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Research Article

Best Proximity Points for Relatively u-Continuous Mappings in Banach and Hyperconvex Spaces

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We prove some best proximity point results for relatively u-continuous mappings in Banach and hyperconvex metric spaces. Our results generalize and extend some recent results to relatively u-continuous mappings and to general spaces.

1. Introduction

Let A, B be nonempty subsets of a Banach space $(M, \|\cdot\|)$. In [1], Eldred et al. considered the best proximity point problem for mappings $T : A \cup B \to A \cup B$ with $T(A) \subset B$ and $T(B) \subset A$ or $T(A) \subset A$ and $T(B) \subset B$, respectively; that is, they sought conditions on the subsets A, B, the space M, and the mapping T that assure existence of points $x_0 \in A$, $y_0 \in B$ such that

$$\|x_0 - T(x_0)\| = \|y_0 - T(y_0)\| = \operatorname{dist}(A, B), \tag{1}$$

or

$$\begin{aligned} x_0 &= T(x_0), \\ y_0 &= T(y_0), \\ \|x_0 - y_0\| &= \operatorname{dist}(A, B), \end{aligned} \tag{2}$$

respectively. In solving this problem they considered a new class of mappings.

Definition 1 (see [1]). Let A, B be nonempty subsets of a metric space (M, d). Then a mapping $T : A \cup B \to A \cup B$ is said to be *relatively nonexpansive* if

$$d(T(x), T(y)) \le d(x, y) \quad \text{for } x \in A,\ y \in B. \tag{3}$$

The assumption that a mapping is relatively nonexpansive is weaker than the assumption that it is nonexpansive and does not even imply continuity [1].

Introducing a geometric condition for Banach spaces called *proximal normal structure*, they obtained the following result.

Theorem 2 (see [1]). *Let (A, B) be a nonempty weakly compact convex pair in a Banach space $(M, \|\cdot\|)$. Let $T : A \cup B \to A \cup B$ be a relatively nonexpansive mapping such that $T(A) \subset B$ and $T(B) \subset A$, and suppose that (A, B) has proximal normal structure. Then there exists $(x_0, y_0) \in A \times B$ such that*

$$\|x_0 - T(x_0)\| = \|y_0 - T(y_0)\| = \operatorname{dist}(A, B). \tag{4}$$

With the goal of generalizing relatively nonexpansive mappings, Eldred et al. [2] introduced the notion of a relatively u-continuous mapping in Banach spaces, which we state here for a metric space.

Definition 3 (see [2]). Let A, B be nonempty subsets of a metric space (M, d). A mapping $T : A \cup B \to A \cup B$ is said to be *relatively u-continuous* if for each $\epsilon > 0$, there exists $\delta > 0$ such that $d(T(x), T(y)) < \varepsilon + \operatorname{dist}(A, B)$ whenever

$$d(x, y) < \delta + \operatorname{dist}(A, B), \quad \forall x \in A,\ y \in B. \tag{5}$$

Every relatively nonexpansive mapping is relatively u-continuous. For an example showing that the converse is not true see [2, Example 2.1].

Eldred et al. [2] were able to extend some of the results of [1] to include the class of relatively u-continuous mappings.

Theorem 4 (see [2]). *Let A, B be nonempty compact convex subsets of a strictly convex Banach space X, and let $T : A\cup B \to A\cup B$ be a relatively u-continuous mapping such that $T(A) \subset B$ and $T(B) \subset A$. Then there exists*

$$(x_0, y_0) \in A \times B$$
$$\text{such that } \|x_0 - T(x_0)\| = \|y_0 - T(y_0)\| = \operatorname{dist}(A, B). \tag{6}$$

In this paper we show that Theorem 4 holds for any Banach space without the assumption of strict convexity as follows.

Theorem 5. *Let $(M, \|\cdot\|)$ be a Banach space, and let A, B be nonempty compact convex subsets of M. If $T : A\cup B \to A\cup B$ is relatively u-continuous such that $T(A) \subset B$ and $T(B) \subset A$, then there exist points $x \in A$ and $y \in B$ such that $\|x - T(x)\| = \|y - T(y)\| = \operatorname{dist}(A, B)$.*

Some interesting best proximity point theorems for various kinds of mappings have been accomplished in [3–8]. Other related results on cyclical mappings can be found in [9, 10].

The aim of this paper is to prove some best proximity point results for relatively u-continuous mappings in Banach and hyperconvex metric spaces. Our results generalize and extend some recent results to relatively u-continuous mappings and to general spaces.

2. Preliminaries

Let A and B be nonempty subsets of a metric space (M, d). Define

$$\operatorname{dist}(A, B) = \inf\{d(x, y) : x \in A, y \in B\},$$
$$A_0 = \{x \in A : d(x, y) = \operatorname{dist}(A, B) \text{ for some } y \in B\},$$
$$B_0 = \{y \in B : d(x, y) = \operatorname{dist}(A, B) \text{ for some } x \in A\}. \tag{7}$$

Definition 6. A metric space (M, d) is *hyperconvex* if given any family $\{x_\alpha : \alpha \in I\}$ of points in M and any family $\{r_\alpha\}$ of nonnegative real numbers satisfying $d(x_\alpha, x_\beta) \le r_\alpha + r_\beta$ for all $\alpha, \beta \in I$, then $\cap B(x_\alpha; r_\alpha) \ne \emptyset$, where

$$B(x; r) = \{y \in M : d(x, y) \le r\}. \tag{8}$$

Definition 7. The *admissible* subsets of M are sets of the form $\cap B(x_\alpha; r_\alpha)$, that is, the family of ball intersections in M. For a subset X of M, $N_\varepsilon(X)$ denotes the closed ε-hull of X; that is, $N_\varepsilon(X) = \{x \in M : \operatorname{dist}(x, X) \le \varepsilon\}$, where $\operatorname{dist}(x, X) = \inf\{d(x, y) : x \in X\}$.

If X is an admissible set, then $N_\varepsilon(X)$ is also an admissible set [11]. For recent progress in hyperconvex metric spaces, we refer the reader to [12].

Definition 8. Let (M, d) be a metric space and $F : M \to 2^M$ a multivalued mapping with nonempty values. Then F is said to be *almost lower semicontinuous* at a point $x \in M$ if for each $\varepsilon > 0$ there is an open neighborhood $U(x)$ of x and a point $z \in M$ such that, for $y \in U(x)$,

$$B(z; \varepsilon) \cap F(y) \ne \emptyset. \tag{9}$$

In establishing existence of best proximity points for relatively u-continuous mappings in Banach and hyperconvex spaces, we apply the following continuous selection and fixed point theorems.

Theorem 9 (see [13]). *Let X be a paracompact space and Y a normed linear space. Let $F : X \to 2^Y$ be a multivalued mapping with nonempty closed convex values. Then F is an almost lower semicontinuous mapping if and only if for each $\epsilon > 0$, F has a continuous ϵ-approximate selection; that is, a function $f : X \to Y$ such that for every $x \in X$, $\operatorname{dist}(f(x), F(x)) < \epsilon$.*

Theorem 10 (see [14]). *Let X be a paracompact topological space, (M, d) a hyperconvex metric space, and $F : X \to 2^M$ an almost lower semicontinuous mapping with admissible values. Then F has a continuous selection; that is, there is a continuous mapping $f : X \to M$ such that $f(x) \in F(x)$ for each $x \in X$.*

Theorem 11 (see [15, 16]). *Let (M, d) be a compact hyperconvex metric space and $f : M \to M$ a continuous mapping. Then f has a fixed point.*

3. Best Proximity Points in Banach Spaces

The following theorem extends the best proximity point result of Eldred et al. [2, Theorem 3.1] for strictly convex Banach spaces to any Banach space.

Proof of Theorem 5. Since A, B are compact convex subsets, A_0, B_0 are nonempty compact convex subsets. By [2, Proposition 3.1] $T(A_0) \subset B_0$ and $T(B_0) \subset A_0$.

By u-continuity of T, for any $x \in A$, $y \in B$ such that $\|x - y\| = \operatorname{dist}(A, B)$ and any positive integer n there is a $\delta_n > 0$ and a neighborhood of x in A_0 defined as

$$U(x, \delta_n) = \{u \in A_0 : \|u - x\| < \delta_n\}, \tag{10}$$

such that $u \in U(x, \delta_n)$ implies that

$$\|T(u) - T(y)\| \le \left(\frac{1}{n}\right) + \operatorname{dist}(A, B). \tag{11}$$

For each positive integer n, define a multivalued mapping $F_n : A_0 \to 2^{A_0}$ by

$$F_n(v) = B\left(T(v); \left(\frac{1}{n}\right) + \operatorname{dist}(A, B)\right) \cap A_0, \tag{12}$$

for $v \in A_0$. Since $T(v) \in B_0$, $F_n(v)$ is nonempty. As the intersection of closed convex sets, each $F_n(v)$ is also closed convex.

By (11), $T(y) \in F_n(u)$ for each $u \in U(x, \delta_n)$, which implies that the mapping F_n is almost lower semicontinuous. By the approximate selection result of Deutsch et al. [13]

(see Theorem 9), for any $\alpha > 0$, F_n has a continuous α-approximate selection; that is, there is a continuous $f_n : A_0 \to A_0$ such that $\operatorname{dist}(f_n(v), F_n(v)) \le \alpha$. Choosing $\alpha = 1/n$, by the definition of F_n the selection f_n satisfies

$$\|T(v) - f_n(v)\| \le \left(\frac{2}{n}\right) + \operatorname{dist}(A, B), \quad \text{for } v \in A_0. \tag{13}$$

Since the mapping f_n is continuous and A_0 is a compact convex subset of a Banach space, the Schauder fixed point theorem implies that f_n has a fixed point x_n; that is, there is a point $x_n \in A_0$ such that $x_n = f_n(x_n)$.

By (13), $\|T(x_n) - x_n\| \to \operatorname{dist}(A, B)$, and by compactness of A_0 and B_0, we can assume that $x_n \to x \in A_0$ and $T(x_n) \to p \in B_0$. Therefore, $\|x - p\| = \operatorname{dist}(A, B)$, and by u-continuity of T, $\|T(x_n) - T(p)\| \to \operatorname{dist}(A, B)$. It follows that

$$\begin{aligned}\operatorname{dist}(A, B) &\le \|p - T(p)\| \\ &\le \|p - T(x_n)\| + \|T(x_n) - T(p)\| \\ &\longrightarrow \operatorname{dist}(A, B),\end{aligned} \tag{14}$$

which implies that $\|p - T(p)\| = \operatorname{dist}(A, B)$. □

The following proposition follows by a slight change in the proof in [2, Proposition 3.1].

Proposition 12. *Let A, B be nonempty subsets of a normed linear space M, and let $T : A \cup B \to A \cup B$ be a relatively u-continuous mapping such that $T(A) \subset A$ and $T(B) \subset B$. Then $T(A_0) \subset A_0$ and $T(B_0) \subset B_0$.*

Proposition 13 (see [17]). *Let $(M, \|\cdot\|)$ be a strictly convex Banach space, A a nonempty compact convex subset of M, and B a nonempty closed convex subset of M. Let $\{x_n\}$ be a sequence in A and $y \in B$. If*

$$\|x_n - y\| \longrightarrow \operatorname{dist}(A, B), \quad \textit{then } x_n \longrightarrow P_A(y). \tag{15}$$

In [1] a best proximity result was given for relatively nonexpansive mappings in a uniformly convex space. The following result is a version of that result for relatively u-continuous mappings in a strictly convex space.

Theorem 14. *Let $(M, \|\cdot\|)$ be a strictly convex Banach space, and let A, B be compact convex subsets of M. If $T : A \cup B \to A \cup B$ is relatively u-continuous such that $T(A) \subset A$ and $T(B) \subset B$, then there exist points $x_0 \in A$ and $y_0 \in B$ such that $x_0 = T(x_0)$, $y_0 = T(y_0)$ and $\|x_0 - y_0\| = \operatorname{dist}(A, B)$.*

Proof. Since A, B are compact convex sets, A_0 and B_0 are nonempty compact convex sets, and by Proposition 12, $T(A_0) \subset A_0$ and $T(B_0) \subset B_0$.

By u-continuity of T, for any positive integer n there is a $\delta_n > 0$ such that

$$\|x - y\| \le \delta_n + \operatorname{dist}(A, B) \tag{16}$$

implies that $\|T(x) - T(y)\| < (1/n) + \operatorname{dist}(A, B)$, for $x \in A$ and $y \in B$. For $x \in A_0$ define $U(x, \delta_n) = \{u \in A_0 : \|u - x\| < \delta_n\}$, and let $y = P_B(x)$. Then $u \in U(x, \delta_n)$ implies that

$$\|u - y\| \le \|u - x\| + \|x - y\| < \delta_n + \operatorname{dist}(A, B), \tag{17}$$

and therefore, by u-continuity of T,

$$\|T(u) - T(y)\| \le \left(\frac{1}{n}\right) + \operatorname{dist}(A, B). \tag{18}$$

For each positive integer n, define a map $F_n : A_0 \to 2^{B_0}$ by

$$F_n(v) = B\left(T(v); \left(\frac{1}{n}\right) + \operatorname{dist}(A, B)\right) \cap B_0, \tag{19}$$

for $v \in A_0$. As the intersection of closed convex sets, $F_n(v)$ is also closed convex. By (18), $T(y) \in F_n(u)$ for $u \in U(x, \delta_n)$, which implies that $F_n(u)$ is nonempty and also that F_n is an almost lower semicontinuous mapping.

Since M is a normed linear space, by Theorem 9 for any $\alpha > 0$, F_n has a continuous α-approximate selection; that is, there is a continuous $f_n : A_0 \to B_0$ such that $\operatorname{dist}(f_n(v), F_n(v)) \le \alpha$, for $v \in A_0$. Choosing $\alpha = 1/n$, by the definition of F_n the selection f_n satisfies

$$\|T(v) - f_n(v)\| \le \left(\frac{2}{n}\right) + \operatorname{dist}(A, B), \tag{20}$$

for $v \in A_0$.

Consider the metric projection operator $P_A : M \to A$. Since $f_n(A_0) \subset B_0$ and $P_A(B_0) \subset A_0$, the map $P_A \circ f_n$ sends A_0 into A_0. Since $P_A \circ f_n$ is continuous and A_0 is compact and convex, by the Schauder fixed point theorem there is a fixed point $x_n = P_A \circ f_n(x_n) \in A_0$. Let $y_n = f_n(x_n) \in B_0$, and assume by compactness that x_n, y_n converge to $x_0 \in A_0$, $y_0 \in B_0$, respectively. By continuity of P_A, $x_0 = P_A(y_0)$.

By definition of the map f_n, $\|T(x_n) - y_n\| \le (2/n) + \operatorname{dist}(A, B)$, and since $y_n \to y_0$ we have

$$\begin{aligned}&\|T(x_n) - y_0\| \\ &\quad \le \|T(x_n) - y_n\| + \|y_n - y_0\| \longrightarrow \operatorname{dist}(A, B).\end{aligned} \tag{21}$$

Therefore, by Proposition 13,

$$T(x_n) \longrightarrow P_A(y_0). \tag{22}$$

By u-continuity of T, for any $\epsilon > 0$ there is a $\delta > 0$ such that

$$\begin{aligned}&\|T(x_n) - T(y_0)\| \\ &\quad < \epsilon + \operatorname{dist}(A, B) \ \text{ provided } \ \|x_n - y_0\| < \delta + \operatorname{dist}(A, B).\end{aligned} \tag{23}$$

Since $x_n \to x_0$, choose n sufficiently large that $\|x_n - x_0\| < \delta$. Then

$$\begin{aligned}&\|x_n - y_0\| \\ &\quad \le \|x_n - x_0\| + \|x_0 - y_0\| < \delta + \operatorname{dist}(A, B),\end{aligned} \tag{24}$$

which implies that

$$\begin{aligned} &\operatorname{dist}(A,B) \\ &\quad \le \|T(x_n) - T(y_0)\| < \epsilon + \operatorname{dist}(A,B). \end{aligned} \tag{25}$$

Since ϵ is arbitrary,

$$\|T(x_n) - T(y_0)\| \longrightarrow \operatorname{dist}(A,B). \tag{26}$$

Therefore, by Proposition 13,

$$T(x_n) \longrightarrow P_A(T(y_0)). \tag{27}$$

By the relations (22) and (27), $T(x_n)$ converges to both $P_A(y_0)$ and $P_A(T(y_0))$. Therefore, $x_0 = P_A(y_0) = P_A(T(y_0))$. Since $y_0, T(y_0) \in B_0$, $\|x_0 - y_0\| = \|x_0 - T(y_0)\| = \operatorname{dist}(A,B)$, and by strict convexity of M, $y_0 = T(y_0)$.

Since $\|x_0 - y_0\| = \operatorname{dist}(A,B)$, we have by u-continuity of T that $\|T(x_0) - T(y_0)\| = \operatorname{dist}(A,B)$. Therefore, $T(x_0) = P_A(T(y_0))$, and since $x_0 = P_A(T(y_0))$, this implies that $x_0 = T(x_0)$. □

4. Best Proximity Points in Hyperconvex Spaces

The following is a best proximity point result for relatively u-continuous mappings in hyperconvex metric spaces. Best proximity point/pair results were obtained in the setting of hyperconvex spaces by some authors in [18–21].

Theorem 15. *Let A, B be admissible subsets of a hyperconvex metric space (M,d), let A_0 be a compact subset of M and let $T : A \cup B \to A \cup B$ be a relatively u-continuous mapping such that $T(A) \subset B$, and $T(B) \subset A$. Then there is an $x_0 \in A_0$ such that $d(x_0, T(x_0)) = \operatorname{dist}(A,B)$.*

Proof. By a result of Kirk et al. [18], the sets A_0 and B_0 are nonempty and hyperconvex. For $x \in A_0$, choose $y \in B_0$ such that $d(x,y) = \operatorname{dist}(A,B)$. Then, by u-continuity of T, for any $\varepsilon > 0$ there is a $\delta > 0$ such that for $u \in A$, $v \in B$,

$$\begin{aligned} &d(u,v) < \delta + \operatorname{dist}(A,B) \\ &\text{implies that } d(T(u), T(v)) < \varepsilon + \operatorname{dist}(A,B). \end{aligned} \tag{28}$$

It follows that $d(T(x), T(y)) = \operatorname{dist}(A,B)$. This implies that $T(x) \in B_0$ for $x \in A_0$.

Define an open neighborhood of x in A_0 by $U(x) = \{u \in A_0 : d(u,x) < \delta\}$.

Then $u \in U(x)$ implies that

$$d(u,y) \le d(u,x) + d(x,y) < \delta + \operatorname{dist}(A,B), \tag{29}$$

and therefore, by u-continuity of T,

$$d(T(u), T(y)) < \varepsilon + \operatorname{dist}(A,B). \tag{30}$$

Define a multivalued $F : A_0 \to 2^{A_0}$ by

$$F(v) = B(T(v); \operatorname{dist}(A,B)) \cap A, \tag{31}$$

for $v \in A_0$. Since $T(v) \in B_0$ for $v \in A_0$, $F(v)$ is a nonempty subset of A_0, and since A is admissible, $F(v)$ is also admissible.

We show that F is almost lower semicontinuous by establishing that $B(T(y);\varepsilon) \cap F(u) \neq \emptyset$ for $u \in U(x)$. By (30) and the hyperconvexity of M, for $u \in U(x)$,

$$B(T(y);\varepsilon) \cap B(T(u); \operatorname{dist}(A,B)) \neq \emptyset. \tag{32}$$

Since $T(u) \in B_0$, we have

$$B(T(u); \operatorname{dist}(A,B)) \cap A \neq \emptyset. \tag{33}$$

Any point p in the intersection (33) is in A_0 since $d(p, T(u)) = \operatorname{dist}(A,B)$. Therefore,

$$B(T(u); \operatorname{dist}(A,B)) \cap A \subset A_0. \tag{34}$$

By (32), (33), and the fact that $T(y) \in A_0$, the sets $B(T(y);\varepsilon)$, $B(T(u); \operatorname{dist}(A,B))$, and A have pairwise nonempty intersection. Since all of these sets are ball intersections, the hyperconvexity of the space M implies that

$$B(T(y);\varepsilon) \cap B(T(u); \operatorname{dist}(A,B)) \cap A \neq \emptyset. \tag{35}$$

Further, by (34), the intersection in (35) is contained in A_0. It follows from (35) that $B(T(y);\varepsilon) \cap F(u) \neq \emptyset$ for $u \in U(x)$. This implies that the mapping F is almost lower semicontinuous.

By the selection theorem in Markin [14] (see Theorem 10), an almost lower semicontinuous mapping on a hyperconvex space with nonempty admissible values has a continuous selection; that is, there is a continuous $f : A_0 \to A_0$ such that $f(x) \in F(x)$ for $x \in A_0$. By Theorem 11, a continuous self-mapping on a compact hyperconvex space has a fixed point. Therefore, there is a $w \in A_0$ such that $w = f(w) \in F(w)$. By the definition of F,

$$d(w, T(w)) = \operatorname{dist}(A,B). \tag{36}$$

□

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References

[1] A. A. Eldred, W. A. Kirk, and P. Veeramani, "Proximal normal structure and relatively nonexpansive mappings," *Studia Mathematica*, vol. 171, no. 3, pp. 283–293, 2005.

[2] A. A. Eldred, V. S. Raj, and P. Veeramani, "On best proximity pair theorems for relatively u-continuous mappings," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 74, no. 12, pp. 3870–3875, 2011.

[3] M. A. Al-Thagafi and N. Shahzad, "Convergence and existence results for best proximity points," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 10, pp. 3665–3671, 2009.

[4] S. S. Basha and N. Shahzad, "Best proximity point theorems for generalized proximal contractions," *Fixed Point Theory and Applications*, vol. 2012, article 42, 9 pages, 2012.

[5] S. S. Basha, N. Shahzad, and R. Jeyaraj, "Best proximity points: approximation and optimization," *Optimization Letters*, vol. 7, no. 1, pp. 145–155, 2013.

[6] S. S. Basha, N. Shahzad, and R. Jeyaraj, "Best proximity point theorems for reckoning optimal approximate solutions," *Fixed Point Theory and Applications*, vol. 2012, article 202, 9 pages, 2012.

[7] S. S. Basha, N. Shahzad, and R. Jeyaraj, "Common best proximity points: global optimal solutions," *Journal of Optimization Theory and Applications*, vol. 148, no. 1, pp. 69–78, 2011.

[8] M. Gabeleh and N. Shahzad, "Existence and convergence theorems of best proximity points," *Journal of Applied Mathematics*, vol. 2013, Article ID 101439, 6 pages, 2013.

[9] W. A. Kirk, P. S. Srinivasan, and P. Veeramani, "Fixed points for mappings satisfying cyclical contractive conditions," *Fixed Point Theory*, vol. 4, no. 1, pp. 79–89, 2003.

[10] S. Karpagam and S. Agrawal, "Existence of best proximity points of p-cyclic contractions," *Fixed Point Theory*, vol. 13, no. 1, pp. 99–105, 2012.

[11] R. Sine, "Hyperconvexity and approximate fixed points," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 13, no. 7, pp. 863–869, 1989.

[12] R. Espínola and P. Lorenzo, "Metric fixed point theory on hyperconvex spaces: recent progress," *Arabian Journal of Mathematics*, vol. 1, no. 4, pp. 439–463, 2012.

[13] F. Deutsch, V. Indumathi, and K. Schnatz, "Lower semicontinuity, almost lower semicontinuity, and continuous selections for set-valued mappings," *Journal of Approximation Theory*, vol. 53, no. 3, pp. 266–294, 1988.

[14] J. T. Markin, "A selection theorem for quasi-lower semicontinuous mappings in hyperconvex spaces," *Journal of Mathematical Analysis and Applications*, vol. 321, no. 2, pp. 862–866, 2006.

[15] M. A. Khamsi, "KKM and Ky Fan theorems in hyperconvex metric spaces," *Journal of Mathematical Analysis and Applications*, vol. 204, no. 1, pp. 298–306, 1996.

[16] W. A. Kirk and S. S. Shin, "Fixed point theorems in hyperconvex spaces," *Houston Journal of Mathematics*, vol. 23, no. 1, pp. 175–188, 1997.

[17] V. S. Raj and P. Veeramani, "Best proximity pair theorems for relatively nonexpansive mappings," *Applied General Topology*, vol. 10, no. 1, pp. 21–28, 2009.

[18] W. A. Kirk, S. Reich, and P. Veeramani, "Proximinal retracts and best proximity pair theorems," *Numerical Functional Analysis and Optimization*, vol. 24, no. 7-8, pp. 851–862, 2003.

[19] J. Markin and N. Shahzad, "Best approximation theorems for nonexpansive and condensing mappings in hyperconvex spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 6, pp. 2435–2441, 2009.

[20] A. Abkar and M. Gabeleh, "Results on the existence and convergence of best proximity points," *Fixed Point Theory and Applications*, vol. 2010, Article ID 386037, 10 pages, 2010.

[21] A. Amini-Harandi, A. P. Farajzadeh, D. O'Regan, and R. P. Agarwal, "Coincidence point, best approximation, and best proximity theorems for condensing set-valued maps in hyperconvex metric spaces," *Fixed Point Theory and Applications*, vol. 2008, Article ID 543154, 8 pages, 2008.