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Research Article

Blow-Up in a Slow Diffusive p-Laplace Equation with the Neumann Boundary Conditions

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We study a slow diffusive p-Laplace equation in a bounded domain with the Neumann boundary conditions. A natural energy is associated to the equation. It is shown that the solution blows up in finite time with the nonpositive initial energy, based on an energy technique. Furthermore, under some assumptions of initial data, we prove that the solutions with bounded initial energy also blow up.

1. Introduction

In this paper, we consider a slow diffusive p-Laplace equation:

$$\begin{gathered} u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = |u|^{q-1}u - ⨍_{\Omega}|u|^{q-1}u\,dx, \\ (x,t) \in \Omega \times (0,T), \\ \frac{\partial u}{\partial n} = 0, \quad (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad x \in \Omega \end{gathered} \tag{1}$$

with $⨍_{\Omega} u_0 dx = 0$, where Ω is a bounded smooth domain $\Omega \subset \mathbb{R}^N$, $p > 2$, $q > p-1$, and $u_0 \in L^{\infty}(\Omega) \cap W^{1,p}(\Omega)$, $u_0 \not\equiv 0$, and denote $⨍_{\Omega} f\,dx = (1/|\Omega|)\int_{\Omega} f\,dx$. It is easy to check that $\int_{\Omega} u\,dx = 0$; that is, the mass of u is conserved.

The problem (1) with $p = 2$ can be used to model phenomena in population dynamics and biological sciences where the total mass of a chemical or an organism is conserved [1, 2]. If $p > 2$, the problem (1) is the degenerate parabolic equation and appears to be relevant in the theory of non-Newtonian fluids (see [3]). Here, we are mainly interested in the case $p > 2$, namely, the degenerate one. When $p = 2$, (1) becomes the heat equation which has been deeply studied in [4, 5]. When $1 < p < 2$, (1) is singular, which can be handled similar to that of [6].

As an important feature of many evolutionary equations, the properties of blow-up solution have been the subject of intensive study during the last decades. Among those investigations in this area, it was Fujita [7] who first established the so-called theory of critical blow-up exponents for the heat equation with reaction sources in 1966, which can be, of course, regarded as the elegant description for either blow-up or global existence of solutions. From then on, there has been increasing interest in the study of critical Fujita exponents for different kinds of evolutionary equations; see [8, 9] for a survey of the literature. In recent years, special attention has been paid to the blow-up property to nonlinear degenerate or singular diffusion equations with different nonlinear sources, including the inner sources, boundary flux, or multiple sources; see, for example, [3, 10, 11].

In some situations, we have to deal with changing sign solutions. For instance, the changing sign solutions were considered in [1] for the nonlocal and quadratic equation

$$u_t = \Delta u + u^2 - ⨍_{\Omega} u^2 dx \tag{2}$$

with the Neumann boundary condition. The study in [5] for

$$u_t = \Delta u + |u|^p - \fint_\Omega |u|^p dx, \tag{3}$$

a natural generalization of (2), proposed with $1 < p \leq 2$ a global existence result (for small initial data) and a new blow-up criterion (based on the partial maximum principle and a Gamma-convergence argument). The authors also conjectured that the solutions blow up when $p > 2$, which was then proved with a positive answer [4]. The changing sign solutions to the reaction-diffusion equation

$$u_t = \Delta u + f(u, k(t)) \tag{4}$$

were discussed in [2], with such as $f(u, k(t)) = |u|^{p-1}u - k(t)$. The blow-up of solutions was obtained even under the source with $\fint_\Omega f\, dx = 0$. The semilinear parabolic equation [12]

$$u_t = \Delta u + |u|^{p-1}u - \fint_\Omega |u|^{p-1}u\, dx \tag{5}$$

with a homogeneous Neumann's boundary condition is studied. A blow-up result for the changing sign solution with positive initial energy is established. In [6], a fast diffusive p-Laplace equation with the nonlocal source

$$\begin{gathered} u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = |u|^q - \fint_\Omega |u|^q dx, \\ (x,t) \in \Omega \times (0,T), \\ \frac{\partial u}{\partial n} = 0, \quad (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad x \in \Omega, \end{gathered} \tag{6}$$

was considered. The authors showed that a critical blow-up criterion was determined for the changing sign weak solutions, depending on the size of q and the sign of the natural energy associated. The relationship between the finite time blow-up and the nonpositivity of initial energy was discussed, based on an energy technique.

Notice that (1) is degenerate if $p > 2$ at points where $\nabla u = 0$; therefore, there is no classical solution in general. For this, a weak solution for problem (1) is defined as follows.

Definition 1. A function $u \in L^\infty(\Omega \times (0,T)) \cap L^p(0,T, W^{1,p}(\Omega))$ with $u_t \in L^2(\Omega \times (0,T))$ is called a weak solution of (1) if

$$\begin{aligned} &\int_0^t \int_\Omega \left[u\frac{\partial\varphi}{\partial s} - |\nabla u|^{p-2}\nabla u \cdot \nabla\varphi + \left(|u|^q - \fint_\Omega |u|^q\right)\varphi\right] dx\, ds \\ &\quad = \int_\Omega u(x,t)\varphi(x,t)\, dx - \int_\Omega u_0(x)\varphi(x,0)\, dx \end{aligned} \tag{7}$$

holds for all $\varphi \in C^1(\overline{\Omega} \times [0,T])$.

The local existence of the weak solutions can be obtained via the standard procedure of regularized approximations [10]. Throughout the paper, we always assume that the weak solution is appropriately smooth for convenience of arguments, instead of considering the corresponding regularized problems.

This paper is organized as follows. In Section 2, we show that the solutions to (1) blow up with nonpositive initial energy. In Section 3, under some assumptions of initial data, we prove that the solutions with bounded initial energy also blow up in finite time.

2. Nonpositive Initial Energy Case

The technique used here is the same as in [4]; define the energy functional by

$$E(t) = \frac{1}{p}\int_\Omega |\nabla u|^p dx - \frac{1}{q+1}\int_\Omega |u|^{q+1} dx. \tag{8}$$

and denote

$$M(t) = \frac{1}{2}\int_\Omega u^2(x,t)\, dx, \qquad H(t) = \int_0^t M(s)\, ds. \tag{9}$$

Theorem 2. *Assume that $p > 2$, $q > p-1$, and $u_0 \in L^\infty(\Omega) \cap W^{1,p}(\Omega)$, $u_0 \not\equiv 0$, and let the initial energy*

$$E(0) = \frac{1}{p}\int_\Omega |\nabla u_0|^p dx - \frac{1}{q+1}\int_\Omega |u_0|^{q+1} dx \tag{10}$$

be nonpositive. Then, there exists T_0 with $0 < T_0 < \infty$, such that

$$\lim_{t \to T_0} M(t) = +\infty. \tag{11}$$

We need three lemmas for the functionals $E(t)$, $M(t)$, and $H(t)$, respectively.

Lemma 3. *The energy $E(t)$ is a nonincreasing function and*

$$E(t) = E(0) - \int_0^t \int_\Omega (u_t)^2 dx\, ds. \tag{12}$$

Proof. A direct computation using (1) and by parts yields

$$\begin{aligned} \frac{d}{dt}E(t) &= \int_\Omega \left(|\nabla u|^{p-2}\nabla u \cdot \nabla u_t - |u|^{q-1}uu_t\right) dx \\ &= \int_\Omega \left(-\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) - |u|^{q-1}u\right) u_t dx \\ &= \int_\Omega \left(-u_t - \fint_\Omega |u|^{q-1}u\, dx\right) u_t dx \\ &= -\int_\Omega (u_t)^2 dx. \end{aligned} \tag{13}$$

Integrate from 0 to t to get (12). □

Lemma 4. *Assume that $p > 2$, $q > p-1$, and $E(0) \leq 0$. Then, $M(t)$ satisfies the following inequality:*

$$M'(t) \geq (q+1)\int_0^t \int_\Omega (u_t)^2 dx\, ds. \tag{14}$$

Proof. An easy computation using (1) and the fact $\int_\Omega u\,dx = 0$ and by parts shows that

$$\begin{aligned} M'(t) &= \int_\Omega u u_t dx \\ &= \int_\Omega u\left(\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + |u|^{q-1}u - \fint_\Omega |u|^{q-1}u\,dx\right) \\ &= -\int_\Omega |\nabla u|^p dx + \int_\Omega |u|^{q+1}dx \\ &= -(q+1)E(t) + \frac{q+1-p}{p}\int_\Omega |\nabla u|^p dx. \end{aligned} \tag{15}$$

The last equality implies

$$\begin{aligned} M'(t) &\geq -(q+1)E(t) \\ &= -(q+1)E(0) + (q+1)\int_0^t\int_\Omega (u_t)^2 dx\,ds \\ &\geq (q+1)\int_0^t\int_\Omega (u_t)^2 dx\,ds, \end{aligned} \tag{16}$$

because of (12) of Lemma 3 and the assumption $E(0) \leq 0$. □

Lemma 5. *Assume that $p > 2$, $q > p-1$, and $E(0) \leq 0$. Then, $H(t)$ satisfies*

$$\frac{q+1}{2}\left(H'(t) - H'(0)\right)^2 \leq H(t)H''(t). \tag{17}$$

Proof. Note the definition of $M(t)$ and $H(t)$, and a simple calculation shows that

$$\begin{aligned} &H'(t) - H'(0) \\ &\quad = M(t) - M(0) \\ &\quad = \int_0^t M'(s)\,ds = \int_0^t\int_\Omega u u_t dx\,ds \\ &\quad \leq \left(\int_0^t\int_\Omega u^2 dx\,ds\right)^{1/2}\left(\int_0^t\int_\Omega (u_t)^2 dx\,ds\right)^{1/2} \\ &\quad \leq \left(\frac{2}{q+1}\right)^{1/2}(H(t))^{1/2}\left(M'(t)\right)^{1/2} \\ &\quad = \left(\frac{2}{q+1}\right)^{1/2}(H(t))^{1/2}\left(H''(t)\right)^{1/2}. \end{aligned} \tag{18}$$

Furthermore,

$$\begin{aligned} H'(t) - H'(0) &= \int_0^t M'(s)\,ds \\ &\geq (q+1)\,t\int_0^t\int_\Omega (u_t)^2 dx\,ds \geq 0. \end{aligned} \tag{19}$$

Therefore,

$$\frac{q+1}{2}\left(H'(t) - H'(0)\right)^2 \leq H(t)H''(t). \tag{20}$$

□

Proof of Theorem 2. Assume for contradiction that the solution u exists for all $t > 0$. We claim that

$$\int_0^{t_0}\int_\Omega (u_t)^2 dx\,ds > 0 \tag{21}$$

for any $t_0 > 0$. Otherwise, there exists $t_0 > 0$ such that

$$\int_0^{t_0}\int_\Omega (u_t)^2 dx\,ds = 0, \tag{22}$$

and hence $u_t = 0$ for a.e. $(x,t) \in \Omega \times (0, t_0]$. Therefore, noticing $E(t) \leq 0$ by Lemma 3, we have from (15) that

$$\int_\Omega |\nabla u|^p dx = 0 \tag{23}$$

for a.e. $t \in (0, t_0]$. Using the Poincaré inequality with $\int_\Omega u\,dx = 0$, we have $u = 0$ for a.e. $(x,t) \in \Omega \times (0, t_0]$. This contradicts $u_0 \not\equiv 0$.

Integrating (14) from t_0 to t, we have

$$M(t) \geq M(t_0) + (q+1)\int_{t_0}^t\int_0^\tau\int_\Omega (u_t)^2 dx\,ds\,d\tau, \tag{24}$$

which implies that

$$\lim_{t\to\infty} H'(t) = \lim_{t\to\infty} M(t) = +\infty. \tag{25}$$

Thus, there exists $t^* \geq t_0$ such that for all $t \geq t^*$

$$\frac{3q+5}{4}\left(H'(t)\right)^2 \leq (q+1)\left[H'(t) - H'(0)\right]^2. \tag{26}$$

Thus, combining (17), we further have

$$\frac{3q+5}{4}\left(H'(t)\right)^2 \leq 2H(t)H''(t) \tag{27}$$

for all $t \geq t^*$. Now, we consider the function $G(t) = (H(t))^{-((q-1)/4)}$. Combining with the above inequality and a simple calculation shows that

$$\begin{aligned} G''(t) &= \frac{q-1}{4}(H(t))^{(-q-7)/4} \\ &\quad \times\left(\frac{q+3}{4}\left(H'(t)\right)^2 - H(t)H''(t)\right) \\ &\leq -\frac{(q-1)^2}{32}(H(t))^{(-q-7)/4}\left(H'(t)\right)^2 \leq 0 \end{aligned} \tag{28}$$

for all $t \geq t^*$. However, since

$$\lim_{t\to\infty} H(t) = \lim_{t\to\infty} M(t) = \infty, \tag{29}$$

we also have

$$\lim_{t\to\infty} G(t) = 0, \tag{30}$$

which is a contradiction. □

3. Bounded Initial Energy Case

Define

$$W(\Omega)=\left\{u\in W^{1,p}(\Omega)\mid\int_\Omega u\,dx=0\right\}\tag{31}$$

with the norm $\|u\|=(\int_\Omega|\nabla u|^p dx)^{1/p}$. Let B be the optimal constant of the embedding inequality

$$\|u\|_{q+1}\le B\|\nabla u\|_p,\tag{32}$$

where $p-1<q\le(Np/(N-p)_+)-1$. Set

$$\begin{aligned}\alpha_1&=B^{-(q+1)/(q-p+1)},\\ E_1&=\left(\frac{1}{p}-\frac{1}{q+1}\right)B^{-p(q+1)/(q-p+1)}>0.\end{aligned}\tag{33}$$

Theorem 6. *Assume that $p>2$, $p-1<q\le(Np/(N-p)_+)-1$. Let the initial data u_0 satisfying $E(0)\le E_1$ and $\|\nabla u_0\|_p>\alpha_1$. Then, there exists T_1 with $0<T_1<\infty$, such that*

$$\lim_{t\to T_1}M(t)=+\infty.\tag{34}$$

First, we prove the following two Lemmas, similar to the idea in [13].

Lemma 7. *Assume that u is a solution of the system* (1). *If $E(0)<E_1$ and $\|\nabla u_0\|_p>\alpha_1$. Then, there exists a positive constant $\alpha_2>\alpha_1$, such that*

$$\|\nabla u\|_p\ge\alpha_2,\quad \text{for any } t\ge0,\tag{35}$$

$$\|u\|_{q+1}\ge B\alpha_2,\quad \text{for any } t\ge0.\tag{36}$$

Proof. Let $\|\nabla u\|_p=\alpha$ and by (32), we have

$$\begin{aligned}E(t)&=\frac{1}{p}\int_\Omega|\nabla u|^p dx-\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\\ &\ge\frac{1}{p}\|\nabla u\|_p^p-\frac{1}{q+1}B^{q+1}\|\nabla u\|_p^{q+1}\\ &=\frac{1}{p}\alpha^p-\frac{1}{q+1}B^{q+1}\alpha^{q+1}.\end{aligned}\tag{37}$$

For convenience, we define

$$g(\alpha)=\frac{1}{p}\alpha^p-\frac{1}{q+1}B^{q+1}\alpha^{q+1}.\tag{38}$$

It is easy to find that g increases if $0<\alpha<\alpha_1$ and decreases if $\alpha>\alpha_1$. Moreover, $g(\alpha)\to-\infty$ as $\alpha\to\infty$ and $g(\alpha_1)=E_1$. Due to $E(0)<E_1$, there exists $\alpha_2>\alpha_1$ such that $g(\alpha_2)=E(0)$. Let $\|\nabla u_0\|_p=\alpha_0$; thus $\alpha_0>\alpha_1$. Then by (37) and (38), we have $g(\alpha_0)\le E(0)=g(\alpha_2)$, which implies that $\alpha_0\ge\alpha_2$. For contradiction to establish (35), we assume that there exists $t_0>0$ such that

$$\alpha_1<\left\|\nabla u(\cdot,t_0)\right\|_p<\alpha_2.\tag{39}$$

It follows from (37) and (38) that

$$E(t_0)\ge g\left(\left\|\nabla u(\cdot,t_0)\right\|_p\right)>g(\alpha_2)=E(0),\tag{40}$$

which is in contradiction with Lemma 3. Hence, (35) is established.

Next to prove (36),

$$E(t)=\frac{1}{p}\int_\Omega|\nabla u|^p dx-\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\le E(0),\tag{41}$$

which implies that

$$\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\ge\frac{1}{p}\int_\Omega|\nabla u|^p dx-E(0)\ge\frac{1}{p}\alpha_2^p-g(\alpha_2).\tag{42}$$

Therefore, (36) is concluded. □

Define

$$F(t)=E_1-E(t),\quad \text{for any } t\ge0.\tag{43}$$

Then, we have the following.

Lemma 8. *Assume that u is a solution of the system* (1). *If $E(0)<E_1$ and $\|\nabla u_0\|_p>\alpha_1$. Then for all $t\ge0$,*

$$0<F(0)\le F(t)\le\frac{1}{q+1}\int_\Omega|u|^{q+1}dx.\tag{44}$$

Proof. By Lemma 3, we know that $F'(t)\ge0$. Thus,

$$F(t)\ge F(0)=E_1-E(0)>0.\tag{45}$$

According to (35) of Lemma 7, a simple computation shows that

$$\begin{aligned}F(t)&=E_1-\frac{1}{p}\int_\Omega|\nabla u|^p dx+\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\\ &\le E_1-\frac{1}{p}B^{-p(q+1)/(q-p+1)}+\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\\ &=-\frac{1}{q+1}B^{-p(q+1)/(q-p+1)}+\frac{1}{q+1}\int_\Omega|u|^{q+1}dx\\ &\le\frac{1}{q+1}\int_\Omega|u|^{q+1}dx,\end{aligned}\tag{46}$$

which guarantees the conclusion of the lemma. □

At the end, let us finish the proof of Theorem 6.

Proof of Theorem 6. According to (15), we have

$$\begin{aligned}M'(t)&=-\int_\Omega|\nabla u|^p dx+\int_\Omega|u|^{q+1}dx\\ &=\int_\Omega|u|^{q+1}dx-pE(t)-\frac{p}{q+1}\int_\Omega|u|^{q+1}dx\\ &=\frac{q+1-p}{q+1}\int_\Omega|u|^{q+1}dx-pE_1+pF(t).\end{aligned}\tag{47}$$

By using (33) and (36), we obtain

$$\begin{aligned} pE_1 &= \left(1 - \frac{p}{q+1}\right) B^{-p(q+1)/(q+1-p)} \\ &= \frac{\alpha_1^{q+1}}{\alpha_2^{q+1}} \frac{q+1-p}{q+1} B^{q+1} \alpha_2^{q+1} \\ &\leq \frac{\alpha_1^{q+1}}{\alpha_2^{q+1}} \frac{q-p+1}{q+1} \int_\Omega |u|^{q+1} dx. \end{aligned} \tag{48}$$

Combining (47) and (48), we get

$$\begin{aligned} M'(t) &\geq \left(1 - \frac{\alpha_1^{q+1}}{\alpha_2^{q+1}}\right) \frac{q+1-p}{q+1} \int_\Omega |u|^{q+1} dx + pF(t) \\ &\geq \left(1 - \frac{\alpha_1^{q+1}}{\alpha_2^{q+1}}\right) \frac{q+1-p}{q+1} |\Omega|^{(1-q)/2} M^{(q+1)/2}. \end{aligned} \tag{49}$$

Since $q > p - 1 > 1$, $M(t)$ blows up at a finite time. The proof of Theorem 6 is complete. □

Remark 9 (behavior of the energy $E(t)$). Similar to Theorem 1.3 of [5], it is easy to be proved. Let $p > 2$, $p - 1 < q \leq (Np/(N-p)_+) - 1$, and let u be a weak solution of (1). If there exists a constant $C_0 > 0$ and a time $T_0' > 0$, such that the solution u exists on $[0, T_0')$ and satisfies $E(t) \geq -C_0$ on $[0, T_0')$, then $F(t)$ is bounded on $[0, T_0')$. Thus, the above result and Theorem 6 reveal that even though the initial energy could be chosen as positive, the energy $E(t)$ needs to become negative at a certain time and then goes to $-\infty$. Otherwise, $E(t)$ has a lower bound on $[0, +\infty)$; thus $F(t)$ is bounded on $[0, +\infty)$. It is in contradiction with Theorem 6.

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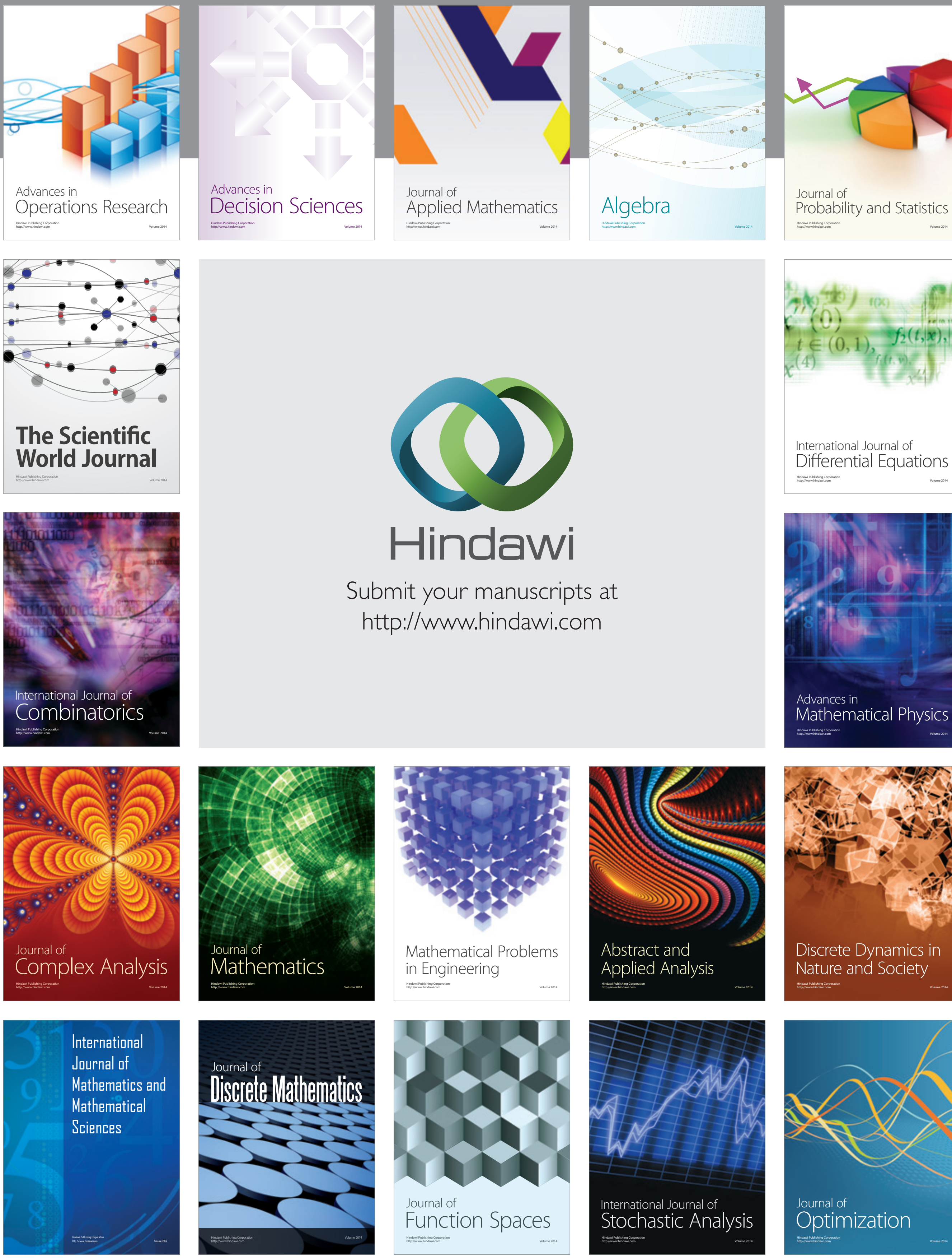
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