

Research Article

Synchronization of Chaotic Neural Networks with Leakage Delay and Mixed Time-Varying Delays via Sampled-Data Control

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This paper investigates the synchronization problem for neural networks with leakage delay and both discrete and distributed time-varying delays under sampled-data control. By employing the Lyapunov functional method and using the matrix inequality techniques, a delay-dependent LMIs criterion is given to ensure that the master systems and the slave systems are synchronous. An example with simulations is given to show the effectiveness of the proposed criterion.

1. Introduction

Since the pioneering works of Pecora and Carroll [1], the synchronization of chaotic systems has received considerable attention due to its potential applications in biology, chemistry, secret communication, cryptography, nonlinear oscillation synchronization, and some other nonlinear fields [2]. It has been shown that the neural networks can exhibit chaotic behavior [3]. Therefore, it has a wider significance to study the problem on the synchronization of chaotic neural networks.

In the past decades, some works dealing with the synchronization of neural networks have also appeared; for example, see [4–22] and references therein. In [4], authors discussed the synchronization and computation in a chaotic neural network. In [7], the local synchronization and global exponential stability for an array of linearly coupled identical connected neural networks with delays were investigated without assuming that the coupling matrix is symmetric or irreducible; the linear matrix inequality approach was used to judge synchronization with global convergence property. In [8], authors presented an adaptive synchronization scheme between two different kinds of delayed chaotic neural networks with partly unknown parameters. An adaptive controller was designed to guarantee the global asymptotic

synchronization of state trajectories for two different chaotic neural networks with time delay. In [9], the concept of μ -synchronization was introduced; some sufficient conditions were derived for the global μ -synchronization for the linearly coupled neural networks with delayed couplings, where the intrinsic systems are recurrently connected neural networks with unbounded time-varying delays, and the couplings include instant couplings and unbounded delayed couplings. In [10], authors proposed a general array model of coupled delayed neural networks with hybrid coupling, which is composed of constant coupling, discretedelay coupling, and distributed-delay coupling. Based on the Lyapunov functional method and Kronecker product properties, several sufficient conditions were established to ensure global exponential synchronization based on the design of the coupling matrices, the inner linking matrices, and/or some free matrices representing the relationships between the system matrices. The conditions are expressed within the framework of linear matrix inequalities, which can be easily computed by the interior-point method. In addition, a typical chaotic cellular neural network was used as the node in the array to illustrate the effectiveness and advantages of the theoretical results. In [11], the globally robust synchronization problem was investigated for an array of coupled neural networks with uncertain parameters and

time delays. Both the cases of linear coupling and nonlinear coupling were simultaneously taken into account. Several criteria for checking the robust exponential synchronization were given for the considered coupled neural networks. In [12], authors presented a new linear matrix inequality-based approach to an H^{∞} output feedback control problem of master-slave synchronization of artificial neural networks with uncertain time delay. In [13], the problem of feedback controller design to achieve synchronization for neural network of neutral type with stochastic perturbation was considered. Based on Lyapunov method and LMI framework, a criterion for master-slave synchronization was obtained. In [16], authors investigated the globally exponential synchronization for linearly coupled neural networks with timevarying delay and impulsive disturbances. Since the impulsive effects discussed were regarded as disturbances, the impulses should not happen too frequently. The concept of average impulsive interval was used to formalize this phenomenon. By referring to an impulsive delay differential inequality, a criterion for the globally exponential synchronization of linearly coupled neural networks with impulsive disturbances was given. In [18], the projective synchronization between two continuous-time delayed neural systems with timevarying delay was investigated. A sufficient condition for synchronization of the coupled systems with modulated delay was presented analytically with the help of the Krasovskii-Lyapunov approach. In [19], the problem of guaranteed cost control for exponential synchronization of cellular neural networks with interval nondifferentiable and distributed time-varying delays via hybrid feedback control was considered. Several delay-dependent sufficient conditions for the exponential synchronization were obtained. In [20], authors studied the synchronization in an array of coupled neural networks with Markovian jumping and random coupling strength. By designing a novel Lyapunov function and using inequality techniques and the properties of random variables, several delay-dependent synchronization criteria were derived for the coupled networks of continuous-time version. Discrete-time analogues of the continuous-time networks were also formulated and studied. In [21], authors considered adaptive synchronization of chaotic Cohen-Grossberg neural networks with mixed time delays. In [22], pth moment exponential synchronization for stochastic delayed Cohen-Grossberg neural networks with Markovian switching was investigated.

On the other hand, with the development of networked control systems, sampled-data control in the presence of a constant input delay has been an important research area in recent years, because networked control systems are usually modeled as sampled-data systems under variable sampling with an additional network-induced delay [23]. There are some results dealing with the synchronization problem using sampled-data control; for example, see [23–38] and references therein. In [23], a new approach, the input delay approach, has been proposed that can deal well with the sampled-data control problems. The main idea of this approach is to convert the considered sampling period into a time-varying but bounded delay and then accomplish the sampled-data control or state estimation tasks by using

the existing theory of time-delayed systems. In [24], the synchronization of chaotic system using a sampled-data fuzzy controller was studied. In [25–27], the synchronization for chaotic Lur'e systems using sampled-data control was investigated; several criteria were given to ensure that the master systems synchronize with the slave systems by using Lyapunov-Krasovskii functional and LMI approach. In [28– 35], authors discussed the synchronization of chaotic system and complex networks by using sampled-data control. In [36–38], the synchronization of neural networks with timevarying delays was considered; by using sampled-data control method, several criteria for checking the synchronization were obtained. To the best of the authors knowledge, there is no results on the problem of the synchronization for neural networks with leakage delay and both discrete and distributed time-varying delays [38]. Therefore, there is a need to further extend the synchronization results reported in [38].

Motivated by the previous discussions, the objective of this paper is to study the synchronization for neural networks with leakage delay and both discrete and distributed timevarying delays by using sampled-data control approach. The obtained sufficient conditions do not require the differentiability of time-varying delays and are expressed in terms of linear matrix inequalities, which can be checked numerically using the effective LMI toolbox in MATLAB. An example is given to show the effectiveness and less conservatism of the proposed criterion.

Notations. The notations are quite standard. Throughout this paper, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote, respectively, the *n*-dimensional Euclidean space and the set of all $n \times m$ real matrices. $\|\cdot\|$ refers to the Euclidean vector norm. A^T represents the transpose of matrix *A* and the asterisk "*" in a matrix is used to represent the term which is induced by symmetry. *I* is the identity matrix with compatible dimension. X > Y means that *X* and *Y* are symmetric matrices and that X - Y is positive definite. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Model Description and Preliminaries

Consider the following neural networks with leakage delay and mixed time-varying delays:

$$\dot{x}(t) = -Dx(t - \delta) + Af(x(t)) + Bf(x(t - \tau(t))) + C \int_{t-\sigma(t)}^{t} f(x(s)) ds + J(t),$$
(1)

for $t \ge 0$, where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$ is the state vector of the network at time t, n corresponds to the number of neurons, $D \in \mathbb{R}^{n \times n}$ is a positive diagonal matrix, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n}$, and $C \in \mathbb{R}^{n \times n}$ are the connection weight matrix, the discretely delayed connection weight matrix, respectively, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T \in \mathbb{R}^n$ denotes the neuron activation at time $t, J(t) = (J_1(t), J_2(t), \dots, J_n(t))^T \in \mathbb{R}^n$ is an external input vector, and $\delta, \tau(t)$, and $\sigma(t)$ denote the

leakage delay, discrete time-varying delay and the distributed time-varying delay, respectively.

In this paper, system (1) is regarded as the master system and a slave system for (1) can be described by the following equation:

$$\dot{y}(t) = -Dy(t - \delta) + Af(y(t)) + Bf(y(t - \tau(t))) + C \int_{t-\sigma(t)}^{t} f(y(s)) ds + u(t) + J(t),$$
(2)

where $u(t) \in \mathbb{R}^n$ is the appropriate control input that will be designed in order to obtain a certain control objective.

Throughout this paper, we make the following assumptions.

(H1) For any $j \in \{1, 2, ..., n\}$, there exist constants F_j^- and F_j^+ such that

$$F_j^- \le \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \le F_j^+, \tag{3}$$

for all $\alpha_1 \neq \alpha_2$.

(H2) The leakage delay δ , the discrete time-varying delays $\tau(t)$, and the distributed time-varying delay $\sigma(t)$ satisfy the following conditions:

$$0 \le \delta, \qquad 0 \le \tau(t) \le \tau, \qquad 0 \le \sigma(t) \le \sigma, \qquad (4)$$

where δ , τ , and σ are constants.

By defining the error signal as e(t) = y(t) - x(t), the error system for (1) and (2) can be represented as follows:

$$\dot{e}(t) = -De(t - \delta) + Ag(e(t)) + Bg(e(t - \tau(t))) + C \int_{t-\sigma(t)}^{t} g(e(s)) ds + u(t),$$
(5)

where g(e(t)) = f(y(t)) - f(x(t)).

In many real-world applications, it is difficult to guarantee that the state variables transmitted to controllers are continuous. In addition, in order to make full use of modern computer technique, the sampled-data feedback control is applied to synchronize delayed neural networks [36]. In this paper, the following sampled-data controller is adopted [38]:

$$u(t) = Ke(t_k), \quad t_k \le t < t_{k+1}, \tag{6}$$

where *K* is sampled-data feedback controller gain matrix to be determined, t_k denotes the sampling instant and satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k \to +\infty} t_k = +\infty$. Moreover, the sampling period under consideration is assumed to be bounded by a known constant *h*; that is,

$$t_{k+1} - t_k \le h,\tag{7}$$

for any integer $k \ge 0$, where *h* is a positive scalar and represents the largest sampling interval.

Substituting control law (6) into the error system (5) yields

$$\dot{e}(t) = -De(t - \delta) + Ag(e(t)) + Bg(e(t - \tau(t))) + C \int_{t-\sigma(t)}^{t} g(e(s)) \, ds + Ke(t_k).$$
(8)

Clearly, it is difficult to analyze the synchronization of neural networks based on error system (8) because of the discrete term $e(t_k)$. Therefore, the input delay approach [23] is applied; that is, a sawtooth function is defined as follows:

$$\gamma(t) = t - t_k, \quad t_k \le t < t_{k+1}.$$
 (9)

It can be found from (7) and (9) that $0 \le \gamma(t) < h$ and $\dot{\gamma}(t) = 1$ for $t \ne t_k$.

By substituting (9) into (8), we get that

$$\dot{e}(t) = -De(t - \delta) + Ag(e(t)) + Bg(e(t - \tau(t))) + C \int_{t-\sigma(t)}^{t} g(e(s)) \, ds + Ke(t - \gamma(t)).$$
(10)

The main purpose of this paper is to design controller with the form (6) to ensure that master system (1) synchronizes with slave system (2). In other words, we are interested in finding a feedback gain matrix K such that error system (10) is stable.

To prove our result, the following lemmas that can be found in [39] are necessary.

Lemma 1 (see [39]). For any constant matrix $W \in \mathbb{R}^{m \times m}$, W > 0, scalar 0 < h(t) < h, and vector function $\omega(\cdot) : [0,h] \rightarrow \mathbb{R}^m$ such that the integrations concerned are well defined; then

$$\left(\int_{0}^{h(t)} \omega(s) \, ds\right)^{T} W\left(\int_{0}^{h(t)} \omega(s) \, ds\right)$$

$$\leq h(t) \int_{0}^{h(t)} \omega^{T}(s) \, W\omega(s) \, ds.$$
(11)

Lemma 2 (see [39]). Given constant matrices P, Q, and R, where $P^T = P, Q^T = Q$, then

$$\begin{bmatrix} P & R \\ R^T & -Q \end{bmatrix} < 0 \tag{12}$$

is equivalent to the following conditions:

$$Q > 0, P + RQ^{-1}R^T < 0.$$
 (13)

3. Main Results

Theorem 3. Suppose that (H1) and (H2) hold. If there exist seven symmetric positive definite matrices P_i (i = 1, 2, 3, 4, 5, 6, 7), four positive diagonal matrices W_1 , W_2 , R_1 , and R_2 , and nine matrices X_{11} , X_{12} , X_{22} , Q_1 , Q_2 , Q_3 , Q_4 , Q_5 , and Z such that the following LMIs hold:

$$X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0,$$
 (14)

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{15}$$

 Ω_{17}

0

 $0 \quad Z$

 $0 \quad \Omega_{19} \quad Q_1 B \quad Q_1 C$

 $\Omega_{29} \quad Q_1 B \quad Q_1 C$

 $-P_{7}$

0

(16)

in which $\Omega_{11} = X_{12} + X_{12}^T + P_1 + \delta^2 P_2 + P_3 + P_5 - Q_3 - Q_3^T - Q_5 - Q_5^T - F_3 R_1, \Omega_{12} = X_{11} - F_1 W_1 + F_2 W_2 - Q_1, \Omega_{13} = -X_{12} - Q_1 D_T, \Omega_{17} = -X_{12} - Q_1 D_T$ $Z + Q_5, \Omega_{19} = Q_1 A + F_4 R_1, \Omega_{22} = \tau P_4 + h P_6 - Q_1 - Q_1^T, \Omega_{29} = W_1 - W_2 + Q_1 A, \Omega_{55} = -Q_2 - Q_2^T - F_3 R_2, \Omega_{77} = -Q_4 - Q_4^T, and$ $\Omega_{99} = \sigma^2 P_7 - R_1$, then master system (1) and slave system (2) are synchronous. Moreover, the desired controller gain matrix K in (6) can be given by

$$K = Q_1^{-1} Z. (17)$$

 Ω_{11} Ω_{12}

 $[Q_3 \ Q_5 \ 0]$

 Ω_{13}

 $-Q_1D$

0

0 0 0

0

 X_{22}

 X_{12}

 Q_3

0

Proof. From assumption (H1), we know that

$$\int_{0}^{e_{i}(t)} (g_{i}(s) - F_{i}^{-}s) ds \ge 0,$$

$$F_{i}^{e_{i}(t)}(F_{i}^{+}s - g_{i}(s)) ds \ge 0, \quad i = 1, 2, \dots, n.$$
(18)

Let $W_1 = \text{diag}(w_{11}, w_{12}, \dots, w_{1n})$ and $W_2 = \text{diag}(w_{21}, w_{22}, \dots, w_{2n})$ w_{2n}), and consider the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t),$$
(19)

where

$$\begin{split} V_{1}(t) &= \left[\int_{t-\delta}^{t} \frac{e(t)}{e(s) \, ds} \right]^{T} \left[\begin{matrix} X_{11} & X_{12} \\ * & X_{22} \end{matrix} \right] \left[\int_{t-\delta}^{t} \frac{e(t)}{e(s) \, ds} \right], \\ V_{2}(t) &= 2 \sum_{i=1}^{n} w_{1i} \int_{0}^{e_{i}(t)} \left(g_{i}(s) - F_{i}^{-}s \right) ds \\ &+ 2 \sum_{i=1}^{n} w_{2i} \int_{0}^{e_{i}(t)} \left(F_{i}^{+}s - g_{i}(s) \right) ds, \\ V_{3}(t) &= \int_{t-\delta}^{t} e^{T}(s) P_{1}e(s) \, ds \\ &+ \delta \int_{-\delta}^{0} \int_{t+\xi}^{t} e^{T}(s) P_{2}e(s) \, ds d\xi, \\ V_{4}(t) &= \int_{t-\tau}^{t} e^{T}(s) P_{3}e(s) \, ds \\ &+ \int_{-\tau}^{0} \int_{t+\xi}^{t} \dot{e}^{T}(s) P_{4}\dot{e}(s) \, ds d\xi, \\ V_{5}(t) &= \int_{t-h}^{t} e^{T}(s) P_{5}e(s) \, ds \\ &+ \int_{-h}^{0} \int_{t+\xi}^{t} \dot{e}^{T}(s) P_{6}\dot{e}(s) \, ds d\xi, \\ V_{6}(t) &= \sigma \int_{-\sigma}^{0} \int_{t+\xi}^{t} g^{T}(e(s)) P_{7}g(e(s)) \, ds d\xi. \end{split}$$

Calculating the time derivative of $V_i(t)$ (i = 1, 2, 3, 4, 5, 6), we obtain

$$\dot{V}_{1}(t) = 2 \begin{bmatrix} e(t) \\ \int_{t-\delta}^{t} e(s) \, ds \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t) - e(t-\delta) \end{bmatrix}^{T}$$

$$= e^{T}(t) \left(X_{12} + X_{12}^{T} \right) e(t) + 2e^{T}(t) X_{11} \dot{e}(t)$$

$$- 2e^{T}(t) X_{12} e(t-\delta) + 2e^{T}(t) X_{22} \int_{t-\delta}^{t} e(s) \, ds$$

$$+ 2\dot{e}^{T}(t) X_{12} \int_{t-\delta}^{t} e(s) \, ds$$

$$- 2e^{T}(t-\delta) X_{22} \int_{t-\delta}^{t} e(s) \, ds,$$
(21)

$$\begin{split} \dot{V}_{2}(t) &= 2\dot{e}^{T}(t) W_{1}\left(g\left(e\left(t\right)\right) - F_{1}e\left(t\right)\right) \\ &+ 2\dot{e}^{T}(t) W_{2}\left(F_{2}e\left(t\right) - g\left(e\left(t\right)\right)\right) \\ &= 2e^{T}(t)\left(-F_{1}W_{1} + F_{2}W_{2}\right)\dot{e}\left(t\right) \\ &+ 2\dot{e}^{T}(t)\left(W_{1} - W_{2}\right)g\left(e\left(t\right)\right), \end{split}$$
(22)
$$&+ 2\dot{e}^{T}(t)\left(P_{1} + \delta^{2}P_{2}\right)e\left(t\right) - e^{T}\left(t - \delta\right)P_{1}e\left(t - \delta\right) \\ &- \delta \int_{t-\delta}^{t} e^{T}\left(s\right)P_{2}e\left(s\right)ds \\ &\leq e^{T}\left(t\right)\left(P_{1} + \delta^{2}P_{2}\right)e\left(t\right) - e^{T}\left(t - \delta\right)P_{1}e\left(t - \delta\right) \\ &- \left(\int_{t-\delta}^{t} e\left(s\right)ds\right)^{T}P_{2}\left(\int_{t-\delta}^{t} e\left(s\right)ds\right), \end{aligned}$$
(23)
$$&- \left(\int_{t-\delta}^{t} e\left(s\right)ds\right)^{T}P_{2}\left(\int_{t-\delta}^{t} e\left(s\right)ds\right), \end{aligned}$$
(24)
$$&+ \tau \dot{e}^{T}\left(t\right)P_{4}\dot{e}\left(t\right) - \int_{t-\tau}^{t} \dot{e}^{T}\left(s\right)P_{4}\dot{e}\left(s\right)ds, \end{aligned}$$
(25)

$$\dot{V}_{6}(t) = \sigma^{2} g^{T}(e(t)) P_{7}g(e(t))$$

$$-\sigma \int_{t-\sigma}^{t} g^{T}(e(s)) P_{7}g(e(s)) ds$$

$$\leq \sigma^{2} g^{T}(e(t)) P_{7}g(e(t))$$

$$-\sigma(t) \int_{t-\sigma(t)}^{t} g^{T}(e(s)) P_{7}g(e(s)) ds$$

$$\leq \sigma^{2} g^{T}(e(t)) P_{7}g(e(t))$$

$$-\left(\int_{t-\sigma(t)}^{t} g(e(s)) ds\right)^{T} P_{7}\left(\int_{t-\sigma(t)}^{t} g(e(s)) ds\right).$$
(26)

In deriving inequalities (23) and (26), we have made use of Lemma 1. It follows from inequalities (21)–(26) that

 \dot{V}

$$\begin{aligned} (t) &\leq e^{T} (t) \left(X_{12} + X_{12}^{T} + P_{1} + \delta^{2} P_{2} + P_{3} + P_{5} \right) e(t) \\ &+ 2e^{T} (t) \left(X_{11} - F_{1} W_{1} + F_{2} W_{2} \right) \dot{e}(t) \\ &- 2e^{T} (t) X_{12} e(t - \delta) + 2e^{T} (t) X_{22} \\ &\times \int_{t-\delta}^{t} e(s) ds + \dot{e}^{T} (t) (\tau P_{4} + h P_{6}) \dot{e}(t) + 2\dot{e}^{T} (t) \\ &\times X_{12} \int_{t-\delta}^{t} e(s) ds + 2\dot{e}^{T} (t) (W_{1} - W_{2}) g(e(t)) \\ &- e^{T} (t - \delta) P_{1} e(t - \delta) - 2e^{T} (t - \delta) X_{22} \\ &\times \int_{t-\delta}^{t} e(s) ds - e^{T} (t - \tau) P_{3} e(t - \tau) \\ &- \left(\int_{t-\delta}^{t} e(s) ds \right)^{T} P_{2} \left(\int_{t-\delta}^{t} e(s) ds \right) \\ &- e^{T} (t - h) P_{5} e(t - h) \\ &- \int_{t-\tau}^{t} \dot{e}^{T} (s) P_{4} \dot{e}(s) ds - \int_{t-h}^{t} \dot{e}^{T} (s) P_{6} \dot{e}(s) ds \\ &+ \sigma^{2} g^{T} (e(t)) P_{7} g(e(t)) \\ &- \left(\int_{t-\sigma(t)}^{t} g(e(s)) ds \right)^{T} P_{7} \left(\int_{t-\sigma(t)}^{t} g(e(s)) ds \right). \end{aligned}$$

From model (10), we have

$$0 = 2 \left(e^{T}(t) + \dot{e}^{T}(t) \right) Q_{1}$$

$$\times \left[-\dot{e}(t) - De(t - \delta) + Ag(e(t)) + Bg(e(t - \tau(t))) + C \int_{t - \sigma(t)}^{t} g(e(s)) \, ds + Ke(t - \gamma(t)) \right].$$
(28)

From Newton-Leibniz formulation and assumption (H2), we have

$$\begin{aligned} 0 &= -2e^{T} (t - \tau (t)) \\ &\times Q_{2} \left(e (t - \tau (t)) - e (t - \tau) - \int_{t - \tau}^{t - \tau(t)} \dot{e} (s) \, ds \right) \\ &\leq -2e^{T} (t - \tau (t)) \, Q_{2} e (t - \tau (t)) \\ &+ 2e^{T} (t - \tau (t)) \, Q_{2} e (t - \tau) + \tau e^{T} (t - \tau (t)) \\ &\times Q_{2} P_{4}^{-1} Q_{2}^{T} e (t - \tau (t)) + \int_{t - \tau}^{t - \tau(t)} \dot{e}^{T} (s) \, P_{4} \dot{e} (s) \, ds, \end{aligned}$$

$$0 = -2e^{T}(t) Q_{3}\left(e(t) - e(t - \tau(t)) - \int_{t-\tau(t)}^{t} \dot{e}(s) ds\right)$$

$$\leq -2e^{T}(t) Q_{3}e(t) + 2e^{T}(t) Q_{3}e(t - \tau(t))$$

$$+ \tau e^{T}(t) Q_{3}P_{4}^{-1}Q_{3}^{T}e(t) + \int_{t-\tau(t)}^{t} \dot{e}^{T}(s) P_{4}\dot{e}(s) ds,$$

$$0 = -2e^{T}(t - \gamma(t))$$

$$\times Q_{4}\left(e(t - \gamma(t)) - e(t - h) - \int_{t-h}^{t-\gamma(t)} \dot{e}(s) ds\right)$$

$$\leq -2e^{T}(t - \gamma(t)) Q_{4}e(t - \gamma(t))$$

$$+ 2e^{T}(t - \gamma(t)) Q_{4}e(t - \gamma(t))$$

$$+ he^{T}(t - \gamma(t)) Q_{4}P_{6}^{-1}Q_{4}^{T}e(t - \gamma(t))$$

$$+ \int_{t-h}^{t-\gamma(t)} \dot{e}^{T}(s) P_{6}\dot{e}(s) ds,$$

$$0 = -2e^{T}(t) Q_{5}\left(e(t) - e(t - \gamma(t)) - \int_{t-\gamma(t)}^{t} \dot{e}(s) ds\right)$$

$$\leq -2e^{T}(t) Q_{5}e(t) + 2e^{T}(t) Q_{5}e(t - \gamma(t))$$

$$+ he^{T}(t) Q_{5}P_{6}^{-1}Q_{5}^{T}e(t) + \int_{t-\gamma(t)}^{t} \dot{e}^{T}(s) P_{6}\dot{e}(s) ds.$$
(29)

In addition, for positive diagonal matrices $R_1 > 0$ and $R_2 > 0$, we can get from assumption (H1) that [40]

$$\begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix}^{T} \begin{bmatrix} F_{3}R_{1} & -F_{4}R_{1} \\ -F_{4}R_{1} & R_{1} \end{bmatrix} \begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix} \leq 0, \quad (30)$$

$$\begin{bmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{bmatrix}^{T} \begin{bmatrix} F_{3}R_{2} & -F_{4}R_{2} \\ -F_{4}R_{2} & R_{2} \end{bmatrix}$$

$$\times \begin{bmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{bmatrix} \leq 0. \quad (31)$$

It follows from (27)-(31) that

$$\begin{split} \dot{V}(t) &\leq e^{T}(t) \left(2X_{12} + P_{1} + \delta^{2}P_{2} + P_{3} + P_{5} \right. \\ &\quad - 2Q_{3} + \tau Q_{3}P_{4}^{-1}Q_{3}^{T} - 2Q_{5} \\ &\quad + hQ_{5}P_{6}^{-1}Q_{5}^{T} - F_{3}R_{1} \right) e(t) \\ &\quad + 2e^{T}(t) \left(X_{11} - F_{1}W_{1} + F_{2}W_{2} - Q_{1} \right) \dot{e}(t) \\ &\quad - 2e^{T}(t) \left(X_{12} + Q_{1}D \right) e(t - \delta) \\ &\quad + 2e^{T}(t) X_{22} \int_{t-\delta}^{t} e(s) \, ds + 2e^{T}(t) Q_{3}e(t - \tau(t)) \\ &\quad + 2e^{T}(t) \left(Q_{1}K + Q_{5} \right) e(t - \gamma(t)) \end{split}$$

$$\begin{aligned} + 2e^{T}(t) (Q_{1}A + F_{4}R_{1}) g(e(t)) \\ + 2e^{T}(t) Q_{1}Bg(e(t - \tau(t))) \\ + 2e^{T}(t) Q_{1}C \int_{t-\sigma(t)}^{t} g(e(s)) ds \\ + e^{T}(t) (\tau P_{4} + hP_{6} - 2Q_{1}) \dot{e}(t) \\ - 2\dot{e}^{T}(t) Q_{1}De(t - \delta) + 2\dot{e}^{T}(t) X_{12} \\ \times \int_{t-\delta}^{t} e(s) ds + 2\dot{e}^{T}(t) Q_{1}Ke(t - \gamma(t)) \\ + 2\dot{e}^{T}(t) (W_{1} - W_{2} + Q_{1}A) g(e(t)) \\ + 2\dot{e}^{T}(t) Q_{1}Bg(e(t - \tau(t))) + 2\dot{e}^{T}(t) Q_{1}C \\ \times \int_{t-\sigma(t)}^{t} g(e(s)) ds - e^{T}(t - \delta) P_{1}e(t - \delta) \\ - 2e^{T}(t - \delta) X_{22} \int_{t-\delta}^{t} e(s) ds - \left(\int_{t-\delta}^{t} e(s) ds\right)^{T} \\ \times P_{2} \left(\int_{t-\delta}^{t} e(s) ds\right) + e^{T}(t - \tau(t)) \\ \times (-2Q_{2} + \tau Q_{2}P_{4}^{-1}Q_{2}^{T} - F_{3}R_{2}) e(t - \tau(t)) \\ + 2e^{T}(t - \tau(t)) Q_{2}e(t - \tau) + 2e^{T}(t - \tau(t)) \\ \times F_{4}R_{2}g(e(t - \tau(t))) - e^{T}(t - \tau) P_{3}e(t - \tau) \\ + e^{T}(t - \gamma(t)) (-2Q_{4} + hQ_{4}P_{6}^{-1}Q_{4}^{T}) e(t - \gamma(t)) \\ + 2e^{T}(t - \gamma(t)) Q_{2}e(t - h) - e^{T}(t - h) P_{5}e(t - h) \\ + g^{T}(e(t)) (\sigma^{2}P_{7} - R_{1}) g(e(t)) \\ - g^{T}(e(t - \tau(t))) R_{2}g(e(t - \tau(t))) \\ - \left(\int_{t-\sigma(t)}^{t} g(e(s)) ds\right)^{T} P_{7} \left(\int_{t-\sigma(t)}^{t} g(e(s)) ds\right) \end{aligned}$$

where $\xi(t) = (e^{T}(t), e^{T}(t), e^{T}(t - \delta), \int_{t-\delta}^{t} e^{T}(s)ds, e^{T}(t - \tau(t)), e^{T}(t - \tau), e^{T}(t - \gamma(t)), e^{T}(t - h), g^{T}(e(t)), g^{T}(e(t - \tau(t))), \int_{t-\sigma(t)}^{t} g^{T}(e(s))ds)^{T}$ and

with $\Pi_{11} = X_{12} + X_{12}^T + P_1 + \delta^2 P_2 + P_3 + P_5 - Q_3 - Q_3^T + \tau Q_3 P_4^{-1} Q_3^T - Q_5 - Q_5^T + hQ_5 P_6^{-1} Q_5^T - F_3 R_1, \Pi_{55} = -Q_2 - Q_2^T + \tau Q_2 P_4^{-1} Q_2^T - F_3 R_2, \Pi_{77} = -Q_4 - Q_4^T + hQ_4 P_6^{-1} Q_4^T.$ By using Lemma 2 and noting $K = Q_1^{-1} Z$, it is easy to varied the equivalence of $\Pi < 0$ and $Q_3 < 0$. Thus, one can

By using Lemma 2 and noting $K = Q_1^{-1}Z$, it is easy to verify the equivalence of $\Pi < 0$ and $\Omega < 0$. Thus, one can derive from (15) and (32) that

$$\dot{V}(t) \le 0, \tag{34}$$

which implies that error-state system (10) is global and asymptotically stable; that is, master system (1) and slave system (2) are synchronous. The proof is completed. \Box

4. Numerical Example

To verify the effectiveness of the theoretical result of this paper, consider the following example.

Example 1. Consider master system (1) and slave system (2) with the following parameters:

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 1.8 & 0.1 \\ -4.3 & 2.9 \end{pmatrix},$$
$$B = \begin{pmatrix} -1.6 & -0.1 \\ -0.2 & -2.7 \end{pmatrix}, \qquad C = \begin{pmatrix} -0.3 & 0.1 \\ 0.1 & -0.2 \end{pmatrix},$$
$$f_1(\alpha) = f_2(\alpha) = \tanh(\alpha), \qquad J_1(t) = J_2(t) = 0,$$
$$\delta = 0.3, \qquad \tau(t) = 0.5 |\sin t|, \qquad \sigma(t) = 0.2 |\cos(2t)|.$$
(35)

The chaotic behaviors of master system (1) and slave system (2) with u(t) = 0 are given in Figure 1 and Figure 2, respectively, with the initial states chosen as $x(s) = (-0.1, 0.1)^T$, $y(s) = (-0.5 \sin(23t), -0.6 \cos(5t))^T$, and $s \in [-0.5, 0]$.

It can be verified that assumptions (H1) and (H2) are satisfied, and $F_1 = 0$, $F_2 = I$, $F_3 = 0$, $F_4 = \text{diag}\{0.5, 0.5\}$, $\tau = 0.5$, and $\sigma = 0.2$.

Further, the sampling period is taken as h = 0.7; by using the MATLAB LMI Control Toolbox, a solution to the LMIs in (14)-(15) is found as follows:

$$\begin{split} P_1 &= 10^{-8} \begin{bmatrix} 0.5575 & -0.0458 \\ -0.0458 & 0.0813 \end{bmatrix}, \\ P_2 &= 10^{-7} \begin{bmatrix} 0.1589 & -0.0521 \\ -0.0521 & 0.1440 \end{bmatrix}, \\ P_3 &= 10^{-8} \begin{bmatrix} 0.7818 & -0.0275 \\ -0.0275 & 0.0804 \end{bmatrix}, \\ P_4 &= 10^{-9} \begin{bmatrix} 0.5382 & -0.0061 \\ -0.0061 & 0.0069 \end{bmatrix}, \\ P_5 &= 10^{-8} \begin{bmatrix} 0.1046 & -0.0257 \\ -0.0257 & 0.0731 \end{bmatrix}, \\ P_6 &= 10^{-8} \begin{bmatrix} 0.2393 & -0.0215 \\ -0.0215 & 0.0690 \end{bmatrix}, \\ P_7 &= 10^{-7} \begin{bmatrix} 0.3430 & -0.0452 \\ -0.0452 & 0.3334 \end{bmatrix}, \\ Q_1 &= 10^{-9} \begin{bmatrix} 0.3710 & -0.0167 \\ -0.0064 & 0.0142 \end{bmatrix}, \\ Q_2 &= 10^{-9} \begin{bmatrix} 0.9695 & -0.0121 \\ -0.0121 & 0.0465 \end{bmatrix}, \\ Q_3 &= 10^{-8} \begin{bmatrix} 0.1136 & -0.0020 \\ -0.0013 & 0.0066 \end{bmatrix}, \\ Q_4 &= 10^{-7} \begin{bmatrix} 0.4169 & -0.0372 \\ -0.0403 & 0.1167 \end{bmatrix}, \\ Q_5 &= 10^{-7} \begin{bmatrix} 0.2032 & -0.0197 \\ -0.0199 & 0.0622 \end{bmatrix}, \\ Z &= 10^{-9} \begin{bmatrix} -0.3765 & 0.0456 \\ 0.0973 & -0.1631 \end{bmatrix}, \\ W_1 &= 10^{-10} \begin{bmatrix} 0.3365 & 0 \\ 0 & 0.3390 \end{bmatrix}, \end{split}$$



FIGURE 1: The chaotic behavior of master system (1).



FIGURE 2: The chaotic behavior of slave system (2) with u(t) = 0.

$$\begin{split} W_2 &= 10^{-9} \begin{bmatrix} 0.7363 & 0 \\ 0 & 0.0811 \end{bmatrix}, \\ R_1 &= 10^{-8} \begin{bmatrix} 0.6542 & 0 \\ 0 & 0.4249 \end{bmatrix}, \\ R_2 &= 10^{-8} \begin{bmatrix} 0.2055 & 0 \\ 0 & 0.0173 \end{bmatrix}, \\ X_{11} &= 10^{-10} \begin{bmatrix} 0.9837 & -0.2378 \\ -0.2378 & 0.5765 \end{bmatrix}, \\ X_{12} &= 10^{-9} \begin{bmatrix} -0.0813 & 0.0340 \\ 0.0424 & -0.1056 \end{bmatrix}, \\ X_{22} &= 10^{-8} \begin{bmatrix} 0.2687 & -0.0933 \\ -0.0933 & 0.2625 \end{bmatrix}. \end{split}$$



FIGURE 3: State trajectory of $e_1(t)$ and $e_2(t)$ of error system (8).

Subsequently, we can obtain from $K = Q_1^{-1}Z$ that

$$K = \begin{bmatrix} -0.7228 & -0.3994\\ 6.5033 & -11.6367 \end{bmatrix}.$$
 (37)

According to Theorem 3, master system (1) and slave system (2) are synchronous under sampled-data controller (6). Figure 3 depicts the synchronization errors of state variables between master system (1) and slave system (2). The numerical simulations clearly verify the effectiveness of the developed sampled-data control approach in the synchronization of two chaotic neural networks with discrete and distributed time-varying delays as well as leakage delay.

5. Conclusions

(36)

In this paper, we have dealt with the synchronization problems for chaotic neural networks with leakage delay and both discrete and distributed time-varying delays. Based on the sampled-data control techniques, Lyapunov stability theory, and the matrix inequality techniques, a delay-dependent criterion sufficient condition has been developed to guarantee synchronization of the considered coupled neural networks. An example has been provided to demonstrate the effectiveness of the proposed criterion since the feasible solutions to the given LMIs criterion in this paper have been found.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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