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Research Article

New Iterative Manner Involving Sunny Nonexpansive Retractions for Pseudocontractive Mappings

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Iterative methods for pseudocontractions have been studied by many authors in the literature. In the present paper, we firstly propose a new iterative method involving sunny nonexpansive retractions for pseudocontractions in Banach spaces. Consequently, we show that the suggested algorithm converges strongly to a fixed point of the pseudocontractive mapping which also solves some variational inequality.

1. Introduction

Let C be a nonempty closed convex subset of a real Banach space E. A mapping $T : C \to C$ is said to be nonexpansive, if

$$\|Tx - Ty\| \leq \|x - y\|, \tag{1.1}$$

for all $x, y \in C$.

Now we know that the involved operators in the many practical applications can be reduced to the nonexpansive mappings, that is, there are a large number of applied areas which are closely related to the nonexpansive mappings, for example, inverse problem, partial differential equations, image recovery, and signal processing. Based on these facts, recently, iterative methods for finding fixed points of nonexpansive mappings have received vast investigations. For related works, please see [1–26] and the references therein.

In the present paper, we focus on a class of strictly pseudocontractive mappings which strictly includes the class of nonexpansive mappings. Recall that a mapping $T : C \to C$ is said to be strictly pseudocontractive if there exists a constant $\lambda > 0$ and $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \lambda\|x - y - (Tx - Ty)\|^2, \tag{1.2}$$

for all $x, y \in C$. We use $\mathrm{Fix}(T)$ to denote the set of fixed points of T.

We know that the strict pseudocontractions have more powerful applications than nonexpansive mappings in solving inverse problems. There are some related references in the literature for strictly pseudocontractive mappings; see, for example, [27–30]. Motivated and inspired by the works in the literature, in the present paper, we firstly propose a new iterative method involving sunny nonexpansive retractions for pseudocontractions in Banach spaces. Consequently, we show that the suggested algorithm converges strongly to a fixed point of the pseudocontractive mapping which also solves some variational inequality.

2. Preliminaries

Let E^* be the dual space of a Banach space E. Let J_q $(q > 1)$ be the generalized duality mapping from E into 2^{E^*} given by

$$J_q(x) = \left\{g \in E^* : \langle x, g\rangle = \|x\|\|g\|, \|g\| = \|x\|^{q-1}\right\}, \tag{2.1}$$

where $\langle\cdot,\cdot\rangle$ denotes the generalized duality pairing. In particular, J_2 is called the normalized duality mapping and it is usually denoted by j. It is well known that E is a uniformly smooth Banach space if and only if J_q is single valued and uniformly continuous on any bounded subset of E.

Let C be a nonempty closed convex subset of a Banach space E, and let D be a nonempty subset of C. Recall that a mapping $Q : C \to D$ is called a retraction from C onto D provided $Q(x) = x$ for all $x \in D$. A retraction $Q : C \to D$ is sunny provided $Q(x + t(x - Q(x))) = Q(x)$ for all $x \in C$ and $t \geq 0$ whenever $x + t(x - Q(x)) \in C$. A sunny nonexpansive retraction is a sunny retraction which is also nonexpansive. Sunny nonexpansive retractions are characterized as follows.

Lemma 2.1. *If E is a smooth Banach space, then $Q : C \to D$ is a sunny nonexpansive retraction if and only if there holds the inequality*

$$\langle x - Qx, J(y - Qx)\rangle \leq 0, \tag{2.2}$$

for all $x \in C$ and $y \in D$.

Lemma 2.2 (see [31]). *Let E be a real q-uniformly smooth Banach space, and let $1 < q \leq 2$. Then, one has*

$$\|x+y\|^q \leq \|x\|^q + q\langle y, J_q(x)\rangle + 2\|Ky\|^q, \tag{2.3}$$

for all $x, y \in E$.

Lemma 2.3 (see [16]). *Let $\{x_n\}$ and $\{z_n\}$ be two bounded sequences in Banach spaces, and let $\{\beta_n\}$ be a sequence in $[0,1]$ with $0 < \liminf_{n\to\infty}\beta_n \leq \limsup_{n\to\infty}\beta_n < 1$. Suppose that $x_{n+1} = (1-\beta_n)x_n + \beta_n z_n$ for all $n \geq 0$ and $\limsup_{n\to\infty}(\|z_{n+1}-z_n\| - \|x_{n+1}-x_n\|) \leq 0$. Then $\lim_{n\to\infty}\|z_n - x_n\| = 0$.*

Lemma 2.4 (see [14]). *Let C be a nonempty closed convex subset of a real q-uniformly smooth and uniformly convex Banach space E. Let $T : C \to C$ be a strictly pseudocontractive mapping. Then $I - T$ is demiclosed.*

Lemma 2.5 (see [32]). *Assume $\{a_n\}$ is a sequence of nonnegative real numbers such that $a_{n+1} \leq (1-\gamma_n)a_n + \gamma_n\delta_n$ for $n \geq 0$ where $\{\gamma_n\}$ is a sequence in $(0,1)$ and $\{\delta_n\}$ is a sequence in R such that*

(i) $\sum_{n=0}^{\infty}\gamma_n = \infty$;

(ii) $\limsup_{n\to\infty}\delta_n \leq 0$ *or* $\sum_{n=0}^{\infty}|\delta_n\gamma_n| < \infty$.

Then $\lim_{n\to\infty}a_n = 0$.

3. Main Results

In this section, we will give our main results. In the sequel, we assume the following:

(C1) E is a uniformly convex and 2-uniformly smooth Banach space which admits a weakly sequentially continuous duality mapping j from E to E^*;

(C2) C is a nonempty closed convex subset of E;

(C3) Q_C is a sunny nonexpansive retraction from E onto C;

(C4) $T : C \to C$ is a λ-strict pseudocontraction;

(C5) $\vartheta : E \to E$ is a ρ-contraction;

(C6) $A : E \to E$ is strongly positive (i.e., $\langle Ax, j(x)\rangle \geq \gamma\|x\|^2$ for some $0 < \gamma < 1$) and linear bounded operator with $\|(1-\zeta)I - \theta A\| \leq 1 - \zeta - \theta$ for all $\zeta > 0, \theta > 0$ and $0 < \zeta + \theta < 1$;

(C7) $\mathrm{Fix}(T) \neq \emptyset$.

First, we consider the following VI: finding $x^\dagger \in \mathrm{Fix}(T)$ such that

$$\left\langle (A - \delta\vartheta)x^\dagger, j\left(\tilde{x} - x^\dagger\right) \right\rangle \geq 0, \quad \tilde{x} \in \mathrm{Fix}(T). \tag{3.1}$$

The set of solutions of (3.1) is denoted by $VI(\mathrm{Fix}(T), A)$. In the sequel, we assume that $VI(\mathrm{Fix}(T), A) \neq \emptyset$. Note that (3.1) has the unique solution.

Next, we propose our algorithm.

Algorithm 3.1. For the initial point $x_0 \in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = \alpha_n\delta\vartheta(x_n) + \left[(1-\eta)I - \alpha_n A\right]x_n + \eta\left[(1-k)Q_C x_n + kTQ_C x_n\right], \quad n \geq 0, \tag{3.2}$$

where $\{\alpha_n\}$ is a sequence in $(0,1)$ and $\delta > 0,\ 0 < \eta < 1,\ \delta\rho < \gamma,\ 0 < k < \lambda/K^2$.

Theorem 3.2. *If the sequence* $\{\alpha_n\}$ *satisfies* $\lim_{n\to\infty}a_n = 0$ *and* $\sum_{n=0}^{\infty}\alpha_n = \infty$*, then the sequence* $\{x_n\}$ *generated by* (3.2) *converges strongly to the unique solution* $x^\dagger$ *of VI* (3.1)*.*

Proof. First, by using Lemma 2.2, we know that $(1-k)I + kT$ is nonexpansive. Q_C is sunny nonexpansive. Thus, $(1-k)Q_C + kTQ_C$ is nonexpansive. Let $x^\dagger \in \mathrm{Fix}(T)$. From (3.2), we have

$$\begin{aligned}
\left\|x_{n+1}-x^\dagger\right\| &= \left\|\alpha_n\delta\vartheta(x_n) + [(1-\eta)I-\alpha_nA]x_n + \eta[(1-k)Q_Cx_n + kTQ_Cx_n] - x^\dagger\right\| \\
&= \Big\|\alpha_n\left(\delta\vartheta(x_n) - Ax^\dagger\right) + [(1-\eta)I-\alpha_nA]\left(x_n-x^\dagger\right) \\
&\quad + \eta\left[(1-k)Q_Cx_n + kTQ_Cx_n - x^\dagger\right]\Big\| \\
&\le \alpha_n\left\|\delta\vartheta(x_n) - Ax^\dagger\right\| + \|(1-\eta)I-\alpha_nA\|\left\|x_n-x^\dagger\right\| \\
&\quad + \eta\left\|(1-k)Q_Cx_n + kTQ_Cx_n - x^\dagger\right\| \\
&\le \alpha_n\delta\left\|\vartheta(x_n) - \vartheta\left(x^\dagger\right)\right\| + \alpha_n\left\|\delta\vartheta\left(x^\dagger\right) - Ax^\dagger\right\| + (1-\eta-\alpha_n\gamma)\left\|x_n-x^\dagger\right\| \\
&\quad + \eta\left\|x_n-x^\dagger\right\| \\
&\le \alpha_n\delta\rho\left\|x_n-x^\dagger\right\| + \alpha_n\left\|\delta\vartheta\left(x^\dagger\right) - Ax^\dagger\right\| + (1-\eta-\alpha_n\gamma)\left\|x_n-x^\dagger\right\| \\
&\quad + \eta\left\|x_n-x^\dagger\right\| \\
&= [1-(\gamma-\delta\rho)\alpha_n]\left\|x_n-x^\dagger\right\| + \alpha_n\left\|\delta\vartheta\left(x^\dagger\right) - Ax^\dagger\right\|.
\end{aligned} \tag{3.3}$$

Thus,

$$\left\|x_{n+1}-x^\dagger\right\| \le \max\left\{\left\|x_n-x^\dagger\right\|, \frac{\|\delta\vartheta(x^\dagger) - Ax^\dagger\|}{\gamma-\delta\rho}\right\} \le \max\left\{\left\|x_0-x^\dagger\right\|, \frac{\|\delta\vartheta(x^\dagger) - Ax^\dagger\|}{\gamma-\delta\rho}\right\}. \tag{3.4}$$

This indicates that $\{x_n\}$ is bounded.

We write $x_{n+1} = (1-\eta)x_n + \eta u_n$ for all $n \ge 0$. So,

$$\begin{aligned}
u_n &= \frac{x_{n+1}-(1-\eta)x_n}{\eta} \\
&= \frac{\alpha_n\delta\vartheta(x_n)}{\eta} - \frac{\alpha_nAx_n}{\eta} + (1-k)Q_Cx_n + kTQ_Cx_n \\
&= \alpha_n\frac{\delta\vartheta - A}{\eta}x_n + [(1-k)Q_C + kTQ_C]x_n.
\end{aligned} \tag{3.5}$$

Hence,

$$u_{n+1} - u_n = \alpha_{n+1}\frac{\delta\vartheta - A}{\eta}x_{n+1} + [(1-k)Q_C + kTQ_C]x_{n+1} - \alpha_n\frac{\delta\vartheta - A}{\eta}x_n \\ - [(1-k)Q_C + kTQ_C]x_n. \tag{3.6}$$

It follows that

$$\begin{aligned}\|u_{n+1} - u_n\| &\le \alpha_{n+1}\left\|\frac{\delta\vartheta - A}{\eta}x_{n+1}\right\| + \alpha_n\left\|\frac{\delta\vartheta - A}{\eta}x_n\right\| \\ &\quad + \|[(1-k)Q_C + kTQ_C]x_{n+1} - [(1-k)Q_C + kTQ_C]x_n\| \\ &\le (\alpha_n + \alpha_{n+1})M + \|x_{n+1} - x_n\|,\end{aligned} \tag{3.7}$$

where $M > 0$ is a constant satisfying $\sup\{\|((\delta\vartheta - A)/\eta)x_n\|\} \le M$. This implies that

$$\limsup_{n\to\infty}(\|u_{n+1} - u_n\| - \|x_{n+1} - x_n\|) \le 0. \tag{3.8}$$

By Lemma 2.3, we deduce

$$\lim_{n\to\infty}\|u_n - x_n\| = 0. \tag{3.9}$$

Therefore,

$$\lim_{n\to\infty}\|x_{n+1} - x_n\| = \lim_{n\to\infty}\eta\|u_n - x_n\| = 0. \tag{3.10}$$

Note that $\mathrm{Fix}(T) = \mathrm{Fix}(TQ_C)$. As a matter of fact, if $p \in \mathrm{Fix}(T)$, that is $p = Tp$, then $p \in \mathrm{Fix}(TQ_C)$. Since T is a self-mapping, it is clear that $p \in C$. So, $Q_Cp = p$. Therefore, $TQ_Cp = Tp = p$. Conversely, if $q \in \mathrm{Fix}(TQ_C)$, that is $q = TQ_Cq$, we also have $q \in C$. Thus, $q = Tq$. Set $S = (1-k)I + kT$. We observe that $\mathrm{Fix}(S) = \mathrm{Fix}(T) = \mathrm{Fix}(TQ_C) = \mathrm{Fix}(SQ_C)$. Next, we estimate $\|x_n - SQ_Cx_n\|$.

Since

$$\begin{aligned}\|x_n - SQ_Cx_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - SQ_Cx_n\| \\ &\le \|x_n - x_{n+1}\| + (1-\eta)\|x_n - SQ_Cx_n\| + \eta\|u_n - SQ_Cx_n\| \\ &\le \|x_n - x_{n+1}\| + (1-\eta)\|x_n - SQ_Cx_n\| + \alpha_n\|\delta\vartheta(x_n) - Ax_n\|,\end{aligned} \tag{3.11}$$

we have

$$\|x_n - SQ_Cx_n\| \le \frac{1}{\eta}\|x_{n+1} - x_n\| + \frac{\alpha_n}{\eta}\|\delta\vartheta(x_n) - Ax_n\| \longrightarrow 0. \tag{3.12}$$

Next, we show

$$\limsup_{n\to\infty}\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \le 0, \tag{3.13}$$

where $x^{\dagger}\in VI(\mathrm{Fix}(T),A)$.

First, we have

$$\limsup_{n\to\infty}\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle = \lim_{i\to\infty}\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n_i+1}-x^{\dagger}\right)\right\rangle. \tag{3.14}$$

Since the sequence $\{x_n\}$ is bounded, hence $\{x_{n_i}\}$ is bounded. Thus, we can take a subsequence $\{x_{n_{i_j}}\}$ of $\{x_n\}$ such that $x_{n_{i_j}}\to\tilde{x}$ weakly. Without loss of generality, we may assume that $x_{n_i}\to\tilde{x}$ weakly. Note that SQ_C is nonexpansive and $\|x_{n_i}-SQ_Cx_{n_i}\|\to 0$. By using the demiclosed principle of nonexpansive mappings (see Lemma 2.4), we get $\tilde{x}\in\mathrm{Fix}(SQ_C)=\mathrm{Fix}(T)$. At the same time, j is weakly sequentially continuous. Therefore,

$$\begin{aligned}\limsup_{n\to\infty}\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle &= \lim_{i\to\infty}\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n_i+1}-x^{\dagger}\right)\right\rangle \\ &= \left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(\tilde{x}-x^{\dagger}\right)\right\rangle \le 0.\end{aligned} \tag{3.15}$$

Finally we show that $x_n\to x^{\dagger}$. From (3.2), we have

$$\begin{aligned}\left\|x_{n+1}-x^{\dagger}\right\|^2 &= \Big\|\alpha_n\left(\delta\vartheta(x_n)-Ax^{\dagger}\right)+((1-\eta)I-\alpha_nA)\left(x_n-x^{\dagger}\right) \\ &\quad +\eta\left((1-k)Q_Cx_n+kTQ_Cx_n-x^{\dagger}\right)\Big\|^2 \\ &\le \left\|((1-\eta)I-\alpha_nA)\left(x_n-x^{\dagger}\right)+\eta\left((1-k)Q_Cx_n+kTQ_Cx_n-x^{\dagger}\right)\right\|^2 \\ &\quad +2\alpha_n\left\langle \delta\vartheta(x_n)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \\ &\le \left(\left\|((1-\eta)I-\alpha_nA)\left(x_n-x^{\dagger}\right)\right\|^2+\left\|\eta\left((1-k)Q_Cx_n+kTQ_Cx_n-x^{\dagger}\right)\right\|\right)^2 \\ &\quad +2\alpha_n\delta\left\langle \vartheta(x_n)-\vartheta\left(x^{\dagger}\right), j\left(x_{n+1}-x^{\dagger}\right)\right\rangle+2\alpha_n\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \\ &\le \left[(1-\eta-\alpha_n\gamma)\left\|x_n-x^{\dagger}\right\|+\eta\left\|x_n-x^{\dagger}\right\|\right]^2+2\alpha_n\delta\rho\left\|x_n-x^{\dagger}\right\|\left\|x_{n+1}-x^{\dagger}\right\| \\ &\quad +2\alpha_n\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \\ &\le (1-\alpha_n\gamma)^2\left\|x_n-x^{\dagger}\right\|^2+\alpha_n\delta\rho\left(\left\|x_n-x^{\dagger}\right\|^2+\left\|x_{n+1}-x^{\dagger}\right\|^2\right) \\ &\quad +2\alpha_n\left\langle \delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger}, j\left(x_{n+1}-x^{\dagger}\right)\right\rangle.\end{aligned} \tag{3.16}$$

It follows that

$$\begin{aligned}
\left\|x_{n+1}-x^{\dagger}\right\|^2 &\le \frac{1-2\alpha_n\gamma+\alpha_n^2\gamma^2+\alpha_n\delta\rho}{1-\alpha_n\delta\rho}\left\|x_n-x^{\dagger}\right\|^2 \\
&\quad+\frac{2\alpha_n}{1-\alpha_n\delta\rho}\left\langle\delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger},j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \\
&=\left[1-\frac{2(\gamma-\delta\rho)\alpha_n}{1-\alpha_n\delta\rho}\right]\left\|x_n-x^{\dagger}\right\|^2+\frac{\alpha_n^2\gamma^2}{1-\alpha_n\delta\rho}\left\|x_n-x^{\dagger}\right\|^2 \\
&\quad+\frac{2\alpha_n}{1-\alpha_n\delta\rho}\left\langle\delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger},j\left(x_{n+1}-x^{\dagger}\right)\right\rangle \\
&=\left[1-\frac{2(\gamma-\delta\rho)\alpha_n}{1-\alpha_n\delta\rho}\right]\left\|x_n-x^{\dagger}\right\|^2+\frac{2(\gamma-\delta\rho)\alpha_n}{1-\alpha_n\delta\rho} \\
&\quad\times\left\{\frac{\alpha_n\gamma^2\|x_n-x^{\dagger}\|^2}{2(\gamma-\delta\rho)}+\frac{1}{\gamma-\delta\rho}\left\langle\delta\vartheta\left(x^{\dagger}\right)-Ax^{\dagger},j\left(x_{n+1}-x^{\dagger}\right)\right\rangle\right\}.
\end{aligned}\tag{3.17}$$

It can be checked easily that $\sum_n(2(\gamma-\delta\rho)\alpha_n/(1-\alpha_n\delta\rho))=\infty$ and $\limsup_{n\to\infty}(\alpha_n\gamma^2\|x_n-x^{\dagger}\|^2/2(\gamma-\delta\rho))+(1/(\gamma-\delta\rho))\langle\delta\vartheta(x^{\dagger})-Ax^{\dagger},j(x_{n+1}-x^{\dagger})\rangle\le 0$. From Lemma 2.5, we deduce $x_n\to x^{\dagger}$. This completes the proof. □

Algorithm 3.3. For the initial point $x_0\in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1}=\alpha_n\delta\vartheta(x_n)+(1-\eta-\alpha_n)x_n+\eta[(1-k)Q_Cx_n+kTQ_Cx_n],\quad n\ge 0,\tag{3.18}$$

where $\{\alpha_n\}$ is a sequence in $(0,1)$ and $\delta>0,\ 0<\eta<1,\ \delta\rho<1,\ 0<k<\lambda/K^2$.

Corollary 3.4. *If the sequence $\{\alpha_n\}$ satisfies $\lim_{n\to\infty}a_n=0$ and $\sum_{n=0}^{\infty}\alpha_n=\infty$, then the sequence $\{x_n\}$ generated by* (3.18) *converges strongly to the unique solution $x^{\dagger}$ of VI: finding $x^{\dagger}\in \mathrm{Fix}(T)$ such that*

$$\left\langle(I-\delta\vartheta)x^{\dagger},j\left(\tilde{x}-x^{\dagger}\right)\right\rangle\ge 0,\quad \tilde{x}\in\mathrm{Fix}(T).\tag{3.19}$$

Algorithm 3.5. For the initial point $x_0\in C$ and $u\in E$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1}=\alpha_nu+(1-\eta-\alpha_n)x_n+\eta[(1-k)Q_Cx_n+kTQ_Cx_n],\quad n\ge 0,\tag{3.20}$$

where $\{\alpha_n\}$ is a sequence in $(0,1)$ and $0<\eta<1,\ 0<k<\lambda/K^2$.

Corollary 3.6. *If the sequence* $\{\alpha_n\}$ *satisfies* $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=0}^{\infty}\alpha_n = \infty$, *then the sequence* $\{x_n\}$ *generated by* (3.20) *converges strongly to the unique solution* $x^\dagger$ *of VI: finding* $x^\dagger \in \mathrm{Fix}(T)$ *such that*

$$\left\langle (I-u)x^\dagger, j\left(\tilde{x}-x^\dagger\right) \right\rangle \geq 0, \quad \tilde{x} \in \mathrm{Fix}(T). \tag{3.21}$$

Algorithm 3.7. For the initial point $x_0 \in C$, we generate a sequence $\{x_n\}$ via the following manner:

$$x_{n+1} = (1-\eta-\alpha_n)x_n + \eta[(1-k)Q_C x_n + kTQ_C x_n], \quad n \geq 0, \tag{3.22}$$

where $\{\alpha_n\}$ is a sequence in $(0,1)$ and $0<\eta<1,\ 0<k<\lambda/K^2$.

Corollary 3.8. *If the sequence* $\{\alpha_n\}$ *satisfies* $\lim_{n\to\infty}\alpha_n = 0$ *and* $\sum_{n=0}^{\infty}\alpha_n = \infty$, *then the sequence* $\{x_n\}$ *generated by* (3.22) *converges strongly to* $Q_{\mathrm{Fix}(T)}(0)$.

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