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Research Article

Controlling Chaos in Permanent Magnet Synchronous Motor Control System via Fuzzy Guaranteed Cost Controller

Yi-You Hou

Department of Electrical Engineering, Far East University, Tainan 74448, Taiwan

Correspondence should be addressed to Yi-You Hou, hou.yi_you@msa.hinet.net

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This paper investigates the guaranteed cost control of chaos problem in permanent magnet synchronous motor (PMSM) via Takagi-Sugeno (T-S) fuzzy method approach. Based on Lyapunov stability theory and linear matrix inequality (LMI) technique, a state feedback controller is proposed to stabilize the PMSM systems. An illustrative example is provided to verify the validity of the results developed in this paper.

1. Introduction

The permanent magnet synchronous motor (PMSM) is an important role in industrial applications due to its simple structure, high efficiency, high power density, and low maintenance cost [1–3]. However, the dynamic characteristics and stability analysis of PMSM has emerged as a new and attractive research field, such as bifurcation, chaos, and limit cycle dynamic behaviors [4–9]. Moreover, many profound theories and methodologies [10–16] have been developed to deal with this issue. In [4], the adaptive dynamic surface control (DSC) of PMSM has been presented. In [5, 6], the authors had derived some feedback control design methods for stability of PMSM in their results. Some control methods had studied to stabilize the PMSM systems, such as sliding mode control (SMC) [7], differential geometry method [8], passivity control [9, 10], sensorless control [11–14], Lyapunov exponents (LEs) placement [15], and fuzzy control [16].

Takagi-Sugeno (T-S) fuzzy concept was introduced by the pioneering work of Takagi and Sugeno [17] and has been successfully and effectively used in complex nonlinear systems [18]. The main feature of T-S fuzzy model is that a nonlinear system can be approximated by a set of T-S linear models. The overall fuzzy model of complex nonlinear systems is achieved by fuzzy "blending" of the set of T-S linear models. Therefore, the controller design and the stability analysis of nonlinear systems can be analyzed via T-S fuzzy models and the so-called parallel distributed compensation (PDC) scheme [19, 20].

This paper contributes to the development of the state-feedback control design for PMSM. Based on Lyapunov stability theory and LMI technique, the stability conditions of PMSM are analyzed. Finally, an example is given to illustrate the usefulness of the obtained results.

2. Problem Formulation and Main Results

Based on d-q axis, the dynamics of permanent synchronous motor plant can be described by the following differential equation [15]:

$$\begin{aligned}\frac{di_d}{dt} &= \frac{u_d - R_1 i_d + wL_q i_q}{L_d},\\ \frac{di_q}{dt} &= \frac{u_q - R_1 i_q + wL_q i_q - w\psi_r}{L_q},\\ \frac{dw}{dt} &= \frac{n_p\psi_r i_q + n_p(L_d - L_q)i_d i_q - T_L - \beta w}{J},\end{aligned} \tag{2.1}$$

where i_d, i_q, and w are state variables, which denote d, q axis stator currents, and w is motor angular speed, respectively. T_L, u_d, and u_q are the external load torque, the direct- and quadrature-axis stator voltage components, respectively. J is the polar moment of inertia, β is the viscous damping coefficient, R_1 is the stator winding resistance, and L_d and L_q are the direct- and quadrature-axis stator inductors, respectively. ψ_r is the permanent magnet flux, and n_p is the number of pole pairs. By applying the affine transformation $x = \lambda\tilde{x}$, $t = \tau\tilde{t}$, $x = [i_d \;\; i_q \;\; w]^T$, $\tilde{x} = [\tilde{i}_d \;\; \tilde{i}_q \;\; \tilde{w}]^T$, $b = L_q/L_d$, $k = \beta/(n_p\tau\psi_r)$, $\tau = L_q/R$, and $\lambda = \mathrm{diag}[\lambda_d \;\; \lambda_q \;\; \lambda_w] = \mathrm{diag}[bk \;\; k \;\; 1/\tau]$, system (2.1) can be transformed as follows:

$$\begin{aligned}\frac{d\tilde{i}_d}{dt} &= -\tilde{i}_d + \tilde{i}_q\tilde{w} + \tilde{u}_d,\\ \frac{d\tilde{i}_q}{dt} &= -\tilde{i}_q - \tilde{i}_d\tilde{w} + \gamma\tilde{w} + \tilde{u}_q,\\ \frac{d\tilde{w}}{dt} &= \sigma\left(\tilde{i}_q - \tilde{w}\right) - \tilde{T}_L,\end{aligned} \tag{2.2}$$

where $\tilde{u}_d = u_d/kR$, $\gamma = -\psi_r/kL_q$, $\tilde{u}_q = u_q/kR$, $\sigma = \beta\tau/J$, and $\tilde{T}_L = \tau^2 T_L/J$.

In the system (2.2), the external inputs are set to zero, namely, $\tilde{T}_L = \tilde{u}_d = \tilde{u}_q = 0$. Then, the system (2.2) becomes

$$\begin{aligned}
\frac{d\tilde{i}_d}{dt} &= -\tilde{i}_d + \tilde{i}_q\tilde{w}, \\
\frac{d\tilde{i}_q}{dt} &= -\tilde{i}_q - \tilde{i}_d\tilde{w} + \gamma\tilde{w}, \\
\frac{d\tilde{w}}{dt} &= \sigma\left(\tilde{i}_q - \tilde{w}\right),
\end{aligned} \tag{2.3}$$

or

$$\begin{aligned}
\dot{x}_1(t) &= -x_1(t) + x_2(t)x_3(t), \\
\dot{x}_2(t) &= -x_2(t) - x_1(t)x_3(t) + \gamma \cdot x_3(t), \\
\dot{x}_3(t) &= \sigma \cdot [x_1(t) - x_3(t)],
\end{aligned} \tag{2.4}$$

where $x_1 = \tilde{i}_d$, $x_2 = \tilde{i}_q$, $x_3 = \tilde{w}$.

To investigate the control design of system (2.4), let the system's state vector $x(t) = [x_1 \;\; x_2 \;\; x_3]^T$ and the control input vector be $u(t)$. Then, the state equations of PMSM can be represented as follows:

$$\dot{x}(t) = A(x(t))x(t) + Bu(t), \tag{2.5}$$

where $x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t)]^T$, $A(x(t)) = \begin{bmatrix} -1 & x_3(t) & 0 \\ -x_3(t) & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$.

The continuous fuzzy system was proposed to represent a nonlinear system [17]. The system dynamics can be captured by a set of fuzzy rules which characterize local correlations in the state space. Each local dynamic described by the fuzzy IF-THEN rule has the property of linear input-output relation. Based on the T-S fuzzy model concept, the nonlinear PMSM system can be expressed as follows.

Model rule i:

If $z_1(t)$ is M_{i1} and $\cdots$ $z_r(t)$ is M_{ir}, then

$$\dot{x}(t) = A_i x(t) + B_i u(t), \tag{2.6}$$

where $z_1(t), z_2(t), \ldots, z_r(t)$ are known premise variables, $M_{ij}, i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, r\}$ is the fuzzy set, and m is the number of model rules; $x(t)$ is the state vector, and $u(t)$ is input vector. The matrices A_i and B are known constant matrices with appropriate dimensions. Given a pair of $(x(t), u(t))$, the final outputs of the fuzzy system are inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{m} w_i(z(t)) \cdot \{A_i x(t) B_i u(t)\}}{\sum_{i=1}^{m} w_i(z(t))} \\ &= \sum_{i=1}^{m} \eta_i(z(t)) \cdot \{A_i x(t) + B_i u(t)\},\end{aligned} \tag{2.7}$$

where $z(t) = [z_1(t)\ \ z_2(t)\ \ \cdots\ \ z_r(t)]$, $w_i(z(t)) = \prod_{j=1}^{r} M_{ij}(z_j(t))$, $\eta_i(z(t)) = w_i(z(t))/\sum_{i=1}^{m} w_i(z(t))$. The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. In this paper, we assume that $w_i(z(t)) \geq 0$, $i \in \{1, 2, \ldots, m\}$, and $\sum_{i=1}^{m} w_i(z(t)) > 0$. Therefore, we have $\eta_i(z(t)) \geq 0$, $i \in \{1, 2, \ldots, m\}$ and $\sum_{i=1}^{m} \eta_i(z(t)) = 1$, for all $t \geq 0$.

To derive the main results, we first introduce the cost fuction of system (2.4) as follows:

$$J = \int_0^{\infty} \left[x^T(s) \cdot S_1 \cdot x(s) + u^T(s) \cdot S_2 \cdot u(s)\right] ds, \tag{2.8}$$

where S_1 and S_2 are two given positive definite symmetric matrices with appropriate dimensions. Associated with cost function (2.8), the fuzzy guaranteed cost control is defined as follows.

Definition 2.1 (see [21]). Consider the T-S fuzzy PMSM system (2.6); if there exist a control law $u(t)$ and a positive scalar J^* such that the closed-loop system is stable and the value of cost function (2.8) satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost and $u(t)$ is said to be a guaranteed cost control law for the T-S fuzzy PMSM system (2.6).

This paper aims at designing a guaranteed cost control law for the asymptotic stabilization of the T-S fuzzy PMSM system (2.6). To achieve this control goal, we utilize the concept of PDC [17] scheme and select the fuzzy guaranteed cost controller via state feedback as follows.

Control rule *j*:

If $z_1(t)$ is M_{j1} and $\cdots z_r(t)$ is M_{jr}, then

$$u(t) = -K_j x(t), \quad t \geq 0,\ j = \{1, 2, \ldots, m\}, \tag{2.9}$$

where K_j, $j \in \{1, 2, \ldots, m\}$ are the state feedback gains. Hence, the overall state feedback control law is represented as follows:

$$u(t) = -\sum_{j=1}^{m} \eta_j(z(t)) \cdot K_j x(t), \quad t \geq 0. \tag{2.10}$$

Before proposing the main theorem for determining the feedback gains K_j $(j = 1, 2, \ldots, m)$, a lemma is introduced.

Lemma 2.2 (see [22] (Schur complement)). *For a given matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ *with* $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, *then the following conditions are equivalent:*

(1) $S < 0$,

(2) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Now we present a asymptotic stabilization condition for T-S fuzzy PMSM system (2.6).

Theorem 2.3. *If there exist some positive definite symmetric matrices $\overline{P}$ and matrices $\widehat{K}_j$, $j \in \{1, 2, \ldots, m\}$ such that the following LMI condition holds for all $i, j \in \{1, 2, \ldots, m\}$:*

$$\widetilde{\Phi}_{ij} = \begin{bmatrix} A_i\overline{P} + \overline{P}A_i^T - B_i\widehat{K}_j - \widehat{K}_j^T B_i^T & \overline{P} & \widehat{K}_j^T \\ * & -S_1^{-1} & 0 \\ * & * & -S_2^{-1} \end{bmatrix} < 0, \tag{2.11}$$

then system (2.6) *is asymptotically stabilizable by controller* (2.10). *The stabilizing feedback control gain is given by $K_j = \widehat{K}_j\overline{P}^{-1}$, and the system performance* (2.8) *is bounded by*

$$J \leq J^* = x^T(0)Px(0), \tag{2.12}$$

where $P = \overline{P}^{-1}$.

Proof. Define the Lyapunov functional

$$V(x_t) = x^T(t)Px(t), \tag{2.13}$$

where $V(x_t)$ is a legitimate Lyapunov functional candidate, and P is positive definte symmetric matrices . By the system (2.6) with $\sum_{i=1}^m \eta_i(z(t)) = 1$, the time derivatives of $V(x_t)$, along the trajectories of system (2.6) with (2.8) and (2.10), satisfy

$$\begin{aligned} &\dot{V}(x_t) - \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t))\eta_j(z(t))x^T(t)\left(S_1 + K_j^T S_2 K_j\right)x(t) \\ &= \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t))\eta_j(z(t))x^T(t)\left(PA_i + A_i^T P - K_j^T B_i^T P - PB_iK_j\right)x(t) \\ &= \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t))\eta_j(z(t))x^T(t)\left(PA_i + A_i^T P - K_j^T B_i^T P - PB_iK_j + S_1 + K_j^T S_2 K_j\right)x(t) \\ &\leq \sum_{i=1}^m \sum_{j=1}^m \eta_i(z(t))\eta_j(z(t))x^T(t)\Phi_{ij}x(t). \end{aligned} \tag{2.14}$$

In order to guarantee $\dot{V}(x_t) - \sum_{i=1}^{m}\sum_{j=1}^{m}\eta_i(z(t))\eta_j(z(t))x^T(t)(S_1 + K_j^T S_2 K_j)x(t) < 0$, we need to satisfy $\Phi_{ij} < 0$. By Lemma 2.2, premultiplying, and postmultiplying the Φ_{ij} in (2.14) by $P^{-1} > 0$, $\Phi_{ij} < 0$ are equivalent to $\widetilde{\Phi}_{ij} < 0$ in (2.11), then we can obtain the following:

$$\begin{aligned}\dot{V}(x_t) &\leq -\sum_{i=1}^{m}\sum_{j=1}^{m}\eta_i(z(t))\eta_j(z(t))x^T(t)\left(S_1 + K_j^T S_2 K_j\right)x(t)\\ &= -\left(x^T(t)\cdot S_1\cdot x(t) + u(t)\cdot S_2\cdot u(t)\right) < 0.\end{aligned} \tag{2.15}$$

From the inequality (2.15), $\dot{V}(x_t) < 0$, we conclude that system (2.6) with (2.8) is asymptotically stable. Integrating (2.12) from 0 to ∞, we have

$$\int_0^\infty \dot{V}(x_s)ds = \lim_{t\to\infty} V(x_t) - V(x_0) \leq -\int_0^\infty \left[x^T(s)\cdot S_1\cdot x(s) + u^T(s)\cdot S_2\cdot u(s)\right]ds. \tag{2.16}$$

Since that the system (2.6) with (2.8) is asymptotically stable, we can obtain the following results:

$$\lim_{t\to\infty} V(x_t) = 0. \tag{2.17}$$

Consequently, $J = \int_0^\infty [x^T(s)\cdot S_1\cdot x(s) + u^T(s)\cdot S_2\cdot u(s)]ds \leq x^T(0)Px(0) = V(x_0) = J^*$. This completes the proof. □

3. Numerical Simulation and Analysis

In this section, a numerical example is presented to demonstrate and verify the performance of the proposed results. Consider a PMSM as given in (2.1) with the following parameters [23]: $L_d = L_q = L = 14.25\,\mathrm{mH}$, $R_1 = 0.9\,\Omega$, $\psi_r = 0.031\,\mathrm{Nm/A}$, $n_p = 1$, $J = 4.7 \times 10^{-5}\,\mathrm{kg\,m^2}$, $\beta = 0.0162\,\mathrm{N/rad}$, $\gamma = 20$, and $\sigma = 5.46$.

From the simulation result, we can get that $x_3(t)$ is bounded in interval $[-12\ \ 12]$. By solving the equation, M_1 and M_2 are obtained as follows:

$$M_1(x_3(t)) = \frac{1}{2}\left(1 + \frac{x_3(t)}{d}\right), \qquad M_2(x_3(t)) = 1 - M_1(x_3(t)) = \frac{1}{2}\left(1 - \frac{x_3(t)}{d}\right). \tag{3.1}$$

M_1 and M_2 can be interpreted as membership functions of fuzzy sets. Using these fuzzy sets, the nonlinear system with time-varying delays can be expressed by the following T-S fuzzy models.

Rule 1. IF $x_3(t)$ is M_1, then

$$\dot{x}(t) = A_1 x(t) + B_1 u(t). \tag{3.2}$$

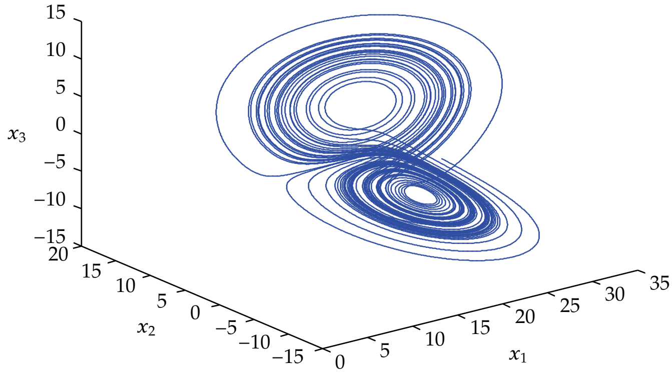

Figure 1: The chaotic attractor of the PMSM system.

Rule 2. IF $x_3(t)$ is M_2, then

$$\dot{x}(t) = A_2 x(t) + B_2 u(t), \tag{3.3}$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T, \qquad A_1 = \begin{bmatrix} -1 & -12 & 0 \\ 12 & -1 & 20 \\ 5.46 & 0 & -5.46 \end{bmatrix}, \tag{3.4}$$

$$A_2 = \begin{bmatrix} -1 & 12 & 0 \\ -12 & -1 & 20 \\ 5.46 & 0 & -5.46 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{3.5}$$

By the theorem, the stabilizing fuzzy control gains are given by $K_1 = K_2 = [3.968 \quad 19.902 \quad 77.990]$.

Consequently, the minimal guaranteed cost is $J^* = 5.443 \times 10^{-11}$. The simulation was done with a four-order Runge-Kutta integration algorithm in MATLAB 7 with a step size of 0.0001. The simulation results with initial conditions $x(0) = [13.5 \ -5 \ -5]^T$ are shown in Figures 1-2. The chaotic attractor of PMSM system is given in Figure 1. The frequency power spectrum of the PMSM system variables is illustrated in Figure 2. The system state responses trajectory of controller design is shown in Figure 3. Figure 4 depicts the time responses of the control input of $u(t)$. When $t = 20$ sec, it is obvious that the feedback control gain can guarantee stability of PMSM systems. From the simulation results, it is shown that the proposed controller works well to guarantee stability.

4. Conclusion

We have presented the solutions to the guaranteed cost control of chaos problem via the Takagi-Sugeno fuzzy control for PMSM system. Based on Lyapunov stability theory and LMI technique, the guaranteed cost control gains can be easily obtained through a convex

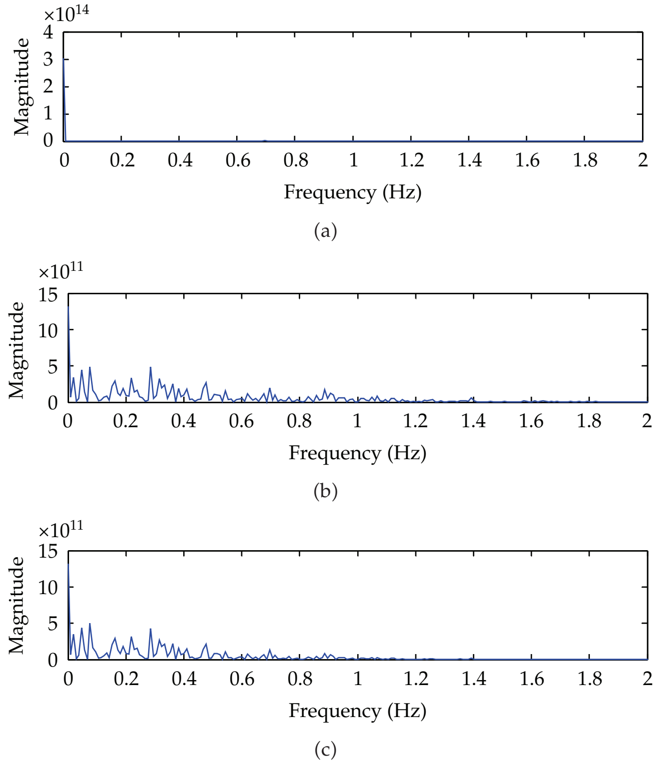

Figure 2: Power spectrum of the state variables $x_1(t)$, $x_2(t)$, and $x_3(t)$ of the PMSM system.

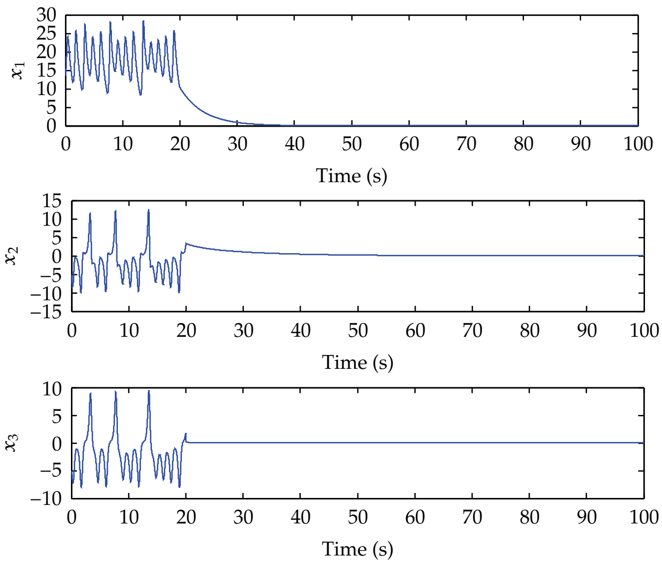

Figure 3: The state responses of the controlled PMSM system.

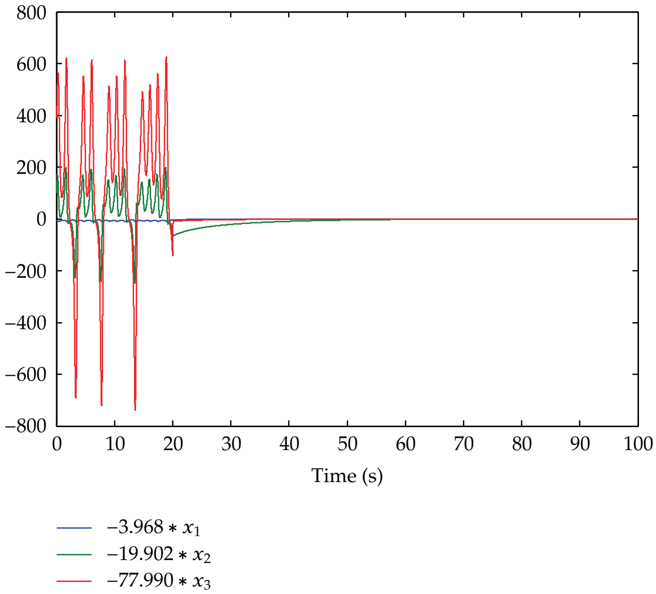

Figure 4: Time response of control input $u(t)$.

optimization problem. Finally, a numerical example shows the validity and superiority of the developed result. The future work will extend the proposed method to the underlying systems with noise and disturbances effects, like noise or disturbances, uncertainties effects, and robustness to time-varying delays. Also, the future application in the experiment will be included.

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