ON THE EXTERIOR MAGNETIC FIELD AND SILENT SOURCES IN MAGNETOENCEPHALOGRAPHY

GEORGE DASSIOS AND FOTINI KARIOTOU

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Two main results are included in this paper. The first one deals with the leading asymptotic term of the magnetic field outside any conductive medium. In accord with physical reality, it is proved mathematically that the leading approximation is a quadrupole term which means that the conductive brain tissue weakens the intensity of the magnetic field outside the head. The second one concerns the orientation of the silent sources when the geometry of the brain model is not a sphere but an ellipsoid which provides the best possible mathematical approximation of the human brain. It is shown that what characterizes a dipole source as "silent" is not the collinearity of the dipole moment with its position vector, but the fact that the dipole moment lives in the Gaussian image space at the point where the position vector meets the surface of the ellipsoid. The appropriate representation for the spheroidal case is also included.

1. The magnetic field

The mathematical theory of magnetoencephalography (MEG) is governed by the equations of quasistatic theory of electromagnetism [11, 14, 15, 19, 20]. If we denote by V^- the region occupied by the conductive brain tissue, with conductivity $\sigma > 0$ and magnetic permeability $\mu_0 > 0$, then, as Geselowitz has shown [3, 9, 10], the magnetic field in the exterior of V^- region, V^+, due to the internal electric dipole current

$$\mathbf{J}^p(\mathbf{r}) = \mathbf{Q}\delta(\mathbf{r}-\mathbf{r}_0), \quad \mathbf{r}_0 \in V^-, \tag{1.1}$$

assumes the representation

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\mathbf{Q} \times \frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|^3} - \frac{\mu_0\sigma}{4\pi}\int_{\partial V^-} u^-(\mathbf{r}')\hat{\mathbf{n}}' \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\,ds(\mathbf{r}'), \tag{1.2}$$

where $\mathbf{r} \in V^+$ and $\mathbf{Q}$ stands for the electric dipole moment.

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The scalar field u^- in the integrand of (1.2), over the boundary ∂V^- of V^-, describes the interior electric potential and solves the interior Neumann problem

$$\sigma \Delta u^-(\mathbf{r}) = \nabla \cdot \mathbf{J}^p(\mathbf{r}), \quad \mathbf{r} \in V^-, \tag{1.3a}$$

$$\frac{\partial}{\partial n} u^-(\mathbf{r}) = 0, \quad \mathbf{r} \in \partial V^-, \tag{1.3b}$$

where $\mathbf{J}^p$ is given by (1.1) and the boundary ∂V^- is assumed to be smooth.

Note that the solution of the boundary value problem (1.3) is unique up to an additive constant. Hence, the general solution of (1.3) has the form

$$u_c^-(\mathbf{r}) = c + u^-(\mathbf{r}), \quad \mathbf{r} \in V^-, \tag{1.4}$$

where u^- satisfies (1.3).

What we are going to show in the sequel is that, no matter what the shape of the smooth bounded boundary ∂V^- is, the leading term of the multipole expansion of (1.2) is not a dipole but a quadrupole term. Observe that an expansion of the source term, in (1.2) in terms of inverse powers of r, offers the leading dipole term

$$\frac{\mu_0}{4\pi} \mathbf{Q} \times \frac{\mathbf{r} - \mathbf{r}_0}{|\mathbf{r} - \mathbf{r}_0|^3} = \frac{\mu_0}{4\pi} \frac{\mathbf{Q} \times \hat{\mathbf{r}}}{r^2} + O\left(\frac{1}{r^3}\right), \quad r \longrightarrow \infty, \tag{1.5}$$

where $\mathbf{r} = r\hat{\mathbf{r}}$.

Similarly, the surface integral in (1.2) provides the expansion

$$\begin{aligned} &-\frac{\mu_0 \sigma}{4\pi} \int_{\partial V^-} u^-(\mathbf{r}')\hat{\mathbf{n}}' \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} ds(\mathbf{r}') \\ &\quad = -\frac{\mu_0 \sigma}{4\pi} \int_{\partial V^-} u^-(\mathbf{r}')\hat{\mathbf{n}}' ds(\mathbf{r}') \times \frac{\hat{\mathbf{r}}}{r^2} + O\left(\frac{1}{r^3}\right), \quad r \longrightarrow \infty. \end{aligned} \tag{1.6}$$

We will show that

$$\mathbf{Q} = \sigma \int_{\partial V^-} u^-(\mathbf{r})\hat{\mathbf{n}} ds(\mathbf{r}). \tag{1.7}$$

To this end we consider the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{V^-} \mathbf{J}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dv(\mathbf{r}'), \quad \mathbf{r} \in V^+, \tag{1.8}$$

where the total current $\mathbf{J}$ is written as

$$\mathbf{J}(\mathbf{r}') = \mathbf{J}^p(\mathbf{r}') + \sigma \mathbf{E}^-(\mathbf{r}') = \mathbf{Q}\delta(\mathbf{r}' - \mathbf{r}_0) - \sigma \nabla_{\mathbf{r}'} u^-(\mathbf{r}') \tag{1.9}$$

and

$$\mathbf{E}^- = -\nabla u^- \tag{1.10}$$

is the interior electric field. The quasistatic form of the Ampere-Maxwell equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{1.11}$$

implies that the total current is a solenoidal field, that is,

$$\nabla \cdot \mathbf{J} = 0. \tag{1.12}$$

Then condition (1.12) is used to prove the dyadic identity

$$\nabla \cdot (\mathbf{J} \otimes \mathbf{r}) = (\nabla \cdot \mathbf{J})\mathbf{r} + \mathbf{J} \cdot \nabla \otimes \mathbf{r} = \mathbf{J}, \tag{1.13}$$

in view of which

$$\begin{aligned}
\mathbf{B}(\mathbf{r}) &= \frac{\mu_0}{4\pi} \int_{V^-} \mathbf{J}(\mathbf{r}') \times \left(-\nabla_{\mathbf{r}} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) dv(\mathbf{r}') \\
&= \frac{\mu_0}{4\pi} \nabla_{\mathbf{r}} \times \int_{V^-} \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv(\mathbf{r}') \\
&= \frac{\mu_0}{4\pi} \nabla_{\mathbf{r}} \times \left[\frac{1}{r} \int_{V^-} \mathbf{J}(\mathbf{r}') dv(\mathbf{r}') + O\left(\frac{1}{r^2} \right) \right] \\
&= \frac{\mu_0}{4\pi} \nabla_{\mathbf{r}} \times \left[\frac{1}{r} \int_{V^-} \nabla_{\mathbf{r}'} \cdot (\mathbf{J}(\mathbf{r}') \otimes \mathbf{r}') dv(\mathbf{r}') + O\left(\frac{1}{r^2} \right) \right] \\
&= \frac{\mu_0}{4\pi} \nabla_{\mathbf{r}} \times \left[\frac{1}{r} \int_{\partial V^-} \hat{\mathbf{n}}' \cdot \mathbf{J}(\mathbf{r}') \otimes \mathbf{r}' ds(\mathbf{r}') + O\left(\frac{1}{r^2} \right) \right] \\
&= -\frac{\mu_0}{4\pi} \frac{\hat{\mathbf{r}}}{r^2} \times \int_{\partial V^-} \hat{\mathbf{n}}' \cdot \mathbf{J}(\mathbf{r}') \otimes \mathbf{r}' ds(\mathbf{r}') + O\left(\frac{1}{r^3} \right).
\end{aligned} \tag{1.14}$$

The fact that $\mathbf{r}_0 \in V^-$, the expression (1.9) for the current $\mathbf{J}$, and the boundary condition (1.3b) on ∂V^- imply that

$$\hat{\mathbf{n}}' \cdot \mathbf{J}(\mathbf{r}') = 0, \quad \mathbf{r}' \in \partial V^-. \tag{1.15}$$

Consequently, (1.14) concludes that

$$\mathbf{B}(\mathbf{r}) = O\left(\frac{1}{r^3} \right), \quad r \longrightarrow \infty. \tag{1.16}$$

In other words, the leading term of $\mathbf{B}$ in the exterior of V^- is a quadrupole for any smooth boundary ∂V^-. This result is compatible with physical reality.

Note that in the absence of conductive material, surrounding the source dipole current at $\mathbf{r}_0$, the expansion of $\mathbf{B}$ starts with a dipole term, that is , a term of order r^{-2}. But, in the presence of conductive material, the corresponding expansion starts with a quadrupole term, that is, a term of order r^{-3}. Hence, the conductive material partially “hides” the dipole.

As far as MEG measurements are concerned, this means that the conductive brain tissue weakens the intensity of the magnetic field exterior to the head.

This result is in accord with what is known for the special cases, where ∂V^- is a sphere [12, 17], a spheroid [1, 4, 5, 6, 7, 13], or an ellipsoid [2].

2. Silent sources

For the case of a sphere [17], where a complete expression for the magnetic field outside the sphere is known in the form

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}(\mathbf{Q}\times\mathbf{r}_0)\cdot\frac{[\tilde{\mathbf{I}}-\mathbf{r}\otimes\nabla]F(\mathbf{r})}{F^2(\mathbf{r})} \tag{2.1}$$

with

$$F(\mathbf{r}) = r\left|\mathbf{r}-\mathbf{r}_0\right|^2 + \mathbf{r}\cdot(\mathbf{r}-\mathbf{r}_0)\left|\mathbf{r}-\mathbf{r}_0\right|, \tag{2.2}$$

it is obvious that if $\mathbf{Q}$ is collinear to $\mathbf{r}_0$, then $\mathbf{B}$ vanishes. This is then characterized as a *silent source* since it represents a nontrivial activity of the brain that is not detectable in the exterior to the head space.

Unfortunately, the complete expression for $\mathbf{B}$, when ∂V^- is an ellipsoid, is not known and it seems far from being possible with the present knowledge of ellipsoidal harmonics. On the other hand, since the human brain is actually shaped in the form of an ellipsoid, with average semiaxes 6, 6.5, and 9 cm [18], even the leading analytic approximation [2] is of value.

In fact, the quadrupole term of $\mathbf{B}$ for a sphere, a prolate spheroid, and an ellipsoid can be written as

$$\mathbf{B}^q(\mathbf{r}) = \lim_{r\to\infty} r^3\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{8\pi}\mathbf{d}\cdot\tilde{\mathbf{G}}(\mathbf{r}), \tag{2.3}$$

where $\mathbf{d}$ is a vector which involves the location, the intensity, and the orientation of the source and $\tilde{\mathbf{G}}$ is a dyadic which is solely dependent on the geometry of the conductive medium. Hence, $\mathbf{d}$ represents the source and $\tilde{\mathbf{G}}$ represents the geometry.

In particular, if ∂V^- is a sphere of radius α, then

$$\mathbf{d} = \mathbf{d}_{\text{sr}} = \mathbf{Q}\times\mathbf{r}_0, \tag{2.4}$$

$$\tilde{\mathbf{G}}_{\text{sr}}(\mathbf{r}) = \frac{1}{r^3}(\tilde{\mathbf{I}} - 3\hat{\mathbf{r}}\otimes\hat{\mathbf{r}}). \tag{2.5}$$

If ∂V^- is the prolate spheroid

$$\frac{x_1^2}{\alpha_1^2} + \frac{x_2^2+x_3^2}{\alpha_2^2} = 1, \quad \alpha_2 < \alpha_1, \tag{2.6}$$

then

$$\mathbf{d} = \mathbf{d}_{\text{sd}} = (\mathbf{Q}\times\mathbf{r}_0)\cdot\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1 + 2\mathbf{Q}\cdot\tilde{\mathbf{S}}\times\mathbf{r}_0\cdot(\tilde{\mathbf{I}}-\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1) \tag{2.7}$$

with

$$\tilde{\mathbf{S}} = \frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2}\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1 + \frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2}(\tilde{\mathbf{I}}-\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1), \tag{2.8}$$

and $\hat{\mathbf{G}}_{\text{sd}}$ is some complicated dyadic function given in [13].

Finally, if ∂V^- is the triaxial ellipsoid

$$\frac{x_1^2}{\alpha_1^2}+\frac{x_2^2}{\alpha_2^2}+\frac{x_3^2}{\alpha_3^2}=1,\qquad \alpha_3<\alpha_2<\alpha_1, \tag{2.9}$$

then

$$\mathbf{d}=\mathbf{d}_{\mathrm{el}}=2(\mathbf{Q}\cdot\tilde{\mathbf{M}}\times\mathbf{r}_0)\cdot\tilde{\mathbf{N}} \tag{2.10}$$

with

$$\begin{aligned}\tilde{\mathbf{M}}&=\alpha_1^2\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1+\alpha_2^2\hat{\mathbf{x}}_2\otimes\hat{\mathbf{x}}_2+\alpha_3^2\hat{\mathbf{x}}_3\otimes\hat{\mathbf{x}}_3,\\ \tilde{\mathbf{N}}&=\frac{\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1}{\alpha_2^2+\alpha_3^2}+\frac{\hat{\mathbf{x}}_2\otimes\hat{\mathbf{x}}_2}{\alpha_1^2+\alpha_3^2}+\frac{\hat{\mathbf{x}}_3\otimes\hat{\mathbf{x}}_3}{\alpha_1^2+\alpha_2^2},\end{aligned} \tag{2.11}$$

where again $\tilde{\mathbf{G}}_{\mathrm{el}}$ is given in terms of elliptic integrals and complicated expressions which can be found in [2].

Note that the dyadic $\tilde{\mathbf{M}}$ specifies the ellipsoid in the sense that the equation

$$\mathbf{r}\cdot\tilde{\mathbf{M}}^{-1}\cdot\mathbf{r}=1 \tag{2.12}$$

coincides with the ellipsoid (2.9), while the dyadic $\tilde{\mathbf{N}}$ characterizes the principal moments of inertia of the ellipsoid since

$$\tilde{\mathbf{N}}=\frac{m}{5}\tilde{\mathbf{L}}^{-1}, \tag{2.13}$$

where $\tilde{\mathbf{L}}$ is the inertia dyadic of the ellipsoid (2.9) and m is its total mass.

Obviously, the ellipsoid is considered to be homogeneous, in which case its inertia dyadic reflects its geometrical characteristics.

It is worth noticing that the dyadic $\tilde{\mathbf{S}}$ divides the space into the 1D axis of revolution represented by $\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1$ and its 2D orthogonal complement represented by

$$\tilde{\mathbf{I}}-\hat{\mathbf{x}}_1\otimes\hat{\mathbf{x}}_1=\hat{\mathbf{x}}_2\otimes\hat{\mathbf{x}}_2+\hat{\mathbf{x}}_3\otimes\hat{\mathbf{x}}_3, \tag{2.14}$$

where all directions are equivalent (2D isotropy).

In the limit, as $\alpha_1\to\alpha$ and $\alpha_2\to\alpha$,

$$\begin{aligned}\tilde{\mathbf{S}}&\longrightarrow\frac{1}{2}\tilde{\mathbf{I}},\\ \mathbf{d}_{\mathrm{sd}}&\longrightarrow\mathbf{Q}\times\mathbf{r}_0=\mathbf{d}_{\mathrm{sr}}.\end{aligned} \tag{2.15}$$

Similarly, the complete geometrical anisotropy, carried by the ellipsoid, is expressed via the dyadics $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{N}}$, which dictate the characteristics of each principal direction in space.

In the limit, as $\alpha_1 \to \alpha$, $\alpha_2 \to \alpha$, and $\alpha_3 \to \alpha$, the following limits are obtained

$$\begin{gathered}\tilde{\mathbf{M}} \longrightarrow \alpha^2 \tilde{\mathbf{I}}, \qquad \tilde{\mathbf{N}} \longrightarrow \frac{1}{2\alpha^2}\tilde{\mathbf{I}}, \\ \mathbf{d}_{\mathrm{el}} \longrightarrow \mathbf{Q} \times \mathbf{r}_0 = \mathbf{d}_{\mathrm{sr}},\end{gathered} \tag{2.16}$$

so that the spherical behavior is recovered.

Obviously, the vector $\mathbf{d}_{\mathrm{sd}}$ for the spheroid and the vector $\mathbf{d}_{\mathrm{el}}$ for the ellipsoid incorporate the modifications of the cross product (2.4) that are imposed by the particular geometry.

If the quadrupole contribution $\mathbf{B}^q$ is known, then

$$\mathbf{d} = \frac{8\pi}{\mu_0} \mathbf{B}^q(\mathbf{r}) \cdot \tilde{\mathbf{G}}^{-1}(\mathbf{r}), \tag{2.17}$$

where $\tilde{\mathbf{G}}$ is also known if the geometry is given.

This means that, if the spherical model is considered, then $\mathbf{Q}$ and $\mathbf{r}_0$ belong to the plane, through the origin, which is perpendicular to $\mathbf{d}_{\mathrm{sd}}$.

For the case of the ellipsoid,

$$\mathbf{d}_{\mathrm{el}} \cdot \tilde{\mathbf{N}}^{-1} = 2\mathbf{Q} \cdot \tilde{\mathbf{M}} \times \mathbf{r}_0, \tag{2.18}$$

which means that the modified $\mathbf{d}_{\mathrm{el}}$ vector, that is, the vector $\mathbf{d}_{\mathrm{el}} \cdot \tilde{\mathbf{N}}^{-1}$, defines a perpendicular plane on which both the modified moment $\mathbf{Q} \cdot \tilde{\mathbf{M}}$ and the position vector $\mathbf{r}_0$ lie.

The intermediate case of the spheroid shows that if $\mathbf{d}_{\mathrm{sd}}$ is known, then we can extract information about the x_1-component of $\mathbf{Q} \times \mathbf{r}_0$ and the projection of $2\mathbf{Q} \cdot \tilde{\mathbf{S}} \times \mathbf{r}_0$ on the orthogonal complement of $\hat{\mathbf{x}}_1$.

This geometric analysis of the $\mathbf{d}$'s identifies the orientation of the silent sources.

For the simplest case of the sphere, a silent source is a dipole with a radial moment [17]. For the general case of the ellipsoid a silent source is a dipole with a modified moment $\mathbf{Q} \cdot \tilde{\mathbf{M}}$ parallel to $\mathbf{r}_0$. Then, since $\tilde{\mathbf{M}}^{-1}$ represents the Gaussian map [16], which takes a position vector on the surface of the ellipsoid to a vector in the normal to the surface direction at that point, it follows that $\mathbf{Q}$ will be silent if it is parallel to the normal of the ellipsoid in the direction of $\mathbf{r}_0$.

This silent direction for $\mathbf{Q}$ becomes parallel to $\mathbf{r}_0$ for the case of a sphere, but it is now clear that it is the normal to the surface direction, and not the collinearity with $\mathbf{r}_0$, that characterizes a dipole as silent.

Finally, we consider the spheroidal case. From (2.7), it follows that the vanishing of $\mathbf{d}_{\mathrm{sd}}$ comes from the simultaneous solvability of the system

$$(\mathbf{Q} \times \mathbf{r}_0) \cdot \hat{\mathbf{x}}_1 = 0, \tag{2.19}$$

$$(\mathbf{Q} \cdot \tilde{\mathbf{S}} \times \mathbf{r}_0) \cdot \hat{\mathbf{x}}_2 = 0, \tag{2.20}$$

$$(\mathbf{Q} \cdot \tilde{\mathbf{S}} \times \mathbf{r}_0) \cdot \hat{\mathbf{x}}_3 = 0. \tag{2.21}$$

Condition (2.19) holds whenever the projections of $\mathbf{Q}$ and $\mathbf{r}_0$ on the x_2x_3-plane are parallel, while (2.20) and (2.21) hold whenever the projections of $\mathbf{Q} \cdot \tilde{\mathbf{S}}$ and $\mathbf{r}_0$ on the x_1x_3 and on the x_1x_2 planes are also parallel.

From (2.20) and (2.21), we obtain

$$\frac{\alpha_1^2}{\alpha_2^2}\frac{Q_1}{Q_2} = \frac{x_{01}}{x_{02}}, \qquad \frac{\alpha_1^2}{\alpha_2^2}\frac{Q_1}{Q_3} = \frac{x_{01}}{x_{03}}, \tag{2.22}$$

where $\mathbf{Q} = (Q_1, Q_2, Q_3)$ and $\mathbf{r}_0 = (x_{01}, x_{02}, x_{03})$.

Taking the ratio of (2.22), we obtain

$$\frac{Q_2}{Q_3} = \frac{x_{02}}{x_{03}}, \tag{2.23}$$

which is exactly what comes out of (2.19). Interpreting everything in geometrical language, we see that the vectors $\mathbf{Q}$ and $\mathbf{r}_0$ should be coplanar and they should lie on the meridian plane specified by $\mathbf{r}_0$. Then $\mathbf{Q}$ should point in the direction of the normal to the ellipse on this meridian plane in the direction of $\mathbf{r}_0$. We see, once more, that $\mathbf{Q}$ should be normal to the surface of the spheroid in the direction of $\mathbf{r}_0$. The only difference with the ellipsoid is that, as a consequence of the rotational symmetry, both $\mathbf{Q}$ and $\mathbf{r}_0$ always lie on a meridian plane.

As a final conclusion we remark that modeling the human brain, which is a genuine triaxial ellipsoid, by a sphere, the MEG measurements are misinterpreted, since detectable sources are considered as silent while at the same time information is lost from detectable sources that we think they are silent.

For a complete characterization of silent electromagnetic activity within the brain, which concerns not only a single dipole but any current distribution inside a spherical conductor, we refer to the work of Fokas, et al. [8].

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References

[1] B. N. Cuffin and D. Cohen, *Magnetic fields of a dipole in special volume conductor shapes*, IEEE Trans. Biomedical Eng. **24** (1977), no. 4, 372–381.

[2] G. Dassios and F. Kariotou, *Magnetoencephalography in ellipsoidal geometry*, J. Math. Phys. **44** (2003), no. 1, 220–241.

[3] ———, *On the Geselowitz formula in biomagnetics*, Quart. Appl. Math. **61** (2003), no. 2, 387–400.

[4] J. C. de Munck, *The potential distribution in a layered anisotropic spheroidal volume conductor*, J. Appl. Phys. **64** (1988), no. 2, 464–470.

[5] T. Fieseler, A. Ioannides, M. Liu, and H. Nowak, *Model studies of the accuracy of the conducting sphere model in MEG using the spheroid*, Biomagnetism: Fundamental Research and Clinical Applications (Proceedings of the 9th International Conference on Biomagnetism, Vienna, 1993) (C. Baumgartner, L. Deecke, G. Stroink, and S. J. Williamson, eds.), Studies in Applied Electromagnetics and Mechanics, vol. 7, IOS Press, Amsterdam, 1995, pp. 445–449.

[6] ———, *A numerically stable approximation for the magnetic field of the conducting spheroid close to the symmetry axis*, Biomag 96: Proceedings of the 10th International Conference Biomagnetism (Santa Fe, 1996), Springer-Verlag, New york, 2000, pp. 209–212.

[7] T. Fieseler, A. Ioannides, and H. Nowak, *Influence of the global volume conductor curvature on point and distributed inverse solutions studied with the spheroid model.*, Models for Biomagnetic Inverse/Forward Problem (11th International Conference on Biomagnetism, Sendai, 1998) (T. Yoshimoto et al., eds.), Tohok University Press, Sendai, 1999, pp. 185–188.

[8] A. S. Fokas, I. M. Gelfand, and Y. Kurylev, *Inversion method for magnetoencephalography*, Inverse Problems **12** (1996), no. 3, L9–L11.

[9] D. B. Geselowitz, *Multipole representation for an equivalent cardiac generator*, Proc. IRE **48** (1960), no. 1, 75–79.

[10] ———, *On the magnetic field generated outside an inhomogeneous volume conductor by internal current sources*, IEEE Trans. Magn. **6** (1970), no. 2, 346–347.

[11] M. Hämäläinen, R. Hari, R. J. Ilmoniemi, J. Knuutila, and O. Lounasmaa, *Magnetoencephalography—theory, instrumentation, and applications to noninvasive studies of the working human brain*, Rev. Modern Phys. **65** (1993), no. 2, 413–497.

[12] R. J. Ilmoniemi, M. S. Hämäläinen, and J. Knuutila, *The forward and inverse problems in the spherical model*, Biomagnetism: Applications and Theory (H. Weinberg, G. Stroink, and T. Katila, eds.), Pergamon Press, New York, 1985, pp. 278–282.

[13] F. Kariotou, *Magnetoencephalography in spheroidal geometry*, Bulletin of the Greek Mathematical Society **47** (2003), 117–135.

[14] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Course of Theoretical Physics, vol. 8, Pergamon Press, London, 1960.

[15] J. Malmivuo and R. Plonsey, *Bioelectromagnetism*, Oxford University Press, New York, 1995.

[16] B. O' Neill, *Elementary Differential Geometry*, Academic press, 1997.

[17] J. Sarvas, *Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem*, Phys. Med. Biol. **32** (1987), no. 1, 11–22.

[18] W. S. Snyder, H. L. Fisher Jr., M. R. Ford, and G. G. Warner, *Estimates of absorbed fractions for monoenergetic photon sources uniformly distributed in various organs of a heterogeneous phantom*, Journal of Nuclear Medicine **10** (1969), no. Suppl. 3, 7–52.

[19] A. Sommerfeld, *Electrodynamics. Lectures on Theoretical Physics, Vol. III*, Academic Press, New York, 1952.

[20] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.

George Dassios: Division of Applied Mathematics, Department of Chemical Engineering, University of Patras, 26504 Patras, Greece

E-mail address: dassios@chemeng.upatras.gr

Fotini Kariotou: Institute of Chemical Engineering and High Temperature Chemical Processes (ICE/HT), Foundation for Research Technology-Hellas (FORTH), 26504 Patras, Greece

E-mail address: fkario@chemeng.upatras.gr