac{\pi}{4}\right) \\ Z\left(A - \frac{\pi}{4}\right) \\ Z\left(A - \frac{3\pi}{4}\right) \\ Z\left(A - \frac{3\pi}{4}\right) \\ Z\left(A + \frac{3\pi}{4}\right) \end{bmatrix},$$
(2)
$$\begin{bmatrix} ex \\ ey \end{bmatrix} = \begin{bmatrix} 0 & \tan\left(y\right) & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}.$$
(3)

Constantikes [9] and Xue et al. [10] proposed elements of a track-alidade model, and the behavior of a four-wheel supported antenna is illustrated in equations (2) and (3). In the model, Z(A) represents the track height profile, A denotes the azimuth angle, r denotes the radius of the track, and h denotes the height of the alidade. ϕ_x , ϕ_y , and ϕ_z represent the pointing error rotation angles along the three coordinate axes, respectively. According to the model, ϕ_x and ϕ_y result from the tilting of the entire alidade, whereas ϕ_z arises from the twist of the alidade.

$$Z(A) = a_0 + \sum_{n=1}^{\infty} Z_n(A), Z_n(A) = a_n * \cos(n * A + \varphi_n)$$
(4)

The track-alidade model generates both tilt and twist effects on telescope pointing. To determine the appropriate effect for astronomical observations, the track height profile, Z(A), is expressed through Fourier expansion in equation (4), where the series index *n* represents the spatial frequency. According to equation (2), the calculation results of Fourier series terms in equation (4) can be classified into four categories depending on the remainder of n/4, as shown in Table 1. Typically, low-frequency components of track height profile possess relatively larger amplitudes. For instance, the pointing model equation (1), which corrects only spatial frequency n < 3 azimuth errors, has proven effective on lots of radio telescopes. Research on the Tianma Radio Telescope (TMRT) 65 m radio telescope indicated that the amplitude of the spatial frequency n > 4 components are significantly lower than the low-frequency components [18]. In addition, measurements of track unevenness on the Sardinia Radio Telescope (SRT) showed that the lower frequency components have greater amplitudes [19]. Therefore, it can be inferred that high-frequency components are less important for most antennas, and then only series terms with n < 4 are considered in the following:

- (1) The n=0 component represents the overall track height, which does not contribute to pointing deviation.
- (2) The n = 1 component corresponds to the tilt of the azimuth axis, which is not considered as local unevenness and is corrected by P₅ and P₆ in pointing model equation (1).
- (3) The n = 2 component results only in the ϕ_z rotation, with an amplification of 2 h/r in magnitude compared to the typical tilt angle Z(A)/r. The relationship to the azimuth angle is in the form of cos (2*x*), corresponding to P₁₇ and P₁₈ in pointing model equation (1).
- (4) The n = 3 component results in the ϕ_x and ϕ_y rotations, with the same magnitude as Z(A)/r. The relationship to the azimuth angle is in the form of cos (3*x*), which are not included in pointing model equation (1).

In most radio telescopes, the alidade height (h) is greater than the track radius (r). As a result, the pointing effect for n = 2 is at least twice that of n = 3. Wen et al. [13] measured

TABLE 1: Pointing effects resulting from different spatial frequency components of the track height.

п	ϕ_x	ϕ_y	ϕ_{z}
4i	0	0	0
4 <i>i</i> + 1	$Z_n(A + \varphi'_n)/r$	$Z_n(A+\varphi_n'')/r$	0
4 <i>i</i> + 2	0	0	$-2h/r^2 * Z_n(A)$
4 <i>i</i> + 3	$Z_n(A+\varphi'_n)/r$	$Z_n(A+\varphi_n'')/r$	0

the track height profile of the NSRT 26 m radio telescope and made a prediction of the pointing effect using the trackalidade model. The prediction shows that the azimuth pointing error mainly exhibits a cos (2x) characteristic with an amplitude of approximately 5", while the elevation error mainly exhibits a cos (3x) characteristic of approximately 2". These results agree with the analysis above.

The n=2 and n=3 components are both suited for astronomical observations. The n=2 component has a higher amplitude, making it less difficult in detection. However, the encoder errors also exhibit a similar cos (2x) characteristic, which necessitates additional measurement techniques (such as the OPT technique described in the next section) to distinguish the effect of alidade twist from other effects. On the other hand, the n=3 component is easier to distinguish in radio pointing data, but its amplitude may be smaller and hence more challenging to detect. Moreover, a higher spatial frequency requires a denser sampling, leading to increased time costs.

2.3. Optical Pointing Telescope Aided Measurement. The OPT has been adopted by several radio telescopes, for example, ALMA [3, 20], Medicina 32 m [21], CCOSMA, and DATE5 scaled prototype [22]. The OPT measures its' pointing direction by capturing star images in the optical band. The direction of the platform where the OPT is installed aligns with the OPT direction, and thus is measured simultaneously. The effectiveness of utilizing an OPT to measure pointing errors of radio telescopes has been demonstrated at several observatories as cited above; the benefits of this method include its speed and the availability of a sufficient number of pointing targets for the OPT. However, those works primarily focused on final pointing models of radio telescopes, rather than on local structure deformation effects. Consequently, the OPTs were mounted above the elevation shaft to maintain alignment with the radio telescope beam.

The twist ϕ_z of the alidade, according to equation (2), is proportional to the height (*h*), thus it should not affect positions where the height is zero. Figure 1 illustrates the twist difference at the top and bottom of the alidade. By placing an OPT at the bottom of alidade, it is possible to capture pointing data free of the alidade twist effects, and at the same time with encoder errors remaining in the data. This OPT pointing data can serve as a reference for obtaining the alidade twist effect ϕ_z from radio pointing data. The difference between the radio pointing and this OPT pointing will remain only the ϕ_z effect. With the coupling between the effect of alidade twist and encoder errors solved, this study focuses on the n = 2 component due to its higher amplitude.



FIGURE 1: The difference in twist between top and bottom positions of the alidade according to the track-alidade model. The ideal and twisted states of the alidade are depicted in black and red colors, respectively. The dashed lines at the top denote the elevation axis, while the arrow vertically above the axis indicates the pointing direction of the instruments above the elevation axis. The arrow at the alidade's bottom indicates the measuring direction of the OPT.

Since the P_{17} and P_{18} parameters in equation (1) encompass all pointing errors of the n=2 component, the determination of the n=2 twist effect can be achieved by comparing the differences in parameter values between the radio and OPT pointing models rather than comparing the raw pointing data. To ensure consistency in parameter definitions between the radio and OPT pointing models, it is desirable for their initial pointing directions to be parallel. As the OPT system does not move along with the antenna in the elevation direction, there will always be an antenna elevation angle that aligns the radio beam with the OPT elevation, thereby eliminating any specific requirement for the OPT elevation angle. However, in azimuth direction, since the OPT moves along with the antenna, the initial installation angle between the radio beam and OPT will always exist. Therefore, we employ certain methods described in Section 3.2 to minimize this azimuth angle between the OPT and radio beam.

3. Experiment

3.1. The Radio Telescope. The NSRT telescope was used in this study, which has a 26-meter aperture and an AltAz mount. It is equipped with receivers in L, S/X, C, K, and Q bands [23]. The measured pointing RMS of the telescope is between 8" and 20", which is influenced by the individual measurement errors at different observing bands. The pointing model of the telescope is a subset of equation (1), with P₂, P₁₀, P₁₅, P₁₆, P₂₁, and P₂₂ masked for empirical reasons. The telescope utilizes the cross-scan observation

method on the calibration source to acquire (x, y, ex, ey) data in the direction of observation. By conducting multiple observations at various directions, multiple sets of (x, y, ex,ey) data can be obtained. These data are utilized for fitting the pointing parameters P1~P22 in equation (1) in subsequent observations, the pointing parameters and equation (1) pointing model are employed to generate the pointing corrections. The telescope's pointing model in service during the experiment was generated from X-band cross-scan data on 02-JUN-2020, which involved 198 effective (x, y, ex, ey) data. The residual of the pointing data after the pointing model fitted is 14.33 arc seconds in RMS, which provides an estimation of the uncertainty of the P1~P22 parameters derived from the fitting process. A Monte Carlo experiment base on this 14.33 arc seconds RMS demonstrated that the uncertainty of the P1~P22 parameters ranges between 0.7 and 2.5 arcseconds.

3.2. The Optical Pointing Telescope Device. In previous works, the OPTs were installed on the elevation structures, thereby shared the tracking capability of the antenna's AltAz mount to ensure continuous tracking of a specific celestial object. However, when positioned as depicted in Figure 1, the OPT is unable to utilize the antenna's elevation motion and consequently cannot track celestial objects' movements. Therefore, an OPT that does not require tracking a specific object would be a suitable choice for this study. Increasing the FOV can increase the numbers of targets detected, thereby eliminating the need for specific target tracking. However, this also results in an increase in plate scale (arcsecond/pixel), which amplifies pixel-related errors and consequently diminishes the pointing measurement accuracy of the OPT. Nevertheless, despite the diminished accuracy, some wide-FOV OPTs still provide a precision of approximately 1" [24, 25], which is sufficient for radio telescope pointing measurements. Inspired by these wide-FOV OPTs, we developed an OPT system with a 12 * 7 degrees FOV, which includes a large aperture lens, a global shutter camera, a GPS module, and a control module. This system uses a 1920 * 1200 pixels CMOS image sensor, achieving a plate scale of about 23 arcsecond/pixel. The system has the capability to capture more than 100 stars on average in an exposure of 0.1 seconds. Using all observed stars in FOV to solve the direction, the final pointing measurement uncertainty can be far less than the plate scale of the imaging system.

The data processing flow is depicted in Figure 2:

- The Python SEP (https://sep.readthedocs.io/en/v1.1. x/tutorial.html) package was utilized to extract the image coordinates (u, v) of each star point.
- (2) A 2D polynomial was employed to characterize the distortion between the actual camera and an ideal camera. This distortion model transforms the measured (u, v) values to those under an ideal camera, denoted as (u', v').
- (3) The astrometry.net (https://astrometry.net/ summary.html) package was utilized to determine

the corresponding catalog target for each imaged star based on an array (u', v') * n containing all image coordinates of star points within the field of view (FOV). Subsequently, the J2000 coordinates (a, d) of the matched catalog target were recorded after each (u', v') data.

- (4) The ad2uv projection uses a rotation matrix and a TAN projection to construct a J2000 coordinate to image coordinate transformation relationship. Using this relationship, the J2000 coordinate of the matched catalog target (a, d) is converted to image coordinate, denoted as (u, v)_catalog. Then the least squares method is used to optimize the rotation and projection parameters to minimize the residual between the (u, v)_catalog and (u', v'). The optimized rotation and projection parameters are used to generate a function uv2ad that can convert image coordinates to J2000 coordinates.
- (5) Select an appropriate image coordinate (u0, v0) as a representative of the optical pointing and then calculate the corresponding (a0, d0) coordinates using the uv2ad function.
- (6) The eq2hor coordinate transformation utilizes the shutter action timestamps by the GPS system to convert the optical pointing from J2000 coordinate system to AltAz coordinate system.

Figure 3(a) shows the location of the OPT, which is installed on a steel plate structure near one of the supporting wheels of the alidade. There is no restriction on the elevation angle of the OPT, so a middle elevation angle is chosen to avoid the atmosphere affect in the lower elevation and the SFL (Samson-Flamsteed) projection affect at higher elevation. The OPT was aligned parallel to the radio beam in azimuth using two methods. Firstly, the OPT was attached to a steel plate that lies approximately in the plane defined by the side A-shaped frame of the alidade. This ensured that the direction of the OPT was approximately perpendicular to the elevation shaft and had almost the same azimuth angle as that of the radio beam. Secondly, a refined parallel alignment between them was achieved through data processing method, as shown in Figure 3(b). The red Pix_0 is the center pixel of the image and was commonly considered as the OPT's pointing direction. This direction inevitably has some deviation from the beam azimuth angle due to the structure and installation errors. So a new pixel, denoted as the green Pix₁ in Figure 3(b), is selected as the representative pixel (u0, v0) when applying the uv2ad function. This new representative pixel further minimized the deviation from the radio beam azimuth angle.

3.3. Observation and Results. In the experiment, the OPT was used to measure 12 uniformly distributed directions spanning the 360-degrees azimuth range. The directions are presented in the "Encoder Az" column of Table 2. The antenna Az angle was steered to each direction and stand still for a few seconds. Then, the OPT was controlled to perform 20 measurements at a rate of 10 Hz. The statistical mean

value and variance of the measurements were derived, and they are presented in the "OPT Az₀," "OPT El₀," "OPT Az", and "OPT El" columns of Table 2, respectively. The "OPT Az_0 ," "OPT El₀" data were derived with the center Pix₀, while the "OPT Az" and "OPT El" were derived with the Pix₁. By comparing with the "Encoder Az" data, it can be found that the newly selected Pix₁ successfully minimize the angle deviation to less than 1 degree, while the result of center Pix₀ remains about 5 degrees. The difference between the "OPT Az" column and the "Encoder Az" column is denoted as *ex*, and the difference between the "OPT El" column and its mean value as *ey*, the values are presented in the last two columns of Table 2. This *ex* and *ey* values are expected to include most of the pointing errors expect for the alidade twist effect, as discussed in Section 2.3.

By letting *x* denote the "Encoder Az" column and *y* denote the average value of the "OPT El" column, the pointing parameters of the OPT can be obtained through the fitting of the pointing model equation (1). The OPT pointing model and the radio pointing model exhibit a high similarity in terms of overall trend, as depicted in Figure 4. A comparison of parameters related to the alidade, including P5, P6, P12, P13, and P14, are presented in Table 3. The P5 and P6 correspond to azimuth axis tilt errors, while the other parameters mainly relate to azimuth encoder errors. In theory, these parameters should be identical in both the radio and OPT pointing models, implying the "Difference" column should be zero. However, considering measurement errors, the "Difference" column can be assumed to follow a normal distribution $N(0, \text{sigma}^2)$, where the sigma parameter represents an estimate of the measurement uncertainty associated with this OPT measurement method. The sigma parameter of the distribution can be obtained by calculating the standard deviation of the "Difference" column, resulting in a value of 1.33 arcseconds. Taking into account the uncertainties associated with the P1~P22 parameters in Section 3.1, this obtained sigma value appears reasonable.

The P₁₇ and P₁₈ parameters correspond to various pointing effects characterized by n = 2 spatial frequency. In the radio pointing model, these effects include alidade twist, encoder errors, and other factors, while in the OPT pointing model; they represent other errors except for alidade twist. Therefore, the difference between the two pointing models is the effect of alidade twist, and is just the measured result by the proposed method in this study. Wen et al. [13] measured the azimuth track height profile of NSRT 26 m antenna and predicted the ex curve based on the track-alidade mode. This ex curve can serve as a model-predicted twist value for comparison with the measured results in this work. As presented in Table 4, the columns labelled "Radio model" and "OPT model" represent P₁₇ and P₁₈ parameters fitted using radio and OPT pointing data through equation (1) pointing model. The "Measured twist" column is the difference value between radio and OPT, and "Model predicted twist" column is the fitting result of the model-predicted ex curve in the form of $P_{17} * \cos(2x) + P_{18} * \sin(2x)$. Figure 5 visualizes the "Radio model," "Measured twist" columns of Table 4, along with the model-predicted ex curve. The measured amplitude of the twist pointing effect is approximately 3.6 arcseconds, with a discrepancy of only about 1



FIGURE 2: The data processing pipeline of the OPT.



FIGURE 3: The installation position of the OPT on the radio telescope (a) and the fine parallel data processing technique (b).

arcsecond compared to the model-predicted value. Considering that the estimated uncertainty for this measurement method in the previous section amounts to 1.33 arcseconds, it can be inferred that the amplitude surpasses this level of uncertainty while the discrepancy remains below it. These findings indicate a significance of the measured alidade twist and a generally consistency of measured and modelpredicted twist within an acceptable range of accuracy. Visual similarity between the two curves can also be found in Figure 5, suggesting that the track-alidade model can be effectively applied to the NSRT 26 m antenna.

By comparing the parameters of the "Radio model" and "Model predicted twist" in Table 4, it can be found that there is a discrepancy of approximately 6 arc seconds in total amplitude, which is mainly caused by the encoder errors and is measured by the OPT. This disparity is clearly depicted in Figure 5 as the amplitude and phase deviation exhibited by the red curve when compared to the other two curves. Consequently, it becomes evident that utilizing the parameters from the radio model alone cannot serve as an accurate measurement of the alidade twist, hence necessitating an OPT reference method.

4. Discussion

Our method utilizes the OPT to provide an intermediate reference frame that enables us to make targeted measurements of alidade twist in the presence of various pointing effects such as encoder errors. Previous works about the alidade twist, such as the DSN 34 m, studied the significant features of the jumps at the track segments' gaps, which are unlikely to couple with other pointing effects and are hence easy to be isolated. In contrast, our approach does not depend on significant features in the pointing effects, making it more universal and suitable for measuring alidade deformation in other radio telescopes.

In this work, we chose to focus on measuring the n=2 frequency component of the track height profile, which has the highest theoretical amplitude under the assumption of high-frequency attenuation, to reduce the challenge of detection. The primary objective of this study on n=2 is to validate the reliability of the track-alidade model, for its application on more complex track profile. The pointing errors induced by the n=2 track unevenness has already been corrected through terms P₁₇ and P₁₈ in equation (1).

TABLE 2: Results of OPT pointing observations.

Encoder Az	OPT Az ₀	OPT El ₀	OPT Az	OPT El	ex ('')	ey ('')
0°	$355^{\circ}.5597 \pm 0.44''$	$53^{\circ}.2649 \pm 0.29''$	$0^{\circ}.0096 \pm 0.68''$	$53^{\circ}.2672 \pm 0.32''$	34.6	-34.3
30°	$25^{\circ}.5530 \pm 0.49''$	$53^{\circ}.2665 \pm 0.24''$	$30^{\circ}.0032 \pm 0.92''$	$53^{\circ}.2692 \pm 0.28''$	11.5	-27.1
60°	$55^{\circ}.5444 \pm 0.43''$	$53^{\circ}.2652 \pm 0.23''$	$59^{\circ}.9944 \pm 0.67''$	$53^{\circ}.2682 \pm 0.28''$	-20.2	-30.7
90°	$85^{\circ}.5391 \pm 0.58''$	$53^{\circ}.2700 \pm 0.21''$	$89^{\circ}.9895 \pm 0.71''$	$53^{\circ}.2732 \pm 0.22''$	-37.8	-12.7
120°	$115^{\circ}.5393 \pm 0.50''$	$53^{\circ}.2737 \pm 0.19''$	$119^{\circ}.9902 \pm 0.66''$	$53^{\circ}.2770 \pm 0.23''$	-35.3	1.0
150°	$145^{\circ}.5422 \pm 0.36''$	$53^{\circ}.2812 \pm 0.29''$	$149^{\circ}.9938 \pm 0.59''$	$53^{\circ}.2844 \pm 0.31''$	-22.3	27.7
180°	$175^{\circ}.5473 \pm 0.45''$	$53^{\circ}.2820 \pm 0.31''$	$179^{\circ}.9991 \pm 0.76''$	$53^{\circ}.2850 \pm 0.30''$	-3.2	29.8
210°	$205^{\circ}.5551 \pm 0.56''$	$53^{\circ}.2821 \pm 0.31''$	$210^{\circ}.0070 \pm 0.93''$	$53^{\circ}.2847 \pm 0.38''$	25.2	28.7
240°	$235^{\circ}.5638 \pm 0.48''$	$53^{\circ}.2830 \pm 0.27''$	$240^{\circ}.0156 \pm 0.70''$	$53^{\circ}.2851 \pm 0.27''$	56.2	30.2
270°	$265^{\circ}.5631 \pm 0.37''$	$53^{\circ}.2784 \pm 0.29''$	$270^{\circ}.0144 \pm 0.41''$	$53^{\circ}.2804 \pm 0.30''$	51.8	13.3
300°	$295^{\circ}.5620 \pm 0.38''$	$53^{\circ}.2742 \pm 0.26''$	$300^{\circ}.0128 \pm 0.49''$	$53^{\circ}.2763 \pm 0.33''$	46.1	-1.5
330°	$325^{\circ}.5651 \pm 0.41''$	$53^{\circ}.2679 \pm 0.22''$	$330^{\circ}.0152 \pm 0.54''$	$53^{\circ}.2699 \pm 0.24''$	54.7	-24.5



FIGURE 4: A preview of the radio and OPT pointing data, along with the pointing models fitted by the data. The (a) displays the azimuth pointing error, and the (b) the elevation pointing error. The red triangles indicate the *ex* and *ey* data of OPT in Table 2, and the corresponding red line is the pointing model based on that data. The black dots are the radio pointing data observed on 02-JUN-2020. The black line is the radio pointing model on the elevation of OPT observation.

TABLE 3: Alidade-related parameters of radio pointing model and OPT pointing model.

Pointing parameter	Radio model ('')	OPT model ('')	Difference ('')
P ₅	33.23	31.48	1.75
P ₆	11.27	12.92	-1.65
P ₁₂	0.89	-0.45	1.34
P ₁₃	1.24	1.10	0.14
P_{14}	1.00	2.13	-1.13

TABLE 4: n = 2 spatial frequency-related parameters.

Pointing parameter	Radio model ('')	OPT model ('')	Measured twist $('')$	Model predicted twist ('')
P ₁₇	-5.27	-4.92	-0.35	-0.52
P ₁₈	-8.23	-4.68	-3.55	-4.56

Hence, there is no need for further pointing correction after the measurement of this work. The n=3 and higher frequency components are not included in equation (1) pointing model. Their incorporation into the pointing model is expected to improve the pointing performance. In our future work, we intend to investigate the n = 3 components using OPT; however, prior to that, we plan to address the time-consuming issue associated with this method. The current method requires the antenna to decelerate until it reaches a standstill for each measuring point before the OPT



FIGURE 5: Measured and model predicted values of the alidade twist. The red solid line shows the *ex* values obtained by extracting the P_{17} and P_{18} terms from the overall pointing model, where the P_{17} and P_{18} coefficient values were obtained from the "Radio model" column of Table 4. The blue solid line represents the measured alidade twist values obtained from the pointing model using P_{17} and P_{18} parameters in the "measured twist" column of Table 4. The blue solid line represents the provide 4. The black dashed line represents the predicted *ex* value by the track-alidade model, based on the height profile measured by Wen et al. [13].

measurement, resulting in a relatively significant time cost (about $10\sim30$ s depending on different antennas). The high spatial frequency component with $n \ge 3$ demands more measuring points, thereby increasing the time spent on deceleration. Therefore, we aim to develop a measurement method in the future that does not rely on the standstill state and subsequently apply this approach to investigate the effect of the $n \ge 3$ component.

This method could also be used to measure other types of alidade deformations, as well as the deformations of other parts of the antenna. For instance, the P_8 term in equation (1) represents the elastic deformation caused by gravity and shares the same $\cos(y)$ form as the P_{10} term, which accounts for elevation encoder error. This coupling phenomenon is similar to the alidade twist measurement discussed in this work. An OPT installed on the elevation axis can decouple the effects of elevation encoders and gravitational deformations. The use of OPT-assisted measurements is beneficial not only for improving pointing performance requirements but also for structural and deformation studies of radio telescopes.

5. Conclusions

This work analysed the characteristics of the track-alidade model and proposed a method to measure the alidade twist by adding reference pointing measurements with the OPT at the bottom of the alidade. The alidade twist effect was determined with astronomical observations by comparing these OPT reference pointing measurements with radio pointing measurements. We were able to successfully test this method on the NSRT 26 m and measure the pointing errors resulting from the track-induced twisting of the alidade. The measurement results were consistent with the model-predicted curve of the previous study with an accuracy of 1.33". Due to its low time cost and versatility, this method has the potential to be very useful for other large radio telescopes seeking to mitigate pointing errors resulting from track-related twisting of the alidade.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- R. M. Prestage, K. T. Constantikes, T. R. Hunter et al., "The Green Bank Telescope," *Proceedings of the IEEE*, vol. 97, no. 8, pp. 1382–1390, 2009.
- [2] E. White, F. D. Ghigo, R. M. Prestage et al., "Green Bank Telescope: overview and analysis of metrology systems and pointing performance," *Astronomy and Astrophysics*, vol. 659, p. A113, 2022.
- [3] J. G. Mangum, J. W. M. Baars, A. Greve et al., "Evaluation of the ALMA prototype antennas," *Publications of the Astronomical Society of the Pacific*, vol. 118, no. 847, pp. 1257–1301, 2006.
- [4] N. Wang, Q. Xu, J. Ma et al., "The Qitai radio telescope," *Science China Physics, Mechanics and Astronomy*, vol. 66, no. 8, Article ID 289512, 2023.
- [5] W. Gawronski, "Control and pointing challenges of large antennas and telescopes," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 2, pp. 276–289, 2007.
- [6] J.-Q. Wang, L.-F. Yu, R.-B. Zhao et al., "High precision pointing error detection method for large radio telescope," *Scientia Sinica Physica, Mechanica & Astronomica*, vol. 47, no. 12, Article ID 129504, 2017.
- [7] L. Fu, J.-Q. Wang, Y.-B. Jiang et al., "Improvement of the pointing precision of the Tianma radio telescope with an inclinometer measurement system," *Experimental Astronomy*, vol. 48, no. 1, pp. 49–64, 2019.
- [8] A. Attoli, F. Stochino, F. Buffa et al., "Sardinia radio telescope structural behavior under solar thermal load," *Structures*, vol. 39, no. 5, pp. 901–916, 2022.
- [9] C. Kim, "The GBT pointing model," Precision Telescope Control System, National Radio Astronomy Observatory Green Bank PTCS Project Note, vol. 53, 2006.

- [10] S. Xue, K.-T. Ma, and C.-S. Wang, "A conceptual investigation of a large radio telescope support point number effect on its pointing accuracy," *International Journal of Antennas and Propagation*, vol. 2020, Article ID 2706309, 16 pages, 2020.
- [11] N. Li, J. Wu, B.-Y. Duan, and C.-S. Wang, "Effect of the rail unevenness on the pointing accuracy of large radio telescope," *Acta Astronautica*, vol. 132, pp. 13–18, 2017.
- [12] B. Lew, "Improving pointing of Torun 32-m radio telescope: effects of rail surface irregularities," *Experimental Astronomy*, vol. 45, no. 1, pp. 81–105, 2018.
- [13] H.-X. Wen, Q. Xu, and N. Wang, "Measurement of NSRT 26m antenna track unevenness and influence on pointing accuracy," *Astronomical Research and Technology*, vol. 16, no. 02, p. 158, 2019.
- [14] P. T. Wallace, "A rigorous algorithm for telescope pointing," Advanced Telescope and Instrumentation Control Software II, Volume 4848 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, SPIE, St Bellingham, WA, USApp. 125–136, 2002.
- [15] M. L. Meeks, J. A. Ball, and A. B. Hull, "The pointing calibration of the Haystack antenna," *IEEE Transactions on Antennas and Propagation*, vol. 16, no. 6, pp. 746–751, 1968.
- [16] A. Greve, J. F. Panis, and C. Thum, "The pointing of the IRAM 30-m telescope," A&AS, vol. 115, p. 379, 1996.
- [17] B. Hou, B. Zhou, X. Li et al., "Periodic nonlinear error analysis and compensation of a single-excited petal-shaped capacitive encoder to achieve high-accuracy measurement," *Sensors*, vol. 19, no. 10, p. 2412, 2019.
- [18] J. Wang, Y. Jiang, W. Gou, R. Zhao, L. Yu, and Y. Jiang, "Using inclinometer to improve the pointing accuracy of TM65 m radio telescope," *Scientia Sinica Physica, Mechanica* & Astronomica, vol. 49, no. 12, Article ID 129502, 2019.
- [19] T. Pisanu, F. Buffa, S. Poppi et al., "The SRT inclinometer for monitoring the rail and the thermal gradient effects on the alidade structure," *Ground-based and Airborne Telescopes V*, *Volume 9145 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, p. 91454R, SPIE, St Bellingham, WA, USA, 2014.
- [20] N. Ukita, B. Ikenoue, and M. Saito, "Optical seeing measurements with an optical telescope on a radio antenna," *Publications of the National Astronomical Observatory of Japan*, vol. 11, pp. 1–11, 2008.
- [21] S. Poppi, C. Pernechele, T. Pisanu, and M. Morsiani, "Highprecision pointing with the Sardinia radio telescope," *Ground*based and Airborne Telescopes III, Volume 7733 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, SPIE, St Bellingham, WA, USA, 2010.
- [22] Y. I. N. G. Xiao-ke, L. O. U. Zheng, L. Wei, and Z. U. O. Yingxi, "Study on the pointing error model of terahertz telescopes with optically assisted pointing measurements," *Chinese Astronomy and Astrophysics*, vol. 46, no. 2, pp. 105–120, 2022.
- [23] Q. Xu, L. Lin, and N. Wang, "Review of the refurbishment project for NSRT," Ground-based and Airborne Telescopes VII, Volume 10700 of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, SPIE, St Bellingham, WA, USA, 2018.
- [24] C. C. Liebe, "Accuracy performance of star trackers- a tutorial," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38, no. 2, pp. 587–599, 2002.
- [25] M. Sánchez-Portal, A. Marston, B. Altieri et al., "The pointing system of the Herschel space observatory. Description, Calibration, Performance and improvements," *Experimental Astronomy*, vol. 37, no. 2, pp. 453–479, 2014.