Hindawi
Advances in Astronomy
Volume 2024, Article ID 8446071, 16 pages
https://doi.org/10.1155/2024/8446071

*Research Article*

# A Study of the Early Cosmic Dynamics in a Multifield Model of Inflation and Curvature Perturbations

**Muhammad Zahid Mughal**[1] **Feroz Khan**[2] **Fuad A. Awwad**[3] **and Emad A. A. Ismail**[3]

[1]*Department of Mathematics, Grand Asian University Sialkot, Sialkot 51310, Punjab, Pakistan*
[2]*Department of Mathematics, HITEC University Taxila, Taxila, Punjab, Pakistan*
[3]*Department of Quantitative Analysis, College of Business Administration, King Saud University, P.O. Box 71115, Riyadh 11587, Saudi Arabia*

Correspondence should be addressed to Muhammad Zahid Mughal; muhammad.zahid@gaus.edu.pk

Received 8 December 2023; Revised 1 March 2024; Accepted 15 March 2024; Published 17 April 2024

Academic Editor: Kovacs Tamas



In this work, we study the cosmological dynamics of the early universe by employing a small field potential in the context of multifield inflation. By investigating analytically the cosmological observables, such as the number of e-folds $N$, potential slow-roll parameters $\epsilon_V$, $\eta_V$, and spectral index $n_s$ which carry significant information, we show that they impact the inflationary universe considerably. The tensor-to-scalar ratio concerning the curvature perturbation is worked out for the potential under consideration which is another significant observable and comes out to be nonnegative. In multifield models of inflation, both types of curvature and isocurvature perturbations exist, while, in the present work, during the slow-roll, isocurvature perturbations are suppressed and therefore can be neglected. The surviving perturbations which are due to the curvature only can be tackled suitably by a mechanism developed by Sasaki and Stewart known as Sasaki–Stewart formalism. The effect of augmenting the number of e-folds on the power spectrum through the spectral index and its impact on the observable parameters of the slow-roll inflationary phase is observed by carrying out the analysis. It is observed analytically that the spectrum of multifield inflation is effectively different than its corresponding single-field inflation. The field values and their masses affect the results profoundly at the time of horizon-crossing.

## 1. Introduction

Inflation as an exponentially growing phase in the history of the very early universe was introduced in the framework of the standard model of cosmology which entails the big bang origin of the observable universe. It is believed to have resolved a number of enigmatic issues encountered in the standard model. It sets the initial conditions for the big bang origin on one hand and explains the growth of cosmic structure on the other hand. There is a large number of inflationary models proposed with single as well as with multiple scalar fields. The inflationary models with more than one light field are referred to as multifield models. Obtaining the inflationary era in multifield models with two or more fields has comparatively more perspectives, but less predictive power of observables [1–4]. A notable difference between single-field and multifield models is their sensitivity to respond to the initial conditions. Multifield models increase the characterization and features of the adiabatic spectrum by creating isocurvature or entropy perturbations that have an impact on anisotropies of the CMB [1, 3, 5–8]. The generation of density perturbations in some multifield models is treated in such a way as if to decouple it from the dynamics of the inflationary era.

At the end of inflation in a multifield scenario, primordial density perturbations can be created on account of the inhomogeneous phase of reheating or modulated hybrid inflation if the decaying of dark energy turns to be sensitive to the local values of multifields except for the inflation [9–11]. The curvaton inflation on the other hand traverses its path averse to it [12–14] where, at some point, after inflation a weakly coupled field produces primordial density perturbations during its decay into radiation. Isocurvature density perturbations are also a relict from the curvaton scenario but the abundance of it comes out in many distinct ways in relation to thermal equilibrium abundance at the time when the curvaton sets to decay [15]. The realization of inflation can also be availed through multifield scenarios comprising the double and hybrid models as well. The inflationary theory [16] resolves the flatness, homogeneity, and monopole problems in a very natural way and leads to predict the density perturbations which show approximately the scale invariance and are compatible with the current observational data available.

Cosmic inflationary models propound that the early universe underwent an incipient phase of accelerated expansion driven by the dynamics of single, double, or more scalar fields [17, 18], while the brief phase of accelerated expansion to be called inflation takes the responsibility of seeding all structure formation of the universe we observe today in the form of quantum fluctuations accompanying it in the very tiny fraction of the first second. Quantum fluctuations [19] are considered to intertwine with the exponential expansion; however, they are supposed to be frozen in Hubble radius while they horizon-cross it. Nonetheless, once the inflationary phase comes to an end, they stretch to cosmological scales and grow out to the scale of the present-day universe we do observe it [20].

Inflationary dynamics are backed up by the contribution of generic entropic perturbations to the adiabatic one. In single-field models of inflation, adiabatic perturbations are produced when in the models of multifield both types of perturbations are generated, namely, entropic and adiabatic [21]. Inflation has thus transformed into the robust paradigmatic theory for understanding the properties and characteristics of our entire observable universe. Nonetheless, inflation models consisting of a single field are characterized by a few fine-tuning problems in general, particularly on the parametric scales. For instance, the masses of the fields and their couplings with one another, in addition to the values of the fields , make it imperative to substantially transform the theories concerning the high-energy regime more realistic.

On taking a number of fields altogether into consideration, it was observed that they can function and operate in coordination with one another in order to give rise to a brief era of inflation with the help of the assisted inflation mechanism proposed by Liddle et al. [22], although neither of these fields has the ability to sustain the inflationary era individually and alone. The multifield inflationary models diminish the enigmatic problems faced by single-field inflationary models which can be thought of as a fascinating feature to substantiate and materialize the inflationary scenario. The evolution of the universe faces problems when we use a single tachyonic field to derive inflation because, in this case, a larger anisotropy is likely to be generated. Piao studied a model of assisted inflation [23] by taking multi-tachyon fields to derive an inflationary period. The spectrum of curvature perturbations of multifield inflation with a small field potential was studied [24] by Ahmad et al. They put to use the Sasaki–Stewart formalism and reached the results which were obtained with the assumption that isocurvature, *i.e.*, entropy perturbations can, nonetheless, be neglected. Piao investigated that [25] primordial density perturbations can be possibly generated by taking into account the sufficient number of e-folds and by making the use of the decaying speed of sound in a gradually expanding phase of the universe. Some interesting and appealing topics concerning the applications can be studied in [26–31].

Cai et al. investigated the entropy perturbations in inflation and computed the entropy corrections to the power spectrum of curvature perturbations [32] by finding out a transfer coefficient analytically. He described a correlation function between entropic and curvature perturbations for this purpose. The mechanism of relating the power spectrum to the slow-roll parameters is described in [33, 34] with a detailed account presented there. The governing evolutionary equations in the background for the process of driving the primordial power spectrum [35] are given by $\dot{H} = -\dot{\phi}^2/2$ and $\ddot{\phi} = -3H\dot{\phi} - V,_{\phi}(\phi)$. There are appealing related discussions with regard to the application and implementation of these ideas [36–42].

Avgoustidis et al. investigated [43] the importance of slow-roll corrections in multifield inflationary models when the evolution of cosmological perturbations in the form of quantum fluctuations takes place. They studied the evolution of curvature and isocurvature perturbations to the next order in the regime of slow-roll inflation. Cosmological observables are sensitive during the time of reheating phase in multifield inflationary models. A study was carried out by Hotinli et al. to examine [44] the observables during this phase by devising a method that permits a method, implementing the semi-analytic remedying of the effect of perturbative reheating on cosmological perturbations, and using the technique of abrupt decay approximation. They further showed that the rate at which the scalar fields decay into radiation affects the tensor-to-scalar ratio "$r$" and scalar spectral index "$n_s$." A method was presented by Frazer [45] for deriving the analytical expression of the density function of cosmological observables in multifield models of inflation using semiseparable potentials. Frazer found that the sharp peak of the density function is very faintly sensitive to the distribution of initial conditions which means inflationary models of multifield may possess a density function for the observables that peaked sharply.

The dynamics of the exact multifield scenarios have been investigated in the classical style in [46] for the case of the hybrid inflationary model. Asadi and Nozari investigated a multifield model with two fields to study its reheat phase in order to have some constraints in the parametric space. They found the number of e-folds and the temperature during this era of the reheating phase of their model [47]. A class of multifield models based on those fields that decay or get stabilized in a staggered style during inflation was explored by Battefeld and Battefeld [48]. They observed that fields remain flat before marching towards a steep downfall in assisted inflation, and when these fields face such a decrease, their decay rate is measured dynamically and the transfer of energy takes place in the other degrees of freedom. A further decrease in potential energy caused by the decay of the fields contributes to the observables such as spectral index and tensor-to-scalar ratio. The number of e-folds is bounded for the acceleratedly expanding universe that emerges out from the de Sitter epoch asymptotically [49], and the multifield model of dark energy is investigated.

In this work, we intend to investigate an inflationary model with multifields in the context of their number of e-folds, slow-roll parameters, and spectral indices. The multifield inflationary models possess some remarkably interesting signatures which the single-field models digress due to taking into consideration a single field and have more perspectives for the observational evidence which provides motivation to study these models theoretically. The study of inflationary phase driven combinedly by multifields usually by axions spaced sparsely is of great interest. The curvature perturbations are an inflationary relic that seeds the structure formation specifically. The investigations of the spectrum of these perturbations in multifield inflation are carried out enormously. For the case of equal and unequal masses by considering the suitable initial conditions, these are investigated [50–53]. When we use the power-law potential for multifields, the spectrum for these perturbations comes out to be redder than it is when a single scalar field is employed [54–59]. Spontaneous symmetry breaking naturally gives rise to the small field models with multifields where the fields usually begin with unstable equilibrium about the origin and roll down towards a stable minimum. Multifield inflation could also lead to figuring out a mechanism to understand the quantum gravity, an interplay between gravity and quantum field theory.

The remaining part of the paper is organized as follows: In Section 2, we perform calculations to determine the expressions for the number of e-folds and spectral index, following the formalism developed by Sasaki and Stewart. Specifically, we focus on the case where $p > 2$. Additionally, this section includes an explanatory, albeit brief discussion of the resulting outcomes.In Section 3, we examine the scenario where $p = 2$. We observe that assigning a negative value to p yields nonsensical results for the number of e-folds and spectral index, while assigning a value of $-2$ provides results that are interpretable. Section 3 also includes graphical representations of the results and accompanying remarks that elucidate the relationships among the derived cosmological observables. The final section, Section 4, summarizes the overall findings of the study and their implications. By considering different values of $p$, we derive predictions regarding the number of e-folds and spectral index.

## 2. Driving Multifield Inflation Based on Small Field Potential and the Spectrum of Curvature Perturbation

*2.1. Introduction to the Multifield Potential.* While the universe undergoes an inflationary phase, the geometry of spacetime tends to be flat, so while addressing the multifield model of inflation we consider cosmic geometry to be flat. So, after proposing the geometry to be flat, we begin by considering the following potential for investigation in the present work:

$$V = \sum_i V_i(\phi_i) = \sum_i \Lambda_i \left[1 - \left(\frac{\phi_i}{\mu_i}\right)^{-p}\right], \tag{1}$$

where the subscript "$i$" in the potential in question stands for the $i$ th field in addition to the entities related to it. Furthermore, $\Lambda_i$ is the mass scale and $\mu_i$ is a parameter which describes the height and tilt of the potential of the $i$ th field, respectively. The parameters $p$ and $\mu_i$ are free variables to choose suitably from the underlying conditions. The potential given in equation (1) is the multifield version of the brane inflationary potential $V(\phi) = \Lambda(1 - (\phi/\mu)^{-p})$ used in the brane model of inflation. In the brane model, the inflation is proposed to engender by the motion of branes in the extra dimensions. The effective Lagrangian for such a system leads to the following expression: $\mathscr{L} = -1/2\,(\partial_t\phi)^2 - 2T_3 r_o^4/r_{\mathrm{UV}}^4 (1 - (T_3^2 r_o^4/N\phi^4))$, where $T_3$ is the tension of a light brane and $r$ is related to the distance between the two branes. Other parameters are defined for the system on the same lines. The effective Lagrangian for the brane inflation could be worked out to have the form of the potential expressed in equation (1) for the case $i = 1$ with an arbitrary value of $p$ [60]. The case $i = 1$ related to the potential has already been frequently studied in the literature (e.g., see reference [60] and the references furnished therein in the corresponding section). The potential can be considered a generalized version of the small field models of inflation as discussed in [60–66] for the negative values of $p$. Various potentials bearing resemblance to such models in many aspects are also investigated in [67–69].

Inflationary cosmology by posing an ultrafast phase of cosmic expansion in its early evolution solves many problems, while single-field inflation despite showing extraordinary consistency to the observational data available today leaves room for considering the multifield models of inflation. Being motivated by Lyth bound, in addition, in particle physics, multifield models come to the scene naturally, especially in the realm of high energy physics beyond the standard models of particle physics such as supersymmetry (SUSY), supergravity (SUGRA), and string theory where generally many fields are considered to be present. In these theories, inflation is thought to be driven by the presence of more than one scalar field where these fields may interact similar to particle interactions. The presence of more than one field leads to predictions that could quietly be revolutionary in comparison to single-filed inflation. The choice of the potential in equation (1) is motivated by the presence of multiple scalars in the context of the axions of string theory where brane inflation is thought to be caused by branes. In the context of superstring theory, by compactifying six dimensions, the model incorporates multiple scalars such as axion, dilaton, and spin two modes of tensor perturbations in its low states of energy. Thus, we see that physically or cosmologically, the choice of the potential in equation (1) is well justified and this model has a correspondence in elementary particle physics. On plotting the potential, we see that it demonstrates an increasing function of the field, so that the inflation field advances from the right-hand direction to the left. The field would disappear for $\phi = \mu$ or $\phi/\mu = 1$ and it might, therefore work in the domain $\phi/\mu > 1$. The study of this model hence should be carried out only in the region lying within the limit $\phi/\mu > 1$. In addition, the brane inflation conforms to the condition $\mu/M_{pl} \ll 1$ and occurs following this condition. In Figure 1, the plot of the potential is demonstrated, where the potential and logarithm of the potential are plotted for $i = 1$ and $p = 2$.

The dynamics in the background of a multifield model of inflation can be realized and understood by describing them in terms of dimensionless slow-roll parameters $\varepsilon$, $\eta_{\|}$, and $\eta_{\perp}$, which is similar to the situation of a single-field model, however, the second slow-roll parameter $\eta$ is required to be modified in the scenario due to multifield inflation likely to be confronted with the eta problem. The parameter $\varepsilon$ is the first slow-roll parameter and $\eta_{\|}$ and $\eta_{\perp}$ give the slow-roll rate of the fields along the perpendicular direction of the motion of the fields. The parameter $\eta_{\perp}$ gives the turn rate of the fields along the perpendicular direction of motion. The slow roll would last as long as $\varepsilon \ll 1$ and $|\eta_{\|}| \ll 1$, whereas the parameter $\eta_{\perp}$ gives the turn rate of the fields perpendicular to the motion of the fields. A comparatively larger value of $\eta_{\perp}$ may pose as an interesting phenomena to the multifield scenario, however, it does not imply that it will necessarily violate the slow-roll conditions and will destroy it altogether as is described for multifield inflation. It is also manifested from the slow-roll parameters defined for the multifield inflation that Hubble parameter $H$ and the field derivative $\partial_t \phi$ would grow gradually.

*2.2. Analytical Analysis for the Case Related to $p > 2$.* Considering the potential used usually for the brane inflationary phase concerning $p > 2$, this potential was found to be relevant and useful in many situations occurring in the viable phenomena ensuing from the real world [70–75]. It is interesting and significant to note that when we assign the value $p = 2$, then the model under consideration could be thought to be the degenerate version of the inflationary model in the small field realm. The potential we consider here is with $-p$. This is the profile representing small field inflation and can be regarded as the lowest order of Taylor series expansion of an arbitrary potential about the origin of maxima and minima of it. The equation of motion of the scalar field $\phi_i$ while it slow-rolls during the slow-roll phase is

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_i'(\phi_i) = 0, \tag{2}$$

where during the slow-roll phase, $\ddot{\phi}_i \approx 0$ and accordingly from equation (2), we are left with the following expression which dominates the phase:

$$\dot{\phi}_i = -\frac{V_i'(\phi_i)}{3H}, \tag{3}$$

and the number of e-folds $N$ during the phase of the inflationary scenario can be calculated by the usual formula as

$$N = \int_{t_i}^{t_f} \mathrm{Hdt} = -M_P^{-2} \sum_i \int_{\phi_i^s}^{\phi_i^e} \frac{V_i}{V_i{}'} d\phi_i. \tag{4}$$

The lower limit $\phi_i^s$ in the integral marks the point of time at the beginning of the inflationary phase when the corresponding perturbations cross the horizon and the upper limit $\phi_i^e$ in the integral corresponds to the point in time when the inflationary phase terminates. It is noticeable, however, that $\phi_i^s < \phi_i^e$ in general and the coming of inflation to an end is consistent with the condition $\phi_i^e \le \mu_i$. In addition, the model of small field potential under consideration satisfies the constraint $\mu_i \le M_P$. For further evaluation of equation (4), we have the following from equation (1):

$$\sum_i V_i'(\phi_i) = \sum_i \frac{\Lambda_i}{\mu_i}\left(\frac{\phi_i}{\mu_i}\right)^{-(p+1)} = p\sum_i \frac{\Lambda_i}{\mu_i}\left(\frac{\phi_i}{\mu_i}\right)^{-(p+1)}. \tag{5}$$

Furthermore, we have

$$\sum_i \frac{V_i(\phi_i)}{V_i'(\phi_i)} = \frac{V_i}{V_i{}'} = \sum_i \frac{\Lambda_i\left[1 - (\phi_i/\mu_i)^{-p}\right]}{\Lambda_i/\mu_i p\,(\phi_i/\mu_i)^{-(p+1)}}, \tag{6}$$

which simplifies to

$$\frac{V_i}{V_i'} = \frac{1}{p}\left(-\phi_i + \frac{\phi_i{}^{p+1}}{\mu_i^p}\right), \tag{7}$$

and by substituting equation (7) into (4), we get

$$N = \frac{1}{pM_p^2}\sum_i \left[\begin{array}{c} \dfrac{1}{\mu_i^p (p+2)}\left((\phi_i^s)^{p+2} - (\phi_i^e)^{p+2}\right) \\ -0.5\left((\phi_i^e)^2 - (\phi_i^s)^2\right) \end{array}\right]. \tag{8}$$

For the small field inflation, it turns out that the value of $\mu_i$ is less than the Planck's mass $M_p$, *i.e.*, $\mu_i \le M_p$ and the inflation comes to an end for $\phi_i^e \le \mu_i$. This causes the quadratic terms, *i.e.*, $(\phi_i^s)^2$ and $(\phi_i^e)^2$ to disappear due to $\phi_i^s \le \phi_i^e \le M_p$. Then, from equation (8), we have

$$N = \sum_i \left[\begin{array}{c} \dfrac{1}{\mu_i^p p(p+2)}\left(\left(\dfrac{\phi_i^s}{M_p}\right)^{p+2} - \left(\dfrac{\phi_i^e}{M_p}\right)^{p+2}\right) \\ -\dfrac{0.5}{p}\left(\left(\dfrac{\phi_i^e}{M_p}\right)^2 - \left(\dfrac{\phi_i^s}{M_p}\right)^2\right) \end{array}\right], \tag{9}$$

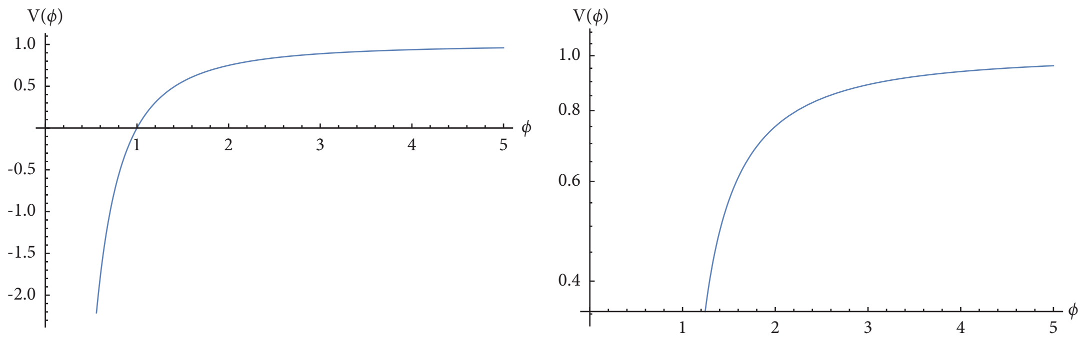
or

$$N = \frac{1}{p(p+2)\mu_i^p}\sum_i\left(\left(\frac{\phi_i^s}{M_p}\right)^{p+2} - \left(\frac{\phi_i^e}{M_p}\right)^{p+2}\right), \tag{10}$$

which is further simplified to

$$N = \frac{\mu_i^2}{p(p+2)M_{pl}^2}\sum_i\left[\frac{\phi_i^s}{\mu_i}\left(1 - \left(\frac{\phi_i^s}{\phi_i^e}\right)\right)\right]^{p+2}. \tag{11}$$

If we approximate the expression $1 - (\phi_i^s/\phi_i^e)^{p+2}$ to unity, then we are left with




(a) (b)

Figure 1: The plot of the potential for the brane inflationary scenario. (a) the potential is plotted in simple scale, whereas on the (b) it is plotted after taking its logarithm as a function of $\phi/\mu$ for $p = 2$.

$$N = \frac{1}{p(p+2)M_{pl}^2} \sum_i \mu_i^2 \left(\frac{\mu_i}{\phi_i^s}\right)^{-(p+2)}. \tag{12}$$

Curvature perturbations, as well as isocurvature perturbations both, exist in multifield inflation models; however, to keep things simpler, it is considered here that during the slow-roll phase, isocurvature perturbations are suppressed and can be neglected. The remaining perturbations which are due to the curvature only can be tackled suitably by a mechanism developed by Sasaki and Stewart known as Sasaki–Stewart formalism [7, 8, 17, 76–81]. Thus, we see that the magnitude of these curvature perturbations at the end of inflation could be worked out on the spatial hypersurfaces of constant density denoted by $\zeta$ as

$$\zeta = R - \frac{H}{\dot{\rho}}\delta\rho, \tag{13}$$

where $\delta p_{na} = \delta p - \dot{p}/\dot{\rho}\delta\rho$. We know that curvature perturbation is a gauge invariant quantity and therefore has an arbitrary nature and leads to temperature fluctuations in CMB and spawns the seeds for cosmic structure. Differentiating with respect to time and using the energy conservation equation give

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{na} + \frac{k^2}{3}\left(\frac{\delta q}{a^2(\rho + p)} + \sigma_g\right). \tag{14}$$

It reduces to the following on superhorizon scales ($k \ll aH$):

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{na}, \tag{15}$$

which on superhorizon scales vanishes in single-field inflation. These perturbations, however, in multiple scalars' case are measured by means of the power spectrum and its related parameter bispectrum.

$$\begin{aligned} \left\langle \zeta_{k_1}, \zeta_{k_2} \right\rangle &= (2\pi)^3 \delta^3 (k_1 + k_2) P_\zeta(\kappa), \\ \left\langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \right\rangle &= (2\pi)^3 \delta^3 (k_1 + k_2 + k_3) \mathrm{B}_\zeta (\kappa_1 + \kappa_2 + \kappa_3), \end{aligned} \tag{16}$$

where $\kappa = |k_1| = |k_2| = |k_3|$ and the power spectrum is worked out to be $P_\zeta(\kappa) = 2\pi^2/\kappa^3 P_\zeta(\kappa)$ and the spectral index $n_s - 1$ using this formalism is given as

$$n_s - 1 = \left[\sum_i \left(\frac{V_i'}{V_i}\right)^2 + 2\sum_i \left(\frac{V_i'}{V_i}\right)^{-2} - 2\sum_i \frac{1}{V_i}\frac{(V_i/V_i')^2 V''}{\left(V_j/V_i'\right)^2}\right] M_{pl}^2, \tag{17}$$

In equation (12), we replace $\mu_i/\varphi_i^s$ by $\omega_{i,}$ *i.e.*,

$$\frac{\mu_i}{\varphi_i^s} \simeq \omega_i, \tag{18}$$

then equation (12) can be reexpressed in the following form:

$$N = \frac{1}{p(p+2)M_{pl}^2} \sum_i \mu_i^2 \omega_i^{-(p+2)}. \tag{19}$$

Now, we substitute from equation (19) and get

$$\sum_i \mu_i^2 \omega_i^{-(p+2)} = B_1, \tag{20}$$

then equation (19) is written as

$$N = \frac{1}{p(p+2)M_{pl}^2} B_1. \tag{21}$$

The reduced Planck mass can be expressed in terms of the newly defined constant $B_1$ as

$$M_{pl}^2 = \frac{B_1}{p(p+2)}\frac{1}{N}. \tag{22}$$

Now, we calculate all three terms in the Sasaki–Stewart formalism from equation (1) by squaring both sides as

$$V^2 = \sum_i (V_i(\varphi_i))^2 = \sum_i \Lambda_i^2\left[1 + \left(\frac{\varphi_i}{\mu_i}\right)^{-2p} - 2\left(\frac{\varphi_i}{\mu_i}\right)^{-p}\right]. \quad (23)$$

The second and third terms can be neglected as the constraint $\varphi_i^e \leq \mu_i$ is satisfied for the inflation to come to an end. In addition, the model of small field potential under consideration satisfies the condition $\mu_i \leq M_P$ and equation (23) also has to satisfy $\varphi_i^s \leq \varphi_i^s \leq \mu_i \leq M_P$ which again motivates to ignore the terms that may result in quadratic form. The same will be applicable for $\mu_i/\varphi_i^s \simeq \omega_i$ in equation (18) in conjunction with equations (44) and (49), wherever applicable.

$$V^2 = \sum_i (V_i(\varphi_i))^2 = \sum_i \Lambda_i^2. \quad (24)$$

Now, let us consider that

$$\sum_i \Lambda_i = B_2, \quad (25)$$

then equation (24) takes the following form:

$$V^2 = \sum_i (V_i(\varphi_i))^2 = B_2^2. \quad (26)$$

Now, from equation (5), by squaring both sides and employing equation (18), we obtain the following equation:

$$\sum_i (V_i')^2 = p^2 \sum_i \frac{\Lambda_i^2}{\mu_i^2} \frac{1}{\omega_i^{-2(p+1)}}. \quad (27)$$

Now, let us take the expression

$$\sum_i \frac{\Lambda_i^2}{\mu_i^2} \frac{1}{\omega_i^{-2(p+1)}} = B_3, \quad (28)$$

then, equation (27) becomes

$$\sum_i (V_i')^2 = p^2 B_3. \quad (29)$$

Now, from equation (6), the simplification after squaring both sides and by using equation (18) gives

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 = \frac{1}{p^2} \sum_i \left(\varphi_i^2 + \varphi_i^2\left(\frac{\mu_i}{\varphi_i}\right)^{-2p} - 2\varphi_i^2\left(\frac{\mu_i}{\varphi_i}\right)^{-p}\right). \quad (30)$$

By using the definition from equation (18) in equation (30), we obtain

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 = \frac{\varphi_i^2}{p^2} \sum_i \left(1 + \omega_i^{-2p} - 2\omega_i^{-2p}\right), \quad (31)$$

and by absorbing $\varphi_i^2$ into the definition of $\omega_i$ as given in equation (18), and after simplification, we obtain

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 = \frac{1}{p^2} \sum_i \left(\omega_i^{-2} + \omega_i^{-2(p+1)} - 2\omega_i^{-4}\right)\mu_i^2, \quad (32)$$

Let us now take

$$\sum_i \left(\omega_i^{-2} + \omega_i^{-2(p+1)} - 2\omega_i^{-4}\right)\mu_i^2 = B_4. \quad (33)$$

Now, equation (33) takes the following form by using the above-defined constant:

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 = \frac{1}{p^2} B_4. \quad (34)$$

Then, equation (5) by differentiating once again gives

$$\sum_i V_i''(\varphi_i) = -p(p+1)\sum_i \frac{\Lambda_i}{\mu_i^2}\left(\frac{\varphi_i}{\mu_i}\right)^{-(p+2)}. \quad (35)$$

Hence, by using equation (18), we have

$$\sum_i V_i''(\varphi_i) = -p(p+1)\sum_i \frac{\Lambda_i}{(\omega_i)^{-(p+2)}\mu_i^2}. \quad (36)$$

By finding out the product of equations (30) and (35) and by using equation (18), we get

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 V_i''(\varphi_i) = -\frac{p+1}{p} \sum_i \Lambda_i\left(\omega_i^p + \omega_i^{-p} - 2\right), \quad (37)$$

and

$$\sum_i \Lambda_i\left(\omega_i^p + \omega_i^{-p} - 2\right) = B_5, \quad (38)$$

then, equation (37) becomes

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 V_i''(\varphi_i) = -\frac{p+1}{p} B_5, \quad (39)$$

with

$$\sum_i \left(\frac{V_i(\varphi_i)}{V_i'(\varphi_i)}\right)^2 = \sum_i \left(\frac{V_j(\varphi_j)}{V_j'(\varphi_j)}\right)^2. \quad (40)$$

On substituting from equations (22), (24)–(26), (29), (34), and (39) in equation (17), we determine the following equation as follows:

$$n_s - 1 = \left[\frac{p^2 B_3}{B_2^2} - 2\frac{1}{\left(B_4/p^2\right)} + 2\frac{1}{B_2}\frac{(-(p+1/p)B_5)}{\left(B_4/p^2\right)}\right]\left(\frac{B_1}{p(p+2)}\frac{1}{N}\right). \quad (41)$$

Equation (41) gives the following expression after simplifying it:

$$n_s - 1 = -\frac{p}{p+2}\frac{1}{N}\left(\frac{B_1B_3}{B_2^2}\right) - 2\left(\frac{p}{p+2}\right)\frac{1}{N}\left(\frac{B_1}{B_4}\right) - 2\left(\frac{p+1}{p+2}\right)\frac{1}{N}\left(\frac{B_1B_5}{B_2B_4}\right). \quad (42)$$

By multiplying and dividing the 1st and 2nd terms in the abovementioned expression with $p+1$ and by simplifying, we get

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(\frac{1}{2}\frac{p}{p+2}\frac{B_1B_3}{B_2^2} + \frac{p}{p+1}\frac{B_1}{B_4} + \frac{B_1B_5}{B_2B_4}\right). \quad (43)$$

In equation (43), on the right-hand side, the 1st and 2nd terms inside the parenthesis vanish due to $\omega_i^{-np\pm n} \simeq 0$, and the remaining part is represented as

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(\frac{B_1B_5}{B_2B_4}\right). \quad (44)$$

We further write down the abovementioned equation in a suitable form by adding and subtracting 1 on the right-hand side within the parenthesis as

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(\frac{B_1B_5}{B_2B_4} + 1 - 1\right), \quad (45)$$

or

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(1 + \frac{B_1B_5 - B_2B_4}{B_2B_4}\right). \quad (46)$$

Let us replace the given equation as

$$\frac{B_1B_5 - B_2B_4}{B_2B_4} = R(\omega_i), \quad (47)$$

then equation (46) has the following form:

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(1 + R(\omega_i)\right). \quad (48)$$

For $R(\omega_i) \simeq 0$, equation (48) serves to calculate the spectral index for a single scalar field. However, the term $R(\omega_i)$ adds in for the case when we are considering multifields. Therefore, it is important to determine this factor. We will use the definitions of involved constants to find out the approximate value of the $R(\omega_i)$ for larger values of $p$ than 2. From equations (20), (25), (28), (33), and (38) by substituting for the constants $B_1, B_2, B_4, B_5$ in equation (47) to determine $R(\omega_i)$ we get

$$R(\omega_i) = \sum_i \frac{2\left(\omega_i^p - \omega_i^2\right)\omega_i^p}{\omega_i^{2(p+1)} - 2\omega_i^{2p} + \omega_i^2}. \quad (49)$$

From equation (36), we have the expression for $\omega_i^p$ and by considering $\sum_i V_i''(\varphi_i) = V_i''(\varphi_i) = m_i^2$, we have

$$\omega_i^p = -\frac{1}{p(p+1)}\frac{\mu_i^2 m_i^2}{\Lambda_i\omega_i^2}, \quad (50)$$

Then, by using the value of $\omega_i^p$ in equation (49), we get

$$R(\omega_i) = \sum_i \frac{2\left(\mu_i^2 m_i^2 + \omega_i^4\Lambda_i p(p+1)\right)}{\mu_i^2 m_i^2\left(\omega_i^2 - 1\right) - \omega_i^4\Lambda_i p(p+1)}. \quad (51)$$

The expression of $R(\omega_i)$ found in equation (51) is due to the consideration of multifields instead of a single field. Equation (48) now takes the following form:

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}\left(1 + \sum_i \frac{2\left(\mu_i^2 m_i^2 + \omega_i^4\Lambda_i p(p+1)\right)}{\mu_i^2 m_i^2\left(\omega_i^2 - 1\right) - \omega_i^4\Lambda_i p(p+1)}\right), \quad (52)$$

and with $R(\omega_i) \simeq 0$, equation (48) comes out to be

$$n_s - 1 = -2\frac{(p+1)}{p+2}\frac{1}{N}. \quad (53)$$

The expression in equation (52) represents the spectral index corresponding to multifields, while equation (53) demonstrates the value of the spectral index conforming to the case when a single scalar field is taken into account. In this case, the masses of all the fields considered are of the same value at the time of horizon-crossing. This poses the case when the spectral index of the multifield is the same and corresponds to the spectral index of the single scalar field [50]. It is also clear that the term $R(\omega_i)$ appears due to the consideration of multifields. It can also be observed in this regard that the value of $R(\omega_i)$ will be positive for $\omega_i < \omega_{i+1}$ and $m_i^2 < m_{i+1}^2$. However, it will turn into a negative for $\omega_i < \omega_{i+1}$ and $m_i^2 > m_{i+1}^2$. The positive value of $R(\omega_i)$ is interpreted to be its spectrum which is redder for multifields as compared to its corresponding spectrum emerging from a single field. While the nagative value attached to that implies the corresponding spectrum would be less redder comparatively. However, a stringent condition begs further work to develop.

*2.3. Analytical Analysis for the Case Related to* $p = -2, +2$. Now, we will discuss some specific cases for the values of $p$. We will investigate for $p = 2, -2$ and will observe what effect it bores upon the expressions of number of e-folds and spectral indices.

Let us first take the case when $p = 2$. From equations (1), (5), and (31), we have

$$\sum_i V_i(\phi_i) = \sum_i \Lambda_i\left[1-\left(\frac{\phi_i}{\mu_i}\right)^{-2}\right],$$

$$\sum_i V_i'(\phi_i) = 2\sum_i \frac{\Lambda_i}{\mu_i}\left(\frac{\phi_i}{\mu_i}\right)^{-3}, \tag{54}$$

$$\sum_i V_i''(\phi_i) = -6\sum_i \frac{\Lambda_i}{\mu_i^2}\left(\frac{\phi_i}{\mu_i}\right)^{-4}.$$

From equation (12), for the number of e-folds with $p = 2$, we get

$$N = \frac{1}{8M_{pl}^2}\sum_i \mu_i^2\left(\frac{\mu_i}{\varphi_i^s}\right)^{-4}. \tag{55}$$

Equation (55) leads to the absurd result which is

$$N = \frac{1}{8\mu_i^2 M_{pl}^2}\sum_i\left[\left(\frac{\varphi_i^e}{M_{pl}}\right)^2 + \left(\frac{\varphi_i^s}{M_{pl}}\right)^2\right] \longrightarrow 0. \tag{56}$$

Similarly, the spectral index $n_s - 1$ from equation (52) for $p = 2$ can be calculated as

$$n_s - 1 = -\frac{3}{2}\frac{1}{N}\left(1 + \sum_i \frac{2\left(\mu_i^2 m_i^2 + 6\omega_i^4\Lambda_i\right)}{\mu_i^2 m_i^2\left(\omega_i^2 - 1\right) - 6\omega_i^4\Lambda_i}\right). \tag{57}$$

For $N \simeq 0$, as we have an absurd result from the abovementioned equation, equation (57) leads to the value of the spectral index $n_s - 1$ approaching to $\infty$ which is again meaningless seemingly. For $p = -2$, equation (1) takes the following form:

$$\sum_i V_i(\varphi_i) = \sum_i \Lambda_i\left[1-\left(\frac{\varphi_i}{\mu_i}\right)^2\right]. \tag{58}$$

Equations (5) and (35) are reduced to the following expressions:

$$\sum_i V_i'(\phi_i) = V_i'(\phi_i) = -2\sum_i \frac{\Lambda_i}{\mu_i^2}\phi_i,$$
$$\sum_i V_i''(\phi_i) = V_i''(\phi_i) = -2\sum_i \frac{\Lambda_i}{\mu_i^2}, \tag{59}$$

and the expression for the number of e-folds $N$ is given by

$$N = -\frac{1}{2M_{pl}^2}\sum_i \ln\left(\frac{\varphi_i^s}{\varphi_i^e}\right)\mu_i^2. \tag{60}$$

Finding the values of the following from equation (58) results in:

$$\sum_i \left(V_i'\right)^2 = 4\sum_i \frac{\Lambda_i^2}{\omega_i^2\mu_i^2}, \tag{61}$$

$$\sum_i \left(\frac{V_i}{V_i'}\right)^2 = \frac{1}{4}\sum_i \omega_i^2\mu_i^2. \tag{62}$$

Similarly, we get

$$\sum_j \left(\frac{V_j}{V_j'}\right)^2 = \frac{1}{4}\sum_j \omega_j^2\mu_j^2, \tag{63}$$

and finally, we have

$$\sum_i \left(\frac{V_i}{V_i'}\right)^2 V_i'' = -\frac{1}{2}\sum_i \Lambda_i\omega_i^2. \tag{64}$$

By substituting equations (61)–(64) in equation (17), it produces the following expression for the spectral index:

$$n_s - 1 = -M_{pl}^2\sum_i\left(\frac{\Lambda_i\omega_i^2}{\Lambda_i\left(\mu_j^2\omega_j^2\right)}\right), \tag{65}$$

where we used the condition that $\omega_i^{-2} \simeq 0$ and from equation (3), we have

$$\frac{\dot{\varphi}_i}{V_i'(\varphi_i)} = \frac{\dot{\varphi}_j}{V_j'(\varphi_j)}. \tag{66}$$

Thus, by finding $V_i'(\varphi_i)$ and $V_j'(\varphi_j)$ from equation (58) and by using them in equation (65), we obtain

$$\mu_i^2\frac{\dot{\varphi}_i}{\Lambda_i\varphi_i} = \mu_j^2\frac{\dot{\varphi}_j}{\Lambda_j\varphi_j}. \tag{67}$$

By integrating the abovementioned equation between the limits $\varphi_i^s$ *and*$\varphi_i^e$ for the $i$ th field and between $\varphi_j^s$ and $\varphi_i^e$ for the corresponding $j$ th field, we get

$$\frac{\mu_i^2}{\Lambda_i}\int_{\varphi_i^s}^{\varphi_i^e}\frac{\dot{\varphi}_i}{\varphi_i} = \frac{\mu_j^2}{\Lambda_j}\int_{\varphi_j^s}^{\varphi_j^e}\frac{\dot{\varphi}_j}{\varphi_j}, \tag{68}$$

and after simplification, it gives

$$\sum_i \mu_i^2\ln\left(\frac{\varphi_i^s}{\varphi_i^e}\right) = \sum_k \mu_k^2\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\sum_i\frac{\Lambda_i}{\Lambda_k}. \tag{69}$$

For some fixed value of $k$, the summation sign $\sum_k$ can be dropped out of the expression as

$$\sum_i \mu_i^2\ln\left(\frac{\varphi_i^s}{\varphi_i^e}\right) = \mu_k^2\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\sum_i\frac{\Lambda_i}{\Lambda_k}. \tag{70}$$

In a more simplified form, it can be written as

$$\mu_i^2\ln\left(\frac{\varphi_i^s}{\varphi_i^e}\right) = \mu_k^2\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\frac{\Lambda_i}{\Lambda_k}, \tag{71}$$

The number of e-folds in equation (60) becomes

$$N = -\frac{\mu_k^2}{2M_{pl}^2}\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\sum_i\frac{\Lambda_i}{\Lambda_k}. \tag{72}$$

Thus, from equation (72), we can find the expression for Planck mass in terms of the number of e-folds as

$$M_{pl}^2 = -\frac{1}{2N}\mu_k^2 \ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\sum_i \frac{\Lambda_i}{\Lambda_k}. \tag{73}$$

By using the value of Planck mass $M_{pl}^2$ from equation (73), the expression for spectral index in equation (65) now reads as

$$n_s - 1 = \frac{2}{N}\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\frac{\sum_i (\Lambda_i/\Lambda_k)\omega_i^2}{\sum_j (\mu_j/\mu_k)^2 \omega_j^2}, \tag{74}$$

where for $\Lambda_i = \Lambda_k$, all the fields $\varphi_i$ or $\varphi_k$ will possess the same value of $\Lambda_i$. In this case, the expression for the spectral index in the abovementioned equation reduces to the following form:

$$n_s - 1 = \frac{2}{N}\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right)\frac{\sum_i \omega_i^2}{\sum_j (\mu_j/\mu_k)^2 \omega_j^2}. \tag{75}$$

The results of equations (74) and (75) are independent of the choice of the values of $k$ as are considered to the uncompromised level. In equation (75), if the multifields happen to be such that they can avail the chance of having the same $\mu_i$ and $\mu_j = \mu_k$, then we would have

$$n_s - 1 = \frac{2}{N}\ln\left(\frac{\varphi_k^s}{\varphi_k^e}\right). \tag{76}$$

It can be noted from equation (76) that all the terms included in the $\ln(\varphi_k^s/\varphi_k^e)$ might be equivalent on the basis of equation (72). On the other hand, equation (76) represents the same equation for the corresponding single-field case. The value of $\ln(\varphi_k^s/\varphi_k^e)$ in equation (72) will be smaller for the bigger value of $\mu_i$, when $\Lambda_i$ is taken as equivalent to $\mu_i$. If we consider $\mu_k = \mathrm{Max}(\mu_n)$, where $n$ pertains to natural numbers, then it leads to $\mu_i/\mu_k < 1$ implying that the spectrum is redder than its corresponding spectrum which results from equation (76) for a single scalar field $\varphi_k$. In this case, the value of $\ln(\varphi_k^s/\varphi_k^e)$ would represent almost the smallest value from all the values of $\ln(\varphi_i^s/\varphi_i^e)$. It would accordingly indicate that in the context of equation (76), the value of $k$ approaches nearer to unity in case of a single scalar field $\varphi_k$. On the other hand, if we take into account $\mu_k = \mathrm{Min}(\mu_n)$, where $n$ belongs to natural numbers, then this would give rise to $\mu_i/\mu_k > 1$ which leads to the factual result that the spectrum is less red than its corresponding spectrum which results from equation (76) in the case of a single scalar field $\varphi_k$. In this case, the value of $\ln(\varphi_k^s/\varphi_k^e)$ would represent almost the biggest value out of all the values of $\ln(\varphi_i^s/\varphi_i^e)$ showing that in equation (76) in the case of a single scalar field $\varphi_k$, the value of $k$ shifts away from unity. It means that the value of the scalar spectral index falls between that of a single scalar field in general for the biggest $\mu_k$ and the smallest accordingly. In Tables 1 and 2, the range for the e-folds $N$ that is cosmologically viable as concerns for the early cosmic evolution corresponding to $p$ and that of spectral index $n_s$ corresponding to $N$ is shown, respectively.

Then, in Figures 2 and 3, the spectral index $(n_s)$ is plotted against the e-folding number $N$ for a range of values.

Table 1: Spectral index $(n_s)$ in terms of e-folding number $N$ for a range of values of $p$.

| Values of $p$ | Spectral index $(n_s)$ in terms of e-folds $(N)$ |
|---|---|
| $p=-4$ | $-3/N$ |
| $p=-3$ | $-4/N$ |
| $p=-2$ | $-3/N$ |
| $p=-1$ | Undefined |
| $p=0$ | $-1/N$ |
| $p=1$ | $-4/3N$ |
| $p=2$ | $-3/2N$ |
| $p=3$ | $-8/5N$ |
| $p=4$ | $-5/3N$ |

Table 2: Tabulation of spectral index $(n_s)$ against the number of e-folds $N$.

| Sr. No | Number of e-folds $(N)$ | Spectral index $(n_s)$ |
|---|---|---|
| 1 | 30 | 0.90 |
| 2 | 35 | 0.914 |
| 3 | 40 | 0.925 |
| 4 | 45 | 0.93 |
| 5 | 50 | 0.94 |
| 6 | 55 | 0.945 |
| 7 | 60 | 0.95 |
| 8 | 65 | 0.953 |
| 9 | 70 | 0.96 |

The presence of multiple scalars could lead to a somewhat revolutionary inflationary paradigm such that the dynamics due to these fields are quite naive compared to the single field. For example, specifically, the generation of primordial perturbation spectra and nonadiabatic field perturbations could affect the evolution of curvature perturbations and the detection of non-Gaussianity. The inflation in the present case of potential occurs long enough so that it is sufficiently long that the observable universe comes close to being spatially flat. In order to understand the tensor-to-scalar ratio occurring in the model under consideration, we determine first the spectrum of scalar curvature perturbations using the relation given beneath as it is figured out in references [51, 80].

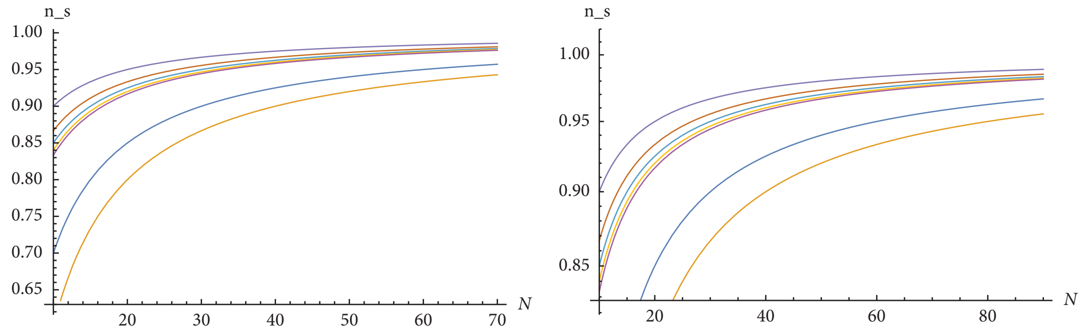
$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \delta_{ij} N_{,\varphi_i} N_{,\varphi_j}, \tag{77}$$

which comes out approximately to be

$$\mathcal{P}_{\mathcal{R}} \approx \frac{V}{12\pi^2}\sum_i \left(\frac{V_i}{V_i'}\right)^2. \tag{78}$$

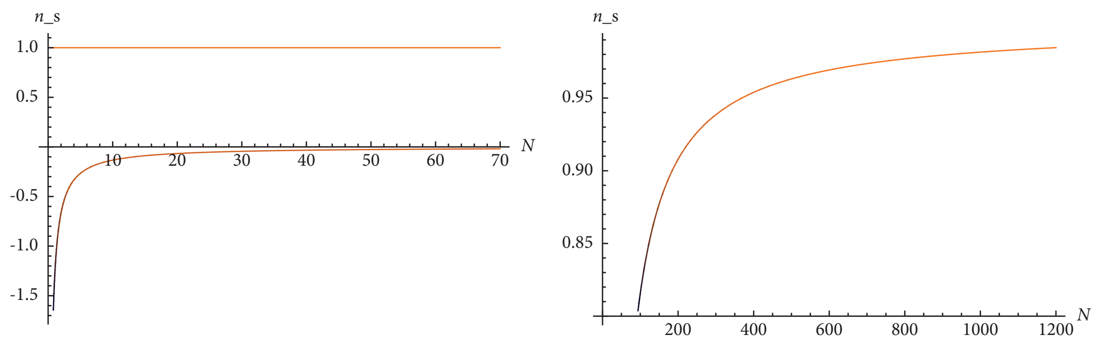
Substituting from equations (1) and (5) into equation (78) and by simplifying, it produces

$$\mathcal{P}_{\mathcal{R}} \approx \frac{V}{12\pi^2} p^2 \left(\frac{\varphi_i}{\mu_i}\right)^{2p+4}, \tag{79}$$




(a) (b)

FIGURE 2: Plot of spectral index $(n_s)$ against the e-folding number $(N)$ for the values of $p$ given in Table 1. (a) the plot is directly drawn between two quantities, (b) the logarithm of the plot is displayed.




(a) (b)

FIGURE 3: Plot of spectral index $(n_s)$ against the e-folding number $(N)$ for the values of $p$ given in Table 1. (a) the plot is simply presented between the parameters, (b) it is drawn after taking the logarithm of $(n_s)$.

or

$$\mathcal{P}_{\mathcal{R}} \approx \frac{V}{12\pi^2} \frac{p^2}{\mu_i^2} \left(\frac{\varphi_i}{\mu_i}\right)^{p+2} \mu_i^2, \tag{80}$$

and with the help of equation (12), it becomes

$$\mathcal{P}_{\mathcal{R}} \approx \frac{V}{12\pi^2} \frac{p^4 (p+2)^2 N^2}{\mu_i^2}, \tag{81}$$

and

$$\mathcal{P}_{\mathcal{R}} \approx \frac{p^4 (p+2)^2 N^2}{12\pi^2 \mu_i^2} \sum_i \Lambda_i \left(1 - \frac{\varphi_i}{\mu_i}\right)^{-p}, \tag{82}$$

where it gives an additional relation that is

$$\sum_i \Lambda_i \left(1 - \frac{\varphi_i}{\mu_i}\right)^{-p} \approx \frac{12\pi^2 \mu_i^2 \mathcal{P}_{\mathcal{R}}}{p^4 (p+2)^2 N^2}. \tag{83}$$

For tensor perturbations in the case of a general multifield model, the relation can be used suitably as given in reference [52].

$$\mathcal{P}_{\mathcal{T}} \approx \frac{2H^2}{\pi^2}. \tag{84}$$

Now, the tensor-to-scalar ratio can be computed as

$$r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = \frac{24\mu_i}{p^4 (p+2)^2 N^2} \frac{H^2}{V}. \tag{85}$$

The recent BICEP results put the following constraint on the tensor-to-scalar ratio as an upper bound $r < 0.034\,(95\,\%\,CL)$ Friedmann evolution equation:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{86}$$

for vanishing the curvature term $k$ and the energy density $\rho \longrightarrow V_i(\varphi_i)$ during the inflationary phase, and this further gives

$$H^2 = \frac{1}{3M_{pl}^2} \sum_i \Lambda_i \left(1 - \frac{\varphi_i}{\mu_i}\right)^{-p}. \tag{87}$$

Equation (85) takes the following form in the light of equations (1) and (87):

$$r = \frac{1}{p^4 (p+2) N^2} \frac{\mu_i}{M_{pl}^2}. \tag{88}$$

It can be seen from equation (88) that the tensor-to-scalar $r$ ratio is dominantly dependent on the number of e-folds $N$ as well as on the distribution of $\mu_i$. This dependence indeed motivates us to stay focused on determining the spectral index corresponding to the model in question.

Now, considering that the fields are uncoupled such that the dynamics during slow-roll inflation are governed by the following equation:

$$\Delta\varphi \simeq \sqrt{\sum_i \left(\frac{\partial_t \varphi_i}{H}\right)^2}. \tag{89}$$

Then, using equations (3), (5), and (86) in equation (89) leads to

$$\Delta\varphi \simeq pM_{pl}^2 \frac{\sqrt{\sum_i \left(\Lambda_i/\mu_i\,(\varphi_i/\mu_i)^{-(p+1)}\right)^2}}{\sum_i \Lambda_i\left(1-(\varphi_i/\mu_i)^{-p}\right)}. \tag{90}$$

The quantum fluctuations during this phase occur as

$$\delta\varphi \simeq \sqrt{\sum_i (\delta\varphi_i)}, \tag{91}$$

where $\delta\varphi_i \sim H/2\pi$ and by making use of equations (1) and (86), it gives

$$\delta\varphi \simeq \frac{1}{\pi M_{pl}}\sqrt{\frac{N}{12}}\sqrt{\sum_i \Lambda_i\left(1-\left(\frac{\varphi_i}{\mu_i}\right)^{-p}\right)}. \tag{92}$$

The critical field values can be reached at $\Delta\varphi \simeq \delta\varphi$, and for $p = 2$, we have

$$p^2 M_{pl}^6 \sum_i \left(\frac{\Lambda_i}{\mu_i}\left(\frac{\varphi_i}{\mu_i}\right)^{-(p+1)}\right)^2 \simeq \frac{N}{12\pi^2}\sum_i \left(\Lambda_i\left(1-\left(\frac{\varphi_i}{\mu_i}\right)^{-p}\right)\right)^3, \tag{93}$$

Now, by using the slow-roll condition $\varepsilon \ll 1$, from equations (1) and (5), we get

$$\frac{1}{2} pM_{pl}^2 \sum_i \left(\frac{\Lambda_i}{\mu_i}\left(\frac{\varphi_i}{\mu_i}\right)^{-(p+1)}\right)^2 \ll \sum_i \left(\Lambda_i\left(1-\left(\frac{\varphi_i}{\mu_i}\right)^{-p}\right)\right)^2, \tag{94}$$

or toward the end of the slow-roll phase, we get

$$\frac{1}{2} pM_{pl}^2 \sum_i \left(\frac{\Lambda_i}{\mu_i}\left(\frac{\varphi_i}{\mu_i}\right)^{-(p+1)}\right)^2 \simeq \sum_i \left(\Lambda_i\left(1-\left(\frac{\varphi_i}{\mu_i}\right)^{-p}\right)\right)^2. \tag{95}$$

Reheating in single-field inflation models could take place by the breakdown of a slow-roll constraint, while in multifield models, it could be geared by an instability when the fields reach a minimum. The reheating phase comes at the end of the inflationary period when the proposed multifields lose energy to transform into other relics such as radiation and particles which grow to the present structure formation. At the end of the multifield inflationary phase, the reheating phase can be understood through isocurvature perturbations. In a flat FLRW background, the line element for the linear perturbations reads as

$$ds^2 = -(1+2\chi)dt^2 + 2aG_i dx^i dt + a(t)^2\delta_{ij}dx^i dx^j, \tag{96}$$

where $\chi$ and $G$ represent lapse function and shear, respectively. Then, we have

$$\delta\varphi_\alpha + \left\{3H + \frac{1}{2}\left(\Gamma_\gamma^\alpha + \Gamma_m^\alpha\right)\right\}\dot{\delta\varphi}_\alpha - \frac{\nabla^2}{a^2}\delta\varphi_\alpha + \sum_\beta U_{,\alpha\beta}\delta\varphi_\beta$$
$$+ \left\{2U_{,\alpha}\frac{1}{2}\dot{\varphi}_\alpha\left(\Gamma_\gamma^\alpha + \Gamma_m^\alpha\right) + 3H\dot{\delta\varphi}_\alpha\right\}\chi + \dot{\varphi}_\alpha\chi + \frac{4\pi G}{H}\dot{\varphi}_\alpha\delta\rho = 0. \quad (97)$$

The adiabatic perturbations for a fluid in general, that is, the adiabatic pressure can be written in expanded form as $\delta P_{na} = \delta P - c_s^2\delta\rho$, where $c_s^2 = \partial_t P/\partial_t\rho$. For the two scalar fields, we have the following:

$$\delta P_{na} = \frac{1}{3H^2 M_{Pl}^2}\left(U_{\varphi_1}\dot{\varphi}_1 + U_{\varphi_2}\dot{\varphi}_2\right)\left(\dot{\varphi}_1\delta\varphi_1 + \dot{\varphi}_2\delta\varphi_2\right)$$
$$- 2\left(U_{\varphi_1}\delta\varphi_1 + U_{\varphi_2}\delta\varphi_2\right) - \frac{2}{3H}\frac{\left(U_{\varphi_1}\dot{\varphi}_1 + U_{\varphi_2}\dot{\varphi}_2\right)}{\varphi_1^2 + \varphi_2^2} \quad (98)$$
$$\times\left\{\dot{\varphi}_1\delta\dot{\varphi}_1 + \dot{\varphi}_2\delta\dot{\varphi}_2 + U_{,\varphi_1}\delta\varphi_1 + U_{,\varphi_2}\delta\varphi_2\right\},$$

which on comparison gives

$$\delta P = \dot{\varphi}_1\delta\dot{\varphi}_1 + \dot{\varphi}_2\delta\dot{\varphi}_2 - \left(\varphi_1^2 + \varphi_2^2\right)\chi - U_{,\varphi_1}\delta\varphi_1 - U_{,\varphi_2}\delta\varphi_2. \quad (99)$$

By reheating when the fields decay into the fluids which transform into radiation and particles, the pressure perturbation of nonadiabatic fluid becomes

$$\delta P_{na} = \frac{1}{3}\delta\rho_\gamma\left(1 - \frac{\partial_t\rho_\gamma}{\partial_t\rho_\gamma + \partial_t\rho_m}\right) - \frac{1}{3}\frac{\partial_t\rho_\gamma\delta\rho_m}{\partial_t\rho_\gamma + 3\partial_t\rho_m}, \quad (100)$$

where $c_s^2 = (\partial_t\rho_\sigma/\partial_t\rho)c_\sigma^2$. Thus, through effective field equations coupling the fields by means of their decay products, the reheating period after a multifield inflation occurs, where the nonadiabatic pressure is inside the few orders of magnitude of the pressure perturbations when the inflation ends. When different mass scales are given, we have to use a robust technique which provides some suitable and viable solution. *R.* Easter and *L.* McAllister devised a very powerful technique to work out the mass scales concerning the multifields [51]. The method is frequently employed in inflation or multiple field scenarios. They suggested a new technique known to be as the law regarding the distribution of mass scales in general and is named after the two inventors as Marcenco-Pastur law. In multifield models of inflation, it is customary to make use of random matrix theory which might play a very basic and important role in the distribution of different masses related to the spectrum. This is accomplished by using some suitable law, the best example is the Mar č enko-Pastur law. In the beginning, the law was employed first in the string theory where the problem of the distribution of masses related to the axion field was being faced. In multifield models, different trajectories of inflation occur and therefore, they become susceptible to the initial conditions as the values of the fields lay in the background dynamics. As in most cases, the inflationary scenarios are based on the hypothesis taken ad hoc which poses the problem of finding the initial conditions not to be much reliable. Thus, based on it, inflationary parameters, in some specific scenarios are predicted not to depend largely on priors of initial conditions [81]. In this work, we utilize the Mar č enko-Pastur law for the distribution of mass scales for the factors $\mu$ and $\Lambda_i$. The Mar č enko-Pastur law puts to use two parameters $\overline{\mu}$ and $\beta$, where $\beta$ stands for the factor $\mu$ expressed as the ratio of rows and columns of the mass. For any mass-scale matrix of order $(n + r) \times n$, we can write it by $\beta = n/n + r$. Now, the values that the parameter $\mu$ can have, which has to be the smallest value on one hand and the largest value on the other hand, can be determined by the following expressions, respectively:

$$\mu_1^2 = x = \overline{\mu}^2\left(1 - \sqrt{\beta}\right)^2,$$
$$\mu_2^2 = y = \overline{\mu}^2\left(1 + \sqrt{\beta}\right)^2, \quad (101)$$

whereas during slow-roll approximation, the field values can be worked out to be

$$\varphi_j(t) \simeq \varphi_j(t_0)[T(t)]^{\mu_1^2/y}, \quad (102)$$

where $T(t) = (\varphi_n(t)/\varphi_n(t_0))$ specifies the ratio of relatively larger field values between $t_0$ and $t$ where they stand for some initial and later times, respectively. Now, we introduce $z = 2\ln[T(t)]/y$ in equations (93) and (95), where $\varphi_j^2$ is replaced by $\varphi_j^2(t_0)e^{z\mu_i^2}$. Straightforwardly, now we can figure out the values of mass distributions on the average within the respective range regardless of the field value distributions in the beginning when the correlation relations are evaded and overlooked between them. Then, by applying the power series expansion, we can find out the average value of the term involving exponentiation.

$$\left\langle e^{z\mu_i^2}\right\rangle = \sum_i\left\langle\mu_i^2\right\rangle\frac{c^j}{j} = \overline{\mu}^{2i}\sum_{j=1}^{i}T(i,j)\beta^{j-1}\frac{c^j}{j}$$
$$= \sum_{i=0}^{\infty}\overline{\mu}^{2i}F_1(1 - i, -i, 2, \beta)\frac{c^j}{j}. \quad (103)$$

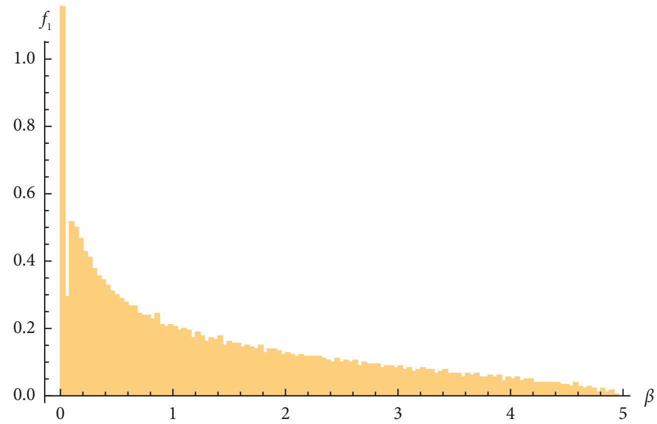
Now, equation (93) can be written as

$$\mu_j^2\varphi_j^2 = n\alpha\overline{\mu}^2\sum_{i=0}^{\infty}\overline{\mu}^{2i}F_1(-i, -i - 1, 2, \beta)\frac{z^i}{i}, \quad (104)$$

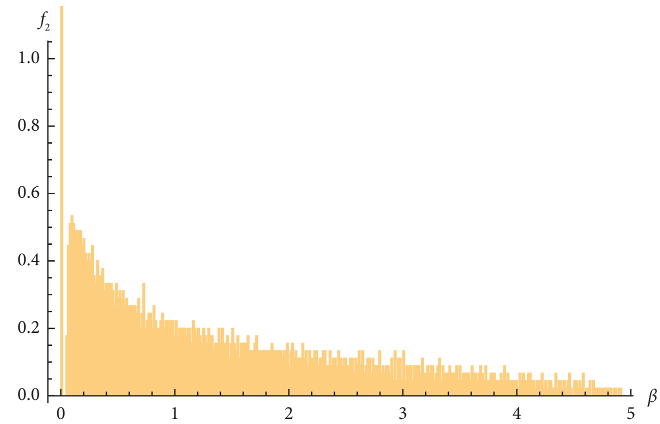
where $\alpha = \langle\zeta_j^2(t_0)\rangle$, moreover, we have

$$\mu_j^4\varphi_j^2 = n\alpha\overline{\mu}^4\sum_{i=0}^{\infty}\overline{\mu}^{2i}F_1(-i - 1, -i - 2, 2, 2, \beta)\frac{z^i}{i}. \quad (105)$$

On substituting equations (104) and (105) in equation (93) in the first place and afterward in equation (95), we obtain the following for $\alpha$:




(a)




(b)

Figure 4: The figure demonstrates the mass distribution according to the Marčhenko–Pastur law as it takes place against the dimensionless mass variables in the case $\beta$ that takes on different values. The parameter $c$ is along the parallel axis when the functions are along vertical axes. It can be noted that the law of large numbers of mass scales ensures that the mass distribution of $N$ fields obeys the distribution probability similar to that of a single field. the (a) plot is simply presented, (b) it is drawn after taking its logarithm.

$$\begin{cases} \alpha = p^2 M_{pl}^6 \sum_i \left( \dfrac{\Lambda_i}{\mu_i} \left( \dfrac{\varphi_i}{\mu_i} \right)^{-(p+1)} \right)^2 f_1(t,\beta) \\ \alpha = \dfrac{1}{2} p M_{pl}^2 \sum_i \left( \dfrac{\Lambda_i}{\mu_i} \left( \dfrac{\varphi_i}{\mu_i} \right)^{-(p+1)} \right)^2 f_2(t,\beta) \end{cases}, \tag{106}$$

where

$$\begin{cases} f_1(t,\beta) = \dfrac{\left( \sum_{i=0}^{\infty} \overline{\mu}^{2i} F_1(-i-1, -i-2, 2, \beta) z^i / i \right)^{3/2}}{\left( \sum_{i=0}^{\infty} \overline{\mu}^{2i} F_1(-i, -i-1, 2, \beta) z^i / i \right)^{5/2}} \\ f_2(t,\beta) = \dfrac{\left( \sum_{i=0}^{\infty} \overline{\mu}^{2i} F_1(-i-1, -i-2, 2, \beta) z^i / i \right)^{1/2}}{\left( \sum_{i=0}^{\infty} \overline{\mu}^{2i} F_1(-i, -i-1, 2, \beta) z^i / i \right)^{2}} \end{cases}, \tag{107}$$

In Figure 4, the distribution of mass scales is plotted according to Mar č enko-Pastur law.

We can have values of the functions $f_1(t,\beta)$ and $f_2(t,\beta)$ corresponding to the distinct values as adapted by or assigned to the parameter $z$. However, for comparatively bigger values of it, the functions behave like a constant as the figures show it. In the case, when values of the fields and mass scales are equivalent, the functions $f_1(t,\beta) = f_2(t,\beta) \simeq 1$ and from equation (106) the value of $\alpha \simeq \varphi^2$ and $\overline{m} = m$, in this case, which leads to regain the values of the concerned fields.

## 3. Comments on Conclusions

In this article, we conducted an investigation into the model of inflationary phase dynamics by considering multifields where a small field potential written in general form $V = \sum_i V_i(\varphi_i) = \sum_i \Lambda_i [1 - (\varphi_i/\mu_i)^{-p}]$ is under consideration. It stands for multifields, however, we figured out the results for up to two fields and presented the outcomes analytically. The model is characterized by two free parameters $p$ and $\mu_i$ which are free to choose as constrained by their predicted range of values. The variable $p$ is importantly negative in the model we worked out and is arbitrarily chosen. The potential upon which the model is based represents the small field inflation and can be regarded as Taylor series expansion about the origin of its minima and maxima in its lowest order. In these models of inflation, the field is usually considered to begin with an unstable equilibrium around the origin and then to roll down along its potential about the origin. As the field expression denotes a generalized potential to stand for multiple scalars connoting the inflationary potential, $i$ denotes an $i$th field taken into account out of multiple fields. The parameters $\Lambda_i$ and $\mu_i$ denote the height and tilt of the $i$th chosen potential in the multiple fields. The spectrum of curvature perturbations that give rise to the growth of cosmic structure is an important relic from inflation. We investigated this spectrum for the potential under consideration here. In the first place, we considered the case for a value of $p$ larger than 2. In this case, in general, when the multifields have the equivalent masses, the equations of motion give rise to those of single-field inflation producing the phase of nonperturbations. This occurs due to relative mass differences in the multifields and it could be observed that the spectrum comes out to be more or less redder in comparison with the corresponding single-field model accordingly. The multifields under consideration as well as their effective masses play a very significant role due to the dependence of the results at the time of horizon-crossing. We noted that the result corresponds to that of a single scalar field when the effective masses of all the fields are taken to be equivalent. The spectrum in this case results to be the same and therefore coincides with the spectrum of a single field. It is concluded that the results for the values of $p > 2$, $p = 2$, and $p = -2$ are different and the behaviors of the field potentials and the corresponding spectrums are distinct as well as different in their nature.

It can be further noted that all the terms included in the factor $\ln(\varphi_k^s/\varphi_k^e)$ might be equivalent on account of the result determined. With some extra terms, the two expressions represent the same equation for the corresponding single-field case. When $\Lambda_i$ are taken to be equivalent to $\mu_i$, the larger value of $\mu_i$ corresponds to the minimum value of $\ln \quad (\varphi_k^s/\varphi_k^e)$. When we consider $\mu_k = \operatorname{Max}(\mu_n)$, where $n$ denotes natural numbers, it leads to $\mu_i/\mu_k < 1$ which implies that the spectrum is redder than its corresponding spectrum resulting from the result of a single scalar field $\varphi_k$. In this case, the value of $\ln(\varphi_k^s/\varphi_k^e)$ would represent almost the smallest value from all the values of $\ln(\varphi_i^s/\varphi_i^e)$ which indicates that in equation (76), expressing the case of a single scalar field $\varphi_k$, the value of $k$ tends to get nearer to unity. On the other hand, when we take the $\mu_k = \operatorname{Min}(\mu_n)$ for $n$ to be a natural number, it gives rise to $\mu_i/\mu_k > 1$, which resultantly leads to the result stating that the spectrum is less red than its corresponding spectrum resulting from the result for a single scalar field $\varphi_k$. In this case, the value of $\ln(\varphi_k^s/\varphi_k^e)$ would represent almost the larger one out of all the values of $\ln(\varphi_k^s/\varphi_k^e)$ which shows that in the case of a single scalar field $\varphi_k$, the value of $k$ shifts away from unity. It means that the value of the scalar spectral index falls between that of a single field in general for the biggest $\mu_k$ and the smallest accordingly.

The results we came across depend on the effective masses and the values of the fields, however, they emerge irrespective of the consideration for the initial conditions. Due to the spectrum being calculated on the time of horizon-crossing, these occur at this time. In order to obtain these results we only require to satisfy the constraints concerning the slow-roll approximation of the fields in the beginning only. The following condition $\delta\varphi_j/\dot{\varphi}_j = \delta\varphi_j/\dot{\varphi}_j$ is required to be imposed so that the isocurvature perturbations can be ignored. By implementing the condition, it seems as though the fields are confined to some specific trajectories. Although the isocurvature perturbation modes look plausible to be taken into account, for the time being, we evaded them to keep the things simple and to stick to the main theme, however, this is underway in our next investigation.

From the investigations conducted with regard to the observable parameters e.g., slow-roll parameters, e-folding number, and spectral index, we see that they effectively influence the inflationary scenario when a host of a large number of scalar fields is taken into account as the multifield case demands. Multifield models might predict a range of values for the spectral index, although the initial values of the multifield scalars depend upon the coefficient $\mu$. In Figures 2 and 3, the spectral index $(n_s)$ is plotted against the e-folding number $N$ for a range of values. It illustrates the behavior and trend of the spectral index against the number of e-folds $N$ where $0.70 < n_s < 0.97$ corresponds to values $N = 20, 30, 40, 50, 60, 70$ for plot (a) in Figure 2 and $0.85 < n_s < 0.98$ corresponds to values $N = 20, 30, 40, 50, 60, 70, 80, 90$ for the logarithm of plot (b) in Figure 2. The range of values of the spectral index against e-folds falls in the viable limit for cosmological evolution. The range of values for spectral index with an increasing number of e-fold is listed in Tables 1 and 2. The scalar field inflationary models in conjunction with the potential in question such as natural inflation, double-well inflationary model, and brane inflationary model are also of concern. The recent Planck results put the stringent constraint on the spectral index $n_s$, that is, $n_s = 0.9649 \pm 0.0042\,(68\,\%\,C.L.)$ which can be used to see as how the model in question contrasts with it.

## Data Availability

The data used to support the findings of the study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

All of the authors shared their contribution to the design and structure of the study concept of the manuscript. First author gave the idea and carried out the calculations, the second assisted in literature review while the third and fourth authors contributed in drafting and reaching the conclusions section with providing financial support.

## Acknowledgments

This study was supported by the Researchers supporting project number (RSPD2023R1060), King Saud University, Riyadh Saudi Arabia.

## References

[1] A. D. Linde, "Generation of isothermal density perturbations in an inflationary universe," *ZhETF Pisma Redaktsiiu*, vol. 40, pp. 496–498, 1984.

[2] L. A. Kofman, A. D. Linde, and A. A. Starobinsky, "Inflationary universe generated by the combined action of a scalar field and gravitational vacuum polarization," *Physics Letters B*, vol. 157, no. 5-6, pp. 361–367, 1985.

[3] L. A. Kofman and A. D. Linde, "Generation of density perturbations in inflationary cosmology," *Nuclear Physics B*, vol. 282, pp. 555–588, 1987.

[4] J. Silk and M. S. Turner, "Double inflation," *Physical Review D*, vol. 35, no. 2, pp. 419–428, 1987.

[5] L. A. Kofman and D. Y. Pogosyan, "Nonflat perturbations in inflationary cosmology," *Physics Letters B*, vol. 214, no. 4, pp. 508–514, 1988.

[6] J. Lesgourgues, "Features in the primordial power spectrum of double D-term inflation," *Nuclear Physics B*, vol. 582, no. 1-3, pp. 593–626, 2000.

[7] D. Polarski and A. A. Starobinsky, "Isocurvature perturbations in multiple inflationary models," *Physical Review D*, vol. 50, no. 10, pp. 6123–6129, 1994.

[8] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, "Adiabatic and entropy perturbations from inflation," *Physical Review D*, vol. 63, no. 2, Article ID 023506, 2000.

[9] G. Dvali, A. Gruzinov, and M. Zaldarriaga, "New mechanism for generating density perturbations from inflation," *Physical Review D*, vol. 69, no. 2, Article ID 023505, 2004.

[10] L. Kofman, "Probing string theory with modulated cosmological fluctuations," 2003, https://arxiv.org/abs/astro-ph/0303614.

[11] F. Bernardeau, L. Kofman, and J. P. Uzan, "Modulated fluctuations from hybrid inflation," *Physical Review D*, vol. 70, no. 8, Article ID 083004, 2004.

[12] A. Linde and V. Mukhanov, "Non-Gaussian isocurvature perturbations from inflation," *Physical Review D*, vol. 56, no. 2, pp. R535–R539, 1997.

[13] D. H. Lyth and D. Wands, "Generating the curvature perturbation without an inflaton," *Physics Letters B*, vol. 524, no. 1-2, pp. 5–14, 2002.

[14] T. Moroi and T. Takahashi, "Effects of cosmological moduli fields on cosmic microwave background," *Physics Letters B*, vol. 522, no. 3-4, pp. 215–221, 2001.

[15] D. H. Lyth, C. Ungarelli, and D. Wands, "Primordial density perturbation in the curvaton scenario," *Physical Review D*, vol. 67, no. 2, Article ID 023503, 2003.

[16] A. H. Guth, "Inflationary universe: a possible solution to the horizon and flatness problems," *Physical Review D*, vol. 23, no. 2, pp. 347–356, 1981.

[17] B. A. Bassett, S. Tsujikawa, and D. Wands, "Inflation dynamics and reheating," *Reviews of Modern Physics*, vol. 78, no. 2, pp. 537–589, 2006.

[18] A. H. Guth and D. I. Kaiser, "Inflationary cosmology: exploring the universe from the smallest to the largest scales," *Science*, vol. 307, no. 5711, pp. 884–890, 2005.

[19] A. N. Taylor and A. Berera, "Perturbation spectra in the warm inflationary scenario," *Physical Review D*, vol. 62, no. 8, Article ID 083517, 2000.

[20] A. Riotto, "Inflation and the theory of cosmological perturbations," 2002, https://arxiv.org/abs/hep-ph/0210162.

[21] P. K. Man, "Slow-roll analysis of double-field axion inflation," *Progress of Theoretical and Experimental Physics*, vol. 2018, no. 11, Article ID 113E04, 2018.

[22] A. R. Liddle, A. Mazumdar, and F. E. Schunck, "Assisted inflation," *Physical Review D*, vol. 58, no. 6, Article ID 061301, 1998.

[23] Y. S. Piao, R. G. Cai, X. Zhang, and Y. Z. Zhang, "Assisted tachyonic inflation," *Physical Review D*, vol. 66, no. 12, Article ID 121301, 2002.

[24] A. Ashoorioon, A. Krause, and K. Turzynski, "Energy transfer in multi field inflation and cosmological perturbations," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 02, p. 014, 2009.

[25] Y. S. Piao, "Primordial density perturbation and decaying speed of sound," *Physical Review D*, vol. 79, no. 6, Article ID 067301, 2009.

[26] M. D. Marsh, J. D. Barrow, and C. Ganguly, "Inhomogeneous initial data and small-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2018, no. 05, p. 026, 2018.

[27] S. H. H. Tye, J. Xu, and Y. Zhang, "Multifield inflation with a random potential," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 04, p. 018, 2009.

[28] T. Kobayashi and S. Mukohyama, "Effects of light fields during inflation," *Physical Review D*, vol. 81, no. 10, Article ID 103504, 2010.

[29] S. V. Ketov, "Multifield versus single-field in the supergravity models of inflation and primordial black holes," *Universe*, vol. 7, no. 5, p. 115, 2021.

[30] L. Senatore and M. Zaldarriaga, "The effective field theory of multifield inflation," *Journal of High Energy Physics*, vol. 2012, no. 4, pp. 24–62, 2012.

[31] O. Özsoy and G. Tasinato, "Inflation and primordial black holes," *Universe*, vol. 9, no. 5, p. 203, 2023.

[32] R. G. Cai, B. Hu, and Y. S. Piao, "Entropy perturbations in N-flation," *Physical Review D*, vol. 80, no. 12, Article ID 123505, 2009.

[33] E. D. Stewart and D. H. Lyth, "A more accurate analytic calculation of the spectrum of cosmological perturbations produced during inflation," *Physics Letters B*, vol. 302, no. 2-3, pp. 171–175, 1993.

[34] E. H. Lieb and J. Yngvason, "The physics and mathematics of the second law of thermodynamics," *Physics Reports*, vol. 310, no. 1, pp. 1–96, 1999.

[35] I. Wolfson and R. Brustein, "Small field models with gravitational wave signature supported by CMB data," *PLoS One*, vol. 13, no. 5, Article ID e0197735, 2018.

[36] C. Longden, "Non-standard hierarchies of the runnings of the spectral index in inflation," *Universe*, vol. 3, no. 1, p. 17, 2017.

[37] P. Avelino, T. Barreiro, C. S. Carvalho et al., "Unveiling the dynamics of the universe," *Symmetry*, vol. 8, no. 8, p. 70, 2016.

[38] V. Kamali, M. Motaharfar, and R. O. Ramos, "Recent developments in warm inflation," *Universe*, vol. 9, no. 3, p. 124, 2023.

[39] D. Cruces, "Review on stochastic approach to inflation," *Universe*, vol. 8, no. 6, p. 334, 2022.

[40] N. Sauber, H. Theisel, and H. P. Seidel, "Multifield-graphs: an approach to visualizing correlations in multifield scalar data," *IEEE Transactions on Visualization and Computer Graphics*, vol. 12, no. 5, pp. 917–924, 2006.

[41] C. T. Byrnes and K. Y. Choi, "Review of local non-gaussianity from multifield inflation," *Advances in Astronomy*, vol. 2010, Article ID 724525, 18 pages, 2010.

[42] P. Carrilho, D. Mulryne, J. Ronayne, and T. Tenkanen, "Attractor behaviour in multifield inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2018, no. 06, p. 032, 2018.

[43] A. Avgoustidis, S. Cremonini, A. C. Davis, R. H. Ribeiro, K. Turzyński, and S. Watson, "The importance of slow-roll corrections during multifield inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2012, no. 02, p. 038, 2012.

[44] S. C. Hotinli, J. Frazer, A. H. Jaffe, J. Meyers, L. C. Price, and E. R. Tarrant, "Effect of reheating on predictions following multiple-field inflation," *Physical Review D*, vol. 97, no. 2, Article ID 023511, 2018.

[45] J. Frazer, "Predictions in multifield models of inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2014, no. 01, p. 028, 2014.

[46] S. Clesse, "Hybrid inflation: multi-field dynamics and cosmological constraints," 2011, https://arxiv.org/abs/1109.5575.

[47] K. Asadi and K. Nozari, "Reheating constraints on a two-field inflationary model," *Nuclear Physics B*, vol. 949, Article ID 114827, 2019.

[48] D. Battefeld and T. Battefeld, "Multifield inflation on the landscape," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 03, p. 027, 2009.

[49] T. Banks and W. Fischler, "An upper bound on the number of e-foldings," 2003, https://arxiv.org/abs/astro-ph/0307459.

[50] L. Alabidi and D. H. Lyth, "Inflation models and observation," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 05, p. 016, 2006.

[51] R. Easther and L. McAllister, "Random matrices and the spectrum of N-flation," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 05, p. 018, 2006.

[52] S. A. Kim and A. R. Liddle, "N-flation: multifield inflationary dynamics and perturbations," *Physical Review D*, vol. 74, no. 2, Article ID 023513, 2006.

[53] R. G. Cai, Z. K. Guo, and S. J. Wang, "Reheating phase diagram for single-field slow-roll inflationary models," *Physical Review D*, vol. 92, no. 6, Article ID 063506, 2015.

[54] Y. S. Piao, "Perturbation spectra of N-flation," *Physical Review D*, vol. 74, no. 4, Article ID 047302, 2006.

[55] J. O. Gong, "End of multifield inflation and the perturbation spectrum," *Physical Review D*, vol. 75, no. 4, Article ID 043502, 2007.

[56] M. E. Olsson, "Inflation assisted by heterotic axions," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 04, p. 019, 2007.

[57] S. Thomas and J. Ward, "IR inflation from multiple branes," *Physical Review D*, vol. 76, no. 2, Article ID 023509, 2007.

[58] S. A. Kim and A. R. Liddle, "N-flation: non-Gaussianity in the horizon-crossing approximation," *Physical Review D*, vol. 74, no. 6, Article ID 063522, 2006.

[59] S. A. Kim and A. R. Liddle, "N-flation: observable predictions from the random matrix mass spectrum," *Physical Review D*, vol. 76, no. 6, Article ID 063515, 2007.

[60] J. Martin, C. Ringeval, and V. Vennin, "Encyclopædia inflationaris," *Physics of the Dark Universe*, vol. 5-6, pp. 75–235, 2014.

[61] K. Freese, J. A. Frieman, and A. V. Olinto, "Natural inflation with pseudo Nambu-Goldstone bosons," *Physical Review Letters*, vol. 65, no. 26, pp. 3233–3236, 1990.

[62] L. Knox and A. Olinto, "Initial conditions for natural inflation," *Physical Review D*, vol. 48, no. 2, pp. 946–949, 1993.

[63] W. H. Kinney and K. T. Mahanthappa, "Inflation at low scales: general analysis and a detailed model," *Physical Review D*, vol. 53, no. 10, pp. 5455–5467, 1996.

[64] L. Covi, "Models of inflation, supersymmetry breaking and observational constraints," in *Dark Matter in Astro-And Particle Physics*, pp. 163–175, Springer, Berlin, Heidelberg, 2001.

[65] K. Tzirakis and W. H. Kinney, "Inflation over a local maximum of a potential," *Physical Review D*, vol. 75, no. 12, Article ID 123510, 2007.

[66] M. Kawasaki and M. Yamaguchi, "Supersymmetric topological inflation model," *Physical Review D*, vol. 65, no. 10, Article ID 103518, 2002.

[67] W. H. Kinney and A. Riotto, "Dynamical supersymmetric inflation," *Astroparticle Physics*, vol. 10, no. 4, pp. 387–395, 1999.

[68] W. H. Kinney and A. Riotto, "A signature of inflation from dynamical supersymmetry breaking," *Physics Letters B*, vol. 435, no. 3-4, pp. 272–276, 1998.

[69] L. Boubekeur and D. H. Lyth, "Hilltop inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 07, p. 010, 2005.

[70] J. L. Cervantes-Cota and H. Dehnen, "Induced gravity inflation in the standard model of particle physics," *Nuclear Physics B*, vol. 442, no. 1-2, pp. 391–409, 1995.

[71] J. O. Gong, "Modular thermal inflation without slow-roll approximation," *Physics Letters B*, vol. 637, no. 3, pp. 149–155, 2006.

[72] G. Lazarides and A. Vamvasakis, "Standard-smooth hybrid inflation," *Physical Review D*, vol. 76, no. 12, Article ID 123514, 2007.

[73] F. Bauer and D. A. Demir, "Higgs–Palatini inflation and unitarity," *Physics Letters B*, vol. 698, no. 5, pp. 425–429, 2011.

[74] A. O. Barvinsky, "Standard Model Higgs inflation: CMB, Higgs mass and quantum cosmology," *Progress of Theoretical Physics Supplement*, vol. 190, pp. 1–19, 2011.

[75] R. Kallosh and A. Linde, "New models of chaotic inflation in supergravity," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 11, p. 011, 2010.

[76] D. Langlois, "Correlated adiabatic and isocurvature perturbations from double inflation," *Physical Review D*, vol. 59, no. 12, Article ID 123512, 1999.

[77] L. Amendola, C. Gordon, D. Wands, and M. Sasaki, "Correlated perturbations from inflation and the cosmic microwave background," *Physical Review Letters*, vol. 88, no. 21, Article ID 211302, 2002.

[78] C. T. Byrnes and D. Wands, "Curvature and isocurvature perturbations from two-field inflation in a slow-roll expansion," *Physical Review D*, vol. 74, no. 4, Article ID 043529, 2006.

[79] M. Sasaki and T. Tanaka, "Super-horizon scale dynamics of multi-scalar inflation," *Progress of Theoretical Physics*, vol. 99, no. 5, pp. 763–781, 1998.

[80] M. Sasaki and E. D. Stewart, "A General analytic formula for the spectral index of the density perturbations produced during inflation," *Progress of Theoretical Physics*, vol. 95, no. 1, pp. 71–78, 1996.

[81] R. Easther, J. Frazer, H. V. Peiris, and L. C. Price, "Simple predictions from multifield inflationary models," *Physical Review Letters*, vol. 112, no. 16, Article ID 161302, 2014.