

Research Article

Stability and Superstability of Ring Homomorphisms on Non-Archimedean Banach Algebras

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Received 11 November 2010; Revised 11 January 2011; Accepted 19 January 2011

Academic Editor: Yuming Shi

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Using fixed point methods, we prove the superstability and generalized Hyers-Ulam stability of ring homomorphisms on non-Archimedean Banach algebras. Moreover, we investigate the superstability of ring homomorphisms in non-Archimedean Banach algebras associated with the Jensen functional equation.

1. Introduction and Preliminaries

In 1897, Hensel [1] has introduced a normed space which does not have the Archimedean property.

During the last three decades theory of non-Archimedean spaces has gained the interest of physicists for their research in particular in problems coming from quantum physics, p -adic strings, and superstrings [2]. Although many results in the classical normed space theory have a non-Archimedean counterpart, their proofs are essentially different and require an entirely new kind of intuition [3–10].

Let \mathbb{K} be a field. A non-Archimedean absolute value on \mathbb{K} is a function $|\cdot| : \mathbb{K} \rightarrow \mathbb{R}$ such that for any $a, b \in \mathbb{K}$ we have

- (i) $|a| \geq 0$ and equality holds if and only if $a = 0$,
- (ii) $|ab| = |a||b|$,
- (iii) $|a + b| \leq \max\{|a|, |b|\}$.

Condition (iii) is called the strict triangle inequality. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integer n . We always assume in addition that $|\cdot|$ is non trivial, that is, that there is an $a_0 \in \mathbb{K}$ such that $|a_0| \notin \{0, 1\}$.

Let X be a linear space over a scalar field \mathbb{K} with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \rightarrow \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

- (NA1) $\|x\| = 0$ if and only if $x = 0$;
- (NA2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;
- (NA3) the strong triangle inequality (ultrametric); namely,

$$\|x + y\| \leq \max\{\|x\|, \|y\|\} \quad (x, y \in X). \quad (1.1)$$

Then $(X, \|\cdot\|)$ is called a non-Archimedean space.

It follows from (NA3) that

$$\|x_m - x_l\| \leq \max\{\|x_{j+1} - x_j\| : l \leq j \leq m-1\} \quad (m > l), \quad (1.2)$$

and therefore a sequence $\{x_m\}$ is Cauchy in X if and only if $\{x_{m+1} - x_m\}$ converges to zero in a non-Archimedean space. By a complete non-Archimedean space we mean one in which every Cauchy sequence is convergent. A non-Archimedean Banach algebra is a complete non-Archimedean algebra \mathcal{A} which satisfies $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in \mathcal{A}$. For more detailed definitions of non-Archimedean Banach algebras, we can refer to [11].

The first stability problem concerning group homomorphisms was raised by S. M. Ulam [12] in 1940 and affirmatively solved by D. H. Hyers [13]. Perhaps T. Aoki was the first author who has generalized the theorem of Hyers (see [14]).

T. M. Rassias [15] provided a generalization of Hyers' theorem which allows the Cauchy difference to be unbounded.

Theorem 1.1 (T. M. Rassias). *Let $f : E \rightarrow E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(x + y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p) \quad (1.3)$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then the limit

$$L(x) = \lim_{n \rightarrow \infty} \frac{f(2^n x)}{2^n} \quad (1.4)$$

exists for all $x \in E$ and $L : E \rightarrow E'$ is the unique additive mapping which satisfies

$$\|f(x) - L(x)\| \leq \frac{2\epsilon}{2-2^p} \|x\|^p \quad (1.5)$$

for all $x \in E$. Also, if for each $x \in E$ the mapping $f(tx)$ is continuous in $t \in \mathbb{R}$, then L is \mathbb{R} -linear.

Moreover, D. G. Bourgin [16] and Găvruta [17] have considered the stability problem with unbounded Cauchy differences (see also [18–23]).

On the other hand, J. M. Rassias [24–29] considered the Cauchy difference controlled by a product of different powers of norm. However, there was a singular case; for this singularity a counterexample was given by Găvruta [30].

Theorem 1.2 (J. M. Rassias [24]). *Let X be a real normed linear space and Y a real complete normed linear space. Assume that $f : X \rightarrow Y$ is an approximately additive mapping for which there exist constants $\theta \geq 0$ and $p, q \in \mathbb{R}$ such that $r = p + q \neq 1$ and f satisfies the inequality*

$$\|f(x + y) - f(x) - f(y)\| \leq \theta \|x\|^p \|y\|^q \tag{1.6}$$

for all $x, y \in X$. Then there exists a unique additive mapping $L : X \rightarrow Y$ satisfying

$$\|f(x) - L(x)\| \leq \frac{\theta}{|2^r - 2|} \|x\|^r \tag{1.7}$$

for all $x \in X$. If, in addition, $f : X \rightarrow Y$ is a mapping such that the transformation $t \mapsto f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$, then L is an \mathbb{R} -linear mapping.

Bourgin [16, 31] is the first mathematician dealing with stability of (ring) homomorphism $f(xy) = f(x)f(y)$. The topic of approximate homomorphisms was studied by a number of mathematicians, see [32–37] and references therein. A function $f : A \rightarrow A$ is a ring homomorphism or additive homomorphism if f is an additive function satisfying

$$f(xy) = f(x)f(y) \tag{1.8}$$

for all $x, y \in A$.

Now we will state the following notion of fixed point theory. For the proof, refer to [38], see also [39, chapter 5]. For an extensive theory of fixed point theorems and other nonlinear methods, the reader is referred to [40, 41]. In 2003, Radu [42] proposed a new method for obtaining the existence of exact solutions and error estimations, based on the fixed point alternative (see also [43–45]).

Let (X, d) be a generalized metric space. An operator $T : X \rightarrow X$ satisfies a Lipschitz condition with Lipschitz constant L if there exists a constant $L \geq 0$ such that $d(Tx, Ty) \leq Ld(x, y)$ for all $x, y \in X$. If the Lipschitz constant L is less than 1, then the operator T is called a strictly contractive operator. Note that the distinction between the generalized metric and the usual metric is that the range of the former is permitted to include the infinity. We recall the following theorem by Margolis and Diaz.

Theorem 1.3 (cf. [38, 42]). *Suppose that one is given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \rightarrow \Omega$ with Lipschitz constant L . Then for each given $x \in \Omega$, either*

$$d(T^m x, T^{m+1} x) = \infty \quad \forall m \geq 0 \tag{1.9}$$

or there exists a natural number m_0 such that

- (i) $d(T^m x, T^{m+1} x) < \infty$ for all $m \geq m_0$,
- (ii) the sequence $\{T^m x\}$ is convergent to a fixed point y^* of T ;
- (iii) y^* is the unique fixed point of T in $\Lambda = \{y \in \Omega : d(T^{m_0} x, y) < \infty\}$;
- (iv) $d(y, y^*) \leq (1/(1-L))d(y, Ty)$ for all $y \in \Lambda$.

Recently, the first author of the present paper [4] established the stability of ring homomorphisms on non-Archimedean Banach algebras. In this paper, using fixed point methods, we prove the generalized Hyers-Ulam stability of ring homomorphisms on non-Archimedean Banach algebras. Moreover, we investigate the superstability of ring homomorphisms on non-Archimedean Banach algebras associated with the Jensen functional equation.

2. Approximation of Ring Homomorphisms in Non-Archimedean Banach Algebras

Throughout this section we suppose that A, B are two non-Archimedean Banach algebras. For convenience, we use the following abbreviation for a given function $f : A \rightarrow B$:

$$\Delta f(x, y) = f(x + y) - f(x) - f(y) \quad (2.1)$$

for all $x, y \in A$.

Theorem 2.1. *Let $f : A \rightarrow B$ be a function for which there exist functions $\varphi, \psi : A \times A \rightarrow [0, \infty)$ such that*

$$\|\Delta f(x, y)\| \leq \varphi(x, y), \quad (2.2)$$

$$\|f(xy) - f(x)f(y)\| \leq \psi(x, y) \quad (2.3)$$

for all $x, y \in A$. If there exists a constant $0 < L < 1$ such that

$$\varphi(2x, 2y) \leq |2|L\varphi(x, y) \quad (2.4)$$

$$\psi(2x, 2y) \leq |2|^2 L\psi(x, y)$$

for all $x, y \in A$, then there exists a unique ring homomorphism $H : A \rightarrow B$ such that

$$\|f(x) - H(x)\| \leq \frac{1}{|2|(1-L)}\varphi(x, x), \quad (2.5)$$

for all $x \in A$.

Proof. It follows from (2.4) that

$$\lim_{n \rightarrow \infty} \frac{1}{|2|^n} \varphi(2^n x, 2^n y) = 0, \quad (2.6)$$

$$\lim_{n \rightarrow \infty} \frac{1}{|2|^{2n}} \varphi(2^n x, 2^n y) = 0 \quad (2.7)$$

for all $x, y \in X$. By (2.6), $\lim_{n \rightarrow \infty} (1/|2|^n) \varphi(0, 0) = 0$. Hence, $\varphi(0, 0) = 0$. Letting $x = y = 0$ in (2.2), we get $\|f(0)\| \leq \varphi(0, 0) = 0$. So $f(0) = 0$.

Let us define Ω to be the set of all mappings $g : A \rightarrow B$ and introduce a generalized metric on Ω as follows:

$$d(g, h) = \inf\{K \in (0, \infty) : \|g(x) - h(x)\| \leq K\varphi(x, x), \forall x \in A\}. \quad (2.8)$$

It is easy to show that (Ω, d) is a generalized complete metric space [44, 45].

Now we consider the function $T : \Omega \rightarrow \Omega$ defined by $Tg(x) = (1/2)g(2x)$ for all $x \in A$ and all $g \in \Omega$. Note that for all $g, h \in \Omega$,

$$\begin{aligned} d(g, h) < K &\implies \|g(x) - h(x)\| \leq K\varphi(x, x), \quad \forall x \in A, \\ &\implies \left\| \frac{1}{2}g(2x) - \frac{1}{2}h(2x) \right\| \leq \frac{1}{|2|}K\varphi(2x, 2x), \quad \forall x \in A, \\ &\implies \left\| \frac{1}{2}g(2x) - \frac{1}{2}h(2x) \right\| \leq LK\varphi(x, x), \quad \forall x \in A, \\ &\implies d(Tg, Th) \leq LK. \end{aligned} \quad (2.9)$$

Hence, we see that

$$d(Tg, Th) \leq Ld(g, h) \quad (2.10)$$

for all $g, h \in \Omega$, that is, T is a strictly self-function of Ω with the Lipschitz constant L .

Putting $y := x$ in (2.2), we have

$$\|f(2x) - 2f(x)\| \leq \varphi(x, x) \quad (2.11)$$

for all $x \in A$. So

$$\left\| f(x) - \frac{1}{2}f(2x) \right\| \leq \frac{1}{|2|}\varphi(x, x) \quad (2.12)$$

for all $x \in A$, that is, $d(f, Tf) \leq 1/|2| < \infty$.

Now, from the fixed point alternative, it follows that there exists a fixed point H of T in Ω such that

$$H(x) = \lim_{n \rightarrow \infty} \frac{1}{2^n} f(2^n x) \quad (2.13)$$

for all $x \in A$, since $\lim_{n \rightarrow \infty} d(T^n f, H) = 0$.

On the other hand it follows from (2.2), (2.6), and (2.13) that

$$\|\Delta H(x, y)\| = \lim_{n \rightarrow \infty} \frac{1}{|2|^n} \|\Delta f(2^n x, 2^n y)\| \leq \lim_{n \rightarrow \infty} \frac{1}{|2|^n} \varphi(2^n x, 2^n y) = 0 \quad (2.14)$$

for all $x, y \in A$. So $\Delta H(x, y) = 0$. This means that H is additive. So it follows from the definition of H , (2.3), (2.7), and (2.13) that

$$\|H(xy) - H(x)H(y)\| = \lim_{n \rightarrow \infty} \frac{1}{|2|^{2n}} \|f(2^{2n} xy) - f(2^n x)f(2^n y)\| \leq \lim_{n \rightarrow \infty} \frac{1}{|2|^{2n}} \varphi(2^{2n} x, 2^{2n} y) = 0 \quad (2.15)$$

for all $x, y \in A$. So $H(xy) = H(x)H(y)$. According to the fixed point alternative, since H is the unique fixed point of T in the set $\Lambda = \{g \in \Omega : d(f, g) < \infty\}$, H is the unique function such that

$$\|f(x) - H(x)\| \leq K\varphi(x, x) \quad (2.16)$$

for all $x \in A$ and $K > 0$. Again using the fixed point alternative, we get

$$d(f, H) \leq \frac{1}{1-L} d(f, Tf) \leq \frac{1}{|2|(1-L)} \quad (2.17)$$

and so we conclude that

$$\|f(x) - H(x)\| \leq \frac{1}{|2|(1-L)} \varphi(x, x) \quad (2.18)$$

for all $x \in A$. This completes the proof. \square

Corollary 2.2. Let θ, p, s be nonnegative real numbers with p , and $s > 1$ and $2s - 2p \geq 1$. Suppose that $f : A \rightarrow B$ is a function such that

$$\begin{aligned} \|\Delta f(x, y)\| &\leq \theta(\|x\|^p \cdot \|y\|^p) \\ \|f(xy) - f(x)f(y)\| &\leq \theta(\|x\|^s \cdot \|y\|^s) \end{aligned} \quad (2.19)$$

for all $x, y \in A$. Then there exists a unique ring homomorphism $H : A \rightarrow B$ satisfying

$$\|f(x) - H(x)\| \leq \frac{\theta}{|2| - |2|^{2p}} \|x\|^{2p}, \tag{2.20}$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.1 by taking

$$\varphi(x, y) := \theta(\|x\|^p \cdot \|y\|^p), \quad \psi(x, y) := \theta(\|x\|^s \cdot \|y\|^s) \tag{2.21}$$

for all $x, y \in A$. Then we can choose $L = |2|^{2p-1}$ and we get the desired results. □

Remark 2.3. Let $f : A \rightarrow B$ be a function for which there exist functions $\varphi, \psi : A \times A \rightarrow [0, \infty)$ satisfying (2.2) and (2.3). Let $0 < L < 1$ be a constant such that $\varphi(x/2, y/2) \leq (L/|2|)\varphi(x, y)$ for all $x, y \in A$. By a similar method to the proof of Theorem 2.1, one can show that there exists a unique ring homomorphism $H : A \rightarrow B$ satisfying

$$\|f(x) - H(x)\| \leq \frac{L}{|2|(1-L)} \varphi(x, x). \tag{2.22}$$

For the case $\varphi(x, y) := \delta + \theta(\|x\|^p \cdot \|y\|^p)$ (where θ, δ are nonnegative real numbers and $0 < 2p < 1$), there exists a unique ring homomorphism $H : A \rightarrow B$ satisfying

$$\|f(x) - H(x)\| \leq \frac{\delta}{|2|^{2p} - |2|} + \frac{\theta}{|2|^{2p} - |2|} \|x\|^{2p} \tag{2.23}$$

for all $x \in A$.

In the following we establish the superstability of ring homomorphisms on non-Archimedean Banach algebras associated with the Jensen functional equation $f((x + y)/2) = (f(x) + f(y))/2$.

Theorem 2.4. *Suppose there exist functions $\varphi, \psi : A \times A \rightarrow [0, \infty)$ such that there exists a constant $0 < L < 1$ such that*

$$\begin{aligned} \varphi(0, 2y) &\leq |2|L\varphi(0, y), \\ \psi(2x, 2y) &\leq |2|^2L\psi(x, y) \end{aligned} \tag{2.24}$$

for all $x, y \in A$. Moreover, assume that $f : A \rightarrow B$ is a function such that

$$\left\| f\left(\frac{x+y}{2}\right) - \frac{f(x) + f(y)}{2} \right\| \leq \varphi(0, y), \tag{2.25}$$

$$\|f(xy) - f(x)f(y)\| \leq \psi(x, y) \tag{2.26}$$

for all $x, y \in A$. Then f is a ring homomorphism.

Proof. Let us define Ω , d and $T : \Omega \rightarrow \Omega$ by the same definitions as in the proof of Theorem 2.1. By the same reasoning as in the proof of Theorem 2.1, one can show that T has a (unique) fixed point H in Ω such that

$$H(x) = \lim_{n \rightarrow \infty} \frac{1}{2^n} f(2^n x) \quad (2.27)$$

for all $x \in A$ and $H : A \rightarrow B$ is a ring homomorphism. On the other hand by the same reasoning as in the proof of Theorem 2.1, we can prove that $\varphi(0,0) = 0$ and $f(0) = 0$. Also, letting $y = 0$ in (2.25), we get $f(x/2) = f(x)/2$ for all $x \in A$ (see [24, 25]). So, by uniqueness property of H , we have $H = f$. It follows that f is a ring homomorphism. \square

Corollary 2.5. *Let θ , p , and s be nonnegative real numbers with $p, s > 1$. Suppose that $f : A \rightarrow B$ is a function such that*

$$\left\| f\left(\frac{x+y}{2}\right) - \frac{f(x)+f(y)}{2} \right\| \leq \theta \|y\|^p, \quad \|f(xy) - f(x)f(y)\| \leq \theta(\|x\|^s \cdot \|y\|^s) \quad (2.28)$$

for all $x, y \in A$. Then f is a ring homomorphism.

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