EXISTENCE AND REGULARITY OF WEAK SOLUTIONS TO THE PRESCRIBED MEAN CURVATURE EQUATION FOR A NONPARAMETRIC SURFACE

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1. Introduction

The prescribed mean curvature equation with Dirichlet condition for a nonparametric surface $X : \Omega \rightarrow \mathbb{R}^3$, $X(u, v) = (u, v, f(u, v))$ is the quasilinear partial differential equation

$$\begin{aligned} (1 + f_u^2)f_{uu} + (1 + f_v^2)f_{vv} - 2f_u f_v f_{uv} &= 2h(u, v, f)(1 + |\nabla f|^2)^{3/2} \quad \text{in } \Omega, \\ f &= g \quad \text{in } \partial\Omega, \end{aligned} \quad (1.1)$$

where Ω is a bounded domain in $\mathbb{R}^2$, $h : \overline{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $g \in H^1(\Omega)$.

We call $f \in H^1(\Omega)$ a weak solution of (1.1) if $f \in g + H_0^1(\Omega)$ and for every $\varphi \in C_0^1(\Omega)$

$$\int_{\Omega} ((1 + |\nabla f|^2)^{-1/2} \nabla f \nabla \varphi + 2h(u, v, f)\varphi) du dv = 0. \quad (1.2)$$

It is known that for the parametric Plateau's problem, weak solutions can be obtained as critical points of a functional (see [2, 6, 7, 8, 10, 11]).

The nonparametric case has been studied for $H = H(x, y)$ (and generally $H = H(x_1, \dots, x_n)$ for hypersurfaces in $\mathbb{R}^{n+1}$) by Gilbarg, Trudinger, Simon, and Serrin, among other authors. It has been proved [5] that there exists a solution for any smooth boundary data if the mean curvature H' of $\partial\Omega$ satisfies

$$H'(x_1, \dots, x_n) \geq \frac{n}{n-1} |H(x_1, \dots, x_n)| \quad (1.3)$$

for any $(x_1, \dots, x_n) \in \partial\Omega$, and $H \in C^1(\overline{\Omega}, \mathbb{R})$ satisfying the inequality

$$\left| \int_{\Omega} H \varphi \right| \leq \frac{1-\epsilon}{n} \int_{\Omega} |D\varphi| \quad (1.4)$$

for any $\varphi \in C_0^1(\Omega, \mathbb{R})$ and some $\epsilon > 0$. They also proved a non-existence result (see [5, Corollary 14.13]): if $H'(x_1, \dots, x_n) < (n/(n-1))|H(x_1, \dots, x_n)|$ for some $(x_1, \dots, x_n)$ and the sign of H is constant, then for any $\epsilon > 0$ there exists $g \in C^\infty(\overline{\Omega})$ such that $\|g\|_\infty \leq \epsilon$ and that Dirichlet's problem is not solvable.

We remark that the solutions obtained in [5] are classical. In this paper, we find weak solutions of the problem by variational methods.

We prove that for prescribed h there exists an associated functional to h , and under some conditions on h and g we find that this functional has a global minimum in a convex subset of $H^1(\Omega)$, which provides a weak solution of (1.1). We denote by $H^1(\Omega)$ the usual Sobolev space, [1].

2. The associated variational problem

Given a function $f \in C^2(\Omega)$, the generated nonparametric surface associated to this function is the graph of f in $\mathbb{R}^3$, parametrized as $X(u, v) = (u, v, f(u, v))$.

The mean curvature of this surface is

$$h(u, v, f) = \frac{1}{2} \frac{Ef_{vv} - 2Ff_{uv} + Gf_{uu}}{(1 + f_u^2 + f_v^2)^{3/2}}, \quad (2.1)$$

where E, F , and G are the coefficients of the first fundamental form [4, 9].

For prescribed h , weak solutions of (1.1) can be obtained as critical points of a functional.

PROPOSITION 2.1. *Let $J_h : H^1(\Omega) \rightarrow \mathbb{R}$ be the functional defined by*

$$J_h(f) = \int_{\Omega} ((1 + |\nabla f|^2)^{1/2} + H(u, v, f)) du dv, \quad (2.2)$$

where $H(u, v, z) = \int_0^z 2h(u, v, t)dt$. Then (1.1) is the Euler Lagrange equation of (2.2).

Remark 2.2. If $f \in T = g + H_0^1(\Omega)$ is a critical point of J_h , then f is a weak solution of (1.1).

Proof. For $\varphi \in C_0^1(\Omega)$, integrating by parts we obtain

$$dJ_h(f)(\varphi) = 2 \int_{\Omega} \left(\frac{1}{2} \frac{Ef_{vv} - 2Ff_{uv} + Gf_{uu}}{(1 + f_u^2 + f_v^2)^{3/2}} - h(u, v, f) \right) \varphi du dv. \quad (2.3)$$

□

3. Behavior of the functional J_h

In this section, we study the behavior of the functional J_h restricted to T . For simplicity we write $J_h(f) = A(f) + B(f)$, with

$$A(f) = \int_{\Omega} (1 + |\nabla f|^2)^{1/2} du dv, \quad B(f) = \int_{\Omega} H(u, v, f) du dv. \quad (3.1)$$

We will assume that h is bounded.

LEMMA 3.1. *The functional $A : T \rightarrow \mathbb{R}$ is continuous and convex.*

Proof. Continuity can be proved by a simple computation. Let $a, b \geq 0$ such that $a + b = 1$. By Cauchy inequality, it follows that

$$\sqrt{1 + |\nabla(af + bf_0)|^2} \leq a\sqrt{1 + |\nabla f|^2} + b\sqrt{1 + |\nabla f_0|^2} \quad (3.2)$$

and convexity holds. $\square$

Remark 3.2. As A is continuous and convex, then it is weakly lower semicontinuous in T .

LEMMA 3.3. *The functional B is weakly lower semicontinuous in T .*

Proof. Since h is bounded, we have

$$|H(u, v, z)| \leq c|z| + d. \quad (3.3)$$

From the compact immersion $H_0^1(\Omega) \hookrightarrow L^1(\Omega)$ and the continuity of Nemytskii operator associated to H in $L^1(\Omega)$, we conclude that B is weakly lower semicontinuous in T (see [3, 12]). $\square$

4. Weak solutions as critical points of J_h

Let us assume that $g \in W^{1,\infty}$, and consider for each $k > 0$, the following subset of T :

$$\overline{M}_k = \{f \in T : \|\nabla(f - g)\|_\infty \leq k\}. \quad (4.1)$$

$\overline{M}_k$ is nonempty, closed, convex, bounded, then it is weakly compact.

Remark 4.1. As $g \in W^{1,\infty}$, taking $p > 2$ we obtain, for any $f \in \overline{M}_k$:

$$\|f - g\|_p \leq c\|\nabla(f - g)\|_p. \quad (4.2)$$

Then, by Sobolev imbedding, $\|f - g\|_\infty \leq c_1\|f - g\|_{1,p} \leq \bar{c}k$ for some constant $\bar{c}$. We deduce that $f \in W^{1,\infty}$ and $f(\Omega) \subset K$ for some fixed compact $K \subset \mathbb{R}$. Thus, the assumption $\|h\|_\infty < \infty$ is not needed.

Let ρ be the slope of J_h in $\overline{M}_k$ defined by

$$\rho(f_0, \overline{M}_k) = \sup \{dJ_h(f_0)(f_0 - f); f \in \overline{M}_k\} \quad (4.3)$$

(see [7, 11]), then the following result holds.

LEMMA 4.2. *If $f_0 \in \overline{M}_k$ verifies*

$$J_h(f_0) = \inf \{J_h(f) : f \in \overline{M}_k\}, \quad (4.4)$$

then $\rho(f_0, \overline{M}_k) = 0$.

Proof.

$$\begin{aligned} dJ_h(f_0)(f - f_0) &= \lim_{\varepsilon \rightarrow 0} \frac{J_h(f_0 + \varepsilon(f - f_0)) - J_h(f_0)}{\varepsilon} \\ &= \lim_{\varepsilon \rightarrow 0} \frac{J_h((1 - \varepsilon)f_0 + \varepsilon f) - J_h(f_0)}{\varepsilon}. \end{aligned} \quad (4.5)$$

When $0 < \varepsilon < 1$ we have that $(1 - \varepsilon)f_0 + \varepsilon f \in \overline{M}_k$, and then $dJ_h(f_0)(f_0 - f) \leq 0$ for all $f \in \overline{M}_k$. As $dJ_h(f_0)(f_0 - f_0) = 0$, we conclude that $\rho(f_0, \overline{M}_k) = 0$. $\square$

Remark 4.3. Let J_h be weakly semicontinuous and let $\overline{M}_k$ be a weakly compact subset of T , then J_h achieves a minimum f_0 in $\overline{M}_k$. By Lemma 4.2, $\rho(f_0, \overline{M}_k) = 0$.

As in [7], if f_0 has zero slope, we call it a ρ -critical point. The following result gives sufficient conditions to assure that if f_0 is a ρ -critical point, then it is a critical point of J_h .

THEOREM 4.4. *Let $f_0 \in \overline{M}_k$ such that $\rho(f_0, \overline{M}_k) = 0$, and assume that one of the following conditions holds:*

- (i) $dJ_h(f_0)(f_0 - g) \geq 0$
- (ii) $\|\nabla(f_0 - g)\|_\infty < k$.

Then $dJ_h(f_0) = 0$.

Proof. As $\rho(f_0, \overline{M}_k) = 0$, we have that $dJ_h(f_0)(f_0 - f) \leq 0$, and then $dJ_h(f_0)(f_0 - g) \leq dJ_h(f_0)(f - g)$ for any $f \in \overline{M}_k$.

We will prove that $dJ_h(f_0)(\varphi) = 0$ for any $\varphi \in C_0^1$. Let $\tilde{\varphi} = k\varphi/2\|\nabla\varphi\|_\infty$, then $\pm\tilde{\varphi} + g \in \overline{M}_k$, and then $dJ_h(f_0)(f_0 - g) \leq \pm dJ_h(f_0)(\tilde{\varphi})$.

Suppose that $dJ_h(f_0)(\tilde{\varphi}) \neq 0$, then $dJ_h(f_0)(f_0 - g) < 0$.

If (i) holds, we immediately get a contradiction. On the other hand, if (ii) holds, there exists $r > 1$ such that $g + r(f_0 - g) \in \overline{M}_k$. Then $dJ_h(f_0)(f_0 - g) \leq rdJ_h(f_0)(f_0 - g)$, a contradiction. $\square$

Examples

Let us assume that $\int_\Omega ((\nabla(f - g)\nabla g)/\sqrt{1 + |\nabla f|^2}) du dv \geq 0$ for any $f \in \overline{M}_k$. Then condition (i) of Theorem 4.4 is fulfilled for example if

(a) $|h(u, v, z)| \leq c(z - g(u, v))_+$ for every $(u, v) \in \Omega$, $z \in \mathbb{R}^3$, for some constant c small enough.

(b) $\int_\Omega h(u, v, f)(f - g) du dv \geq 0$ for every $f \in \overline{M}_k$. As a particular case, we may take $h(u, v, z) = c(z - g(u, v))$ for any $c \geq 0$.

(c) $h(u, v, z) = -c(z - g(u, v))$ for some $c > 0$ small enough.

Indeed, in all the examples the inequality $dJ_h(f)(f-g) \geq 0$ holds for any $f \in \overline{M}_k$, since

$$\begin{aligned} dJ_h(f)(f-g) &= \int_{\Omega} \left(\frac{\nabla f \nabla(f-g)}{\sqrt{1+|\nabla f|^2}} + 2h(u, v, f)(f-g) \right) du dv \\ &= \int_{\Omega} \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(f-g) \right) du dv + \int_{\Omega} \frac{\nabla(f-g) \nabla g}{\sqrt{1+|\nabla f|^2}} du dv \\ &\geq \int_{\Omega} \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(f-g) \right) du dv. \end{aligned} \quad (4.6)$$

Then the result follows immediately in example (b). In examples (a) and (c), being $\|\nabla(f-g)\|_{\infty} \leq k$ we can choose $\tilde{k}$ such that $\sqrt{1+\|\nabla f\|_{\infty}^2} \leq \tilde{k}$. Then

$$\begin{aligned} \int_{\Omega} \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(u, v, f)(f-g) \right) du dv &\geq \int_{\Omega} \left(\frac{|\nabla(f-g)|^2}{\tilde{k}} - 2c(f-g)^2 \right) du dv \\ &\geq \frac{1}{\tilde{k}} \|\nabla(f-g)\|_2^2 - 2cc_1^2 \|\nabla(f-g)\|_2^2 \\ &= \left(\frac{1}{\tilde{k}} - 2cc_1^2 \right) \|\nabla(f-g)\|_2^2, \end{aligned} \quad (4.7)$$

where c_1 is the Poincaré's constant associated to Ω .

Thus, the result holds for $c \leq 1/2\tilde{k}c_1^2$.

Remark 4.5. As in the preceding examples, it can be proved that if $dJ_h(f)(f-g) \geq 0$ for any $f \in \overline{M}_k$, then g is a weak solution of (1.1). Indeed, if $dJ_h(g) \neq 0$, from Theorem 4.4 it follows that $\rho(g, \overline{M}_k) > 0$. As J_h achieves a minimum in every $\overline{M}_k$, we may take $k \geq k_n \rightarrow 0$, and f_n such that $\rho(f_n, \overline{M}_{k_n}) = 0$. As $\overline{M}_{k_n} \subset \overline{M}_k$, condition (i) in Theorem 4.4 holds, and then $dJ_h(f_n) = 0$. It is immediate that $f_n \rightarrow g$ in $W^{1,\infty}$, and then it follows easily that $dJ_h(g) = 0$.

Furthermore, for constant g we can see that if $dJ_h(f)(f-g) \geq 0$ for any $f \in \overline{M}_k$, then g is a global minimum of J_h in $\overline{M}_k$: let us define $\varphi(t) = J_h(tf + (1-t)g)$, then $\varphi'(t) = dJ_h(tf + (1-t)g)(f-g)$. As $0 \leq dJ_h(tf + (1-t)g)(tf + (1-t)g - g) = tdJ_h(tf + (1-t)g)(f-g)$ it follows that $J_h(f) - J_h(g) = \varphi(1) - \varphi(0) = \varphi'(c) \geq 0$.

5. Multiple solutions

In this section, we study the multiplicity of weak solutions of (1.1). Consider

$$\overline{N}_k = \left\{ f \in \overline{M}_k \cap H^2 : \left\| \frac{\partial^2 f}{\partial x_i \partial x_j} \right\|_2 \leq k \right\}, \quad (5.1)$$

$\overline{N}_k$ is a nonempty, closed, bounded, and convex subset of T , therefore $\overline{N}_k$ is weakly compact.

Then we obtain the following theorem, which is a variant of the mountain pass lemma.

THEOREM 5.1. *Let $f_0 \in \overline{N}_k$ be a local minimum of J_h and assume that $J_h(f_1) < J_h(f_0)$ for some $f_1 \in \overline{N}_k$. Let*

$$c = \inf_{\gamma \in \Gamma} \sup_{t \in [0,1]} J_h(\gamma(t)), \quad (5.2)$$

where $\Gamma = \{\gamma \in C([0,1], \overline{N}_k) : \gamma(0) = f_0, \gamma(1) = f_1\}$. Then there exists $f \in \overline{N}_k$ such that $J_h(f) = c$ and $\rho(f, \overline{N}_k) = 0$.

We remark that f is not a local minimum of J_h . This kind of f is called an unstable critical point.

The proof of Theorem 5.1 follows from Theorem 3 in [7] and Lemmas 5.2, 5.3, and 5.4 below.

LEMMA 5.2. *The functional J_h is $C^1(\overline{N}_k)$.*

Proof. Let $f, f_0 \in \overline{N}_k$. Then

$$\begin{aligned} & |dJ_h(f)(\varphi) - dJ_h(f_0)(\varphi)| \\ & \leq \|\varphi\|_{H_0^1} \left(\left\| \frac{\nabla f}{\sqrt{1+|\nabla f|^2}} - \frac{\nabla f_0}{\sqrt{1+|\nabla f_0|^2}} \right\|_2 + \|N_h(f_0) - N_h(f)\|_2 \right), \end{aligned} \quad (5.3)$$

where N_h is the Nemytskii operator associated to h . Let

$$\begin{aligned} \left\| \frac{\nabla f}{\sqrt{1+|\nabla f|^2}} - \frac{\nabla f_0}{\sqrt{1+|\nabla f_0|^2}} \right\|_2 & \leq \left\| \sqrt{1+|\nabla f_0|^2} \nabla f - \sqrt{1+|\nabla f|^2} \nabla f_0 \right\|_2 \\ & \leq \kappa \|f_0 - f\|_{H_0^1} \end{aligned} \quad (5.4)$$

and $N_h : L^2 \rightarrow L^2$ continuous, the result holds. $\square$

LEMMA 5.3. *The slope ρ is H^1 -continuous.*

Proof. Let $f_n \in \overline{N}_k$ such that $f_n \rightarrow f_0$ in H_0^1 . For $\epsilon > 0$ we take $g_n \in \overline{N}_k$ such that $\rho(f_n, \overline{N}_k) - \epsilon/2 < dJ_h(f_n)(f_n - g_n)$. Then

$$\begin{aligned} \rho(f_n, \overline{N}_k) - \rho(f_0, \overline{N}_k) & \leq dJ_h(f_n)(f_n - g_n) + \frac{\epsilon}{2} - dJ_h(f_0)(f_0 - g_n) \\ & \leq \|dJ_h(f_n)\|_{(H_0^1)^*} \|f_n - f_0\|_{H_0^1} \\ & \quad + \|dJ_h(f_n) - dJ_h(f_0)\|_{(H_0^1)^*} \|f_0 - g_n\|_{H_0^1} + \frac{\epsilon}{2} < \epsilon \end{aligned} \quad (5.5)$$

for $n \geq n_0$. Operating in the same way with $\rho(f_0, \overline{N}_k) - \rho(f_n, \overline{N}_k)$, we conclude that $\rho(f_n, \overline{N}_k) \rightarrow \rho(f_0, \overline{N}_k)$. $\square$

LEMMA 5.4 (Palais Smale condition). *Let $(f_n)_{n \in \mathbb{N}} \subset \overline{N}_k$ such that $\lim_{n \rightarrow \infty} \rho(f_n, \overline{N}_k) = 0$. Then $(f_n)_{n \in \mathbb{N}}$ has a convergent subsequence in $H_0^1(\Omega)$.*

Proof. As $f_n \in \overline{N}_k$, we may suppose that $f_n \rightarrow f$ weakly. Let $\Psi_n = f_n - f$. We will see that $\Psi_n \rightarrow 0$. Indeed,

$$\begin{aligned} dJ_h(f_n)(\Psi_n) &= \int_{\Omega} \left(\frac{\nabla f_n}{\sqrt{1+|\nabla f_n|^2}} \nabla \Psi_n + 2h(u, v, f_n) \Psi_n \right) du dv \\ &= \int_{\Omega} \frac{1}{\sqrt{1+|\nabla f_n|^2}} |\nabla \Psi_n|^2 du dv + \int_{\Omega} \frac{\nabla \Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f du dv \quad (5.6) \\ &\quad + \int_{\Omega} 2h(u, v, f_n) \Psi_n du dv. \end{aligned}$$

Then for some constant c

$$c \|\nabla \Psi_n\|_2^2 \leq \rho(f_n, \overline{N}_k) - \int_{\Omega} \frac{\nabla \Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f du dv - \int_{\Omega} 2h(u, v, f_n) \Psi_n du dv. \quad (5.7)$$

By Rellich-Kondrachov theorem $\Psi_n \rightarrow 0$ in $L^2(\Omega)$, and then

$$\left| \int_{\Omega} 2h(u, v, f_n) \Psi_n du dv \right| \leq 2\|h\|_{\infty} |\Omega|^{1/2} \|\Psi_n\|_2 \rightarrow 0, \quad (5.8)$$

$$\begin{aligned} &\left| \int_{\Omega} \frac{\nabla \Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f du dv \right| \\ &= \left| - \int_{\Omega} \frac{\Delta f}{\sqrt{1+|\nabla f_n|^2}} \Psi_n du dv - \int_{\Omega} \Psi_n \nabla (1+|\nabla f_n|^2)^{-1/2} \nabla f du dv \right| \quad (5.9) \\ &\leq \|\Delta f\|_2 \|\Psi_n\|_2 + \|\nabla f_n\|_{\infty} \|\nabla f\|_{\infty} \|D^2 f_n\|_2 \|\Psi_n\|_2 \rightarrow 0. \quad \square \end{aligned}$$

Example 5.5. Now we will show with an example that problem (1.1) may have at least three ρ -critical points in N_k .

Let $g = g_0$ be a constant, and $h(u, v, z) = -c(z - g_0)$ for some constant $c > 0$. Then, g_0 is a minimum of J_h in $\overline{M}_{k_1}$ for k_1 small enough, and a local minimum in M_k for any $k \geq k_1$.

Moreover, taking $\Omega = B_R$, $f(u, v) = g_0 + R^2 - (u^2 + v^2)$, it follows that

$$J_h(f) - J_h(g_0) = 2\pi \left(o(R^3) - \frac{c}{6} R^6 \right), \quad (5.10)$$

and taking $k = 2\sqrt{\pi}R$ it holds that $f \in \overline{N}_k$. Hence, if R is big enough, it follows that g_0 is not a global minimum in $\overline{N}_k$. Furthermore, we see that the proof of Lemma 4.2 may be repeated in $\overline{N}_k$, and then the minimum of J_h in $\overline{N}_k$ is a ρ -critical point. From Theorem 5.1 there is a third ρ -critical point which is not a local minimum of J_h .

6. Regularity

As we proved, problem (1.1) admits (for an appropriate $k > 0$) a weak solution in a subset $\overline{M}(k) = \{f \in T / \|\nabla(f - g)\|_\infty \leq k\}$.

Consider $p > 2$, and $f_0 \in W^{2,p}(\Omega) \hookrightarrow C^1(\overline{\Omega})$ a weak solution of (1.1). Then $L_{f_0} f_0 = 2h(u, v, f_0)(1 + \nabla f_0^2)^{3/2}$ in Ω where for any $f \in C^1(\overline{\Omega})$ $L_f : W^{2,p} \rightarrow L^p$ is the strictly elliptic operator given by

$$L_f \phi = (1 + f_v^2)\phi_{uu} + (1 + f_u^2)\phi_{vv} - 2f_u f_v \phi_{uv}. \quad (6.1)$$

In order to prove the regularity of f_0 , we study equation (6.2)

$$L_{f_0} \phi = 2h(u, v, f_0)(1 + \nabla f_0^2)^{3/2} \quad \text{in } \Omega, \quad \phi = g \text{ in } \partial\Omega. \quad (6.2)$$

PROPOSITION 6.1. *Let us assume that $\partial\Omega \in C^{2,\alpha}$, $g \in C^{2,\alpha}$, and $h \in C^\alpha$ for some $0 < \alpha \leq 1 - 2/p$. Then, if $\phi \in W^{2,p}$ is a strong solution of (6.2), $\phi \in C^{2,\alpha}(\overline{\Omega})$.*

Proof. By Sobolev imbedding $\phi \in C^{1,\alpha}(\overline{\Omega})$. Then $L_{f_0} \phi \in C^\alpha(\overline{\Omega})$ and the coefficients of the operator L_{f_0} belong to C^α . By Theorem 6.14 in [5], the equation $Lw = L_{f_0} \phi$ in Ω , $w = g$ in $\partial\Omega$ is uniquely solvable in $C^{2,\alpha}(\overline{\Omega})$, and the result follows from the uniqueness in Theorem 9.15 in [5]. $\square$

Remark 6.2. As a simple consequence, we obtain that $f_0 \in C^{2,\alpha}(\overline{\Omega})$, by the uniqueness in $W^{2,p}$ given by [5, Theorem 9.15].

COROLLARY 6.3. *Let us assume that $\partial\Omega \in C^{k+2,\alpha}$, $g \in C^{k+2,\alpha}$, and $h \in C^{k,\alpha}$ for some $0 < \alpha \leq 1 - 2/p$. Then $f_0 \in C^{k+2,\alpha}(\overline{\Omega})$.*

Proof. It is immediate from Proposition 2.1 and Theorem 6.19 in [5]. $\square$

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