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Research Article

Application of Sumudu Decomposition Method to Solve Nonlinear System of Partial Differential Equations

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We develop a method to obtain approximate solutions of nonlinear system of partial differential equations with the help of Sumudu decomposition method (SDM). The technique is based on the application of Sumudu transform to nonlinear coupled partial differential equations. The nonlinear term can easily be handled with the help of Adomian polynomials. We illustrate this technique with the help of three examples, and results of the present technique have close agreement with approximate solutions obtained with the help of Adomian decomposition method (ADM).

1. Introduction

Most of phenomena in nature are described by nonlinear differential equations. So scientists in different branches of science try to solve them. But because of nonlinear part of these groups of equations, finding an exact solution is not easy. Different analytical methods have been applied to find a solution to them. For example, Adomian has presented and developed a so-called decomposition method for solving algebraic, differential, integrodifferential, differential-delay and partial differential equations. In the nonlinear case for ordinary differential equations and partial differential equations, the method has the advantage of dealing directly with the problem [1, 2]. These equations are solved without transforming them to more simple ones. The method avoids linearization, perturbation, discretization, or any unrealistic assumptions [3, 4]. It was suggested in [5] that the noise terms appears always for inhomogeneous equations. Most recently, Wazwaz [6] established a necessary condition that is essentially needed to ensure the appearance of "noise terms" in the inhomogeneous equations. In the present paper, the intimate connection between the Sumudu transform theory and decomposition method arises in the solution of nonlinear partial differential equations is demonstrated.

The Sumudu transform is defined over the set of the functions

$$A = \left\{ f(t) : \exists M, \tau_1, \tau_2 > 0, \left| f(t) \right| < M e^{t/\tau_j}, \text{if } t \in (-1)^j \times [0, \infty) \right\} \tag{1.1}$$

by the following formula:

$$\begin{aligned} G(u) &= S\left[f(t); u\right] \\ &= \int_0^\infty f(ut) e^{-t} dt, \quad u \in (-\tau_1, \tau_2). \end{aligned} \tag{1.2}$$

The existence and the uniqueness were discussed in [7], for further details and properties of the Sumudu transform and its derivatives we refer to [8]. In [9], some fundamental properties of the Sumudu transform were established.

In [10], this new transform was applied to the one-dimensional neutron transport equation. In fact one can easily show that there is a strong relationship between double Sumudu and double Laplace transforms, see [7].

Further in [11], the Sumudu transform was extended to the distributions and some of their properties were also studied in [12]. Recently Kılıçman et al. applied this transform to solve the system of differential equations, see [13].

A very interesting fact about Sumudu transform is that the original function and its Sumudu transform have the same Taylor coefficients except a factor $n!$. Thus if

$$f(t) = \sum_{n=0}^{\infty} a_n t^n \tag{1.3}$$

then

$$F(u) = \sum_{n=0}^{\infty} n! a_n t^n, \tag{1.4}$$

see [14].

Similarly, the Sumudu transform sends combinations, $C(m, n)$, into permutations, $P(m, n)$ and hence it will be useful in the discrete systems. Further

$$\begin{aligned} S(H(t)) &= \pounds(\delta(t)) = 1, \\ \pounds(H(t)) &= S(\delta(t)) = \frac{1}{u}. \end{aligned} \tag{1.5}$$

Thus we further note that since many practical engineering problems involve mechanical or electrical systems where action is defined by discontinuous or impulsive forcing terms. Then the Sumudu transform can be effectively used to solve ordinary differential equations as well as partial differential equations and engineering problems. Recently, the Sumudu transform was introduced as a new integral transform on a time scale $\mathbb{T}$ to solve a system of dynamic equations, see [15]. Then the results were applied on ordinary differential equations when $\mathbb{T} = \mathbb{R}$, difference equations when $\mathbb{T} = \mathbb{N}_0$, but also, for q-difference equations when $\mathbb{T} = q^{\mathbb{N}_0}$, where $q^{\mathbb{N}_0} := \{q^t : t \in \mathbb{N}_0 \text{ for } q > 1\}$ or $\mathbb{T} = q^{\overline{\mathbb{Z}}} := q^{\mathbb{Z}} \cup \{0\}$ for $q > 1$ which has important applications in quantum theory and on different types of time scales like $\mathbb{T} = h\mathbb{N}_0$, $\mathbb{T} = \mathbb{N}_0^2$, and $\mathbb{T} = \mathbb{T}_n$ the space of the harmonic numbers. During this study we use the following Sumudu transform of derivatives.

Theorem 1.1. *Let $f(t)$ be in A, and let $G^n(u)$ denote the Sumudu transform of the nth derivative, $f^n(t)$ of $f(t)$, then for $n \geq 1$*

$$G^n(u) = \frac{G(u)}{u^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{u^{n-k}}. \tag{1.6}$$

For more details, see [16].

We consider the general inhomogeneous nonlinear equation with initial conditions given below:

$$LU + RU + NU = h(x,t), \tag{1.7}$$

where L is the highest order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, NU represents the nonlinear terms and $h(x,t)$ is the source term. First we explain the main idea of SDM: the method consists of applying Sumudu transform

$$S[LU] + S[RU] + S[NU] = S[h(x,t)]. \tag{1.8}$$

Using the differential property of Laplace transform and initial conditions we get

$$\frac{1}{u^n}S[U(x,t)] - \frac{1}{u^n}U(x,0) - \frac{1}{u^{n-1}}U'(x,0) - \cdots - \frac{U^{n-1}(x,0)}{u} + S[RU] + S[NU] = S[h(x,t)]. \tag{1.9}$$

By arrangement we have

$$S[U(x,t)] = U(x,0) + uU'(x,0) + \cdots + u^{n-1}U^{n-1}(x,0) - u^n S[RU] - u^n S[NU] + u^n S[h(x,t)]. \tag{1.10}$$

The second step in Sumudu decomposition method is that we represent solution as an infinite series:

$$U(x,t) = \sum_{i=0}^{\infty} U_i(x,t) \tag{1.11}$$

and the nonlinear term can be decomposed as

$$NU(x,t) = \sum_{i=0}^{\infty} A_i, \tag{1.12}$$

where A_i are Adomian polynomials [6] of $U_0, U_1, U_2, \ldots, U_n$ and it can be calculated by formula

$$A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[N\sum_{i=0}^{\infty}\lambda^i U_i\right]_{\lambda=0}, \quad i = 0,1,2,\ldots. \tag{1.13}$$

Substitution of (1.11) and (1.12) into (1.10) yields

$$\begin{aligned} S\left[\sum_{i=0}^{\infty} U_i(x,t)\right] &= U(x,0) + uU'(x,0) + \cdots + u^{n-1}U^{n-1}(x,0) - u^n S[RU(x,t)] \\ &\quad - u^n S\left[\sum_{i=0}^{\infty} A_i\right] + u^n S[h(x,t)]. \end{aligned} \tag{1.14}$$

On comparing both sides of (1.14) and by using standard ADM we have:

$$S[U_0(x,t)] = U(x,0) + uU'(x,0) + \cdots + u^{n-1}U^{n-1}(x,0) + u^n S[h(x,t)] = Y(x,u) \tag{1.15}$$

then it follows that

$$\begin{aligned} S[U_1(x,t)] &= -u^n S[RU_0(x,t)] - u^n S[A_0], \\ S[U_2(x,t)] &= -u^n S[RU_1(x,t)] - u^n S[A_1]. \end{aligned} \tag{1.16}$$

In more general, we have

$$S[U_{i+1}(x,t)] = -u^n S[RU_i(x,t)] - u^n S[A_i], \quad i \geq 0. \tag{1.17}$$

On applying the inverse Sumudu transform to (1.15) and (1.17), we get

$$\begin{aligned} U_0(x,t) &= K(x,t), \\ U_{i+1}(x,t) &= -S^{-1}\left[u^n S[RU_i(x,t)] + u^n S[A_i]\right], \quad i \geq 0, \end{aligned} \tag{1.18}$$

where $K(x,t)$ represents the term that is arising from source term and prescribed initial conditions. On using the inverse Sumudu transform to $h(x,t)$ and using the given condition we get

$$\Psi = \Phi + S^{-1}[h(x,t)], \tag{1.19}$$

where the function Ψ, obtained from a term by using the initial condition is given by

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2 + \Psi_3 + \cdots + \Psi_n, \tag{1.20}$$

the terms $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \ldots, \Psi_n$ appears while applying the inverse Sumudu transform on the source term $h(x,t)$ and using the given conditions. We define

$$U_0 = \Psi_k + \cdots + \Psi_{k+r}, \tag{1.21}$$

where $k = 0, 1, \ldots, n$, $r = 0, 1, \ldots, n-k$. Then we verify that U_0 satisfies the original equation (1.7). We now consider the particular form of inhomogeneous nonlinear partial differential equations:

$$LU + RU + NU = h(x,t) \tag{1.22}$$

with the initial condition

$$U(x,0) = f(x), \qquad U_t(x,0) = g(x), \tag{1.23}$$

where $L = \partial^2/\partial t^2$ is second-order differential operator, NU represents a general non-linear differential operator where as $h(x,t)$ is source term. The methodology consists of applying Sumudu transform first on both sides of (1.10) and (1.23),

$$S[U(x,t)] = f(x) + ug(x) - u^2 S[RU] - u^2 S[NU] + u^2 S[h(x,t)]. \tag{1.24}$$

Then by the second step in Sumudu decomposition method and inverse transform as in the previous we have

$$U(x,t) = f(x) + tg(t) - S^{-1}\left[u^2 S[RU] - u^2 S[NU]\right] + S^{-1}\left[u^2 S[h(x,t)]\right]. \tag{1.25}$$

2. Applications

Now in order to illustrate STDM we consider some examples. Consider a nonlinear partial differential equation

$$U_{tt} + U^2 - U_x^2 = 0, \quad t > 0 \tag{2.1}$$

with initial conditions

$$\begin{aligned} U(x,0) &= 0, \\ U_t(x,0) &= e^x. \end{aligned} \tag{2.2}$$

By taking Sumudu transform for (2.1) and (2.2) we obtain

$$S[U(x,t)] = ue^{x} + u^{2}S\left[U_x^2 - U^2\right]. \tag{2.3}$$

By applying the inverse Sumudu transform for (2.3), we get

$$[U(x,t)] = te^{x} + S^{-1}\left[u^{2}S\left[U_x^2 - U^2\right]\right] \tag{2.4}$$

which assumes a series solution of the function $U(x,t)$ and is given by

$$U(x,t) = \sum_{i=0}^{\infty} U_i(x,t). \tag{2.5}$$

Using (2.4) into (2.5) we get

$$\sum_{i=0}^{\infty} U_i(x,t) = te^{x} + S^{-1}\left[u^{2}S\left[\sum_{i=0}^{\infty} A_i(U) - \sum_{i=0}^{\infty} B_i(U)\right]\right]. \tag{2.6}$$

In (2.6) $A_i(u)$ and $B_i(u)$ are Adomian polynomials that represents nonlinear terms. So Adomian polynomials are given as follows:

$$\begin{aligned} &\sum_{i=0}^{\infty} A_i(U) = U_x^2, \\ &\sum_{i=0}^{\infty} A_i(U) = U^2. \end{aligned} \tag{2.7}$$

The few components of the Adomian polynomials are given as follows:

$$\begin{gathered} A_0(U) = U_{0x}^2, \qquad A_1(U) = 2U_{0x}U_{1x}, \qquad A_i(U) = \sum_{r=0}^{i} U_{rx}U_{i-rx}, \\ B_0(U) = U_0^2, \qquad B_1(U) = 2U_0U_1, \qquad B_i(U)\sum_{r=0}^{i} U_r U_{i-r}. \end{gathered} \tag{2.8}$$

From the above equations we obtain

$$\begin{gathered} U_0(x,t) = te^{x}, \\ U_{i+1}(x,t) = S^{-1}\left[S\left[\sum_{i=0}^{\infty} A_i(U) - \sum_{i=0}^{\infty} B_i(U)\right]\right], \quad n \geq 0. \end{gathered} \tag{2.9}$$

Then the first few terms of $U_i(x,t)$ follow immediately upon setting

$$\begin{aligned}
U_1(x,t) &= S^{-1}\left[u^2 S\left[\sum_{i=0}^{\infty} A_0(U) - \sum_{i=0}^{\infty} B_0(U)\right]\right] \\
&= S^{-1}\left[u^2 S\left[U_{0x}^2 - U_0^2\right]\right] = S^{-1}\left[u^2 S\left[t^2 e^{2x} - t^2 e^{2x}\right]\right] \\
&= S^{-1}\left[u^2 S[0]\right] = 0.
\end{aligned} \tag{2.10}$$

Therefore the solution obtained by LDM is given as follows:

$$U(x,t) = \sum_{i=0}^{\infty} U_i(x,t) = te^{x}. \tag{2.11}$$

Example 2.1. Consider the system of nonlinear coupled partial differential equation

$$\begin{aligned}
U_t(x,y,t) - V_x W_y &= 1, \\
V_t(x,y,t) - W_x U_y &= 5, \\
W_t(x,y,t) - U_x V_y &= 5
\end{aligned} \tag{2.12}$$

with initial conditions

$$\begin{aligned}
U(x,y,0) &= x + 2y, \\
V(x,y,0) &= x - 2y, \\
W(x,y,0) &= -x + 2y.
\end{aligned} \tag{2.13}$$

Applying the Sumudu transform (denoted by S) we have

$$\begin{aligned}
U(x,y,u) &= x + 2y + u + uS[V_x W_y], \\
V(x,y,u) &= x - 2y + 5u + uS[W_x U_y], \\
W(x,y,u) &= -x + 2y + 5u + uS[U_x V_y].
\end{aligned} \tag{2.14}$$

On using inverse Sumudu transform in (2.14), our required recursive relation is given by

$$\begin{aligned}
U(x,y,t) &= x + 2y + t + S^{-1}[uS[V_x W_y]], \\
V(x,y,t) &= x - 2y + 5t + S^{-1}[uS[W_x U_y]], \\
U(x,y,t) &= -x + 2y + 5t + S^{-1}[uS[U_x V_y]].
\end{aligned} \tag{2.15}$$

The recursive relations are

$$\begin{aligned}
U_0(x,y,t) &= t + x + 2y, \\
U_{i+1}(x,y,t) &= S^{-1}\left[uS\left[\sum_{i=0}^{\infty} C_i(V,W)\right]\right], \quad i \geq 0, \\
V_0(x,y,t) &= 5t + x - 2y, \\
V_{i+1}(x,y,t) &= S^{-1}\left[uS\left[\sum_{i=0}^{\infty} D_i(U,W)\right]\right], \quad i \geq 0, \\
W_0(x,y,t) &= 5t - x + 2y, \\
W_{i+1}(x,y,t) &= S^{-1}\left[uS\left[\sum_{i=0}^{\infty} E_i(U,V)\right]\right], \quad i \geq 0,
\end{aligned} \tag{2.16}$$

where $C_i(V,W)$, $D_i(U,W)$, and $E_i(U,V)$ are Adomian polynomials representing the nonlinear terms [1] in above equations. The few components of Adomian polynomials are given as follows

$$\begin{aligned}
C_0(V,W) &= V_{0x}W_{0y}, \\
C_1(V,W) &= V_{1x}W_{0y} + V_{0x}W_{1y}, \\
&\vdots \\
C_i(V,W) &= \sum_{r=0}^{i} V_{rx}W_{i-ry}, \\
D_0(U,W) &= U_{0y}W_{0x}, \\
D_1(U,W) &= U_{1y}W_{0x} + W_{1x}U_{0y}, \\
&\vdots \\
D_i(U,W) &= \sum_{r=0}^{i} W_{rx}U_{i-ry}, \\
E_0(U,V) &= U_{0x}V_{0y}, \\
E_1(U,V) &= U_{1x}V_{0y} + U_{0x}V_{1y}, \\
&\vdots \\
E_i(V,W) &= \sum_{r=0}^{i} U_{rx}V_{i-ry}.
\end{aligned} \tag{2.17}$$

By this recursive relation we can find other components of the solution

$$\begin{aligned}
U_1(x,y,t) &= S^{-1}[uS[C_0(V,W)]] = S^{-1}\left[uS\left[V_{0x}W_{0y}\right]\right] = S^{-1}[uS[(1)(2)]] = 2t,\\
V_1(x,y,t) &= S^{-1}[uS[D_0(U,W)]] = S^{-1}\left[uS\left[W_{0x}U_{0y}\right]\right] = S^{-1}[uS[(-1)(2)]] = -2t,\\
W_1(x,y,t) &= S^{-1}[uS[E_0(U,V)]] = S^{-1}\left[uS\left[U_{0x}V_{0y}\right]\right] = S^{-1}[uS[(1)(-2)]] = -2t,\\
U_2(x,y,t) &= S^{-1}[uS[C_1(V,W)]] = S^{-1}\left[uS\left[V_{1x}W_{0y}+V_{0x}W_{1y}\right]\right] = 0,\\
V_2(x,y,t) &= S^{-1}[uS[D_1(U,W)]] = S^{-1}\left[uS\left[U_{0y}W_{1x}+U_{1y}W_{0x}\right]\right] = 0,\\
W_2(x,y,t) &= S^{-1}[uS[D_1(U,V)]] = S^{-1}\left[uS\left[U_{1x}V_{0y}+U_{0x}V_{1y}\right]\right] = 0.
\end{aligned}\tag{2.18}$$

The solution of above system is given by

$$\begin{aligned}
U(x,y,t) &= \sum_{i=0}^{\infty}U_i(x,y,t) = x+2y+3t,\\
V(x,y,t) &= \sum_{i=0}^{\infty}V_i(x,y,t) = x-2y+3t,\\
W(x,y,t) &= \sum_{i=0}^{\infty}W_i(x,y,t) = -x+2y+3t.
\end{aligned}\tag{2.19}$$

Example 2.2. Consider the following homogeneous linear system of PDEs:

$$\begin{aligned}
U_t(x,t) - V_x(x,t) - (U-V) &= 2,\\
V_t(x,t) + U_x(x,t) - (U-V) &= 2,
\end{aligned}\tag{2.20}$$

with initial conditions

$$U(x,0) = 1+e^x, \qquad V(x,0) = -1+e^x. \tag{2.21}$$

Taking the Sumudu transform on both sides of (2.20), then by using the differentiation property of Sumudu transform and initial conditions, (2.21) gives

$$\begin{aligned}
S[U(x,t)] &= 1+e^x-2u+uS[V_x]+uS[U-V],\\
S[V(x,t)] &= -1+e^x-2u-uS[U_x]+uS[U-V],
\end{aligned}\tag{2.22}$$

$$U_x(x,t) = \sum_{i=0}^{\infty}U_{xi}(x,t), \qquad V_x(x,t) = \sum_{i=0}^{\infty}V_{xi}(x,t). \tag{2.23}$$

Using the decomposition series (2.23) for the linear terms $U(x,t)$, $V(x,t)$ and U_x, V_x, we obtain

$$S\left[\sum_{i=0}^{\infty} U_i(x,t)\right] = 1 + e^x - 2u + uS\left[\sum_{i=0}^{\infty} V_{ix}\right] + uS\left[\sum_{i=0}^{\infty} U_i - \sum_{i=0}^{\infty} V_i\right],$$
$$S\left[\sum_{i=0}^{\infty} V_i(x,t)\right] = -1 + e^x - 2u - uS\left[\sum_{i=0}^{\infty} U_{ix}\right] + uS\left[\sum_{i=0}^{\infty} U_i - \sum_{i=0}^{\infty} V_i\right]. \tag{2.24}$$

The SADM presents the recursive relations

$$\begin{aligned} S[U_0(x,t)] &= 1 + e^x - 2u, \\ S[V_0(x,t)] &= -1 + e^x - 2u, \\ S[U_{i+1}] &= uS[V_{ix}] + uS[U_i - V_i], \quad i \ge 0, \\ S[V_{i+1}] &= -uS[U_{ix}] + uS[U_i - V_i], \quad i \ge 0. \end{aligned} \tag{2.25}$$

Taking the inverse Sumudu transform of both sides of (2.25) we have

$$\begin{aligned} U_0(x,t) &= 1 + e^x - 2t, \\ V_0(x,t) &= -1 + e^x - 2t, \\ U_1 &= S^{-1}[uS[V_{0x}] + uS[U_0 - V_0]] = S^{-1}[ue^x + 2u] = te^x + 2t, \\ V_1 &= S^{-1}[-uS[U_{0x}] + uS[U_0 - V_0]] = S^{-1}[-ue^x + 2u] = -te^x + 2t, \\ U_2 &= S^{-1}\left[u^2 e^x\right] = \frac{t^2}{2!}e^x, \\ V_2 &= S^{-1}\left[u^2 e^x\right] = \frac{t^2}{2!}e^x, \end{aligned} \tag{2.26}$$

and so on for other components. Using (1.11), the series solutions are given by

$$\begin{aligned} U(x,t) &= 1 + e^x\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!}\cdots\right), \\ V(x,t) &= -1 + e^x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!}\cdots\right). \end{aligned} \tag{2.27}$$

Then the solutions follows

$$\begin{aligned} U(x,t) &= 1 + e^{x+t}, \\ V(x,t) &= -1 + e^{x-t}. \end{aligned} \tag{2.28}$$

Example 2.3. Consider the system of nonlinear partial differential equations

$$U_t + VU_x + U = 1,$$
$$V_t - UV_x - V = 1 \tag{2.29}$$

with initial conditions

$$U(x,0) = e^x, \qquad V(x,0) = e^{-x}. \tag{2.30}$$

On using Sumudu transform on both sides of (2.29), and by taking Sumudu transform for the initial conditions of (2.30) we get

$$S[U(x,t)] = e^x + u - uS[VU_x] - uS[U],$$
$$S[V(x,t)] = e^x + u + uS[UV_x] + uS[V]. \tag{2.31}$$

Similar to the previous example, we rewrite $U(x,t)$ and $V(x,t)$ by the infinite series (1.11), then inserting these series into both sides of (2.31) yields

$$S\left[\sum_{i=0}^{\infty} U_i(x,t)\right] = e^x + u - uS\left[\sum_{i=0}^{\infty} A_i\right] - uS\left[\sum_{i=0}^{\infty} U_i\right],$$
$$S\left[\sum_{i=0}^{\infty} V_i(x,t)\right] = e^{-x} + u + uS\left[\sum_{i=0}^{\infty} B_i\right] - uS\left[\sum_{i=0}^{\infty} V_i\right], \tag{2.32}$$

where the terms A_i and B_i are handled with the help of Adomian polynomials by (1.12) that represent the nonlinear terms VU_x and UV_x, respectively. We have a few terms of the Adomian polynomials for VU_x and UV_x which are given by

$$A_0 = U_{0x}V_0, \qquad A_1 = U_{0x}V_1 + U_{1x}V_0,$$
$$A_2 = U_{0x}V_2 + U_{1x}V_1 + U_{2x}V_0,$$
$$\vdots$$
$$B_0 = V_{0x}U_0, \qquad B_1 = V_{0x}U_1 + V_{1x}U_0, \tag{2.33}$$
$$B_2 = V_{0x}U_2 + V_{1x}U_1 + V_{2x}U_0,$$
$$\vdots$$

By taking the inverse Sumudu transform we have

$$U_0 = e^x + t,\qquad V_0 = e^{-x} + t, \tag{2.34}$$

$$U_{i+1} = S^{-1}[u] - S^{-1}[uS[A_i]] - S^{-1}[uS[U_i]],\qquad V_{i+1} = S^{-1}[u] + S^{-1}[uS[B_i]] + S^{-1}[uS[V_i]]. \tag{2.35}$$

Using the inverse Sumudu transform on (2.35) we have

$$\begin{aligned} U_1 &= -t - \frac{t^2}{2!} - te^x - \frac{t^2}{2!}e^x, \\ V_1 &= -t - \frac{t^2}{2!} + te^{-x} - \frac{t^2}{2!}e^{-x}, \\ U_2 &= \frac{t^2}{2!} + \frac{t^2}{2!}e^x \cdots, \\ V_2 &= \frac{t^2}{2!} + \frac{t^2}{2!}e^{-x} \cdots. \end{aligned} \tag{2.36}$$

The rest terms can be determined in the same way. Therefore, the series solutions are given by

$$\begin{aligned} U(x,t) &= e^x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!}\cdots\right), \\ V(x,t) &= e^{-x}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!}\cdots\right). \end{aligned} \tag{2.37}$$

Then the solution for the above system is as follows:

$$U(x,t) = e^{x-t}, \qquad V(x,t) = e^{-x+t}. \tag{2.38}$$

3. Conclusion

The Sumudu transform-Adomian decomposition method has been applied to linear and nonlinear systems of partial differential equations. Three examples have been presented, this method shows that it is very useful and reliable for any nonlinear partial differential equation systems. Therefore, this method can be applied to many complicated linear and nonlinear PDEs.

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