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Research Article

Finite-Time Robust Stabilization for Stochastic Neural Networks

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This paper is concerned with the finite-time stabilization for a class of stochastic neural networks (SNNs) with noise perturbations. The purpose of the addressed problem is to design a nonlinear stabilizator which can stabilize the states of neural networks in finite time. Compared with the previous references, a continuous stabilizator is designed to realize such stabilization objective. Based on the recent finite-time stability theorem of stochastic nonlinear systems, sufficient conditions are established for ensuring the finite-time stability of the dynamics of SNNs in probability. Then, the gain parameters of the finite-time controller could be obtained by solving a linear matrix inequality and the robust finite-time stabilization could also be guaranteed for SNNs with uncertain parameters. Finally, two numerical examples are given to illustrate the effectiveness of the proposed design method.

1. Introduction

Since the first paper of Ott et al. [1], a large number of monographs and papers studying the stabilization of the nonlinear systems without or with delays have been published [2–5]. These publications have developed many control techniques including continuous feedback and discontinuous feedback. Take [4] for example, the authors studied the pinning stabilization problem of linearly coupled stochastic neural networks, where a minimum number of controllers are used to force the NNs to the desired equilibrium point by fully utilizing the structure of the network.

On the other hand, the well-known Hopfield neural networks, Cohen-Grossberg neural networks and cellular neural networks [6–18], and so forth have been extensively studied in the past decades and successfully applied in many areas such as signal processing, combinatorial optimization, and pattern recognition. Specially, the stability of Hopfield neural networks has received much research attention since, when applied, the neural network is sometimes assumed to have only one globally stable equilibrium [7–9, 19, 20].

Until now, the stability analysis issues for many kinds of neural networks in the presence of stochastic perturbations and/or parameter uncertainties have attracted a lot of research attention. The reasons include twofold: (a) in real nervous systems, because of random fluctuations from the release of neurotransmitters, and other probabilistic causes, the synaptic transmission is indeed a noisy process; (b) the connection weights of the neurons depend on certain resistance and capacitance values that always exist uncertainties. Therefore, the robust stability has been studied for neural networks with parameter uncertainties [21–24] or external stochastic perturbations [7, 19, 25, 26]. However, to the best of the authors' knowledge, most literature regarding the stability of neural networks is based on the convergence time being large enough, even though we eagerly want the argued network states to become stable as quickly as possible in practical applications. In order to achieve faster stabilization speed and hope to complete stabilization in finite time rather than merely asymptotically [27], an effective method is using finite-time stabilization techniques, which have also demonstrated better robustness and disturbance rejection properties [28].

In this paper, we will focus on the finite-time robust stabilization for neural networks with both stochastic perturbations and parameter uncertainties. The difference of this paper lies in three aspects. First, based on the finite-time stability theorem of stochastic nonlinear systems [29], a new continuous finite-time stabilizator is proposed for a stochastic neural network (SNN). Moreover, in contrast to [30–33], we prove finite-time stabilization by constructing a suitable Lyapunov function and obtain some criteria which are easy to be satisfied. Second, the gain parameters in finite-time stabilizator are designed by solving a linear matrix inequality. Finally, a robust finite-time stabilizator for SNNs with parameter uncertainties is designed as well. Moreover, two illustrative examples are provided to show the effectiveness of the proposed designing.

The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, meaning that $X - Y$ is positive semidefinite (resp., positive definite). $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximal and minimal eigenvalues of real matrix M. Let $(\mathbf{\Omega}, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and contains all $\mathbf{P}$-null sets). $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x with respect to the given probability measure $\mathbf{P}$. I and 0 represent the identity matrix and a zero matrix, respectively; $\operatorname{diag}(\cdots)$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Model Formulation and Preliminaries

Some preliminary knowledge is presented in this section for the derivation of our main results. The deterministic NN can be described by the following differential equation:

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + J \tag{2.1}$$

or

$$\dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) + J_i, \quad i = 1, 2, \ldots, n, \tag{2.2}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the vector of neuron states; n represents the number of neurons in the network; $A = \operatorname{diag}(a_1, a_2, \ldots, a_n)$ is an $n \times n$ constant diagonal matrix with $a_i > 0$, $i = 1, 2, \ldots, n$; $B = (b_{ij})_{n \times n}$ is an $n \times n$ interconnection matrix; $f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a diagonal mapping, where f_i, $i = 1, 2, \ldots, n$ represents the neuron input-output activation and $J = (J_1, J_2, \ldots, J_n)^T$ is a constant external input vector.

To establish our main results, it is necessary to give the following assumption for system (2.1) or (2.2).

Assumption 2.1. The neuron activation function f of the NN (2.1) satisfies the following Lipschitz condition:

$$\left\| f_i(x) - f_i(y) \right\| \leq M_i \|x - y\|, \quad \forall x, y \in \mathbb{R},\ i = 1, 2, \ldots, n, \tag{2.3}$$

where M_i is a positive constant for $i = 1, 2, \ldots, n$. For convenience, let $M = \operatorname{diag}\{M_1, M_2, \ldots, M_n\}$.

Because of the existence of environmental noise in real neural networks, the stochastic disturbances should be taken into account in the recurrent NN. For this purpose, we modify the system (2.1) as the following SNN:

$$dx(t) = \left[-Ax(t) + Bf(x(t)) + J\right]dt + h(t, x(t))d\omega(t), \tag{2.4}$$

where $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T \in \mathbb{R}^n$ is an n-dimensional Brownian motion defined on the probability space $(\boldsymbol{\Omega}, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P})$ satisfying the usual conditions (i.e., the filtration contains all $\mathbf{P}$-null sets and is right continuous). The white noise $d\omega_i(t)$ is independent of $d\omega_j(t)$ for $i \neq j$. The intensity function h is the noise intensity function matrix satisfying the following condition:

$$\operatorname{trace}\left[h^T(t, x(t)) \cdot h(t, x(t))\right] \leq \|M_h x(t)\|^2, \tag{2.5}$$

where M_h is a known constant matrix with compatible dimensions.

In this paper, we want to control the SNN (2.4) to the desired state x^*, which is an equilibrium point of NN (2.1). Based on the discussions in many other papers, the stochastic perturbation will vanish at this equilibrium point x^*, that is, $h(t, x^*) = 0$. Without loss of generality, one can shift the equilibrium point x^* to the origin by using the translation $y(t) = x(t) - x^*$, which derives the following stochastic dynamical system:

$$dy(t) = \left[-Ay(t) + Bg\left(y(t)\right)\right]dt + h\left(t, y(t)\right)d\omega(t), \tag{2.6}$$

where $g(y(t)) = f(x(t) + x^*) - f(x(t))$.

Consider the SNN (2.6) with parameter uncertainties: the parameter matrices A and B are unknown but bounded, which are assumed to satisfy

$$A \in A_I, \qquad B \in B_I, \tag{2.7}$$

where $A_I = \{A \mid 0 < \underline{a}_i \leq a_i \leq \overline{a}_i\}$, $B_I = \{B \mid \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij}\}$, and $i, j = 1, 2, \ldots, n$.

We denote that $\underline{A} = \operatorname{diag}(\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n)$, $\overline{A} = \operatorname{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$, $\underline{B} = (\underline{b}_{ij})_{n\times n}$, $\overline{B} = (\overline{b}_{ij})_{n\times n}$, $A_0 = (1/2)(\overline{A} + \underline{A})$, $B_0 = (1/2)(\overline{B} + \underline{B})$, $A_1 = (1/2)(\overline{A} - \underline{A}) := \operatorname{diag}(\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n)$, $B_1 = (1/2)(\overline{B} - \underline{B}) := (\widetilde{b}_{ij})_{n\times n}$, $E_A = \operatorname{diag}(\sqrt{\widetilde{a}_1}, \sqrt{\widetilde{a}_2}, \ldots, \sqrt{\widetilde{a}_n})$, $E_B = [\sqrt{\widetilde{b}_{11}}e_1, \ldots, \sqrt{\widetilde{b}_{1n}}e_1, \ldots, \sqrt{\widetilde{b}_{n1}}e_n, \ldots, \sqrt{\widetilde{b}_{nn}}e_n]_{n\times n^2}$, and $F_B = [\sqrt{\widetilde{b}_{11}}e_1, \ldots, \sqrt{\widetilde{b}_{1n}}e_n, \ldots, \sqrt{\widetilde{b}_{n1}}e_1, \ldots, \sqrt{\widetilde{b}_{nn}}e_n]^T_{n^2\times n}$, where $e_i \in \mathbb{R}^n$ is the column vector with the ith element 1 and 0 elsewhere.

For $i, j = 1, 2, \ldots, n$, let

$$\begin{gathered} \mathbf{\Delta} = \{\Delta \in \mathbb{R}^{n\times n} \mid \Delta = \operatorname{diag}(\delta_1, \delta_2, \ldots, \delta_n),\ |\delta_i| \leq 1\}, \\ \mathbf{\Omega} = \left\{\Omega \in \mathbb{R}^{n^2\times n^2} \mid \Omega = \operatorname{diag}(\omega_{11}, \ldots, \omega_{1n}, \ldots, \omega_{n1}, \ldots, \omega_{nn}),\ |\omega_{ij}| \leq 1\right\}. \end{gathered} \tag{2.8}$$

Then, through simple manipulations, one has

$$A_I = \{A = A_0 + E_A \Delta E_A \mid \Delta \in \mathbf{\Delta}\}, \qquad B_I = \{B = B_0 + E_B \Omega F_B \mid \Omega \in \mathbf{\Omega}\}. \tag{2.9}$$

In order to stabilize the SNN (2.4) to the equilibrium point x^*, equivalently, one can stabilize the SNN (2.6) to the origin due to the transformation. Hence, in the remainder of this paper, a controller $u(t)$ will be designed for the stabilization of SNN (2.6) in mean square. The controlled SNN can be described by the following stochastic differential equation (SDE):

$$dy(t) = \left[-Ay(t) + Bg(y(t)) + u(t)\right]dt + h(t, y(t))d\omega(t). \tag{2.10}$$

Similar to [30–33], the controller is designed as follows:

$$u(t) = -k_1 y(t) - k_2 \operatorname{sign}(y(t))\left|y(t)\right|^{\alpha}, \tag{2.11}$$

where $|y(t)|^{\alpha} = (|y_1(t)|^{\alpha}, |y_2(t)|^{\alpha}, \ldots, |y_n(t)|^{\alpha})^T$, $\operatorname{sign}(y(t)) = \operatorname{diag}(\operatorname{sign}(y_1(t)), \operatorname{sign}(y_2(t)), \ldots, \operatorname{sign}(y_n(t)))$, constants k_1, k_2 are gain coefficients to be determined, and the real number α satisfies $0 < \alpha < 1$. In fact, here the continuous function $u(t)$ in the SNN (2.10) is the key point for ensuring the finite-time stabilization.

Obviously, when $0 < \alpha < 1$, the controller $u(t)$ is a continuous function with respect to y, which leads to the continuity of controlled system (2.10) with respect to the state $y(t)$ [30–33]. If $\alpha = 0$, $u(t)$ turns to be a discontinuous one, which has been considered in [34–36]. If $\alpha = 1$ in the controller (2.11), then it becomes the typical stabilization issues which only can realize an asymptotical stabilization in infinite time [3–5].

Similar to the definition of finite-time stability in probability [29], the finite-time stabilization in probability is given through the following definition.

Definition 2.2. The system (2.6) is said to be finite-time stabilized at the original point by the controller (2.11) in probability, that is, the controlled SNN (2.10) is finite-time stable in probability [37] if, for any initial state $x(0)$, there exists a finite-time function T_0 such that

$$\mathbf{P}\{\|y(t)\| = 0\} = 1, \quad \forall t \geq T_0, \tag{2.12}$$

where $T_0 = T_0(y(0), \omega) = \inf\{T \geq 0 : y(t) = 0, \forall t \geq T\}$ is called the stochastic setting time function satisfying $\mathbb{E}[T_0] < \infty$.

The following lemmas are needed for the derivation of our main results in this paper.

Lemma 2.3 (see [38]. (Itô's formula)). *Let $x(t)$ ba an n-dimensional Itô's process on $t \geq 0$ with the stochastic differential*

$$dx(t) = f(t)dt + g(t)d\omega(t). \tag{2.13}$$

Let $V(x(t), t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+; \mathbb{R}^+)$. Then, $V(x(t), t)$ is a real-valued Itô's process with its stochastic differential given by

$$\begin{gathered} dV(x(t), t) = \mathcal{L}V(x(t), t)dt + V_x(x(t), t)g(t)d\omega(t), \\ \mathcal{L}V(x(t), t) = V_t(x(t), t) + V_x(x(t), t)f(t) + \frac{1}{2}\operatorname{trace}\left(g^T(t)V_{xx}(x(t), t)g(t)\right), \end{gathered} \tag{2.14}$$

where $\mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+)$ denotes the family of all real-valued functions $V(x(t), t)$ such that they are continuously twice differentiable in x and t.

Lemma 2.4 (see [29]). *Consider the stochastic differential equation* (2.13) *with $f(0) = 0$ and $g(0) = 0$ and assume system* (2.13) *has a unique global solution. If there exist real numbers $\eta > 0$ and $0 < \alpha < 1$, such that for the function $V(x)$ in Lemma 2.3,*

$$\mathcal{L}V(x) \leq -\eta(V(x))^{\alpha}, \tag{2.15}$$

then the origin of system (2.13) *is globally stochastically finite-time stable, and $\mathbb{E}[T_0] < (V(x_0))^{1-\alpha}/\eta(1-\alpha)$.*

Lemma 2.5 (see [39]). *If $a_1, a_2, \ldots, a_n$ are positive number and $0 < r < p$, then*

$$\left(\sum_{i=1}^{n} a_i^p\right)^{1/p} \leq \left(\sum_{i=1}^{n} a_i^r\right)^{1/r}. \tag{2.16}$$

Lemma 2.6 (Boyd et al. [40]). *If $\mathcal{U}$, $\mathcal{V}(t)$, and $\mathcal{W}$ are real matrices of appropriate dimension with $\mathcal{N}$ satisfying $\mathcal{N} = \mathcal{N}^T$, then*

$$\mathcal{N} + \mathcal{U}\mathcal{V}(t)\mathcal{W} + \mathcal{W}^T\mathcal{V}^T(t)\mathcal{U}^T < 0 \tag{2.17}$$

for all $\mathcal{V}^T(t)\mathcal{V}(t) \leq I$, *if and only if there exists a positive constant* λ, *such that*

$$\mathcal{N} + \lambda^{-1}\mathcal{U}\mathcal{U}^T + \lambda\mathcal{W}^T\mathcal{W} < 0. \tag{2.18}$$

3. Main Results

In this section, we first give some theorems in detail to guarantee that the original point of SNN (2.6) is stabilized in finite time, that is, the controlled system (2.10) with (2.11) is finite-time stable in probability. Then, for SNN (2.6) with parameter uncertainties, we provide a sufficient condition under which the controlled system (2.10) is robust finite-time stable in probability. Finally, the control gains k_1 and k_2 are designed by solving some linear matrix inequalities.

Theorem 3.1. *The controlled system* (2.10) *with* (2.11) *is finite-time stable in probability, if there exist a constant* ε *and a positive-definite matrix* $P \in \mathbb{R}^{n\times n}$ *such that*

$$-2PA - 2k_1P + \varepsilon^{-1}PBB^TP + \varepsilon M^TM + \lambda_{\max}(P)M_h^TM_h < 0. \tag{3.1}$$

Moreover, the upper bound of the stochastic settling time for stabilization can be in terms of the initial errors as $(\lambda_{\max}(P)/\lambda_{\min}(P)) \cdot (\|y(0)\|^{1-\alpha}/k_2(1-\alpha))$.

Proof. Consider the controlled system (2.10) with the controller (2.11), we have

$$dy(t) = \left[-(A + k_1I)y(t) + Bg(y(t)) - k_2\,\mathrm{sign}(y(t))|y(t)|^{\alpha}\right]dt + h(t, y(t))d\omega(t). \tag{3.2}$$

Next, we will prove system (3.2) is finite-time stable in probability based on Definition 2.2. To this end, choose the candidate Lyapunov function $V(y(t)) = y^T(t)Py(t)$ and calculate the time derivative of $V(y(t))$ along the trajectories of the augmented system (3.2). By the Itô's formula, we obtain the stochastic differential as

$$dV(y(t)) = \mathcal{L}V(y(t))dt + 2y^T(t)Ph(t, y(t))d\omega(t), \tag{3.3}$$

where

$$\begin{aligned}
\mathcal{L}V(y(t)) &= 2y^T(t)P\left[(-A - k_1I)y(t) + Bg(t) - k_2\,\mathrm{sign}(y(t))|y(t)|^{\alpha}\right] + \mathrm{trace}\left[h^T(t)Ph(t)\right] \\
&= 2y^T(t)P(-A - k_1I)y(t) + 2y^T(t)PBg(t) + \mathrm{trace}\left[h^T(t)Ph(t)\right] \\
&\quad - 2k_2y^T(t)P\,\mathrm{sign}(y(t))|y(t)|^{\alpha}.
\end{aligned} \tag{3.4}$$

From condition (2.3), using the inequality $x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y$, where $\varepsilon > 0$ is an arbitrary constant, we have

$$\begin{aligned} 2y^T(t)PBg(t) &\le \varepsilon^{-1} y^T(t)PBB^T Py(t) + \varepsilon g^T(t)g(t) \\ &\le \varepsilon^{-1} y^T(t)PBB^T Py(t) + \varepsilon y^T(t)M^T My(t). \end{aligned} \tag{3.5}$$

Combining (2.5), (3.4)-(3.5) results in

$$\begin{aligned} \mathcal{L}V(y(t)) \le{}& y^T(t)\left[-PA - A^T P - 2k_1 P + \varepsilon^{-1} PBB^T P + \varepsilon M^T M + \lambda_{\max}(P) M_h^T M_h\right] \\ &\times y(t) - 2k_2 \lambda_{\min}(P) \sum_{i=1}^{n} \left|y_i(t)\right|^{\alpha+1}. \end{aligned} \tag{3.6}$$

From $0 < \alpha < 1$ and Lemma 2.5, we get

$$\left(\sum_{i=1}^{n} \left|y_i(t)\right|^{\alpha+1}\right)^{1/(\alpha+1)} \ge \left(\sum_{i=1}^{n} \left|y_i(t)\right|^{2}\right)^{1/2}, \tag{3.7}$$

then,

$$\sum_{i=1}^{n} \left|y_i(t)\right|^{\alpha+1} \ge \left(\sum_{i=1}^{n} \left|y_i(t)\right|^{2}\right)^{(\alpha+1)/2} = \left[y^T(t)y(t)\right]^{(\alpha+1)/2}. \tag{3.8}$$

Thus, based on condition (3.1), taking the expectations on both sides of (3.3), we have

$$\begin{aligned} \mathbb{E}\{dV(y(t))\} &\le -2k_2 \lambda_{\min}(P) \mathbb{E}\left\{ \left[y^T(t)y(t)\right]^{(\alpha+1)/2} \right\} \\ &\le -2k_2 \cdot \lambda_{\min}(P) [\lambda_{\max}(P)]^{-(\alpha+1)/2} \mathbb{E}\left\{ V(y(t))^{(\alpha+1)/2} \right\}, \\ \text{and} \quad \mathbb{E}\left\{ V^{(\alpha+1)/2}(y(0)) \right\} &= \left(\mathbb{E}\{V(y(0))\}\right)^{(\alpha+1)/2}. \end{aligned} \tag{3.9}$$

By Lemma 2.4, $V(y(t))$ stochastically converges to zero in a finite time, that is, the controlled system (3.2) is finite-time stable in probability, and the settle time is upper bounded by

$$\begin{aligned} T_P &= \frac{[\lambda_{\max}(P)]^{(\alpha+1)/2} \cdot [V(y(0))]^{(1-\alpha)/2}}{2k_2 \cdot \lambda_{\min}(P) \cdot ((1-\alpha)/2)} \\ &\le \frac{[\lambda_{\max}(P)]^{(\alpha+1)/2} [\lambda_{\max}(P)]^{(1-\alpha)/2} \|y(0)\|_2^{1-\alpha}}{\lambda_{\min}(P) \cdot k_2(1-\alpha)} \\ &= \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \cdot \frac{\|y(0)\|^{1-\alpha}}{k_2(1-\alpha)}. \end{aligned} \tag{3.10}$$

This completes the proof. □

Remark 3.2. The two gain parameters k_1 and k_2 in the controller $u(t)$ play different roles in ensuring the finite-time stability of the controlled system (3.2). We can see from Theorem 3.1 that, whether or not the controlled system (3.2) could realize the finite-time stability mainly depends on the value of k_1 and satisfies condition (3.1) but nothing on k_2. However, the size of the settle time depends on the value of k_2 but unrelated to k_1, the only requirement for the gain k_1 is satisfying condition (3.1).

Remark 3.3. In [31, 32, 35, 41], the candidate Lyapunov function $V(t)$ was chosen as a simple form of $V(t) = y^T(t)y(t)$ and then the upper bound of settle time turns to be $\|y(0)\|^{1-\alpha}/k_2(1-\alpha)$. In this paper, in order ro reduce some conservation of conditions in Theorem 3.1, a positive definite matrix parameter P is introduced such that condition (3.1) is easier to be satisfied. And the previous conclusions could be included by our results if the matrix $P = pI$ is taken, where p is a arbitrary constant, just as shown in the next corollary.

Corollary 3.4. *The controlled system* (3.2) *is finite-time stable in probability, if there exist two constants ε and p such that*

$$-2pA - 2k_1 pI + \varepsilon^{-1}p^2BB^T + \varepsilon M^T M + pM_h^T M_h < 0. \tag{3.11}$$

Moreover, the upper bound of the settle time is

$$\mathbf{T} = \frac{\|y(0)\|^{1-\alpha}}{k_2(1-\alpha)}. \tag{3.12}$$

Our next goal is to deal with the design problem, that is, giving a practical design procedure for the controller gains: k_1 and k_2, such that the inequalities in Theorem 3.1 or Corollary 3.4 are satisfied. Obviously, those inequalities are difficult to solve, since they are nonlinear and coupled. A meaningful approach to tackling such a problem is to convert the nonlinearly coupled matrix inequalities into linear matrix inequalities (LMIs), while the controller gains are designed simultaneously.

Based on the discussion in Remark 3.2, the parameter gain k_2 is one of the primary factors that affect the size of the settle time, which is unrelated to condition (3.11). Hence, in the following discussion, we will fix the gain parameter k_2 and mainly focus on the design of control gain k_1. We claim that the desired controller gain k_1 can be designed if a linear matrix inequality is feasible.

Theorem 3.5. *For a fixed control gain k_2, the finite-time stabilization problem is solvable for the SNN* (2.6)*, if there exist three positive scalars p, K, and ε such that*

$$\begin{pmatrix} -2pA - 2KI + pM_h^T M_h & pB & \varepsilon M^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{pmatrix} < 0. \tag{3.13}$$

Moreover, the control gain coefficient $k_1 = p^{-1}K$.

Proof. The result can be proved by pre- and post-multiplying the inequality (3.13) by the block-diagonal matrix $\operatorname{diag}\{I, \varepsilon^{-1/2}I, \varepsilon^{-1/2}I\}$ and then following from the famous Schur complement lemma and Corollary 3.4 and we omit it here.

Just as mentioned in Introduction, when modelling a dynamic system, one can hardly obtain an exact model. Specially, in practical implementation of neural networks, the firing rates and the weight coefficients of the neurons depend on certain resistance and capacitance values, which are subject to uncertainties. It is thus necessary to take parameter uncertainties into account in the considered neural network. In the following, we consider the robust finite-time stabilization issue for SNN (2.6) under the parametric uncertainties (2.7). □

Theorem 3.6. *The interval SNN* (3.2) *with uncertain parameters* (2.7) *is robust finite-time stable in probability, if there exist three constants ε, λ_1, λ_2 and a positive-definite matrix $P \in \mathbb{R}^{n\times n}$ such that*

$$\begin{pmatrix} \Phi & PB_0 & \varepsilon M^T & PE_A & PE_B \\ \star & -\varepsilon I + \lambda_2 F_B^T F_B & 0 & 0 & 0 \\ \star & \star & -\varepsilon I & 0 & 0 \\ \star & \star & \star & -\lambda_1 I & 0 \\ \star & \star & \star & \star & -\lambda_2 \mathbf{I} \end{pmatrix} < 0, \tag{3.14}$$

where $\Phi = -2PA_0 - 2k_1 I + PM_h^T M_h + \lambda_1 E_A^T E_A$ *and* $\mathbf{I} = \operatorname{diag}(I, I)$.

Proof. From Theorems 3.1 and 3.5, we know that the SNN (3.2) is finite-time stable in probability, if there exist a constant ε and a positive-definite matrix $P \in \mathbb{R}^{n\times n}$ such that the following LMI holds:

$$\begin{pmatrix} -2PA - 2KI + PM_h^T M_h & PB & \varepsilon M^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{pmatrix} < 0. \tag{3.15}$$

Thus, for the uncertain parameters satisfying (2.7), we have

$$\begin{aligned} \Psi &= \begin{pmatrix} -2P(A_0 + E_A \Delta E_A) - 2KI + PM_h^T M_h & P(B_0 + E_B \Omega F_B) & \varepsilon M^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{pmatrix} \\ &= \begin{pmatrix} -2PA_0 - 2KI + PM_h^T M_h & PB_0 & \varepsilon M^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{pmatrix} + \begin{pmatrix} -2PE_A \Delta E_A & PE_B \Omega F_B & 0 \\ \star & 0 & 0 \\ \star & \star & 0 \end{pmatrix} < 0. \end{aligned} \tag{3.16}$$

For the second term in the above equality, it is easy to have

$$\begin{pmatrix} -2PE_A\Delta E_A & PE_B\Omega F_B & 0 \\ \star & 0 & 0 \\ \star & \star & 0 \end{pmatrix} = \begin{pmatrix} PE_A \\ 0 \\ 0 \end{pmatrix}\Delta\begin{pmatrix} E_A & 0 & 0 \end{pmatrix} + \begin{pmatrix} E_A \\ 0 \\ 0 \end{pmatrix}\Delta\begin{pmatrix} E_AP & 0 & 0 \end{pmatrix}$$
$$+ \begin{pmatrix} PE_B \\ 0 \\ 0 \end{pmatrix}\Omega\begin{pmatrix} 0 & F_B & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_B^T \\ 0 \end{pmatrix}\Omega\begin{pmatrix} E_B^TP & 0 & 0 \end{pmatrix}. \tag{3.17}$$

Then, based on Lemma 2.6, (3.16) and (3.17), there exist two constants λ_1 and λ_2 such that

$$\Psi = \begin{pmatrix} -2PA_0 - 2KI + PM_h^TM_h & PB_0 & \varepsilon M^T \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{pmatrix}$$
$$+ \begin{pmatrix} \lambda_1^{-1}PE_AE_AP + \lambda_1E_AE_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \lambda_2^{-1}PE_BE_B^TP & 0 & 0 \\ 0 & \lambda_2F_B^TF_B & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.18}$$
$$< 0.$$

Then the result can be proved by the famous Schur complement lemma and condition (3.14). □

Corollary 3.7. *For a fixed control gain k_2, the finite-time robust stabilization problem is solvable for the SNN* (2.6) *with* (2.7)*, if there exist five positive scalars p, K, ε, λ_1, and λ_2 such that*

$$\begin{pmatrix} \overline{\Phi} & pB_0 & \varepsilon M^T & pE_A & pE_B \\ \star & -\varepsilon I + \lambda_2F_B^TF_B & 0 & 0 & 0 \\ \star & \star & -\varepsilon I & 0 & 0 \\ \star & \star & \star & -\lambda_1 I & 0 \\ \star & \star & \star & \star & -\lambda_2\mathbf{I} \end{pmatrix} < 0, \tag{3.19}$$

where $\overline{\Phi} = -2pA_0 - 2KI + pM_h^TM_h + \lambda_1E_A^TE_A$. Moreover, the control gain coefficient $k_1 = p^{-1}K$.

Proof. Let $P = pI$ and we can prove the result based on Theorem 3.6. □

4. Two Numerical Examples

Example 4.1. Consider the following stochastic neural network:

$$dx(t) = \left[-Ax(t) + Bf(x(t)) + J\right]dt + h(t, x(t))d\omega(t), \tag{4.1}$$

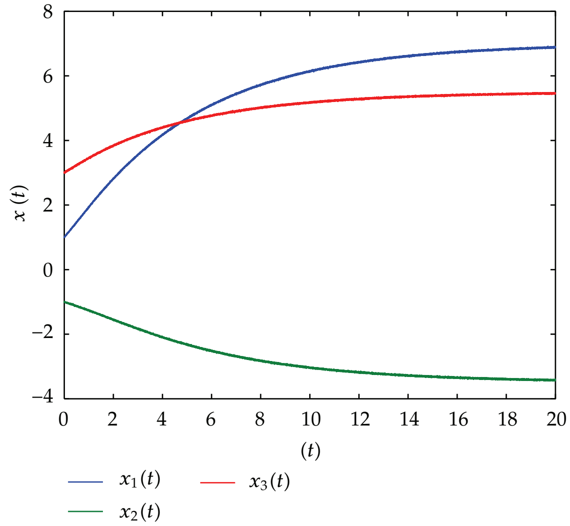

Figure 1: Trajectories of SNN (4.1) without any controller in Example 4.1.

where

$$A = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -0.2 & 0.2 \\ 0.1 & 1 & 0.2 \\ 0.3 & 0.2 & 1 \end{bmatrix}, \qquad J = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{4.2}$$

$h(t, x(t)) = \mathrm{diag}(\tanh(x_1(t)), \tanh(x_2(t)), \tanh(x_3(t)))$, and the activation function is taken as $f(s) = \tanh(s)$. Then, it is obvious that $M = M_h = I_3$, where I_3 is a 3×3 identity matrix. The SNN (4.1) with the above-given parameters is depicted in Figure 1 with initial values $x(0) = [1, -1, 3]^T$.

The stabilization controller is designed as

$$u(t) = -k_1 x(t) - k_2 \,\mathrm{sign}(x(t))|x(t)|^{\alpha}, \tag{4.3}$$

where the parameter α is chosen as 0.5 and the initial value $x(0) = [1, -1, 3]^T$. Then, $\|x(0)\| = 3.3166$.

According to Theorem 3.5 and using Matlab LMI toolbox, we solve the LMI (3.13), and obtain $p = 2.8118$, $K = 10.8900$, and $\varepsilon = 10.1114$. Then by Theorem 3.5, the desired controller parameter can be designed as $k_1 = 3.8730$.

By choosing an arbitrary fixed gain k_2, SNN (4.1) can be stabilized in finite time in probability. Taking $k_2 = 1$, for example, we can obtain the upper bound of the settle time $\mathbf{T} = \|x(0)\|^{1-\alpha}/k_2(1-\alpha) = 3.6423$.

Simulation result is depicted in Figure 2, which shows the states $x_1(t)$, $x_2(t)$, and $x_3(t)$ of the controlled SNN (4.1). The simulation result has confirmed the effectiveness of our main results.

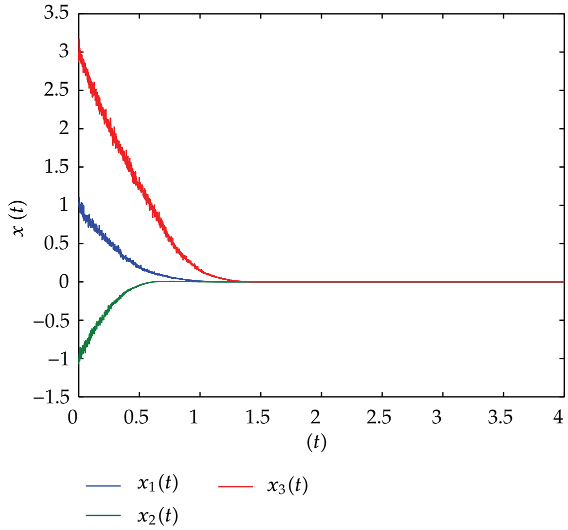

Figure 2: Trajectories of SNN (4.1) under the controller (4.3) with $k_2 = 1$ in Example 4.1.

Example 4.2. Still consider the SNN (4.1) with second-order parameter uncertainties:

$$\underline{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \overline{A} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \qquad \underline{B} = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} 0.4 & 0.3 \\ 0.3 & 0.5 \end{bmatrix}. \tag{4.4}$$

The parameter α in the controller (4.3) is chosen as 0.5 and the initial value $x(0) = [1, -1]^T$. Then, $\|x(0)\| = 1.414$. According to Corollary 3.7 and using Matlab LMI toolbox, we solve the LMI (3.19) and obtain $p = 5.3906$, $K = 10.0457$, $\varepsilon = 12.5372$, $\lambda_1 = 21.7115$, and $\lambda_2 = 20.9350$. Then by Corollary 3.7, the desired controller parameter can be designed as $k_1 = 1.8635$.

By choosing an arbitrary fixed gain k_2, SNN (4.1) can be robustly stabilized in finite time in probability. Taking $k_2 = 1.5$, for example, we can obtain the upper bound of the settle time $\mathbf{T} = \|x(0)\|^{1-\alpha} / k_2(1-\alpha) = 1.5856$.

Simulation result is depicted in Figure 3, which shows the states $x_1(t)$ and $x_2(t)$ of the second-order controlled SNN (4.1). The simulation result has confirmed the effectiveness of our main results.

5. Conclusions

In this paper, we have investigated the issue of finite-time stabilization for SNNs with noise perturbations by constructing a continuous nonlinear stabilizator. Meanwhile, Based on the Lyapunov-Krasovskii functional method combining with the LMI techniques, a sufficient criterion is derived for the states of the augmented system to be global finite-time stable in probability. Subsequently, for SNNs with parameter uncertainties, the robust finite-time stabilizator could be designed well. Finally, two illustrative examples have been used to demonstrate the usefulness of the main results. It is expected that the theory established in

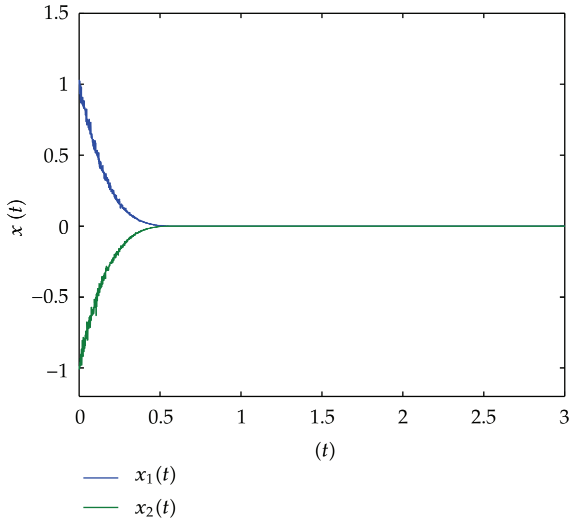

Figure 3: Trajectories of SNN (4.1) under the controller (4.3) with $k_2 = 1.5$ in Example 4.2.

this paper can be widely applied in delayed systems, particularly in those discontinuous cases. It will be an interesting topic in our future research.

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