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Research Article

A Strong Convergence Theorem for Total Asymptotically Pseudocontractive Mappings in Hilbert Spaces

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Demiclosedness principle for total asymptotically pseudocontractive mappings in Hilbert spaces is established. The strong convergence to a fixed point of total asymptotically pseudocontraction in Hilbert spaces is obtained based on demiclosedness principle, metric projective operator, and hybrid iterative method. The main results presented in this paper extend and improve the corresponding results of Zhou (2009), Qin, Cho, and Kang (2011) and of many other authors.

1. Introduction

Throughout this article we assume that H is a real Hilbert space, whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\|\cdot\|$, respectively; C is a nonempty closed convex subset of H; $\mathbb{N}$ and $\mathbb{R}^+$ denote the natural number set and the set of nonnegative real numbers, respectively. Let $T : C \to C$ be a nonlinear mapping; $F(T)$ denotes the set of fixed points of mapping T. We use "$\to$" to stand for strong convergence and "$\rightharpoonup$" for weak convergence.

Recently, the iterative approximation of fixed points for asymptotically pseudocontractive mappings, total asymptotically pseudocontractive mappings in Hilbert, or Banach spaces has been studied extensively by many authors, see for example [1–5].

The asymptotically pseudocontractions and total asymptotically pseudo-contractions are defined as follows.

Definition 1.1 (see [3]). $T : C \to C$ is said to be asymptotically pseudocontraction if there exists a sequence $\{k_n\}$ of positive real numbers with $k_n \geq 1$, $\lim_{n\to+\infty} k_n = 1$, such that

$$\langle T^n x - T^n y, x - y \rangle \leq k_n \|x - y\|^2 \tag{1.1}$$

holds; in [4], $T : C \to C$ is said to be total asymptotically pseudo-contraction if there exists sequences $\{\mu_n\}, \{\nu_n\}$ with $\mu_n, \nu_n \to 0$ as $n \to \infty$ and strictly increasing continuous functions $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\psi(0) = 0$, such that

$$\langle T^n x - T^n y, x - y \rangle \leq \|x - y\|^2 + \mu_n \psi(\|x - y\|) + \nu_n, \quad n \in \mathbb{N} \tag{1.2}$$

holds; Zhou [3, page 3144] have proved the following Theorem.

Theorem Zhou

Let C be a bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian asymptotically pseudo-contraction with a fixed point. Suppose the control sequence α_n is chosen so that $\alpha_n \in [a, b]$, for some $a, b \in (0, 1/(1 + L))$. Let x_n be a sequence generated in the following manner:

$$\begin{gathered} x_0 \in C, \quad \text{chosen arbitrarily; } n \geq 0, \\ y_n = (1 - \alpha_n) x_n + \alpha_n T^n x_n, \\ C_n = \Big\{ z \in C : \alpha_n [1 - (1 + L)\alpha_n] \|x_n - T^n x_n\|^2 \\ \leq \langle x_n - z, y_n - T^n y_n \rangle + (k_n - 1)(\operatorname{diam} C)^2 \Big\}, \\ Q_n = \{ z \in C : \langle x_n - z, x_0 - x_n \rangle \} \geq 0, \\ x_{n+1} = P_{C_n \cap Q_n} x_0. \end{gathered} \tag{1.3}$$

Then the iterative sequence $\{x_n\}$ converges strongly to $P_{F(T)} x_0$.

Qin et al. [5] introduced the class of total asymptotically pseudocontractive mappings in Hilbert spaces and established the following weak convergence theorem of fixed points.

Theorem Qin

Let C be a bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian total asymptotically pseudo-contractive mapping. Assume that $F(T)$ is nonempty and there exist positive constants M and M^* such that $\psi(\lambda) \leq M^* \lambda^2$ for all $\lambda \geq M$. Let x_n be a sequence generated in the following manner:

$$\begin{gathered} x_1 \in C, \\ y_n = (1 - \beta_n) x_n + \beta_n T^n x_n, \\ x_{n+1} = (1 - \alpha_n) x_n + \alpha_n T^n y_n, \quad \forall n \geq 1, \end{gathered} \tag{1.4}$$

where α_n and β_n are sequences in $(0, 1)$. Assume that the following restrictions are satisfied:

(a) $\Sigma_{n=1}^{\infty} \mu_n < \infty$ and $\Sigma_{n=1}^{\infty} \nu_n < \infty$.

(b) $a < \alpha_n < \beta_n < b$ for some $a > 0$ and some $b \in (0, L^{-2}[\sqrt{1 + L^2} - 1])$.

Then the iterative sequence $\{x_n\}$ converges weakly to fixed point of T.

The purpose of this article is to prove the strong convergence for total asymptotically pseudo-contraction in Hilbert spaces. The results presented in the article improve and extend the corresponding results of Zhou [3], Qin et al. [5], and many other authors.

2. Preliminaries

A mapping $T : C \to C$ is said to be uniformly L-Lipschitzian if there exists some $L > 0$ such that

$$\|T^n x - T^n y\| \leq L\|x - y\| \tag{2.1}$$

holds for all $x, y \in C$ and for all $n \in \mathbb{N}$. Let C be a nonempty closed convex subset of a real Hilbert space H. For every point $x \in H$ there exists a unique nearest point in C, denoted by $P_C x$, such that $\|x - P_C x\| \leq \|x - y\|$ holds for all $y \in C$, where P_C is called the metric projection of H onto C.

In order to prove the results of this article, we will need the following lemmas.

Lemma 2.1 (see [6]). *Let C be a nonempty closed convex subset of a real Hilbert space H. Given $x \in H$ and $z \in C$, then $z = P_C x$ if and only if there holds the relation*

$$\langle x - z, y - z \rangle \leq 0, \quad \forall y \in C. \tag{2.2}$$

Lemma 2.2 (see [6]). *Let C be a nonempty closed convex subset of a real Hilbert space H and $P_C : H \to C$ the metric projection from H onto C. Then the following inequality holds:*

$$\|y - P_C x\|^2 + \|x - P_C x\|^2 \leq \|x - y\|^2, \quad \forall x \in H, y \in C. \tag{2.3}$$

3. Main Results

Theorem 3.1 (demiclosedness principle). *Let C be a nonempty bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian and total asymptotically pseudo-contraction. Suppose there exists $M^* > 0$, such that $\psi(\xi_n) \leq M^* \xi_n$, then $I - T$ is demiclosed at zero, where I is the identical mapping.*

Proof. Assume that $\{x_n\} \subset C$, with $x_n \rightharpoonup x$ and $x_n - Tx_n \to 0$ as $n \to \infty$. We want to prove $x \in C$ and $x = Tx$. Since C is a closed convex subset of H, so $x \in C$. In the following we prove $x = Tx$.

Now we choose $\alpha \in (0, 1/(1 + L))$ and let $y_{\alpha,m} = (1 - \alpha)x + \alpha T^m x$ for arbitrary fixed $m \geq 1$. Because T is uniformly L-Lipschitzian, we have

$$\begin{aligned} \|x_n - T^m x_n\| &\leq \|x_n - Tx_n\| + \left\|Tx_n - T^2(t)x_n\right\| + \cdots + \left\|T^{m-1}x_n - T^m x_n\right\| \\ &\leq mL\|x_n - Tx_n\| \\ &\longrightarrow 0, \quad \text{as } n \longrightarrow \infty. \end{aligned} \tag{3.1}$$

Since T is total asymptotically pseudo-contraction, we have

$$\begin{aligned}
&\langle x - y_{\alpha,m}, (I - T^m) y_{\alpha,m} \rangle \\
&\quad = \langle x - x_n, (I - T^m) y_{\alpha,m} \rangle + \langle x_n - y_{\alpha,m}, (I - T^m) y_{\alpha,m} \rangle \\
&\quad = \langle x - x_n, (I - T^m) y_{\alpha,m} \rangle + \langle x_n - y_{\alpha,m}, (I - T^m) x_n \rangle \\
&\qquad + \langle x_n - y_{\alpha,m}, (I - T^m) y_{\alpha,m} - (I - T^m) x_n \rangle \\
&\quad \le \langle x - x_n, (I - T^m) y_{\alpha,m} \rangle + \langle x_n - y_{\alpha,m}, (I - T^m) x_n \rangle \\
&\qquad + \mu_m \psi(\|x_n - y_{\alpha,m}\|) + \nu_m.
\end{aligned} \tag{3.2}$$

By assumption $x_n \rightharpoonup x$, $x_n - Tx_n \to 0$ and $\|x_n - T^m x_n\| \to 0$ as $n \to \infty$, we have

$$\begin{aligned}
&\langle x - y_{\alpha,m}, (I - T^m) y_{\alpha,m} \rangle \\
&\quad \le \mu_m \psi(\|x_n - y_{\alpha,m}\|) + \nu_m \\
&\quad \le \mu_m M^* \|x_n - y_{\alpha,m}\| + \nu_m \\
&\quad \le \mu_m M^* (\operatorname{diam} C) + \nu_m.
\end{aligned} \tag{3.3}$$

By the L-Lipschitz of T and the definition of $y_{\alpha,m}$, we have

$$\begin{aligned}
&\langle x - y_{\alpha,m}, (I - T^m) x - (I - T^m) y_{\alpha,m} \rangle \\
&\quad \le (1 + L) \|x - y_{\alpha,m}\|^2 \\
&\quad \le (1 + L) \alpha^2 \|x - T^m x\|^2.
\end{aligned} \tag{3.4}$$

Thus we have

$$\begin{aligned}
\|x - T^m x\|^2 &= \langle x - T^m x, x - T^m x \rangle = \frac{1}{\alpha} \langle x - y_{\alpha,m}, x - T^m x \rangle \\
&= \frac{1}{\alpha} \langle x - y_{\alpha,m}, x - T^m x - (y_{\alpha,m} - T^m y_{\alpha,m}) \rangle + \frac{1}{\alpha} \langle x - y_{\alpha,m}, (y_{\alpha,m} - T^m y_{\alpha,m}) \rangle \\
&\le \alpha (1 + L) \|x - T^m x\|^2 + \frac{1}{\alpha} \langle x - y_{\alpha,m}, y_{\alpha,m} - T^m y_{\alpha,m} \rangle \\
&\le \alpha (1 + L) \|x - T^m x\|^2 + \frac{1}{\alpha} (\mu_m M^* (\operatorname{diam} C) + \nu_m),
\end{aligned} \tag{3.5}$$

which implies that

$$\alpha [1 - \alpha (1 + L)] \|x - T^m x\|^2 \le \mu_m M^* (\operatorname{diam} C) + \nu_m, \quad \forall m \in \mathbb{N}. \tag{3.6}$$

When $m \to \infty$, μ_m, $\nu_m \to 0$, so we have $\|x - T^m x\| \to 0$, $m \to \infty$, that is, $T^m x \to x$, $m \to \infty$, so $T^{m+1} x \to Tx$, $m \to \infty$. By the continuity of T, we have $Tx = x$. □

Theorem 3.2. *Let C be a nonempty bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian and total asymptotically pseudo-contraction. Suppose there exists $M^* > 0$, such that $\psi(\xi_n) \le M^*\xi_n$, then $F(T)$ is a closed convex subset of C.*

Proof. Since T is uniformly L-Lipschitzian continuous, $F(T)$ is closed. We need to show that $F(T)$ is convex. We let $p_1, p_2 \in F(T)$, and $p = tp_1 + (1-t)p_2$ for $t \in (0,1)$. We take $\alpha \in (0, 1/(1+L))$ and let $y_{\alpha,n} = (1-\alpha)p + \alpha T^n p$, $n \in \mathbb{N}$. Then for any $z \in F(T)$, we have

$$\begin{aligned}
\|p - T^n p\|^2 &= \langle p - T^n p, p - T^n p\rangle = \frac{1}{\alpha}\langle p - y_{\alpha,n}, p - T^n p\rangle \\
&= \frac{1}{\alpha}\langle p - y_{\alpha,n}, p - T^n p - (y_{\alpha,n} - T^n y_{\alpha,n})\rangle + \frac{1}{\alpha}\langle p - y_{\alpha,n}, y_{\alpha,n} - T^n y_{\alpha,n}\rangle \\
&\le \frac{1+L}{\alpha}\|p - y_{\alpha,n}\|^2 + \frac{1}{\alpha}\langle p - z, y_{\alpha,n} - T^n y_{\alpha,n}\rangle + \frac{1}{\alpha}\langle z - y_{\alpha,n}, y_{\alpha,n} - T^n y_{\alpha,n}\rangle \\
&\le \frac{1+L}{\alpha}\|p - y_{\alpha,n}\|^2 + \frac{1}{\alpha}\langle p - z, y_{\alpha,n} - T^n y_{\alpha,n}\rangle + \frac{1}{\alpha}(\mu_n M^*(\operatorname{diam} C) + \nu_n) \\
&\le \alpha(1+L)\|p - T^n p\|^2 + \frac{1}{\alpha}\langle p - z, y_{\alpha,n} - T^n y_{\alpha,n}\rangle + \frac{1}{\alpha}(\mu_m M^*(\operatorname{diam} C) + \nu_m).
\end{aligned} \tag{3.7}$$

This implies that

$$\alpha[1 - \alpha(1+L)]\|p - T^n p\|^2 \le \langle p - z, y_{\alpha,n} - T^n y_{\alpha,n}\rangle + \mu_m M^*(\operatorname{diam} C) + \nu_m. \tag{3.8}$$

Now we take $z = p_1, p_2$, multiplying t and $1-t$ on both sides of above inequality, respectively, and adding up, and we can get

$$\alpha[1 - \alpha(1+L)]\|p - T^n p\|^2 \le \mu_m M^*(\operatorname{diam} C) + \nu_m. \tag{3.9}$$

By $n \to \infty$, we get $T^n p \to p$. Since T is continuous, we have $T^{n+1}p \to Tp$ as $n \to \infty$, so that $p = Tp$. □

Theorem 3.3. *Let C be a bounded and closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a uniformly L-Lipschitzian and total asymptotically pseudo-contraction. Suppose there exists $M^* > 0$, such that $\psi(\xi_n) \le M^*\xi_n$, $F(T) \ne \emptyset$, α_n is a sequence in $[a,b]$, where $a, b \in (0, 1/(1+L))$. Let x_n be a sequence generated by*

$$\begin{gathered}
x_1 = x \in C, \quad \forall n \in \mathbb{N}, \\
y_n = (1-\alpha_n)x_n + \alpha_n T^n x_n, \\
H_n = \left\{z \in C : \alpha_n[1 - (1+L)\alpha_n]\|x_n - T^n x_n\|^2 \le \langle x_n - z, y_n - T^n y_n\rangle + \zeta_n\right\}, \\
W_n = \{z \in C : \langle x_n - z, x - x_n\rangle\} \ge 0, \\
x_{n+1} = P_{H_n \cap W_n} x,
\end{gathered} \tag{3.10}$$

where $\zeta_n = \mu_m M^*(\operatorname{diam} C) + \nu_m$, then the iterative sequence $\{x_n\}$ converges strongly to $P_{F(T)}x$ in C.

Proof. We divide the proof into seven steps.

(I) $P_{F(T)}$ is well defined for every $x \in C$.

By Theorem 3.2, we know $F(T)$ is closed and convex subset of C. Moreover, by our assumption that $F(T)$ is nonempty, therefore, $P_{F(T)}x$ is well defined for every $x \in C$.

(II) H_n and W_n are closed and convex for all $n \in \mathbb{N}$.

From the definitions of W_n and H_n, it is obvious that H_n and W_n are closed and convex for each $n \in \mathbb{N}$. We omit the details.

(III) We prove $F(T) \subset H_n \bigcap W_n$ for each $n \in \mathbb{N}$.

We first show $F(T) \subset H_n$. Let $z \in F(T)$, by (3.10), and the uniform L-Lipschitz continuity of T and the total asymptotical pseudo-contractiveness of T, we have

$$\begin{aligned}
\|x_n - T^n x_n\|^2 &= \langle x_n - T^n x_n, x_n - T^n x_n \rangle \\
&= \frac{1}{\alpha_n}\langle x_n - y_n, x_n - T^n x_n \rangle \\
&= \frac{1}{\alpha_n}\langle x_n - y_n, x_n - T^n x_n - (y_n - T^n y_n) \rangle + \frac{1}{\alpha_n}\langle x_n - y_n, (y_n - T^n y_n) \rangle \\
&\leq \frac{1+L}{\alpha_n}\|x_n - y_n\|^2 + \frac{1}{\alpha_n}\langle x_n - z, y_n - T^n y_n \rangle + \frac{1}{\alpha_n}(\mu_n M^*(\operatorname{diam} C) + \nu_n) \\
&= (1+L)\alpha_n \|x_n - T^n x_n\|^2 + \frac{1}{\alpha_n}\langle x_n - z, y_n - T^n y_n \rangle \\
&\quad + \frac{1}{\alpha_n}(\mu_n M^*(\operatorname{diam} C) + \nu_n).
\end{aligned} \tag{3.11}$$

This implies that

$$\alpha_n[1 - \alpha_n(1+L)]\|x_n - T^n x_n\|^2 \leq \langle x_n - z, y_n - T^n y_n \rangle + (\mu_n M^*(\operatorname{diam} C) + \nu_n). \tag{3.12}$$

This shows that $z \in H_n$ for all $n \in \mathbb{N}$. So $F(T) \subset H_n$ for all $n \in \mathbb{N}$. Next we prove $F(T) \subset W_n$ for all $n \in \mathbb{N}$. By induction, for $n = 1$, we have $F(T) \subset C = W_1$. Assume that $F(T) \subset W_n$. Since x_{n+1} is the projection of x onto $H_n \bigcap W_n$, by Lemma 2.1, we have

$$\langle x_{n+1} - z, x - x_{n+1} \rangle \geq 0, \tag{3.13}$$

for any $z \in H_n \bigcap W_n$, by the definition of W_{n+1}, this shows that $z \in W_{n+1}$. So $F(T) \subset H_n \bigcap W_n$, for all $n \in \mathbb{N}$.

(IV) We prove that $\lim_{n\to\infty}\|x_n - x\|$ exists.

From (3.10) and Lemma 2.1, we have $x_n = P_{w_n}x$, this with $x_{n+1} \in W_{n+1}$ show $\|x_n - x\| \leq \|x_{n+1} - x\|$, for all $n \in \mathbb{N}$. As $z \in F(T) \subset W_n$, we also have $\|x_n - x\| \leq \|z - x\|$, for all $n \in \mathbb{N}$. Consequently, $\lim_{n\to\infty}\|x_n - x\|$ exists and $\{x_n\}$ is bounded.

(V) We prove that $\|x_{n+1} - x_n\| \to 0$ as $n \to \infty$.

By Lemma 2.2, we have

$$\|x_{n+1} - x_n\|^2 \leq \|x_{n+1} - x\|^2 - \|x_n - x\|^2 \longrightarrow 0, \tag{3.14}$$

as $n \to \infty$.

(VI) Now we prove $\|x_n - Tx_n\| \to 0$ as $n \to \infty$.

It follows from $\|x_{n+1} - x_n\| \to 0$ as $n \to \infty$, $x_{n+1} \subset H_n$, $\{y_n\}$ is bounded, $\{T^n y_n\}$ is bounded, and $\alpha_n \in (a, b)$ that

$$\begin{aligned}
&\alpha_n[1 - \alpha_n(1 + L)]\|x_n - T^n x_n\|^2 \\
&\quad \leq \langle x_n - z, y_n - T^n y_n\rangle + (\mu_n M^*(\operatorname{diam} C) + \nu_n) \\
&\quad \leq \|x_n - z\|\|y_n - T^n y_n\| + (\mu_n M^*(\operatorname{diam} C) + \nu_n) \longrightarrow 0, \quad n \longrightarrow \infty.
\end{aligned} \tag{3.15}$$

So $\|x_n - T^n x_n\| \to 0$ as $n \to \infty$. Additional

$$\begin{aligned}
\|x_n - Tx_n\| &\leq \|x_{n+1} - x_n\| + \left\|T^{n+1}x_{n+1} - x_{n+1}\right\| \\
&\quad + \left\|T^{n+1}x_n - T^{n+1}x_{n+1}\right\| + \left\|Tx_n - T^{n+1}x_n\right\| \\
&\leq (L+1)\|x_{n+1} - x_n\| + \left\|T^{n+1}x_{n+1} - x_{n+1}\right\| + L\|x_n - T^n x_n\|.
\end{aligned} \tag{3.16}$$

So $\|x_n - Tx_n\| \to 0$ as $n \to \infty$.

(VII) Finally, we prove $x_n \to P_{F(T)}x$ as $n \to \infty$.

Let x_{n_k} be a subsequence of x_{n_k} such that $x_{n_k} \rightharpoonup \hat{x} \in C$, then by Theorem 3.1, we have $\hat{x} \in F(T)$. We let $\omega \in P_{F(T)}x$. For any $n \in \mathbb{N}$, $x_{n+1} = P_{H_n \bigcap W_n}x$ and $\omega \in P_{F(T)}x \subset H_n \bigcap W_n$, so we get $\|x_{n+1} - x\| \leq \|\omega - x\|$.

On the other hand, from the weak lower semicontinuity of the norm, we have

$$\begin{aligned}
\|\hat{x} - x\|^2 &= \|\hat{x}\|^2 - 2\langle \hat{x}, x\rangle + \|x\|^2 \\
&\leq \liminf_{n\to\infty}\left(\|x_{n_k}\|^2 - 2\langle \|x_{n_k}\|^2, x\rangle + \|x\|^2\right) \\
&= \liminf_{n\to\infty}\|x_{n_k} - x\|^2 \\
&\leq \limsup_{n\to\infty}\|x_{n_k} - x\|^2 \\
&\leq \|\omega - x\|^2.
\end{aligned} \tag{3.17}$$

From the definition of $P_{F(T)}x$, we obtain $\hat{x} = \omega$ and hence $\limsup_{n\to\infty}\|x_{n_k} - x\|^2 = \|\omega - x\|^2$. So we have $\limsup_{k\to\infty}\|x_{n_k}\| = \|\omega\|$. Thus we obtain that x_{n_k} converges strongly to $P_{F(T)}x$. Since x_{n_k} is an arbitrary weakly convergent sequence of x_n, we can conclude that x_n converges strongly to $P_{F(T)}x$. This completes the proof of Theorem 3.3. □

Remark 3.4. Theorem 3.3 extends the main results of Zhou [3] and improves the main results of Qin et al. [5] and of many other authors.

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