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Research Article

Adaptive Observer-Based Fault Estimation for Stochastic Markovian Jumping Systems

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This paper studies the adaptive fault estimation problems for stochastic Markovian jump systems (MJSs) with time delays. With the aid of the selected Lyapunov-Krasovskii functional, the adaptive fault estimation algorithm based on adaptive observer is proposed to enhance the rapidity and accuracy performance of fault estimation. A sufficient condition on the existence of adaptive observer is presented and proved by means of linear matrix inequalities techniques. The presented results are extended to multiple time-delayed MJSs. Simulation results illustrate that the validity of the proposed adaptive faults estimation algorithms.

1. Introduction

Fault detection and isolation (FDI) [1, 2] has been the subject of extensive research since the 1970s and becomes one of the hotspots in control theory presently. With the rising demands of product quality, effectiveness, and safety in modern industries, people expect that they can get the failure information before the fault damages the system. Many techniques have been proposed especially for sensor and actuator failures with application to a wide range of engineering fields. Among these, the most commonly used schemes for fault detection relate to observer-based approaches [2–5]. It should be pointed out that the observer-based approach, which uses a parametric design technique to perform both detection and diagnosis, only works for a small number of sensor faults. In some cases, fault estimation strategies [6–8] are needed to carry on controlling the faulty system. Compared with FDI, fault estimation is a more challenging task because it requires an estimation of the location after the alarm has been set, and the size of the fault should be made. Recently, some results based on

adaptive or robust observers [2–8] for fault estimation have been obtained. However, very few results in the literature consider the fault detection and estimation problem for stochastic systems.

In fact, as a special class of stochastic systems [9, 10] that involves both time-evolving and event-driven mechanisms, Markovian jump systems (MJSs) [11] have received considerable attention. This class of systems includes two components which are the mode and the state and the dynamics of jumping modes and continuous states, which are, respectively, modeled by finite-state Markov chains and differential equations. It can be used to model a variety of physical systems, which may experience abrupt changes in structures and parameters due to, for example, sudden environment changes, subsystem switching, system noises, and failures occurred in components or interconnections and executor faults. Some illustrative applications of MJSs can be found for examples in [3, 4, 6, 12–18] and the references therein. In recent years, the FDI problems for MJSs have regained increasing interest, and some results are also available [3, 4, 16–18]. However, very little is known on the problem of fault estimation for time-delay nonlinear MJSs. This problem forms the main purpose of this paper, where an adaptive technique [1, 19–21] is proposed and modified for the estimation of actuator faults.

In this paper, we studied the problem of fault estimation for a class of time-delay MJSs. By comparing with the presented results of time-delay and linear dynamic systems, it can be shown that a derivative term is added on the basis of fault estimation equation in our design, which renders that the conventional adaptive fault estimation algorithm [21] can be treated as a special case of the fast adaptive fault estimation algorithm. The introduction of the derivative term plays a major role in improving the rapidity of fault estimation. The augmented dynamic system is firstly constructed based on the adaptive fault estimation observer, and the observer parameters are designed on the system modes. Sufficient conditions are subsequently established on the existence of the mode-dependent adaptive fault estimation observer. The design criterions are presented in the form of linear matrix inequalities (LMIs) [22], which can be easily checked. The presented results are then extended to multiple time-delayed MJSs case. Finally, a numerical example is included to illustrate the effectiveness of the developed techniques.

Let us introduce some notations. The symbols $\Re^n$ and $\Re^{n\times m}$ stand for an n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively, A^{T} and A^{-1} denote the matrix transpose and matrix inverse, $\mathrm{diag}\{A \quad B\}$ represents the block-diagonal matrix of A and B, $\sigma_{\max}(C)$ denote the maximal eigenvalue of a positive-define matrix C, $\sum_{i<j}^{N}$ denotes, for example, for $N = 3$, $\sum_{i<j}^{N} a_{ij} \Leftrightarrow a_{12} + a_{13} + a_{23}$, $\|*\|$ denotes the Euclidean norm of vectors, $\mathbf{E}\{*\}$ denotes the mathematics statistical expectation of the stochastic process or vector, $L_2^n(0 \quad \infty)$ is the space of n dimensional square integrable function vector over $(0 \quad \infty)$, $P > 0$ (or $P \geq 0$) stands for a positive-definite (or nonnegative-definite) matrix, I is the unit matrix with appropriate dimensions, 0 is the zero matrix with appropriate dimensions, $*$ means that the symmetric terms in a symmetric matrix.

2. Problem Formulation

Given a probability space (Ω, F, ρ), where Ω is the sample space, F is the algebra of events, and ρ is the probability measure defined on F. Let the random form process $\{r_t, t \geq 0\}$ be

the continuous-time discrete-state Markov stochastic process taking values in a finite set $\Lambda = \{1, 2, \ldots, N\}$ with transition probabilities given by

$$P_r\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{2.1}$$

where $\Delta t > 0$ and $\lim_{\Delta t \downarrow 0} o(\Delta t)/\Delta t \to 0$. $\pi_{ij} \geq 0$ are the transition probability rates from mode i at time t to mode $j (i \neq j)$ at time $t + \Delta t$, and $\sum_{j=1, i \neq j}^{N} \pi_{ij} = -\pi_{ii}$.

Consider the following linear MJSs over the probability space (Ω, F, ρ):

$$\begin{gathered} \dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-d) + B(r_t)u(t) + B_f(r_t)f(t), \\ y(t) = C(r_t)x(t), \\ x(t) = \eta(t), \quad r(t) = r_0, \quad t \in [-d \;\; 0], \; i = 1, 2, \ldots, S, \end{gathered} \tag{2.2}$$

where $x(t) \in \Re^n$ is the state, $y(t) \in \Re^m$ is the measured output, $u(t) \in \Re^l$ is the controlled input, $f(t) \in \Re^p$ is the unknown actuator fault, and we assume that its derivative is the norm bounded with $\|\dot{f}(t)\| \leq f$, wherein $0 \leq f < \infty$. $d > 0$ is the time delay constant, $\eta(t) \in \Re^n$ is a continuous vector-valued initial function assumed to be continuously differentiable on $[-d \;\; 0]$, and r_0 is the initial mode. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $B_f(r_t)$, and $C(r_t)$ are known mode-dependent matrices with appropriate dimensions, $B_f(r_t)$ is of full column rank with $\operatorname{rank}[B_f(r_t)] = p$, and r_t represents a continuous-time discrete state Markov stochastic process with values in the finite set Λ.

For presentation convenience, we denote $x(t-d)$, $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $B_f(r_t)$, and $C(r_t)$ as x_d, A_i, A_{di}, B_i, B_{fi}, C_i and, respectively.

Definition 2.1 (see Mao [23]). Let $V(x(t), r_t, t > 0) = V(x(t), i)$ be the positive stochastic functional and define its weak infinitesimal operator as

$$\Im V(x(t), i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}[\mathbf{E}\{V(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t) \mid x(t), r_t = i\} - V(x(t), i, t)]. \tag{2.3}$$

Refer to observer design [2–4, 7] and consider the following systems:

$$\begin{gathered} \dot{\hat{x}}(t) = A_i\hat{x}(t) + A_{di}\hat{x}_d + B_{fi}\hat{f}(t) + B_i u(t) + L_i[y(t) - \hat{y}(t)] + L_{di}[y_d - \hat{y}_d], \\ \hat{y}(t) = C_i\hat{x}(t), \end{gathered} \tag{2.4}$$

where $\hat{x}(t) \in \Re^n$ is the observer state vector, $\hat{y}(t) \in \Re^m$ is the observer output vector, and $\hat{f}(t) \in \Re^p$ is an estimate of actuator fault of $f(t)$. Denote

$$\begin{gathered} e(t) = x(t) - \hat{x}(t), \\ z(t) = y(t) - \hat{y}(t), \\ e_f(t) = f(t) - \hat{f}(t). \end{gathered} \tag{2.5}$$

Then, we can present the error dynamics (2.4) as

$$\begin{aligned} \dot{e}(t) &= \overline{A}_i e(t) + \overline{A}_{di} e_d + B_{fi} e_f(t), \\ z(t) &= C_i e(t), \end{aligned} \tag{2.6}$$

where $\overline{A}_i = A_i - L_i C_i$, $\overline{A}_{di} = A_{di} - L_{di} C_i$.

In this paper, by comparison with the conventional adaptive fault estimation algorithm [21], we consider the following fast adaptive fault estimation algorithm. In this algorithm, we add a derivative term $\dot{z}(t)$ in estimation equation, that is,

$$\dot{\hat{f}}(t) = \Gamma H_i [\dot{z}(t) + z(t)], \tag{2.7}$$

which can realize $\lim_{t\to\infty} z(t) = 0$, where H_i is a given mode-dependent matrix and $\Gamma = \Gamma^{\mathrm{T}} > 0$ is a prespecified matrix which defines the learning rate for (2.6).

Remark 2.2. In the conventional adaptive fault estimation algorithm $\hat{f}(t) = \Gamma H \int_{t_f}^{t} z(\tau) d\tau$, it is only an integral term in essence. It fails to deal with time-varying faults, that is, $\dot{f}(t) \neq 0$ though it assumes that the constant fault ($\dot{f}(t) = 0$) estimation is unbiased. In this paper, we add a derivative term $\dot{z}(t)$ in estimation equation and improve the conventional adaptive fault estimation algorithm such that the time-varying faults can be considered. For the stochastic modes jumping case, we select the given matrix H_i as a mode-dependent one.

3. Adaptive Fault Estimation Observer Design

Theorem 3.1. *If there exist a set of positive definite symmetric matrices P_i, Q, U and mode-dependent matrices H_i, X_i, and X_{di}, such that the following matrix equations hold for all $i \in \Lambda$:*

$$H_i C_i = B_{fi}^{\mathrm{T}} P_i, \tag{3.1}$$

$$\Pi_i = \begin{bmatrix} \Xi_i & P_i A_{di} - X_{di} C_i & -A_i^{\mathrm{T}} P_i B_{fi} + C_i^{\mathrm{T}} X_i^{\mathrm{T}} B_{fi} \\ * & -Q & -A_{di}^{\mathrm{T}} P_i B_{fi} + C_i^{\mathrm{T}} X_{di}^{\mathrm{T}} B_{fi} \\ * & * & -2B_{fi}^{\mathrm{T}} P_i B_{fi} + U \end{bmatrix} < 0, \tag{3.2}$$

where $\Xi_i = A_i^{\mathrm{T}} P_i + P_i A_i - X_i C_i - C_i^{\mathrm{T}} X_i^{\mathrm{T}} + Q + \sum_{i=1}^{N} \pi_{ij} P_j$. Then the fast adaptive fault estimation algorithm of (2.7) *can be realized. And in the estimated time-interval, it can estimate errors with the uniformly boundedness of the states and faults. Moreover, the observer gains are respectively as*

$$L_i = P_i^{-1} X_i, \qquad L_{di} = P_i^{-1} X_{di}. \tag{3.3}$$

Proof. Let the mode at time t be i, that is, $r_t = i \in \Lambda$. Take the stochastic Lyapunov-Krasovskii function $V(e(t), e_f(t), r_t, t > 0) : \Re^n \times \Re^n \times \Lambda \times \Re_+ \to \Re_+$ as

$$V\big(e(t), e_f(t), i\big) = V_1\big(e(t), e_f(t), i\big) + V_2\big(e(t), e_f(t), i\big) + V_3\big(e(t), e_f(t), i\big), \tag{3.4}$$

where $V_1(e(t), e_f(t), i) = e^{\mathrm{T}}(t)P_i e(t)$, $V_2(e(t), e_f(t), i) = \int_{t-d}^{t} e^{\mathrm{T}}(t)Qe(t)d\tau$, $V_3(e(t), e_f(t), i) = e_f^{\mathrm{T}}(t)\Gamma^{-1}e_f(t)$, in which $P_i \in \Re^{n\times n}$, $Q \in \Re^{n\times n}$ are the given mode-dependent symmetric positive-definite matrix for each modes $i \in \Lambda$.

According to Definition 2.1 and along the trajectories of the error dynamics MJSs (2.6), we can derive the following:

$$\begin{aligned}
\Im V_1\big(e(t), e_f(t), i\big) &= 2e^{\mathrm{T}}(t)P_i\dot{e}(t) + e^{\mathrm{T}}(t)\sum_{i=1}^{N}\pi_{ij}P_j e(t) \\
&= 2e^{\mathrm{T}}(t)P_i\Big[\overline{A}_i e(t) + \overline{A}_{di}(r)e_d + B_{fi}e_f(t)\Big] + e^{\mathrm{T}}(t)\sum_{i=1}^{N}\pi_{ij}P_j e(t) \\
&= e^{\mathrm{T}}(t)\left[\overline{A}_i^{\mathrm{T}}P_i + P_i\overline{A}_i + \sum_{i=1}^{N}\pi_{ij}P_j\right]e(t) + 2e^{\mathrm{T}}(t)P_i\overline{A}_{di}e_d + 2e^{\mathrm{T}}(t)P_iB_{fi}e_f(t), \\
\Im V_2\big(e(t), e_f(t), i\big) &= e^{\mathrm{T}}(t)Qe(t) - e_d^{\mathrm{T}}Qe_d + \sum_{i=1}^{N}\pi_{ij}\int_{t-d}^{t} e^{\mathrm{T}}(t)Qe(t)d\tau \\
&= e^{\mathrm{T}}(t)Qe(t) - e_d^{\mathrm{T}}Qe_d + \left(\sum_{i=1}^{N}\pi_{ij}\right)\left(\int_{t-d}^{t} e^{\mathrm{T}}(t)Qe(t)d\tau\right) \\
&= e^{\mathrm{T}}(t)Qe(t) - e_d^{\mathrm{T}}Qe_d, \\
\Im V_3\big(e(t), e_f(t), i\big) &= 2e_f^{\mathrm{T}}(t)\Gamma^{-1}\dot{e}_f(t) = 2e_f^{\mathrm{T}}(t)\Gamma^{-1}\Big[\dot{f}(t) - \dot{\hat{f}}(t)\Big] \\
&= -2e_f^{\mathrm{T}}(t)H_i[\dot{z}(t) + z(t)] + 2e_f^{\mathrm{T}}(t)\Gamma^{-1}\dot{f}(t) \\
&= -2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i\dot{e}(t) - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i e(t) + 2e_f^{\mathrm{T}}(t)\Gamma^{-1}\dot{f}(t) \\
&= -2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i\overline{A}_i e(t) - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i\overline{A}_{di}e_d - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_iB_{fi}e_f(t) \\
&\quad - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i e(t) + 2e_f^{\mathrm{T}}(t)\Gamma^{-1}\dot{f}(t).
\end{aligned} \tag{3.5}$$

Given a symmetric positive definite matrix U, we can use the following relation:

$$\begin{aligned}
2e_f^{\mathrm{T}}(t)\Gamma^{-1}\dot{f}(t) &\le e_f^{\mathrm{T}}(t)Ue_f(t) + \dot{f}^{\mathrm{T}}(t)\Gamma^{-1}U^{-1}\Gamma^{-1}\dot{f}(t) \\
&\le e_f^{\mathrm{T}}(t)Ue_f(t) + \dot{f}^2\sigma_{\max}\Big(\Gamma^{-1}U^{-1}\Gamma^{-1}\Big).
\end{aligned} \tag{3.6}$$

Then, we can get

$$\begin{aligned}\Im V(e(t),e_f(t),i) &= \Im V_1(e(t),e_f(t),i) + \Im V_2(e(t),e_f(t),i) + \Im V_3(e(t),e_f(t),i)\\ &= e^{\mathrm{T}}(t)\left[\overline{A}_i^{\mathrm{T}}P_i + P_i\overline{A}_i + \sum_{i=1}^{N}\pi_{ij}P_j + Q\right]e(t) + 2e^{\mathrm{T}}(t)P_i\overline{A}_{di}e_d - e_d^{\mathrm{T}}Qe_d\\ &\quad - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i\overline{A}_ie(t) - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_i\overline{A}_{di}e_d - 2e_f^{\mathrm{T}}(t)B_{fi}^{\mathrm{T}}P_iB_{fi}e_f(t)\\ &\quad + e_f^{\mathrm{T}}(t)Ue_f(t) + \dot{f}^2\sigma_{\max}\left(\Gamma^{-1}U^{-1}\Gamma^{-1}\right).\end{aligned} \tag{3.7}$$

By letting $X_i = P_iL_i$ and $X_{di} = P_iL_{di}$, the derivative of $\Im V(e(t),e_f(t),i)$ with respect to time follows that

$$\Im V(e(t),e_f(t),i) \le \zeta^{\mathrm{T}}(t)\Pi_i\zeta(t) + \eta, \tag{3.8}$$

where $\zeta(t) = \begin{bmatrix} e(t) \\ e_d \\ e_f(t)\end{bmatrix}$, $\eta = \dot{f}^2\sigma_{\max}(\Gamma^{-1}U^{-1}\Gamma^{-1})$.

Thus, it concludes that $\Im V(e(t),e_f(t),i) \le -\lambda\|\zeta(t)\|^2 + \eta$, wherein $\lambda = \min_{r\in\Lambda}\sigma_{\min}(-\Pi_i)$. Obviously, we can get $\Im V(e(t),e_f(t),i) < 0$ if $\eta < \lambda\|\zeta(t)\|^2$. According to stochastic Lyapunov-Krasovskii stability theory, the trajectory of $\zeta(t)$ will converge to the small set $\Phi = \{\zeta(t) \mid \|\zeta(t)\|^2 \le \eta/\lambda\}$, though it is outside set S. Therefore, $\zeta(t)$ is ultimately bounded. This completes the proof. □

Remark 3.2. It is necessary to point out that if the presented faults are constant, that is, $\dot{f}(t) = 0$, then the designed adaptive algorithm can achieve asymptotical convergence from (3.8). Then we can get that $\Im V(e(t),e_f(t),i) \le -\lambda\|\zeta(t)\|^2 \le 0$, which proves the stability of the origin $e(t) = 0$, $e_f(t) = 0$ and the uniformly boundedness of $e(t)$ and $e_f(t)$ with $e(t) \in L_2^n(0\ \ \infty)$. Then, $\lim_{t\to\infty}e(t) \to 0$ holds by Barbalat's Lemma.

4. Extension to Multiple Time-Delayed MJSs

Consider the following multiple time-delayed MJSs over the probability space (Ω, F, ρ):

$$\begin{gathered}\dot{x}(t) = A_ix(t) + \sum_{m=1}^{M}A_{dmi}x(t-d_m) + B_iu(t) + B_{fi}f(t),\\ y(t) = C_ix(t),\\ x(t) = \eta(t), \quad r(t) = r_0, \quad t \in \left[-\max(d_m)\ \ 0\right],\end{gathered} \tag{4.1}$$

where d_m, $m = 1,2,\ldots,M$ are multiple time delays with $0 \le d_m < \infty$, and other notations are the same as in Section 2.

Similar to Section 3, the following observer can be constructed:

$$\begin{aligned}\dot{\hat{x}}(t) &= A_i\hat{x}(t) + \sum_{m=1}^{M} A_{dmi}\hat{x}(t-d_m) + B_{fi}\hat{f}(t) + B_i u(t)\\ &\quad + L_i\left[y(t)-\hat{y}(t)\right] + \sum_{m=1}^{M} L_{dmi}\left[y(t-d_m)-\hat{y}(t-d_m)\right],\\ \hat{y}(t) &= C_i\hat{x}(t)\end{aligned} \tag{4.2}$$

where $\hat{x}(t) \in \Re^n$ is the observer state vector, $\hat{y}(t) \in \Re^m$ is the observer output vector, and $\hat{f}(t) \in \Re^p$ is an estimate of actuator fault of $f(t)$.

Then, we can obtain the error dynamics by using the same notations of $e(t)$, $e_f(t)$, and $z(t)$ as follows:

$$\begin{aligned}\dot{e}(t) &= \overline{A}_i e(t) + \sum_{m=1}^{M} \overline{A}_{dmi} e_d(t-d_m) + B_{fi} e_f(t),\\ z(t) &= C_i e(t)\end{aligned} \tag{4.3}$$

where $\overline{A}_i = A_i - L_i C_i$, $\overline{A}_{dmi} = A_{dmi} - L_{dmi} C_i$.

Prior to the design of an adaptive diagnostic law, we can get the following results for multiple time-delayed MJSs (4.1).

Theorem 4.1. *If there exist a set of positive definite symmetric matrices P_i, Q, U and mode-dependent matrices H_i, X_i, and X_{dmi}, $m = 1, 2, \ldots, M$, such that the following matrix equations hold for all $i \in \Lambda$:*

$$H_i C_i = B_{fi}^{\mathrm{T}} P_i, \tag{4.4}$$

$$\Psi_i = \begin{bmatrix} \Xi_i & P_i A_{di} - \sum_{m=1}^{M} X_{dmi} C_i & -A_i^{\mathrm{T}} P_i B_{fi} + C_i^{\mathrm{T}} X_i^{\mathrm{T}} B_{fi} \\ * & -Q & -A_{di}^{\mathrm{T}} P_i B_{fi} + C_i^{\mathrm{T}} \sum_{m=1}^{M} X_{dmi}^{\mathrm{T}} B_{fi} \\ * & * & -2B_{fi}^{\mathrm{T}} P_i B_{fi} + U \end{bmatrix} < 0. \tag{4.5}$$

Then, the fast adaptive fault estimation algorithm of (2.7) *can be realized. And in the estimated time interval, it can estimate errors with the uniformly boundedness of the states and faults. Moreover, the observer gains are respectively as follows:*

$$L_i = P_i^{-1} X_i, \qquad L_{dmi} = P_i^{-1} X_{dmi}, \quad m = 1, 2, \ldots, M. \tag{4.6}$$

When there are difficulties in solving (3.1) or (4.4), we can transform them into the following SDP problems via disciplined convex programming [24]:

$$\begin{aligned}&\min \quad \delta\\ &\text{s.t.} \quad \begin{bmatrix} \delta I & B_{fi}^{\mathrm{T}} P_i - H_i C_i \\ P_i B_{fi} - C_i^{\mathrm{T}} H_i^{\mathrm{T}} & \delta I \end{bmatrix} > 0.\end{aligned} \tag{4.7}$$

In order to make $B_{fi}^{\mathrm{T}}P_i$ approximate to H_iC_i with a satisfactory precision, we can firstly select a sufficiently small scalar $\delta > 0$ to meet (4.7).

Remark 4.2. The solutions of Theorems 3.1 and 4.1 can be obtained by solving an optimization problem with (4.7). By using the Matlab LMI Toolbox, it is straightforward to check the feasibility of LMIs. In order to illustrate the effectiveness of the developed techniques, we will give several numerical examples about fuzzy jump system with time delays in Section 5.

5. Numeral Example

We consider the following time-delayed stochastic MJSs with parameters given by

$$A_1 = \begin{bmatrix} -0.5 & -0.3 \\ -0.1 & -1.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5 & -0.4 \\ -0.1 & -1.06 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.05 & -0.1 \\ 0 & 0.05 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.07 & -0.1 \\ 0 & -0.05 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix}, \quad B_{f1} = B_{f2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.2 & 0.4 \end{bmatrix}. \tag{5.1}$$

The transition rate matrix that relates the two operation modes is given as $\Pi = \left[\begin{smallmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{smallmatrix}\right]$. By solving the LMIs in (3.1), (3.2), and (4.7) with $\delta = 8.9252 \times 10^{-4}$, we can get the following solutions:

$$L_1 = \begin{bmatrix} 4.5853 \\ 2.2033 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -8.3797 \\ 1.6944 \end{bmatrix},$$
$$L_{d1} = \begin{bmatrix} 0.1110 \\ 0.0691 \end{bmatrix}, \quad L_{d2} = \begin{bmatrix} -0.2448 \\ -0.1343 \end{bmatrix}, \tag{5.2}$$
$$H_1 = -0.0061, \quad H_2 = 0.0019.$$

To show the effectiveness of the designed methods, the time-delay d is assumed to be 0.2 s, and we consider two kinds of actuator faults $f_1(t)$ and $f_2(t)$ in the simulation over the finite-time interval $t \in [0 \;\; 10]$:

$$f_1(t) = \begin{cases} 0, & 0 \le t \le 4, \\ 0.5\sin(5t), & 4 < t \le 10, \end{cases}$$
$$f_2(t) = \begin{cases} 0.5, & 2k-1 < t \le 2k,\ k \in \{N^+, 1 \le k \le 5\}, \\ 0, & \text{others}. \end{cases} \tag{5.3}$$

Let $r_0 = 2$ and $\Gamma = 10$, the jumping modes are shown in Figure 1. The estimated faults and estimation errors of $f_1(t)$ and $f_2(t)$ are shown in Figures 2 and 3, respectively. From the simulation results and design algorithm, it can be concluded that the adaptive fault diagnosis observer can enhance the performance of fault estimation for slow and fast time-varying faults.

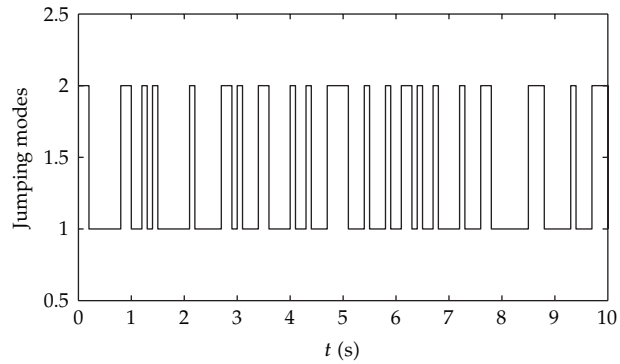

Figure 1: The estimation of changing between modes during the simulation with initial mode 2.

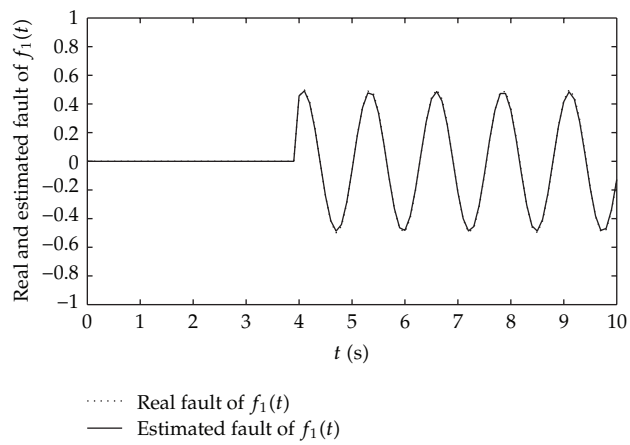

Figure 2: Fault $f_1(t)$ and the estimated $\widehat{f}_1(t)$.

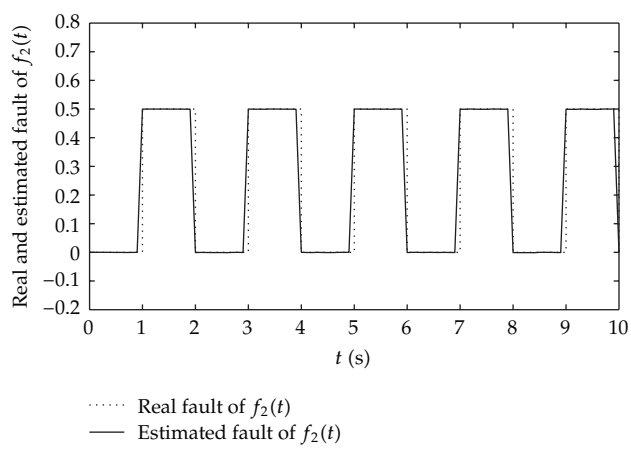

Figure 3: Fault $f_2(t)$ and the estimated $\widehat{f}_2(t)$.

6. Conclusions

In this paper, we have studied the design of adaptive fault estimation observer for time-delayed MJSs. It ensures the rapidity and accuracy performance of fault estimation of the designed observer. By selecting the appropriate Lyapunov-Krasovskii function and applying matrix transformation and variable substitution, the main results are provided in terms of LMIs form and then extended to multiple time-delayed MJSs case. Simulation example demonstrates the effectiveness of the developed techniques.

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