

Research Article

Stability of the n -Dimensional Mixed-Type Additive and Quadratic Functional Equation in Non-Archimedean Normed Spaces

Yang-Hi Lee,¹ Soon-Mo Jung,² and Themistocles M. Rassias³

¹ Department of Mathematics Education, Gongju National University of Education,
Gongju 314-711, Republic of Korea

² Mathematics Section, College of Science and Technology, Hongik University,
339-701 Jochiwon, Republic of Korea

³ Department of Mathematics, National Technical University of Athens, Zografou Campus,
15780 Athens, Greece

Correspondence should be addressed to Soon-Mo Jung, smjung@hongik.ac.kr

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We will prove the stability of the functional equation $2f(\sum_{i=1}^n x_i) + \sum_{1 \leq i, j \leq n, i \neq j} f(x_i - x_j) = (n + 1) \sum_{i=1}^n f(x_i) + (n - 1) \sum_{i=1}^n f(-x_i)$ in non-Archimedean normed spaces.

1. Introduction

A classical question in the theory of functional equations is “when is it true that a function, which approximately satisfies a functional equation, must be somehow close to an exact solution of the equation?” Such a problem, called a *stability problem* of the functional equation, was formulated by Ulam in 1940 (see [1]). In the following year, Hyers [2] gave a partial solution of Ulam’s problem for the case of approximate additive functions. Subsequently, his result was generalized by Aoki [3] for additive functions and by Rassias [4] for linear functions. Indeed, they considered the stability problem for unbounded Cauchy differences. During the last decades, the stability problems of functional equations have been extensively investigated by a number of mathematicians (see [5–23]).

A non-Archimedean field is a field \mathbb{K} equipped with a function (valuation) $|\cdot| : \mathbb{K} \rightarrow [0, \infty)$ such that

(F₁) $|r| = 0$ if and only if $r = 0$;

(F₂) $|rs| = |r||s|$;

(F₃) $|r + s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$.

Clearly, it holds that $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$.

Let X be a vector space over a scalar field \mathbb{K} with a non-Archimedean and nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \rightarrow \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(N₁) $\|x\| = 0$ if and only if $x = 0$;

(N₂) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(N₃) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$.

Then $(X, \|\cdot\|)$ is called a non-Archimedean space. Due to the fact that

$$\|x_n - x_m\| \leq \max_{m \leq i < n} \|x_{i+1} - x_i\| \quad (n > m), \quad (1.1)$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean space. A complete non-Archimedean space is a non-Archimedean space in which every Cauchy sequence is convergent.

Recently, Moslehian and Rassias [24] proved the Hyers-Ulam stability of the Cauchy functional equation

$$f(x + y) = f(x) + f(y), \quad (1.2)$$

and the quadratic functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y) \quad (1.3)$$

in non-Archimedean normed spaces.

We now consider the n -dimensional mixed-type quadratic and additive functional equation

$$2f\left(\sum_{i=1}^n x_i\right) + \sum_{1 \leq i, j \leq n, i \neq j} f(x_i - x_j) = (n+1) \sum_{i=1}^n f(x_i) + (n-1) \sum_{i=1}^n f(-x_i), \quad (1.4)$$

whose solution is called a *quadratic-additive function*.

In 2009, Towanlong and Nakmahachalasint [25] obtained a stability result for the functional equation (1.4), in which they constructed a quadratic-additive function F by composing an additive function A and a quadratic function Q , where A and Q approximate the odd part and the even part of the given function f , respectively.

In this paper, we investigate a general stability problem for the n -dimensional mixed-type quadratic and additive functional equation (1.4) in non-Archimedean normed spaces.

2. Solutions of (1.4)

In this section, we prove the generalized Hyers-Ulam stability of the n -dimensional mixed-type quadratic and additive functional equation (1.4). Assume that H is an additive group and X is a complete non-Archimedean space.

For a given function $f : H \rightarrow X$, we use the abbreviations

$$\begin{aligned} f_e(x) &:= \frac{f(x) + f(-x)}{2}, \\ f_o(x) &:= \frac{f(x) - f(-x)}{2}, \\ Af(x, y) &:= f(x + y) - f(x) - f(y), \\ Qf(x, y) &:= f(x + y) + f(x - y) - 2f(x) - 2f(y), \\ D_nf(x_1, x_2, \dots, x_n) &:= 2f\left(\sum_{i=1}^n x_i\right) + \sum_{1 \leq i, j \leq n, i \neq j} f(x_i - x_j) \\ &\quad - (n+1)\sum_{i=1}^n f(x_i) - (n-1)\sum_{i=1}^n f(-x_i) \end{aligned} \quad (2.1)$$

for all $x, y, x_1, x_2, \dots, x_n \in H$ and for an arbitrarily fixed $n \in \mathbb{N}$.

Theorem 2.1. Assume that $n \geq 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. A function $f : H \rightarrow X$ is a solution of (1.4) if and only if f_e is quadratic, f_o is additive, and $f_e(0) = 0$.

Proof. If a function $f : H \rightarrow X$ is a solution of (1.4), then we have $f_e(0) = 0$,

$$\begin{aligned} Qf_e(x, y) &= f_e(x + y) + f_e(x - y) - 2f_e(x) - 2f_e(y) \\ &= \frac{1}{2}D_nf_e(x, y, 0, \dots, 0) + \frac{1}{2}(n-2)(n+3)f_e(0) \\ &= 0, \\ Af_o(x, y) &= f_o(x + y) - f_o(x) - f_o(y) = \frac{1}{2}D_nf_o(x, y, 0, \dots, 0) = 0 \end{aligned} \quad (2.2)$$

for all $x, y \in H$, that is, f_e is quadratic and f_o is additive.

Conversely, assume that f_e is quadratic, f_o is additive, and $f_e(0) = 0$. We apply an induction on j to prove $D_nf_e(x_1, x_2, \dots, x_n) = 0$ for all $x_1, x_2, \dots, x_n \in H$. For $j = 2$, we have

$$\begin{aligned} D_nf_e(x_1, x_2, 0, \dots, 0) &= 2f_e(x_1 + x_2) + 2f_e(x_1 - x_2) - 4f_e(x_1) - 4f_e(x_2) - (n-2)(n+3)f_e(0) \\ &= 0. \end{aligned} \quad (2.3)$$

If $n > 2$ and $D_n f_e(x_1, x_2, \dots, x_j, 0, \dots, 0) = 0$ for some integer j ($2 \leq j < n$) and for all $x_1, x_2, \dots, x_j \in H$, then a routine calculation yields

$$\begin{aligned}
 & D_n f_e(x_1, x_2, \dots, x_{j+1}, 0, \dots, 0) \\
 &= Qf_e(x_1 + \dots + x_j, x_{j+1} - x_j) + \frac{1}{2} D_n f_e(x_1, \dots, x_{j-1}, 2x_j, 0, \dots, 0) \\
 &+ \frac{1}{2} D_n f_e(x_1, \dots, x_{j-1}, 2x_{j+1}, 0, \dots, 0) - \sum_{k=1}^{j-1} (Qf_e(x_k, x_j) + Qf_e(x_k, x_{j+1})) \\
 &- \frac{j}{2} Qf_e(x_{j+1}, x_{j+1}) - \frac{j}{2} Qf_e(x_j, x_j) \\
 &= 0
 \end{aligned} \tag{2.4}$$

for all $x_1, x_2, \dots, x_{j+1} \in H$. Hence, we conclude that

$$D_n f_e(x_1, x_2, \dots, x_n) = 0 \tag{2.5}$$

for all $x_1, x_2, \dots, x_n \in H$.

Since f_o is additive, a long calculation yields

$$\begin{aligned}
 & D_n f_o(x_1, x_2, \dots, x_n) \\
 &= \sum_{1 \leq i, j \leq n, i \neq j} A f_o(x_i, -x_j) + 2 \sum_{i=1}^{n-1} A f_o\left(\sum_{j=1}^i x_j, x_{i+1}\right) \\
 &= 0.
 \end{aligned} \tag{2.6}$$

Hence, it follows from (2.5) and (2.6) that

$$D_n f(x_1, x_2, \dots, x_n) = D_n f_e(x_1, x_2, \dots, x_n) + D_n f_o(x_1, x_2, \dots, x_n) = 0 \tag{2.7}$$

for all $x_1, x_2, \dots, x_n \in H$; that is, f is a solution of (1.4). \square

3. Generalized Hyers-Ulam Stability of (1.4)

In the following theorem, we will investigate the stability problem of the functional equation (1.4).

Theorem 3.1. Assume that $n \geq 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. Assume that $\varphi : H^n \rightarrow [0, \infty)$ is a function such that

$$\lim_{m \rightarrow \infty} \frac{\varphi(n^m x_1, n^m x_2, \dots, n^m x_n)}{|n|^{2m}} = 0 \tag{3.1}$$

for all $x_1, x_2, \dots, x_n \in H$. Moreover, assume that the limit

$$\tilde{\varphi}(x) := \lim_{m \rightarrow \infty} \max_{0 \leq i < m} \left\{ \frac{\varphi(n^i x, \dots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \dots, -n^i x)}{|4||n|^{2i+2}} \right\} \quad (3.2)$$

exists for each $x \in H$. If a function $f : H \rightarrow X$ satisfies the inequality

$$\|D_n f(x_1, x_2, \dots, x_n)\| \leq \varphi(x_1, x_2, \dots, x_n) \quad (3.3)$$

for any $x_1, x_2, \dots, x_n \in H$, then there exists a unique quadratic-additive function $T : H \rightarrow X$ such that

$$\|f(x) - T(x)\| \leq \tilde{\varphi}(x) \quad (3.4)$$

for each $x \in H$. In particular, T is given by

$$T(x) = \lim_{m \rightarrow \infty} \left(\frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \right) \quad (3.5)$$

for all $x \in H$.

Proof. If we replace x_i in (3.1) with 0 for each $i \in \{1, 2, \dots, n\}$, then we have

$$\lim_{m \rightarrow \infty} \frac{\varphi(0, 0, \dots, 0)}{|n|^{2m}} = 0. \quad (3.6)$$

Since $|n| \leq 1$, it holds that $\varphi(0, 0, \dots, 0) = 0$ and

$$\|(n-1)(n+2)f(0)\| = \|D_n f(0, 0, \dots, 0)\| \leq \varphi(0, 0, \dots, 0) = 0. \quad (3.7)$$

Hence, we conclude that $f(0) = 0$.

Let $J_m f : H \rightarrow Y$ be a function defined by

$$J_m f(x) = \frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \quad (3.8)$$

for all $x \in H$ and $m \in \{0, 1, 2, \dots\}$. A tedious calculation, together with (F_2) , (N_3) , and (3.3), yields

$$\begin{aligned}
\|J_i f(x) - J_{i+1} f(x)\| &= \left\| -\frac{D_n f(n^i x, \dots, n^i x)}{4n^{2i+2}} - \frac{D_n f(-n^i x, \dots, -n^i x)}{4n^{2i+2}} \right. \\
&\quad \left. - \frac{D_n f(n^i x, \dots, n^i x)}{4n^{i+1}} + \frac{D_n f(-n^i x, \dots, -n^i x)}{4n^{i+1}} \right\| \\
&\leq \max \left\{ \frac{\|D_n f(n^i x, \dots, n^i x)\|}{|4||n|^{2i+2}}, \frac{\|D_n f(-n^i x, \dots, -n^i x)\|}{|4||n|^{2i+2}}, \right. \\
&\quad \left. \frac{\|D_n f(n^i x, \dots, n^i x)\|}{|4||n|^{i+1}}, \frac{\|D_n f(-n^i x, \dots, -n^i x)\|}{|4||n|^{i+1}} \right\} \\
&\leq \max \left\{ \frac{\varphi(n^i x, \dots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \dots, -n^i x)}{|4||n|^{2i+2}} \right\}
\end{aligned} \tag{3.9}$$

for all $x \in H$ and $i \in \{0, 1, 2, \dots\}$. It follows from (3.1) and (3.9) that the sequence $\{J_m f(x)\}$ is Cauchy. Since X is complete, we conclude that $\{J_m f(x)\}$ is convergent.

Let us define

$$T(x) := \lim_{m \rightarrow \infty} J_m f(x) \tag{3.10}$$

for any $x \in H$. It follows from (N_3) and (3.9) that

$$\begin{aligned}
\|f(x) - J_m f(x)\| &= \left\| \sum_{i=0}^{m-1} (J_i f(x) - J_{i+1} f(x)) \right\| \\
&\leq \max_{0 \leq i < m} \|J_i f(x) - J_{i+1} f(x)\| \\
&\leq \max_{0 \leq i < m} \left\{ \frac{\varphi(n^i x, \dots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \dots, -n^i x)}{|4||n|^{2i+2}} \right\}
\end{aligned} \tag{3.11}$$

for all $m \in \{0, 1, 2, \dots\}$ and $x \in H$. In view of (3.2), if we let $m \rightarrow \infty$ in (3.11), then we obtain the inequality (3.4).

Replacing x_i in (3.3) with $n^m x_i$ for $i \in \{1, 2, \dots, n\}$ and considering (F_2) and (N_3) , we get

$$\begin{aligned}
\|D_n J_m f(x_1, x_2, \dots, x_n)\| &= \left\| \frac{D_n f(n^m x_1, \dots, n^m x_n) - D_n f(-n^m x_1, \dots, -n^m x_n)}{2n^m} \right. \\
&\quad \left. + \frac{D_n f(n^m x_1, \dots, n^m x_n) + D_n f(-n^m x_1, \dots, -n^m x_n)}{2n^{2m}} \right\|
\end{aligned}$$

$$\leq \max \left\{ \frac{\varphi(n^m x_1, \dots, n^m x_n)}{|2||n|^m}, \frac{\varphi(-n^m x_1, \dots, -n^m x_n)}{|2||n|^m}, \right. \\ \left. \frac{\varphi(n^m x_1, \dots, n^m x_n)}{|2||n|^{2m}}, \frac{\varphi(-2^m x_1, \dots, -2^m x_n)}{|2||n|^{2m}} \right\} \quad (3.12)$$

for all $m \in \{0, 1, 2, \dots\}$ and $x_1, x_2, \dots, x_n \in H$. If we let $m \rightarrow \infty$ in the last inequality, then it follows from the condition (3.1) that $D_n T(x_1, x_2, \dots, x_n) = 0$ for all $x_1, x_2, \dots, x_n \in H$; that is, T is a quadratic-additive function.

Assume that $T' : H \rightarrow X$ is another quadratic-additive function satisfying (3.4). By the definition of D_n , a routine calculation yields

$$-\frac{D_n T'(n^i x, \dots, n^i x)}{4n^{2i+2}} - \frac{D_n T'(-n^i x, \dots, -n^i x)}{4n^{2i+2}} - \frac{D_n T'(n^i x, \dots, n^i x)}{4n^{i+1}} + \frac{D_n T'(-n^i x, \dots, -n^i x)}{4n^{i+1}} \\ = -\frac{1}{2n^{2(i+1)}} \left(T'(n^{i+1} x) + T'(-n^{i+1} x) \right) + \frac{1}{2n^{2i}} \left(T'(n^i x) + T'(-n^i x) \right) \\ - \frac{1}{2n^{i+1}} \left(T'(n^{i+1} x) - T'(-n^{i+1} x) \right) + \frac{1}{2n^i} \left(T'(n^i x) - T'(-n^i x) \right) \quad (3.13)$$

for each $i \in \{0, 1, 2, \dots\}$ and $x \in H$. Hence, it follows from (3.8) that

$$\sum_{i=0}^{k-1} \left(-\frac{D_n T'(n^i x, \dots, n^i x)}{4n^{2i+2}} - \frac{D_n T'(-n^i x, \dots, -n^i x)}{4n^{2i+2}} \right. \\ \left. - \frac{D_n T'(n^i x, \dots, n^i x)}{4n^{i+1}} + \frac{D_n T'(-n^i x, \dots, -n^i x)}{4n^{i+1}} \right) = T'(x) - J_k T'(x) \quad (3.14)$$

for any $k \in \mathbb{N}$ and $x \in H$. Since T' is a solution of (1.4), it follows from the last equality that

$$T'(x) = J_k T'(x) \quad (3.15)$$

for any $k \in \mathbb{N}$ and $x \in H$. Obviously, this equality also holds for T .

Consequently, by considering that $|n| \leq 1$, it follows from (N_3) , (3.1), (3.4), and (3.8) that

$$\|T(x) - T'(x)\| \\ = \lim_{k \rightarrow \infty} \|J_k T(x) - J_k T'(x)\| \\ \leq \lim_{k \rightarrow \infty} \max \{ \|J_k T(x) - J_k f(x)\|, \|J_k f(x) - J_k T'(x)\| \}$$

$$\begin{aligned}
&\leq \lim_{k \rightarrow \infty} |2|^{-1} |n|^{-2k} \max \left\{ \|T(n^k x) - f(n^k x)\|, \|T(-n^k x) - f(-n^k x)\|, \right. \\
&\quad \left. \|f(n^k x) - T'(n^k x)\|, \|f(-n^k x) - T'(-n^k x)\| \right\} \\
&\leq \lim_{k \rightarrow \infty} \lim_{m \rightarrow \infty} \max_{k \leq i < m+k} \left\{ \frac{\varphi(n^i x, \dots, n^i x)}{|8||n|^{2i+2}}, \frac{\varphi(-n^i x, \dots, -n^i x)}{|8||n|^{2i+2}} \right\} \\
&= 0
\end{aligned} \tag{3.16}$$

for all $x \in H$. Therefore, $T = T'$, which proves the uniqueness of T . \square

Corollary 3.2. *Let X and Y be non-Archimedean normed spaces over \mathbb{K} with $|n| < 1$. If Y is complete and $f : X \rightarrow Y$ satisfies the inequality*

$$\|Df(x_1, x_2, \dots, x_n)\| \leq \theta \sum_{i=1}^n \|x_i\|^r \tag{3.17}$$

for all $x_1, x_2, \dots, x_n \in X$ and for some $r > 2$, then there exists a unique quadratic-additive function $T : X \rightarrow Y$ such that

$$\|f(x) - T(x)\| \leq \frac{n\theta}{|4||n|^2} \|x\|^r \tag{3.18}$$

for all $x \in X$.

Proof. Let $\varphi(x_1, x_2, \dots, x_n) = \theta \sum_{i=1}^n \|x_i\|^r$. Since $|n| < 1$ and $r - 2 > 0$, we get

$$\lim_{m \rightarrow \infty} |n|^{-2m} \varphi(n^m x_1, n^m x_2, \dots, n^m x_n) = \lim_{m \rightarrow \infty} |n|^{m(r-2)} \varphi(x_1, x_2, \dots, x_n) = 0 \tag{3.19}$$

for all $x_1, x_2, \dots, x_n \in X$. Therefore, the conditions of Theorem 3.1 are satisfied. Indeed, it is easy to see that $\tilde{\varphi}(x) = n\theta(|4|^{-1}|n|^{-2})\|x\|^r$. By Theorem 3.1, there exists a unique quadratic-additive function $T : X \rightarrow Y$ such that (3.18) holds. \square

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