Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 401562, 9 pages
doi:10.1155/2012/401562

Research Article

Numerical Method for a Markov-Modulated Risk Model with Two-Sided Jumps

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Received 24 August 2012; Accepted 30 October 2012

Academic Editor: Xinguang Zhang

This paper considers a perturbed Markov-modulated risk model with two-sided jumps, where both the upward and downward jumps follow arbitrary distribution. We first derive a system of differential equations for the Gerber-Shiu function. Furthermore, a numerical result is given based on Chebyshev polynomial approximation. Finally, an example is provided to illustrate the method.

1. Introduction

The risk model with two-sided jumps was first proposed by Boucherie et al. [1] and has been further investigated by many authors during the last few years. For example, Kou and Wang [2] studied the Laplace transform of the first passage time and the overshoot for a perturbed compound Poisson model with double exponential jumps. Xing et al. [3] extended the results of Kou and Wang [2] to the case that the surplus process with phase-type downward and arbitrary upward jumps. Zhang et al. [4] assumed that the downward jumps follow arbitrary distribution and the upward jumps have a rational Laplace transform. They derived the Laplace transform of the Gerber-Shiu function by using the roots of the generalized Lundberg equation. Under the assumption that the upward jumps follow Laplace distribution and arbitrary downward jumps, Chi [5] obtained a closed-form expression for the Gerber-Shiu function by applying Wiener-Hopf factorization technique. The applications of the model in finance were also discussed. Jacobsen [6] studied a perturbed renewal risk model with phase-type interclaim times and two-sided jumps, where both the jumps have rational Laplace transforms. Based on the roots of the Cramér-Lundberg equation, the joint Laplace transform on the time to ruin, and the undershoot at ruin were given. However, in all the aforemetioned papers, the topic that the jumps in both directions are arbitrary distributions is still not discussed. The Markov-modulated risk model (Markovian regime switching model)

was first proposed by Asmussen [7] to extend the classical risk model. Since then, it has received remarkable attention in actuarial mathematics, see, for example, Zhu and Yang [8, 9], Zhang et al. [4], Ng and Yang [10], Li and Lu [11], Lu and Tsai [12], and references therein. Motivated by the papers mentioned above, in this paper, we will study the Markov-modulated risk model with two-sided jumps.

Let $\{J(t), t \geq 0\}$ be a homogenous, irreducible, and recurrent Markov process with finite state space $E = \{1, 2, \ldots, n\}$. Denote the intensity matrix of $\{J(t), t \geq 0\}$ by $\mathbf{A} = (\alpha_{ij})_{i,j=1}^{n}$ with $\sum_{j=1}^{n} \alpha_{ij} = 0$ and $\alpha_{ii} := -\alpha_i = -\sum_{j \neq i}^{n} \alpha_{ij}$ for $i \in E$. Let $\{X_i, i = 1, 2, \ldots\}$ be a sequence of independent random variables representing the jumps, and $B(t)$ be a standard Brownian motion with $B(0) = 0$. Here we assume that the premium rates, claim interarrival times, the distributions of the jumps, and the diffusion parameter are all influenced by the environment process $\{J(t), t \geq 0\}$. When $J(t) = i$, the premium rate is c_i, jumps arrive according to a Poisson process with intensity λ_i, the diffusion parameter is $\sigma_i > 0$, and the size of the jumps which arrives at time t follows the distribution F_i with density f_i and finite mean μ_i. Then the Markov-modulated diffusion risk model $\{U(t), t \geq 0\}$ is defined by

$$U(t) = u + \int_0^t c_{J(s)} \mathrm{d}s - \sum_{j=1}^{N(t)} X_j + \int_0^t \sigma_{J(s)} \mathrm{d}B(s), \tag{1.1}$$

where $u \geq 0$ is the initial surplus. If we denote the stationary distribution of $\{J(t), t \geq 0\}$ by $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$, then the positive security loading condition is given by

$$\sum_{i=1}^{n} \pi_i (c_i - \lambda_i \mu_i) > 0. \tag{1.2}$$

In this paper, we further assume that the jumps in (1.1) are two-sided. The upward jumps can be explained as the random income (premium or investment), while the downward jumps are interpreted as the random loss. In this case, the density function is given by

$$f_i(x) = p_i f_{i,d}(x) I(x \geq 0) + q_i f_{i,u}(-x) I(x < 0), \quad \text{for } J(t) = i,\ i = 1, 2, \ldots, n, \tag{1.3}$$

where $0 < p_i \leq 1$, $p_i + q_i = 1$, $I(\cdot)$ is the indicator function, $f_{i,d}$ and $f_{i,u}$ are two arbitrary functions on $[0, \infty)$.

Let $T = \inf\{t \geq 0 : U(t) \leq 0\}$ (∞ otherwise) be the time to ruin. For $\delta \geq 0$, let

$$\phi_i(u) = E\left[e^{-\delta T} \omega(U(T-), |U(\mathrm{T})|) I(T < \infty) \mid J(0) = i, U(0) = u\right], \quad u \geq 0, \tag{1.4}$$

be the Gerber-Shiu function at ruin given that the initial state is i, where $\omega(x_1, x_2)$ is a nonnegative penalty function, $U(T-)$ is the surplus immediately prior to ruin, and $|U(T)|$ is the deficit at ruin. Without loss of generality, we assume that $\omega(0, 0) = 1$. Thus $\phi_i(0) = 1$ for $i = 1, 2, \ldots, n$. When $\omega = 1$, (1.4) reduces to the Laplace transform of the time to ruin

$$\psi_{\delta,i}(u) = E\left[e^{-\delta T} I(T < \infty) \mid J(0) = i, U(0) = u\right], \quad u \geq 0, \tag{1.5}$$

when $\omega = 1$ and $\delta = 0$, (1.4) reduces to the probability of ruin

$$\varphi_i(u) = P(T < \infty \mid J(0) = i, U(0) = u), \quad u \geq 0. \tag{1.6}$$

The purpose of this paper is to present some numerical results on the Gerber-Shiu function for the Markov-modulated diffusion risk model with arbitrary upward and downward jumps. In Section 2 we derive a system of integrodifferential equations and approximate solutions for $\phi_i(u)$. Numerical example is given in the last section.

2. Integrodifferential Equations and Approximate Solution

Theorem 2.1. *For $u \geq 0$, $\phi_i(u)$ $(i = 1, 2, \ldots, n)$ satisfies the following integrodifferential equation*

$$\begin{aligned}
&\frac{\sigma_i^2}{2}\phi_i''(u) + c_i\phi_i'(u) - (\lambda_i + \delta)\phi_i(u) + \sum_{k=1}^{n}\alpha_{ik}\phi_k(u) \\
&\quad = -\lambda_i \int_{-\infty}^{\infty} \phi_i(u-y) f(y) dy \\
&\quad = -\lambda_i p_i \int_0^u \phi_i(u-y) f_{i,d}(y) dy - \lambda_i q_i \int_0^{\infty} \phi_i(u+y) f_{i,u}(y) dy - \omega_i(u),
\end{aligned} \tag{2.1}$$

where

$$\omega_i(u) = \lambda_i p_i \int_u^{\infty} \omega(u, y-u) f_{i,d}(y) dy, \tag{2.2}$$

with boundary conditions

$$\begin{aligned}
\phi_i(0) &= 1, \\
\phi_i(\infty) &= 0.
\end{aligned} \tag{2.3}$$

Proof. Similar to Ng and Yang [10]. □

Remark 2.2. When $n = 1$, (2.1) is identical to (3.2) in Zhang et al. [4].

Clearly, (2.1) is a system of second order linear integrodifferential equations of Fredholm-Volterra type. As is well known, it is very difficult to find analytical solution of this system. Motivated by Akyüz-Dascioglu [13], we will study an alternative system defined on $[0, 1]$ by Chebyshev collocation method. First, we transform the interval $[0, \infty)$ to $[0, 1]$. Following Diko and Usábel [14], we set $u = h(x)$, that is, $h : [0, 1] \to [0, \infty)$. Furthermore, we assume that h is an arbitrary strictly monotone, twice continuously differentiable function throughout the paper.

Theorem 2.3. *Let $h(x)$ be a monotone increase function and $\chi_i(x) = \phi_i(h(x))$ for $x \in [0,1]$. Then $\chi_i(x)$ satisfies the following integrodifferential equation*

$$\begin{aligned}\frac{\sigma_i^2}{2(h'(x))^2}\chi_i''(x) + \left(\frac{c_i}{h'(x)} - \frac{\sigma_i^2 h''(x)}{2(h'(x))^3}\right)\chi_i'(x) - (\lambda_i + \delta)\chi_i(x) + \sum_{k=1}^{n}\alpha_{ik}\chi_k(x) \\ + \int_0^x K_i(x,t)\chi_i(t)dt + \int_0^1 L_i(x,t)\chi_i(t)dt + W_i(x) = 0,\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}K_i(x,t) &= \lambda_i\big(p_i f_{i,d}(h(x) - h(t)) - q_i f_{i,u}(h(t) - h(x))\big)h'(t), \\ L_i(x,t) &= \lambda_i q_i f_{i,u}(h(t) - h(x))h'(t), \\ W_i(x) &= \omega_i(h(x)),\end{aligned} \tag{2.5}$$

with boundary conditions

$$\begin{aligned}\chi_i(0) &= 1, \\ \chi_i(1) &= 0.\end{aligned} \tag{2.6}$$

Proof. By the definitions of function h and χ_i, we have

$$\begin{aligned}p_i \int_0^u \phi_i(u-y) f_{i,d}(y)\mathrm{d}y + q_i \int_0^\infty \phi_i(u+y) f_{i,u}(y)\mathrm{d}y \\ = q_i \int_0^1 \phi_i(h(t)) f_{i,u}(h(t) - h(x))h'(t)\mathrm{d}t \\ + \int_0^x \phi_i(h(t))\big(p_i f_{i,d}(h(x) - h(t)) - q_i f_{i,u}(h(t) - h(x))\big)h'(t)\mathrm{d}t \\ = q_i \int_0^1 \chi_i(t) f_{i,u}(h(t) - h(x))h'(t)\mathrm{d}t \\ + \int_0^x \chi_i(t)\big[p_i f_{i,d}(h(x) - h(t)) - q_i f_{i,u}(h(t) - h(x))\big]h'(t)\mathrm{d}t.\end{aligned} \tag{2.7}$$

Substituting (2.7) and $\chi_i(x) = \phi_i(h(x))$ into (2.1) and simplifying lead to (2.4). The boundary conditions are direct result of the boundary conditions in Theorem 2.1. This completes the proof. □

Remark 2.4. The existence of the solution for the system of integrodifferential equations (2.4) can be found in Fariborzi and Behzadi [15].

According to Akyüz-Dascioglu [13], $\chi_i(x)$ and its derivatives have truncated Chebyshev series expression

$$\chi_i^{(j)}(x) = \sum_{r=0}^{N} a_{ir}^{(j)} T_r^*(x), \quad i = 1, 2, \ldots, n, \; j = 0, 1, 2, \ldots, \; x \in [0, 1], \tag{2.8}$$

where $\chi_i^{(0)}(x) = \chi_i(x)$, $a_{ir}^{(0)} = a_{ir}$, $T_r^*(x)$ are shifted Chebyshev polynomials of the first kind and $a_{ir}^{(j)}$ are the unknown coefficients to be determined.

Let $\mathbf{T}^*(x) = (T_1^*(x), T_2^*(x), \ldots, T_N^*(x))^\top$, $\mathbf{A}_i = (a_{i0}, a_{i1}, \ldots, a_{iN})^\top$, $\mathbf{A}_i^{(j)} = (a_{i0}^{(j)}, a_{i1}^{(j)}, \ldots, a_{iN}^{(j)})^\top$. Then (2.8) can be written in the matrix form

$$\begin{gathered} \chi_i(x) = \mathbf{T}^*(x)\mathbf{A}_i, \\ \chi_i^{(j)}(x) = \mathbf{T}^*(x)\mathbf{A}_i^{(j)} = 4^j \mathbf{T}_r^*(x)\mathbf{M}^n \mathbf{A}_i^{(j)}, \quad j = 1, 2, \ldots, \end{gathered} \tag{2.9}$$

where

$$\mathbf{M} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & \frac{5}{2} & \cdots & \frac{N}{2} \\ 0 & 0 & 2 & 0 & 4 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 3 & 0 & 5 & \cdots & N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{(N+1)\times(N+1)}, \tag{2.10}$$

for odd N, and

$$\mathbf{M} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & \frac{5}{2} & \cdots & 0 \\ 0 & 0 & 2 & 0 & 4 & 0 & \cdots & N \\ 0 & 0 & 0 & 3 & 0 & 5 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & N \end{pmatrix}_{(N+1)\times(N+1)}, \tag{2.11}$$

for even N.

Similarly, the kernel functions $K_i(x,t)$ and $L_i(x,t)$ can be expanded to univariate Chebyshev series

$$\begin{gathered} K_i(x,t) = \boldsymbol{\kappa}_i(x) T_r^*(t), \\ L_i(x,t) = \boldsymbol{\ell}_i(x) T_r^*(t), \end{gathered} \tag{2.12}$$

where

$$\boldsymbol{\kappa}_i(x) = \left(\frac{1}{2}\kappa_{i,1}(x), \kappa_{i,2}(x), \ldots, \frac{1}{2}\kappa_{i,n}(x)\right),$$
$$\boldsymbol{\ell}_i(x) = \left(\frac{1}{2}\ell_{i,1}(x), \ell_{i,2}(x), \ldots, \frac{1}{2}\ell_{i,n}(x)\right), \tag{2.13}$$

with $\kappa_{i,r}$ and $\ell_{i,r}$ are Chebyshev coefficients determined by Clenshaw and Curtis [16].

Theorem 2.5. *For $0 \le x \le 1$, an approximate expression for $\chi_i(x)$ is given by*

$$\chi_i(x) = \mathbf{T}^*(x)\mathbf{A}_i, \quad i = 1, 2, \ldots, n, \tag{2.14}$$

where the column vector $\mathbf{A}_i$ can be determined by the following systems

$$\begin{aligned}
&\frac{2\sigma_i^2}{(h'(x_j))^2}\mathbf{T}^*(x_j)\mathbf{M}\mathbf{A}_i + \left(\frac{c_i}{h'(x_j)} - \frac{\sigma_i^2 h''(x_j)}{2(h'(x_j))^3}\right)\mathbf{T}^*(x_j)\mathbf{A}_i \\
&\quad - (\lambda_i + \delta)\mathbf{T}^*(x_j)\mathbf{A}_i + \sum_{k=1}^{n}\alpha_{ik}\mathbf{T}^*(x_j)\mathbf{A}_k \\
&\quad + \frac{1}{2}\boldsymbol{\kappa}_i(x_j)\mathbf{Z}(2x_j - 1)\mathbf{A}_i + \frac{1}{2}\boldsymbol{\ell}_i(x_j)\mathbf{Z}\mathbf{A}_i + W_i(x_j) = 0, \\
&\mathbf{T}^*(0)\mathbf{A}_i = 1, \\
&\mathbf{T}^*(1)\mathbf{A}_i = 0,
\end{aligned} \tag{2.15}$$

where matrix $\mathbf{Z} = (z_{ij})$ with elements

$$z_{ij} = \begin{cases} \dfrac{1}{1-(i-j)^2} + \dfrac{1}{1-(i+j)^2}, & \text{for even } i+j, \\ 0, & \text{for odd } i+j, \end{cases} \tag{2.16}$$

matrix $\mathbf{Z}(x) = (z_{ij}(x))$ *with elements*

$$z_{ij}(x) = \frac{1}{4}\begin{cases} 2x^2 - 2, & i+j=1, \\ \dfrac{T_{i+j+1}}{i+j+1} - \dfrac{T_{i+j-1}}{i+j-1} - \dfrac{1}{i+j+1} + \dfrac{1}{i+j-1} + x^2 - 1, & |i-j| = 1, \\ \dfrac{T_{i+j+1}}{i+j+1} + \dfrac{T_{1-i-j}}{1-i-j} + \dfrac{T_{1+i-j}}{1+i-j} + \dfrac{T_{1-i+j}}{1-i+j} & \\ \quad +2\left(\dfrac{1}{1-(i+j)^2} + \dfrac{1}{1-(i-j)^2}\right), & \text{for even } i+j, \\ \dfrac{T_{i+j+1}}{i+j+1} + \dfrac{T_{1-i-j}}{1-i-j} + \dfrac{T_{1+i-j}}{1+i-j} + \dfrac{T_{1-i+j}}{1-i+j} & \\ \quad -2\left(\dfrac{1}{1-(i+j)^2} + \dfrac{1}{1-(i-j)^2}\right), & \text{for odd } i+j \end{cases} \tag{2.17}$$

and x_j $(j = 0, 1, 2, \ldots, N-1)$ *are collocations.*

Proof. Using (2.8) and (2.12), one obtains

$$\begin{aligned} \int_0^x K_i(x,t)\chi_i(t)\mathrm{d}t &= \int_0^x \boldsymbol{\kappa}_i(x)(\mathbf{T}^*(t))^\top \mathbf{T}^*(t)\mathbf{A}_i \mathrm{d}t \\ &= \frac{1}{2}\boldsymbol{\kappa}_i(x)\mathbf{Z}(2x-1)\mathbf{A}_i, \\ \int_0^1 L_i(x,t)\chi_i(t)\mathrm{d}t &= \int_0^1 \boldsymbol{\ell}_i(x)(\mathbf{T}^*(t))^\top \mathbf{T}^*(t)\mathbf{A}_i \mathrm{d}t \\ &= \frac{1}{2}\boldsymbol{\ell}_i(x)\mathbf{Z}\mathbf{A}_i, \end{aligned} \tag{2.18}$$

Substituting (2.18) into (2.4), we have

$$\begin{aligned} &\frac{2\sigma_i^2}{(h'(x))^2}\mathbf{T}^*(x)\mathbf{M}\mathbf{A}_i + \left(\frac{c_i}{h'(x)} - \frac{\sigma_i^2 h''(x)}{2(h'(x))^3}\right)\mathbf{T}^*(x)\mathbf{A}_i \\ &\quad - (\lambda_i + \delta)\mathbf{T}^*(x)\mathbf{A}_i + \sum_{k=1}^{n}\alpha_{ik}\mathbf{T}^*(x)\mathbf{A}_k \\ &\quad + \frac{1}{2}\boldsymbol{\kappa}_i(x)\mathbf{Z}(2x-1)\mathbf{A}_i + \frac{1}{2}\boldsymbol{\ell}_i(x)\mathbf{Z}\mathbf{A}_i + W_i(x) = 0, \end{aligned} \tag{2.19}$$

which is identical to (2.15) in form. Substituting the collocations x_j $(j = 0, 1, \ldots, N-1)$ into (2.19) leads to (2.15). $\mathbf{T}^*(0)\mathbf{A}_i = 1$ and $\mathbf{T}^*(1)\mathbf{A}_i = 0$ can be obtained by (2.6). □

Example 2.6. To illustration our method, we use the example of Zhang et al. [4]. Let $n = 1$, $c = 2$, $\sigma^2 = 2$, $\lambda = 1$, $p_1 = 0.6$, $q_1 = 0.4$, $\delta = 0.3$, the downward jumps are exponentially distributed

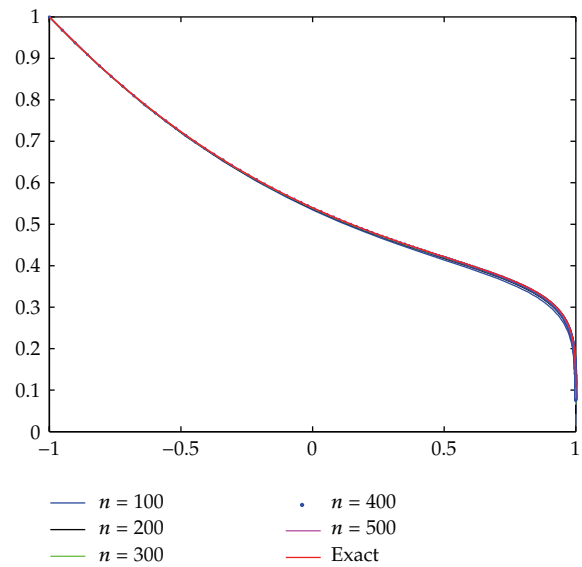

Figure 1: Laplace transform of the time to ruin.

Table 1: Laplace transform of the time to ruin.

u	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	exact
1	0.4656	0.46863	0.47008	0.47107	0.47177	0.4718
2	0.36228	0.36702	0.36918	0.37053	0.37149	0.3720
3	0.30111	0.30774	0.31072	0.31251	0.31371	0.3152
4	0.24968	0.25892	0.26227	0.26427	0.26594	0.2690
5	0.2054	0.21521	0.22034	0.22272	0.22445	0.2297
6	0.16376	0.17695	0.18356	0.18607	0.18856	0.1961
7	0.12624	0.1439	0.15074	0.15459	0.15713	0.1675
8	0.082052	0.11061	0.12152	0.12482	0.129	0.1430

with parameter 0.3, and the upward jump density is given by $f_{1,u}(x) = 0.08e^{-0.4x} + 0.64e^{-0.8x}$. We set $u = h(x) = -\ln(1-x)$ and the collocation points are $x_j = ((1+\cos(s\pi/N))/2)$ $(i = 1, 2, \ldots, N)$.

Figure 1 shows that the approximate solution is very near to the exact solution for any initial surplus u. We remark that the horizontal axis in Figure 1 is $\cos(s\pi/N)$ $(i = 1, 2, \ldots, N)$ and $u = -\ln((1-\cos(s\pi/N))/2)$.

From Table 1 we can see that the errors between the approximate solutions and the exact solutions decrease when N increases. The initial surplus u can also influence the approximate solution: the bigger u need a bigger N to decrease the error.

Acknowledgments

This work is supported by the Natural Science Foundation of Shandong (no. ZR2010AQ015), the Tianyuan fund for Mathematics (no. 11226251), and the Natural Science Foundation of Qufu Normal University (no. 2012ZRB01473).

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