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Research Article

On Exponential Stability for a Class of Uncertain Neutral Markovian Jump Systems with Mode-Dependent Delays

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The exponential stability of neutral Markovian jump systems with interval mode-dependent time-varying delays, nonlinear perturbations, and partially known transition rates is investigated. A novel augmented stochastic Lyapunov functional is constructed, which employs the improved bounding technique and contains triple-integral terms to reduce conservativeness; then the delay-range-dependent and rate-dependent exponential stability criteria are developed by Lyapunov stability theory, reciprocally convex lemma, and free-weighting matrices. The corresponding results are extended to the uncertain case. Finally, numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

Delay differential equations or systems are assuming an increasingly important role in many disciplines like mathematics, science, and engineering. In particular, the stability and stabilization problem for neutral delay differential dynamic systems have received considerable attention during the decades and neutral time-delay systems have been the focus of the research community, which are often encountered in such practical situations as distributed networks, population ecology, processes including steam or heat exchanges [1], and robots in contact with rigid environments [2]. Existing results can be roughly classified into two categories, delay-independent criteria and delay-dependent criteria, where the latter is generally regarded as less conservative. Moreover, since the derivative of the delayed state is involved, it should be pointed out that the stability of neutral time-delay systems is more difficult to tackle, which is identical with singular systems [3, 4]. The stability problem of them is more complicated than that for regular systems because more factors need to be considered. In the past decades, considerable attention has been devoted to the robust delay-independent stability and delay-dependent stability of linear neutral systems, which are mainly obtained based on the Lyapunov-Krasovskii (L-K) method [5–11], and references therein. It should be noted that the delay-partitioning approach is used in [6–8]. Furthermore, when nonlinear perturbations or parameter uncertainties appear in neutral systems, some results on stability analysis have been also presented [12–18]. Various techniques have been proposed in these papers, for example, model transformation techniques, the improved bounding techniques, and matrix decomposition approaches. In particular, He et al. [18] propose a new method for dealing with time-delay systems, which employs free weighting matrices to express the relationships between the terms in the Newton-Leibniz formula and has brought novel results. However, these results have conservativeness to some extent, which exist room for further improvement.

In another line, Markovian jump systems (MJSs) have attracted much attention during the past few decades since its first introduction by Krasovskii and Lidskii in 1961, which can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes. The system parameters usually jump among finite modes, and the mode switching is governed by a Markov process. MJSs have many applications, such as failure prone manufacturing systems, power systems, solar thermal central receivers, robotic manipulator systems, aircraft control systems, and economic systems. A large number of results on estimation and control problems related to such systems have been reported in the literature; see, for example, [19–25] and references therein for more details. However, these lines of literature about the transition probabilities in the jumping process have been assumed to be completely accessible. The ideal assumption on the transition probabilities inevitably limits the application of the traditional Markovian jump system theory. Actually, the likelihood of obtaining such available knowledge is questionable, and the cost may be very expensive. Thus, it is really significant and meaningful, from control perspectives, to further study more general jump systems with partially known transition rates. Recently, many results on the Markovian jump systems with partially known transition rates are obtained [26–31]. Most of these improved results just require some free matrices or the knowledge of the known elements in transition rate matrix, such as the structures of uncertainties, and some else of the unknown elements need not be considered. It is a great progress on the analysis of Markovian jump systems. However, few of these results are concerned with neutral Markovian jump systems with mode-dependent time-varying delays and perturbations. To the best of the authors' knowledge, neutral Markovian jump systems with mode-dependent time-varying delays and partially known transition rates have not been fully investigated, and it is very challenging, especially when nonlinear perturbations exist. Besides, seeking and proposing less conservative delay-range-dependent criterion for uncertain neutral MJSs with nonlinear perturbations and partially known transition rates to desired performance are still open problems. These facts thus motivate our study.

In this paper, the investigated neutral Markovian jump systems are more general than the neutral MJSs with completely known or completely unknown transition rates, which can be viewed as two special cases of the ones tackled here. Specifically, a new augmented stochastic Lyapunov functional containing triple-integral terms is constructed by dividing the delay interval into two subintervals, and then the delay-range-dependent and rate-dependent exponential stability criteria are obtained by reciprocally convex lemma and free weighting matrices. We further extend the criteria to the uncertain case. All the obtained results are presented in terms of LMIs that can be solved numerically. The remainder of the paper is organized as follows. Section 2 presents the problem and preliminaries. Section 3 gives the main results, which are then verified by numerical examples in Section 4. Section 5 concludes the paper.

Notations. The following notations are used throughout the paper. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ matrices. $X < Y$ ($X > Y$), where X and Y are both symmetric matrices, means that $X - Y$ is negative (positive) definite. I is the identity matrix with proper dimensions. For a symmetric block matrix, we use $*$ to denote the terms introduced by symmetry. $\mathscr{E}$ stands for the mathematical expectation, $\|v\|$ is the Euclidean norm of vector v, $\|v\| = (v^T v)^{1/2}$, while $\|A\|$ is spectral norm of matrix A, $\|A\| = [\lambda_{\max}(A^T A)]^{1/2}$. $\lambda_{\max(\min)}(A)$ is the eigenvalue of matrix A with maximum(minimum) real part. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem Statement and Preliminaries

Given a probability space $\{\Omega, \mathscr{F}, \mathbf{P}\}$ where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathscr{F}$. $\{r_t, t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories taking values in a finite set $S = \{1, 2, 3, \ldots, N\}$, with the mode transition probability matrix being

$$P\left(r_{t+\Delta t} = j \mid r_t = i\right) = \begin{cases} \pi_{ij}\Delta t + o\left(\Delta t\right), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o\left(\Delta t\right), & i = j, \end{cases} \tag{1}$$

where $\Delta t > 0$, $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$, $\pi_{ij} \geq 0$ $(i, j \in S, i \neq j)$ is the transition rate from mode i to j and for any state or mode $i \in S$; it satisfies

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}. \tag{2}$$

Since the transition rates of the Markov chain are partially known in this paper, some elements in matrix $\Pi = [\pi_{ij}]_{N\times N}$ are inaccessible. For instance, the system with five operation modes, the jump rates matrix Π may be viewed as

$$\begin{bmatrix} ? & \pi_{12} & ? & ? & \pi_{15} \\ \pi_{21} & ? & ? & \pi_{24} & ? \\ ? & ? & \pi_{33} & \pi_{34} & ? \\ ? & \pi_{42} & ? & ? & ? \\ \pi_{51} & ? & ? & ? & \pi_{55} \end{bmatrix}, \tag{3}$$

where ? represents the unknown element. For notation clarity, we denote $\mathcal{S}^i = \mathcal{S}_k^i \cup \mathcal{S}_{uk}^i$, for all $i \in S$ and

$$\begin{aligned} \mathcal{S}_k^i &\triangleq \left\{j : \pi_{ij} \text{ is known for } j \in S\right\}, \\ \mathcal{S}_{uk}^i &\triangleq \left\{j : \pi_{ij} \text{ is unknown for } j \in S\right\}. \end{aligned} \tag{4}$$

If $\mathcal{S}_k^i \neq \emptyset$, it is further described as

$$\mathcal{S}_k^i = \left\{k_1^i, k_2^i, \ldots, k_m^i\right\}, \quad 1 \leq m \leq N, \tag{5}$$

where k_j^i, $(j = 1, 2, \ldots, m)$ represent the jth known element of the set $\mathcal{S}_k^i$ in the ith row of the transition rate matrix Π. Furthermore, let π_i^m and π_i^M be the lower and upper bound for the diagonal elements of the jump rates matrix Π.

In this paper, the following uncertain neutral Markovian jump systems with mode-dependent interval time-varying delays, nonlinear perturbations, and partially known transition rates over the space $\{\Omega, \mathscr{F}, \mathbf{P}\}$ are considered:

$$\begin{aligned}\dot{x}(t) - C(t,r_t)\dot{x}(t-\tau(t,r_t)) &= A(t,r_t)x(t) + B(t,r_t)x(t-d(t,r_t)) \\ &\quad + D(t,r_t)f_1(x(t),t) \\ &\quad + E(t,r_t)f_2(x(t-d(t,r_t)),t) \\ &\quad + F(t,r_t)f_3(\dot{x}(t-\tau(t,r_t)),t),\end{aligned} \tag{6}$$

$$x(s) = \varphi(s), \quad r_s = r_0, \quad s \in [-\varsigma, 0], \tag{7}$$

where $x(t) \in \mathbb{R}^n$ is the system state and $\tau(t,r_t)$ is mode-dependent interval time-varying neutral delay which satisfies $0 \le \tau_i(t) \le \overline{\tau_i}, \dot{\tau}_i(t) \le \nu_i$ when $r_t = i \in S$. The mode-dependent interval time-varying retarded delay $d(t,r_t)$ is assumed that

$$\begin{aligned}&0 \le d_{1i} \le d_i(t) \le d_{2i}, \quad \max_{i\in S}\{d_{1i}\} \le \min_{j\in S}\{d_{2j}\}, \\ &\dot{d}_i(t) \le \mu_i, \quad \text{if } r_t = i \in S,\end{aligned} \tag{8}$$

where $\overline{\tau_i}, \nu_i \ge 0, d_{1i}, d_{2i}, \mu_i \ge 0$, and $\varsigma = \max_{i\in S}\{\overline{\tau_i}, d_{2i}\}$ are real constant scalars. The initial condition $\varphi(s)$ is a continuously differentiable vector-valued function. $f_1(x(t),t) \in \mathbb{R}^n$, $f_2(x(t-d(t,r_t)),t) \in \mathbb{R}^n$, and $f_3(\dot{x}(t-\tau(t,r_t)),t) \in \mathbb{R}^n$ are unknown nonlinear perturbations which are, with respect to the current state $x(t)$, the delayed state $x(t-d(t,r_t))$ and the neutral delay state $\dot{x}(t-\tau(t,r_t))$, respectively. For all t and $r_t = i \in S$, they are assumed to be bounded in magnitude as

$$\begin{aligned}\|f_1(x(t),t)\| &\le \alpha\|x(t)\|, \\ \|f_2(x(t-d_i(t)),t)\| &\le \beta\|x(t-d_i(t))\|, \\ \|f_3(\dot{x}(t-\tau_i(t)),t)\| &\le \gamma\|\dot{x}(t-\tau_i(t))\|,\end{aligned} \tag{9}$$

where $\alpha \ge 0$, $\beta \ge 0$, and $\gamma \ge 0$ are given constants, for simplicity, $f_1 \triangleq f_1(x(t),t)$, $f_2 \triangleq f_2(x(t-d_i(t)),t)$, and $f_3 \triangleq f_3(\dot{x}(t-\tau_i(t)),t)$.

For notational simplicity further, where $r_t = i \in S$, the parametric matrices $A(t,r_t) \in \mathbb{R}^{n\times n}$, $B(t,r_t) \in \mathbb{R}^{n\times n}$, $C(t,r_t) \in \mathbb{R}^{n\times n}$, $D(t,r_t) \in \mathbb{R}^{n\times n}$, $E(t,r_t) \in \mathbb{R}^{n\times n}$, and $F(t,r_t) \in \mathbb{R}^{n\times n}$ are denoted by $A_i(t)$, $B_i(t)$, $C_i(t)$, $D_i(t)$, $E_i(t)$, and $F_i(t)$, which can be described as

$$\begin{aligned}A_i(t) &= A_i + \Delta A_i(t), \qquad B_i(t) = B_i + \Delta B_i(t), \\ C_i(t) &= C_i, \qquad D_i(t) = D_i + \Delta D_i(t), \\ E_i(t) &= E_i + \Delta E_i(t), \qquad F_i(t) = F_i + \Delta F_i(t),\end{aligned} \tag{10}$$

where A_i, B_i, C_i, D_i, E_i, and F_i are known constant matrices with appropriate dimensions. $\Delta A_i(t) \in \mathbb{R}^{n\times n}$, $\Delta B_i(t) \in \mathbb{R}^{n\times n}$, $\Delta D_i(t) \in \mathbb{R}^{n\times n}$, $\Delta E_i(t) \in \mathbb{R}^{n\times n}$, and $\Delta F_i(t) \in \mathbb{R}^{n\times n}$ are uncertainties. The parametric matrix $\|C_i\| < 1$ and the admissible parametric uncertainties satisfy the following condition:

$$\begin{aligned}&[\Delta A_i(t) \;\; \Delta B_i(t) \;\; \Delta D_i(t) \;\; \Delta E_i(t) \;\; \Delta F_i(t)] \\ &\quad = L_i H_i(t)[N_{Ai} \;\; N_{Bi} \;\; N_{Di} \;\; N_{Ei} \;\; N_{Fi}],\end{aligned} \tag{11}$$

where L_i, N_{Ai}, N_{Bi}, N_{Di}, N_{Ei}, and N_{Fi} are known mode-dependent constant matrices with appropriate dimensions and $H_i(t)$ is an unknown and time-varying matrix satisfying

$$H_i^T(t)H_i(t) \le I, \quad \forall t. \tag{12}$$

Particularly, the following nominal systems can be obtained for $H_i(t) = 0$:

$$\begin{aligned}\dot{x}(t) - C_i\dot{x}(t-\tau_i(t)) &= A_i x(t) + B_i x(t-d_i(t)) \\ &\quad + D_i f_1 + E_i f_2 + F_i f_3.\end{aligned} \tag{13}$$

Before proceeding with the main results, we present the following assumptions, definitions, and lemmas.

Assumption 1. System matrices A_i, (for all $i \in S$), are Hurwitz and all the eigenvalues have negative real parts for each mode. L_i, (for all $i \in S$), is full rank in row.

Assumption 2. The Markov process is irreducible and the system mode r_t is available at time t.

Definition 3 (see [32]). Define operator $\mathfrak{D} : C([-\varsigma,0],\mathbb{R}^n) \to \mathbb{R}^n$ as $\mathfrak{D}(x_t) = x(t) - Cx(t-\tau)$. $\mathfrak{D}$ is said to be stable if the homogeneous difference equation

$$\begin{aligned}&\mathfrak{D}(x_t) = 0, \quad t \ge 0, \\ &x_0 = \psi \in \{\phi \in C([-\varsigma,0],\mathbb{R}^n) : \mathfrak{D}\phi = 0\}\end{aligned} \tag{14}$$

is uniformly asymptotically stable. In this paper, that is, $\|C_i\| + \gamma < 1$.

Definition 4 (see [33]). The system in (6) is exponentially stable with a decay rate ε for all $r_t = i \in S$, if there exist scalars $\varepsilon > 0$ and $\kappa \ge 1$ such that for all $x(t)$,

$$\|x(t)\| \le \kappa \exp\{-\varepsilon(t-t_0)\}\left\|x_{t_0}\right\|_\varsigma, \tag{15}$$

where ε is the exponential decay rate, $\| * \|$ denotes the Euclidean norm, and

$$\begin{aligned}\left\|x_{t_0}\right\|_\varsigma &= \sup_{\theta\in[-\varsigma,0]}\{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\} \\ &= \sup_{s\in[-\varsigma,t_0]}\{\|\varphi(s)\|, \|\dot{\varphi}(s)\|\}.\end{aligned} \tag{16}$$

Definition 5 (see [34]). Define the stochastic Lyapunov-Krasovskii function of system (6) as $V(x(t), r_t = i, t > 0) = V(x_t, i, t)$, where its infinitesimal generator is defined as

$$\begin{aligned}\Gamma V(x(t),i,t) &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\mathscr{E}\{V(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t) \mid x(t) = x,\right. \\ &\qquad \left. r_t = i\} - V(x(t),i,t)\right] \\ &= \frac{\partial}{\partial t}V(x(t),i,t) + \frac{\partial}{\partial x}V(x(t),i,t)\dot{x}(t) \\ &\quad + \sum_{j=1}^{N}\pi_{ij}V(x(t),j,t).\end{aligned} \tag{17}$$

Lemma 6 (see [35]). *Given constant matrices* Ω_1, Ω_2, *and* Ω_3, *where* $\Omega_1 = \Omega_1^T$ *and* $\Omega_2 = \Omega_2^T > 0$. $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ *if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3^T \\ * & \Omega_1 \end{bmatrix} < 0. \tag{18}$$

Lemma 7. *For any constant matrix* $Q = Q^T > 0$, *continuous functions* $0 \le h_1(t) \le h_2(t)$, *constant scalars* $0 \le \tau_1 < \tau_2$, *and constant* $\varepsilon > 0$ *such that the following integrations are well defined,*

(a)

$$\begin{aligned} &\frac{\exp\{2\varepsilon h_2(t)\} - \exp\{2\varepsilon h_1(t)\}}{2\varepsilon} \\ &\quad \times \int_{t-h_2(t)}^{t-h_1(t)} \exp\{2\varepsilon(s-t)\} x^T(s) Q x(s)\, ds \\ &\quad \ge \left[\int_{t-h_2(t)}^{t-h_1(t)} x^T(s)\, ds\right] Q \left[\int_{t-h_2(t)}^{t-h_1(t)} x(s)\, ds\right]. \end{aligned} \tag{19}$$

(b)

$$\begin{aligned} &\frac{\tau_2^2 - \tau_1^2}{2} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q x(s)\, ds\, d\theta \\ &\quad \ge \exp\{-2\varepsilon\tau_2\} \left[\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} x^T(s)\, ds\, d\theta\right] \\ &\quad \times Q \left[\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} x(s)\, ds\, d\theta\right]. \end{aligned} \tag{20}$$

Proof. (a) Is directly obtained from [36]. In addition, from $-\tau_2 \le \theta \le -\tau_1$ and $t + \theta \le s \le t$, it is held that $-\tau_2 \le \theta \le s - t \le 0$. Then

$$\begin{aligned} &\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q x(s)\, ds\, d\theta \\ &\quad \ge \exp\{-2\varepsilon\tau_2\} \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} x^T(s) Q x(s)\, ds\, d\theta. \end{aligned} \tag{21}$$

(b) Is thus true by [37]. □

Lemma 8 (see [38]). *For functions* $\lambda_1(t)$, $\lambda_2(t) \in [0,1]$, $\lambda_1(t) + \lambda_2(t) = 1$, *and* $\eta_1 = 0$ *with* $\lambda_1(t) = 0$ *and* $\eta_2 = 0$ *with* $\lambda_2(t) = 0$, *matrices* $P > 0$, $Q > 0$, *then there exists matrix* T *such that*

$$\begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} > 0 \tag{22}$$

and the following inequality holds:

$$\begin{aligned} &\frac{1}{\lambda_1(t)}\eta_1 P \eta_1^T + \frac{1}{\lambda_2(t)}\eta_2 Q \eta_2^T \\ &\quad \ge \begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix} \begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} \begin{bmatrix} \eta_1^T \\ \eta_2^T \end{bmatrix}. \end{aligned} \tag{23}$$

Lemma 9 (see [39]). *For given matrices* $Q = Q^T$, M, *and* N *with appropriate dimensions,*

$$Q + MF(t)N + N^T F^T(t) M^T < 0 \tag{24}$$

for all $F(t)$ *satisfying* $F^T(t)F(t) \le I$, *if and only if there exists a scalar* $\delta > 0$, *such that*

$$Q + \delta^{-1}MM^T + \delta NN^T < 0. \tag{25}$$

3. Main Results

This section will state the exponential stability analysis for neutral Markovian jump systems with mode-dependent interval time-varying delays, nonlinear perturbations, and partially known transition rates. With creative Lyapunov functional and novel matrix inequalities analysis, delay-range-dependent and rate-dependent exponential stability conditions are presented.

3.1. Exponential Stability for the Nominal Systems

Theorem 10. *For given scalars* π_i^m, π_i^M, α, β, γ, ε, $\overline{\tau_i}$, ν_i, d_{1i}, d_{2i}, μ_i *and constant scalar* d_{mi} *satisfying* $d_{1i} < d_{mi} < d_{2i}$, *the systems described by* (13) *with partially known transition rates are exponentially stable with decay rate* ε *and* $\kappa = \sqrt{\overline{\lambda}/\underline{\lambda}}$ *if* $\|C_i\| + \gamma < 1$ *and there exist symmetric positive matrices* $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_{1i} > 0$, $T_{1i} > 0$, $(i \in S)$, $Q_j > 0$, $(j = 3,4)$, $R_k > 0$, $T_l > 0$, $(k,l = 2,3,4)$, $U_m > 0$, $V_n > 0$, $W_s > 0$, $(m,n,s = 1,2,3,4)$ *and matrices* M_1, M_2, N_1, N_2, N_3, N_4 *for any scalars* ε_1, ε_2, ε_3, *any symmetric matrices* X_i, Y_{1i}, Y_{2i}, Z_{1i}, Z_{2i}, $(i \in S)$ *and any matrices* J_k, $(k = 1,2,\ldots,24)$ *with appropriate dimensions, such that the following linear matrix inequalities hold.*

When $i \in \mathcal{S}_k^i$

$$\begin{gathered} \sum_{j\in\mathcal{S}_k^i, j\ne i} \pi_{ij}\left(Q_{1j} - Y_{1i}\right) - \pi_{ii}Y_{1i} \le 0, \\ \sum_{j\in\mathcal{S}_k^i, j\ne i} \pi_{ij}\left(Q_{2j} - Y_{2i}\right) - \pi_{ii}Y_{2i} \le 0, \\ \sum_{j\in\mathcal{S}_k^i, j\ne i} \pi_{ij}\left(R_{1j} - Z_{1i}\right) - \pi_{ii}Z_{1i} \le 0, \\ \sum_{j\in\mathcal{S}_k^i, j\ne i} \pi_{ij}\left(T_{1j} - Z_{2i}\right) - \pi_{ii}Z_{2i} \le 0, \\ P_j - X_i \le 0, \quad j \in \mathcal{S}_{uk}^i, \\ Q_{1j} - Y_{1i} \le 0, \quad j \in \mathcal{S}_{uk}^i, \\ Q_{2j} - Y_{2i} \le 0, \quad j \in \mathcal{S}_{uk}^i, \\ R_{1j} - Z_{1i} \le 0, \quad j \in \mathcal{S}_{uk}^i, \\ T_{1j} - Z_{2i} \le 0, \quad j \in \mathcal{S}_{uk}^i. \end{gathered} \tag{26}$$

When $i \in \mathcal{S}_{uk}^{i}$

$$\sum_{j\in\mathcal{S}_k^i}\pi_{ij}\left(Q_{1j}-Y_{1i}\right)\le 0;\qquad \sum_{j\in\mathcal{S}_k^i}\pi_{ij}\left(Q_{2j}-Y_{2i}\right)\le 0, \tag{27}$$

$$\sum_{j\in\mathcal{S}_k^i}\pi_{ij}\left(R_{1j}-Z_{1i}\right)\le 0;\qquad \sum_{j\in\mathcal{S}_k^i}\pi_{ij}\left(T_{1j}-Z_{2i}\right)\le 0, \tag{28}$$

$$\begin{gathered}P_j-X_i\le 0,\quad j\in\mathcal{S}_{uk}^i,\ j\ne i,\\ P_j-X_i\ge 0,\quad j\in\mathcal{S}_{uk}^i,\ j=i,\end{gathered} \tag{29}$$

$$\begin{gathered}-\sum_{j\in\mathcal{S}_{uk}^i,j\ne i}\pi_i^M\left(Q_{1j}-Y_{1i}\right)-\pi_i^m Y_{1i}\le 0,\\ -\sum_{j\in\mathcal{S}_{uk}^i,j\ne i}\pi_i^M\left(Q_{2j}-Y_{2i}\right)-\pi_i^m Y_{2i}\le 0,\end{gathered} \tag{30}$$

$$\begin{gathered}-\sum_{j\in\mathcal{S}_{uk}^i,j\ne i}\pi_i^M\left(R_{1j}-Z_{1i}\right)-\pi_i^m Z_{1i}\le 0,\\ -\sum_{j\in\mathcal{S}_{uk}^i,j\ne i}\pi_i^M\left(T_{1j}-Z_{2i}\right)-\pi_i^m Z_{2i}\le 0,\end{gathered} \tag{31}$$

$$\begin{bmatrix}U_1 & M_1\\ M_1^T & U_1\end{bmatrix}>0,\qquad \begin{bmatrix}V_1 & M_2\\ M_2^T & V_1\end{bmatrix}>0, \tag{32}$$

$$\begin{gathered}\text{(i)}\quad \begin{bmatrix}U_3 & N_1\\ N_1^T & U_3\end{bmatrix}>0,\qquad \begin{bmatrix}V_3 & N_2\\ N_2^T & V_3\end{bmatrix}>0,\\ \text{(ii)}\quad \Omega_{i0}^{\varepsilon}+\Omega_{i1}^{\varepsilon}<0,\end{gathered} \tag{33}$$

$$\begin{gathered}\text{(iii)}\quad \begin{bmatrix}U_4 & N_3\\ N_3^T & U_4\end{bmatrix}>0,\qquad \begin{bmatrix}V_4 & N_4\\ N_4^T & V_4\end{bmatrix}>0,\\ \text{(iv)}\quad \Omega_{i0}^{\varepsilon}+\Omega_{i2}^{\varepsilon}<0,\end{gathered} \tag{34}$$

where

$$\begin{aligned}\Omega_{i0}^{\varepsilon}={}&\sum_{m=1}^{24}e_m\Upsilon_m e_m^T+\mathcal{L}(\Xi)\\ &-\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\}-1}\left(e_1-e_5\right)V_2\left(e_1^T-e_5^T\right)\\ &-\exp\{-2\varepsilon\overline{\tau}_i\}\left(\overline{\tau}_i e_1-e_{21}\right)W_1\left(\overline{\tau}_i e_1^T-e_{21}^T\right)\\ &-\exp\{-2\varepsilon d_{1i}\}\left(d_{1i}e_1-e_{11}\right)W_2\left(d_{1i}e_1^T-e_{11}^T\right)\\ &-\exp\{-2\varepsilon d_{mi}\}\left(\varrho_{1i}e_1-e_{14}\right)W_3\left(\varrho_{1i}e_1^T-e_{14}^T\right)\\ &-\exp\{-2\varepsilon d_{2i}\}\left(\varrho_{2i}e_1-e_{15}\right)W_4\left(\varrho_{2i}e_1^T-e_{15}^T\right)\\ &-\frac{2\varepsilon\overline{\tau}_i}{\exp\{2\varepsilon\overline{\tau}_i\}-1}\left[e_{20}\quad e_{21}-e_{20}\right]\\ &\times\begin{bmatrix}U_1 & M_1\\ M_1^T & U_1\end{bmatrix}\begin{bmatrix}e_{20}^T\\ e_{21}^T-e_{20}^T\end{bmatrix}\\ &-\frac{2\varepsilon\overline{\tau}_i}{\exp\{2\varepsilon\overline{\tau}_i\}-1}\left[e_1-e_{18}\quad e_{18}-e_{16}\right]\\ &\times\begin{bmatrix}V_1 & M_2\\ M_2^T & V_1\end{bmatrix}\begin{bmatrix}e_1^T-e_{18}^T\\ e_{18}^T-e_{16}^T\end{bmatrix},\end{aligned} \tag{35}$$

where $\mathcal{L}$ *is a linear operator on* $\mathbb{R}^{n\times n}$ *by*

$$\begin{aligned}\mathcal{L}(G)={}&G+G^T,\quad \forall G\in\mathbb{R}^{n\times n},\\ \Xi={}&-e_1J_1e_2^T+e_1\left(P_iB_i+J_1B_i\right)e_3^T\\ &+e_1\left(P_iC_i\right)e_{19}^T+\sum_{m=2}^{24}e_1\left(A_i^TJ_m^T\right)e_m^T\\ &+e_1P_i\left(D_ie_{22}^T+E_ie_{23}^T+F_ie_{24}^T\right)\\ &+e_2\left(J_2B_i\right)e_3^T-\sum_{m=3}^{24}e_2\left(J_m^T\right)e_m^T\\ &+\sum_{m=4}^{24}e_3\left(B_i^TJ_m^T\right)e_m^T+\sum_{m=1}^{18}e_m\left(J_mC_i\right)e_{19}^T\\ &+\sum_{m=20}^{24}e_{19}\left(C_i^TJ_m^T\right)e_m^T\\ &+\sum_{m=1}^{21}e_mJ_m\left(D_ie_{22}^T+E_ie_{23}^T+F_ie_{24}^T\right)\\ &+e_{22}J_{22}\left(E_ie_{23}^T+F_ie_{24}^T\right)+\sum_{m=23}^{24}e_{22}\left(D_i^TJ_m^T\right)e_m^T\\ &+e_{23}\left(J_{23}F_i\right)e_{24}^T+e_{23}\left(E_i^TJ_{24}^T\right)e_{24}^T,\\ \Upsilon_1={}&A_i^TP_i+P_iA_i+\sum_{j\in\mathcal{S}_k^i}\pi_{ij}\left(P_j-X_i\right)\\ &+2\varepsilon P_i+Q_{1i}+Q_3+R_2+\overline{\tau}_i^2U_1+d_{1i}^2U_2\\ &+\varrho_{1i}^2U_3+\varrho_{2i}^2U_4+\varepsilon_1\alpha^2I+J_1A_i+A_i^TJ_1^T,\\ \Upsilon_2={}&Q_{2i}+Q_4+T_2+\overline{\tau}_i^2V_1+d_{1i}^2V_2\\ &+\varrho_{1i}^2V_3+\varrho_{2i}^2V_4+\frac{\overline{\tau}_i^4}{4}W_1+\frac{d_{1i}^4}{4}W_2\\ &+\varrho_{3i}^2W_3+\varrho_{4i}^2W_4-J_2-J_2^T,\\ \Upsilon_3={}&\varpi\left(\varepsilon,\mu_i\right)R_{1i}+\varepsilon_2\beta^2I+J_3B_i+B_i^TJ_3^T,\\ \Upsilon_4={}&\varpi\left(\varepsilon,\mu_i\right)T_{1i},\\ \Upsilon_5={}&\exp\{-2\varepsilon d_{1i}\}\left(R_{1i}+R_3-R_2\right),\\ \Upsilon_6={}&\exp\{-2\varepsilon d_{mi}\}\left(R_4-R_3\right),\\ \Upsilon_7={}&-\exp\{-2\varepsilon d_{2i}\}R_4,\end{aligned}$$

$$\Upsilon_8 = \exp\{-2\varepsilon d_{1i}\}(T_{1i}+T_3-T_2),$$
$$\Upsilon_9 = \exp\{-2\varepsilon d_{mi}\}(T_4-T_3),$$
$$\Upsilon_{10} = -\exp\{-2\varepsilon d_{2i}\}T_4,$$
$$\Upsilon_{11} = -\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\}-1}U_2,$$
$$\Upsilon_{16} = -\exp\{-2\varepsilon\overline{\tau_i}\}Q_3,$$
$$\Upsilon_{17} = -\exp\{-2\varepsilon\overline{\tau_i}\}Q_4,$$
$$\Upsilon_{18} = -(1-\nu_i)\exp\{-2\varepsilon\overline{\tau_i}\}Q_{1i},$$
$$\Upsilon_{19} = -(1-\nu_i)\exp\{-2\varepsilon\overline{\tau_i}\}Q_{2i} + \varepsilon_3\gamma^2 I + J_{19}C_i + C_i^T J_{19}^T,$$
$$\Upsilon_{22} = -\varepsilon_1 I + J_{22}D_i + D_i^T J_{22}^T,$$
$$\Upsilon_{23} = -\varepsilon_2 I + J_{23}E_i + E_i^T J_{23}^T,$$
$$\Upsilon_{24} = -\varepsilon_3 I + J_{24}F_i + F_i^T J_{24}^T,$$
$$\Upsilon_m = 0, \quad (m = 12, 13, 14, 15, 20, 21),$$
$$\begin{aligned}\Omega_{i1}^{\varepsilon} = &-\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\}-\exp\{2\varepsilon d_{mi}\}}e_{15}U_4e_{15}^T\\ &-\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\}-\exp\{2\varepsilon d_{mi}\}}\\ &\times(e_6-e_7)V_4(e_6^T-e_7^T)\\ &-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}[e_{12}\ \ e_{14}-e_{12}]\\ &\times\begin{bmatrix}U_3 & N_1\\ N_1^T & U_3\end{bmatrix}\begin{bmatrix}e_{12}^T\\ e_{14}^T-e_{12}^T\end{bmatrix}\\ &-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}[e_5-e_3\ \ e_3-e_6]\\ &\times\begin{bmatrix}V_3 & N_2\\ N_2^T & V_3\end{bmatrix}\begin{bmatrix}e_5^T-e_3^T\\ e_3^T-e_6^T\end{bmatrix},\end{aligned}$$
$$\begin{aligned}\Omega_{i2}^{\varepsilon} = &-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}e_{14}U_3e_{14}^T\\ &-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}\\ &\times(e_5-e_6)V_3(e_5^T-e_6^T)\\ &-\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\}-\exp\{2\varepsilon d_{mi}\}}[e_{13}\ \ e_{15}-e_{13}]\\ &\times\begin{bmatrix}U_4 & N_3\\ N_3^T & U_4\end{bmatrix}\begin{bmatrix}e_{13}^T\\ e_{15}^T-e_{13}^T\end{bmatrix}\\ &-\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\}-\exp\{2\varepsilon d_{mi}\}}[e_3-e_7\ \ e_6-e_3]\\ &\times\begin{bmatrix}V_4 & N_4\\ N_4^T & V_4\end{bmatrix}\begin{bmatrix}e_3^T-e_7^T\\ e_6^T-e_3^T\end{bmatrix},\end{aligned} \tag{36}$$

where e_i $\{i = 1, 2, \ldots, 24\}$ *are block entry matrices; that is,*

$$e_3^T = [0\ 0\ I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$$
$$\varpi(\varepsilon,\mu_i) = (\mu_i-1)\exp\{-2\varepsilon d_{1i}\}\quad \textit{with } \mu_i \ge 1, \qquad (\mu_i-1)\exp\{-2\varepsilon d_{2i}\}\quad \textit{with } \mu_i < 1,$$
$$\begin{aligned}\overline{\lambda} = {} & \max_{i\in S}\{\lambda_{\max}(P_i)\} + \frac{1-\exp\{-2\varepsilon\varsigma\}}{2\varepsilon}\left\{\sum_{j=3}^{4}\lambda_{\max}(Q_j)+\sum_{k=2}^{4}\lambda_{\max}(R_k)+\sum_{l=2}^{4}\lambda_{\max}(T_l)\right\}\\ &+\frac{1-\exp\{-2\varepsilon\varsigma\}}{2\varepsilon}\left\{\max_{i\in S}\{\lambda_{\max}(Q_{1i})\}+\max_{i\in S}\{\lambda_{\max}(Q_{2i})\}+\max_{i\in S}\{\lambda_{\max}(R_{1i})\}+\max_{i\in S}\{\lambda_{\max}(T_{1i})\}\right\}\\ &+\frac{\exp\{-2\varepsilon\varsigma\}+2\varepsilon\varsigma-1}{4\varepsilon^2}\left\{\sum_{m=1}^{4}\lambda_{\max}(U_m)+\sum_{n=1}^{4}\lambda_{\max}(V_n)\right\}\\ &+\frac{1-\exp\{-2\varepsilon\varsigma\}+2\varepsilon^2\varsigma^2-2\varepsilon\varsigma}{8\varepsilon^3}\left\{\sum_{s=1}^{4}\lambda_{\max}(W_s)\right\},\end{aligned} \tag{37}$$
$$\underline{\lambda} = \min_{i\in S}\{\lambda_{\min}(P_i)\}, \qquad \varrho_{1i} = d_{mi}-d_{1i}, \qquad \varrho_{2i} = d_{2i}-d_{mi}, \qquad \varrho_{3i} = \frac{1}{2}\left(d_{mi}^2-d_{1i}^2\right), \qquad \varrho_{4i} = \frac{1}{2}\left(d_{2i}^2-d_{mi}^2\right).$$

Proof. Construct the following stochastic Lyapunov functional:

$$V(x(t), i, t) = \sum_{k=1}^{7} V_k(x_t, i), \tag{38}$$

where

$$V_1(x_t, i) = x^T(t) P_i x(t), \tag{39}$$

$$\begin{aligned} V_2(x_t, i) = &\int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_{1i} x(s)\, ds \\ &+ \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_{2i} \dot{x}(s)\, ds \\ &+ \int_{t-\overline{\tau_i}}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_3 x(s)\, ds \\ &+ \int_{t-\overline{\tau_i}}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_4 \dot{x}(s)\, ds, \end{aligned} \tag{40}$$

$$\begin{aligned} V_3(x_t, i) = &\int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} x^T(s) R_{1i} x(s)\, ds \\ &+ \int_{t-d_{1i}}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) R_2 x(s)\, ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} x^T(s) R_3 x(s)\, ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} x^T(s) R_4 x(s)\, ds, \end{aligned} \tag{41}$$

$$\begin{aligned} V_4(x_t, i) = &\int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_{1i} \dot{x}(s)\, ds \\ &+ \int_{t-d_{1i}}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_2 \dot{x}(s)\, ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_3 \dot{x}(s)\, ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_4 \dot{x}(s)\, ds, \end{aligned} \tag{42}$$

$$\begin{aligned} V_5(x_t, i) = &\int_{-\overline{\tau_i}}^{0} \int_{t+\theta}^{t} \overline{\tau_i} \exp\{2\varepsilon(s-t)\} x^T(s) U_1 x(s)\, ds\, d\theta \\ &+ \int_{-d_{1i}}^{0} \int_{t+\theta}^{t} d_{1i} \exp\{2\varepsilon(s-t)\} x^T \\ &\quad \times (s) U_2 x(s)\, ds\, d\theta \\ &+ \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^{t} \varrho_{1i} \exp\{2\varepsilon(s-t)\} x^T \\ &\quad \times (s) U_3 x(s)\, ds\, d\theta \\ &+ \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^{t} \varrho_{2i} \exp\{2\varepsilon(s-t)\} x^T \\ &\quad \times (s) U_4 x(s)\, ds\, d\theta, \end{aligned} \tag{43}$$

$$\begin{aligned} V_6(x_t, i) = &\int_{-\overline{\tau_i}}^{0} \int_{t+\theta}^{t} \overline{\tau_i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_1 \dot{x}(s)\, ds\, d\theta \\ &+ \int_{-d_{1i}}^{0} \int_{t+\theta}^{t} d_{1i} \exp\{2\varepsilon(s-t)\} \dot{x}^T \\ &\quad \times (s) V_2 \dot{x}(s)\, ds\, d\theta \\ &+ \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^{t} \varrho_{1i} \exp\{2\varepsilon(s-t)\} \dot{x}^T \\ &\quad \times (s) V_3 \dot{x}(s)\, ds\, d\theta \\ &+ \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^{t} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{x}^T \\ &\quad \times (s) V_4 \dot{x}(s)\, ds\, d\theta, \end{aligned} \tag{44}$$

$$\begin{aligned} V_7(x_t, i) = &\int_{-\overline{\tau_i}}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \frac{\overline{\tau_i}^2}{2} \exp\{2\varepsilon(s-t)\} \dot{x}^T \\ &\quad \times (s) W_1 \dot{x}(s)\, ds\, d\lambda\, d\theta \\ &+ \int_{-d_{1i}}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \frac{d_{1i}^2}{2} \exp\{2\varepsilon(s-t)\} \\ &\quad \times \dot{x}^T(s) W_2 \dot{x}(s)\, ds\, d\lambda\, d\theta \\ &+ \int_{-d_{mi}}^{-d_{1i}} \int_{\theta}^{0} \int_{t+\lambda}^{t} \varrho_{3i} \exp\{2\varepsilon(s-t)\} \\ &\quad \times \dot{x}^T(s) W_3 \dot{x}(s)\, ds\, d\lambda\, d\theta \\ &+ \int_{-d_{2i}}^{-d_{mi}} \int_{\theta}^{0} \int_{t+\lambda}^{t} \varrho_{4i} \exp\{2\varepsilon(s-t)\} \\ &\quad \times \dot{x}^T(s) W_4 \dot{x}(s)\, ds\, d\lambda\, d\theta. \end{aligned} \tag{45}$$

Remark 11. It should be pointed out that the proposed stochastic augmented Lyapunov functional (38) contains some triple-integral terms, which has not been used in of the existing literature in the same context before. Compared with the existing ones, [37] has shown that such triple-integral terms are very effective in the reduction of conservativeness.

Taking Γ as its infinitesimal generator along the trajectory of system (13), we obtain the following from Definition 5 and (38)–(45):

$$\Gamma V(x(t), i, t) = \sum_{k=1}^{7} \Gamma V_k(x_t, i), \tag{46}$$

$\Gamma V_1(x_t, i)$ can be easily obtained by the following equation:

$$\begin{aligned} \Gamma V_1(x_t, i) = 2\big[&x^T(t) A_i^T + x^T(t - d_i(t)) B_i^T \\ &+ \dot{x}^T(t - \tau_i(t)) C_i^T + f_1^T D_i^T \\ &+ f_2^T E_i^T + f_3^T F_i^T\big] \\ &\times P_i x(t) + x^T(t) \Bigg[\sum_{j \in \mathcal{S}_k^i} \pi_{ij} P_j + \sum_{j \in \mathcal{S}_{uk}^i} \pi_{ij} P_j\Bigg] x(t) \\ &+ 2\varepsilon x^T(t) P_i x(t) - 2\varepsilon V_1(x_t, i). \end{aligned} \tag{47}$$

With regard to $\Gamma V_2(x_t, i)$, the detailed procedures are given as follows.

$$\text{Define}\quad V_2(x_t,i) = V_{21}(x_t,i) + V_{22}(x_t,i) + V_{23}(x_t,i) + V_{24}(x_t,i), \tag{48}$$

where

$$\begin{aligned} V_{21}(x_t,i) &= \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_{1i} x(s)\, ds, \\ V_{22}(x_t,i) &= \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_{2i} \dot{x}(s)\, ds, \\ V_{23}(x_t,i) &= \int_{t-\overline{\tau_i}}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_3 x(s)\, ds, \\ V_{24}(x_t,i) &= \int_{t-\overline{\tau_i}}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_4 \dot{x}(s)\, ds. \end{aligned} \tag{49}$$

By the infinitesimal generator Γ, we obtain

$$\begin{aligned} \Gamma V_{21}(x_t,i) &= \exp\{-2\varepsilon t\} \\ &\quad \times \Gamma\left(\int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds\right) \\ &\quad - 2\varepsilon V_{21}(x_t,i), \end{aligned} \tag{50}$$

where

$$\begin{aligned} &\Gamma\left(\int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds\right) \\ &= \lim_{\Delta\to 0^+} \frac{1}{\Delta}\left\{\mathscr{E}\left[\int_{t+\Delta-\tau(t+\Delta, r_{t+\Delta})}^{t+\Delta} \exp\{2\varepsilon s\} x^T(s) Q_1(r_{t+\Delta}) \right.\right. \\ &\qquad \left. \times x(s)\, ds \mid x(t), r_t = i \right] \\ &\qquad \left. - \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds \right\} \\ &= \lim_{\Delta\to 0^+} \frac{1}{\Delta}\left\{\sum_{j=1}^{N} P(r_{t+\Delta} = j \mid r_t = i) \right. \\ &\qquad \times \left[\int_{t+\Delta-\tau_j(t+\Delta)}^{t+\Delta} \exp\{2\varepsilon s\} x^T(s) Q_{1j} x(s)\, ds\right] \\ &\qquad \left. - \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds \right\} \\ &= \lim_{\Delta\to 0^+} \frac{1}{\Delta}\left[\left(\sum_{j=1}^{N} \pi_{ij}\Delta\right) \int_{t+\Delta-\tau_j(t+\Delta)}^{t+\Delta} \exp\{2\varepsilon s\} x^T(s) \right. \\ &\qquad \times Q_{1j} x(s)\, ds \\ &\qquad + \int_{t+\Delta-\tau_i(t+\Delta)}^{t+\Delta} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds \\ &\qquad \left. - \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1i} x(s)\, ds \right] \\ &= \sum_{j=1}^{N} \pi_{ij} \int_{t-\tau_j(t)}^{t} \exp\{2\varepsilon s\} x^T(s) Q_{1j} x(s)\, ds \\ &\quad + \exp\{2\varepsilon(t)\} x^T(t) Q_{1i} x(t) \\ &\quad - (1-\dot{\tau}_i(t)) \exp\{2\varepsilon(t-\tau_i(t))\} x^T \\ &\quad \times (t-\tau_i(t)) Q_{1i} x(t-\tau_i(t)). \end{aligned} \tag{51}$$

Following the same procedure, $\Gamma V_{22}(x_t, i)$ is also obtained:

$$\begin{aligned} \Gamma V_{22}(x_t,i) &= \dot{x}^T(t) Q_{2i} \dot{x}(t) - (1-\dot{\tau}_i(t)) \exp\{2\varepsilon(t-\tau_i(t))\} \\ &\quad \times \dot{x}^T(t-\tau_i(t)) Q_{2i} \dot{x}(t-\tau_i(t)) \\ &\quad - \sum_{j=1}^{N} \pi_{ij} \int_{t-\tau_j(t)}^{t} \exp\{2\varepsilon s\} \dot{x}^T(s) Q_{2j} \dot{x}(s)\, ds \\ &\quad - 2\varepsilon V_{22}(x_t,i). \end{aligned} \tag{52}$$

Moreover, $\Gamma V_{23}(x_t, i)$, $\Gamma V_{24}(x_t, i)$ are easily calculated as shown in the following:

$$\begin{aligned} \Gamma V_{23}(x_t,i) &= x^T(t) Q_3 x(t) - \exp\{-2\varepsilon\overline{\tau_i}\} \\ &\quad \times x^T(t-\overline{\tau_i}) Q_3 x(t-\overline{\tau_i}) - 2\varepsilon V_{23}(x_t,i), \\ \Gamma V_{24}(x_t,i) &= \dot{x}^T(t) Q_4 \dot{x}(t) - \exp\{-2\varepsilon\overline{\tau_i}\} \\ &\quad \times \dot{x}^T(t-\overline{\tau_i}) Q_4 \dot{x}(t-\overline{\tau_i}) - 2\varepsilon V_{24}(x_t,i). \end{aligned} \tag{53}$$

According to (50), (51), (52), and (53), we can easily obtain the following (54):

$$\begin{aligned} &\Gamma V_2(x_t,i) \\ &\quad = x^T(t)[Q_{1i} + Q_3] x(t) \\ &\qquad + \dot{x}^T(t)[Q_{2i} + Q_4] \dot{x}(t) - 2\varepsilon V_2(x_t,i) \\ &\qquad - (1-\dot{\tau}_i(t)) \exp\{-2\varepsilon\tau_i(t)\} x^T(t-\tau_i(t)) Q_{1i} x \\ &\qquad \times (t-\tau_i(t)) - (1-\dot{\tau}_i(t)) \exp\{-2\varepsilon\tau_i(t)\} \dot{x}^T \\ &\qquad \times (t-\tau_i(t)) Q_{2i} \dot{x}(t-\tau_i(t)) \\ &\qquad - \exp\{-2\varepsilon\overline{\tau_i}\} x^T(t-\overline{\tau_i}) Q_3 x(t-\overline{\tau_i}) \\ &\qquad - \exp\{-2\varepsilon\overline{\tau_i}\} \dot{x}^T(t-\overline{\tau_i}) Q_4 \dot{x}(t-\overline{\tau_i}) \\ &\qquad + \sum_{j\in S} \pi_{ij} \int_{t-\tau_j(t)}^{t} \exp\{2\varepsilon(s-t)\} \\ &\qquad\qquad \times \left[x^T(s) Q_{1j} x(s) + \dot{x}^T(s) Q_{2j} \dot{x}(s)\right] ds \end{aligned}$$

$$
\begin{aligned}
&\leq x^T(t)\left[Q_{1i}+Q_3\right]x(t)\\
&\quad+\dot{x}^T(t)\left[Q_{2i}+Q_4\right]\dot{x}(t)-2\varepsilon V_2(x_t,i)\\
&\quad-\left(1-\dot{\tau}_i(t)\right)\exp\{-2\varepsilon\tau_i(t)\}x^T(t-\tau_i(t))Q_{1i}x\\
&\quad\times(t-\tau_i(t))-\left(1-\dot{\tau}_i(t)\right)\exp\{-2\varepsilon\tau_i(t)\}\dot{x}^T\\
&\quad\times(t-\tau_i(t))Q_{2i}\dot{x}(t-\tau_i(t))\\
&\quad-\exp\{-2\varepsilon\overline{\tau}_i\}x^T(t-\overline{\tau}_i)Q_3x(t-\overline{\tau}_i)\\
&\quad-\exp\{-2\varepsilon\overline{\tau}_i\}\dot{x}^T(t-\overline{\tau}_i)Q_4\dot{x}(t-\overline{\tau}_i)\\
&\quad+\sum_{j\neq i}\pi_{ij}\int_{t-\overline{\tau}_j}^{t}\exp\{2\varepsilon(s-t)\}\\
&\qquad\times\left[x^T(s)Q_{1j}x(s)+\dot{x}^T(s)Q_{2j}\dot{x}(s)\right]ds.
\end{aligned}
\tag{54}
$$

Then, in the same method, $\Gamma V_3(x_t,i)$, $\Gamma V_4(x_t,i)$ can be calculated and the results are given by the following, respectively:

$$
\begin{aligned}
&\Gamma V_3(x_t,i)\\
&\quad=x^T(t)R_2x(t)+\exp\{-2\varepsilon d_{1i}\}x^T\\
&\qquad\times(t-d_{1i})\left[R_{1i}+R_3-R_2\right]x(t-d_{1i})\\
&\qquad+\exp\{-2\varepsilon d_{mi}\}x^T(t-d_{mi})\left[R_4-R_3\right]x(t-d_{mi})\\
&\qquad-\exp\{-2\varepsilon d_{2i}\}x^T(t-d_{2i})R_4x(t-d_{2i})\\
&\qquad-2\varepsilon V_3(x_t,i)-\left(1-\dot{d}_i(t)\right)\\
&\qquad\times\exp\{-2\varepsilon d_i(t)\}x^T(t-d_i(t))R_{1i}x(t-d_i(t))\\
&\qquad+\sum_{j\in S}\pi_{ij}\int_{t-d_j(t)}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}x^T(s)R_{1j}x(s)\,ds\\
&\quad\leq x^T(t)R_2x(t)+\exp\{-2\varepsilon d_{1i}\}x^T(t-d_{1i})\\
&\qquad\times\left[R_{1i}+R_3-R_2\right]x(t-d_{1i})\\
&\qquad+\exp\{-2\varepsilon d_{mi}\}x^T(t-d_{mi})\left[R_4-R_3\right]x\\
&\qquad\times(t-d_{mi})-\exp\{-2\varepsilon d_{2i}\}x^T\\
&\qquad\times(t-d_{2i})R_4x(t-d_{2i})-2\varepsilon V_3(x_t,i)\\
&\qquad-\left(1-\dot{d}_i(t)\right)\exp\{-2\varepsilon d_i(t)\}x^T\\
&\qquad\times(t-d_i(t))R_{1i}x(t-d_i(t))\\
&\qquad+\sum_{j\neq i}\pi_{ij}\int_{t-d_{2j}}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}x^T(s)R_{1j}x(s)\,ds,
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
&\Gamma V_4(x_t,i)\\
&\quad=\dot{x}^T(t)T_2\dot{x}(t)+\exp\{-2\varepsilon d_{1i}\}\dot{x}^T\\
&\qquad\times(t-d_{1i})\left[T_{1i}+T_3-T_2\right]\dot{x}(t-d_{1i})\\
&\qquad+\exp\{-2\varepsilon d_{mi}\}\dot{x}^T(t-d_{mi})\\
&\qquad\times\left[T_4-T_3\right]\dot{x}(t-d_{mi})\\
&\qquad-\exp\{-2\varepsilon d_{2i}\}\dot{x}^T(t-d_{2i})T_4\dot{x}(t-d_{2i})\\
&\qquad-2\varepsilon V_4(x_t,i)-\left(1-\dot{d}_i(t)\right)\exp\{-2\varepsilon d_i(t)\}\dot{x}^T\\
&\qquad\times(t-d_i(t))T_{1i}\dot{x}(t-d_i(t))\\
&\qquad+\sum_{j\in S}\pi_{ij}\int_{t-d_j(t)}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)T_{1j}\dot{x}(s)\,ds\\
&\quad\leq\dot{x}^T(t)T_2\dot{x}(t)+\exp\{-2\varepsilon d_{1i}\}\dot{x}^T(t-d_{1i})\\
&\qquad\times\left[T_{1i}+T_3-T_2\right]\dot{x}(t-d_{1i})\\
&\qquad+\exp\{-2\varepsilon d_{mi}\}\dot{x}^T(t-d_{mi})\left[T_4-T_3\right]\dot{x}(t-d_{mi})\\
&\qquad-\exp\{-2\varepsilon d_{2i}\}\dot{x}^T(t-d_{2i})T_4\dot{x}(t-d_{2i})\\
&\qquad-2\varepsilon V_4(x_t,i)-\left(1-\dot{d}_i(t)\right)\\
&\qquad\times\exp\{-2\varepsilon d_i(t)\}\dot{x}^T(t-d_i(t))T_{1i}\dot{x}(t-d_i(t))\\
&\qquad+\sum_{j\neq i}\pi_{ij}\int_{t-d_{2j}}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)T_{1j}\dot{x}(s)\,ds.
\end{aligned}
\tag{56}
$$

Moreover, $\Gamma V_5(x_t,i)$, $\Gamma V_6(x_t,i)$, and $\Gamma V_7(x_t,i)$ can be directly obtained as follows:

$$
\begin{aligned}
&\Gamma V_5(x_t,i)\\
&\quad=x^T(t)\left[\overline{\tau}_i^2U_1+d_{1i}^2U_2+\varrho_{1i}^2U_3+\varrho_{2i}^2U_4\right]x(t)\\
&\qquad-2\varepsilon V_5(x_t,i)\\
&\qquad-\int_{t-\overline{\tau}_i}^{t}\overline{\tau}_i\exp\{2\varepsilon(s-t)\}x^T(s)U_1x(s)\,ds\\
&\qquad-\int_{t-d_{1i}}^{t}d_{1i}\exp\{2\varepsilon(s-t)\}x^T(s)U_2x(s)\,ds\\
&\qquad-\int_{t-d_{mi}}^{t-d_{1i}}\varrho_{1i}\exp\{2\varepsilon(s-t)\}x^T(s)U_3x(s)\,ds\\
&\qquad-\int_{t-d_{2i}}^{t-d_{mi}}\varrho_{2i}\exp\{2\varepsilon(s-t)\}x^T(s)U_4x(s)\,ds,
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma V_6(x_t,i)\\
&\quad= \dot{x}^T(t)\left[\overline{\tau_i}^2 V_1 + d_{1i}^2 V_2 + \varrho_{1i}^2 V_3 + \varrho_{2i}^2 V_4\right]\dot{x}(t)\\
&\qquad - 2\varepsilon V_6(x_t,i)\\
&\qquad - \int_{t-\overline{\tau_i}}^{t} \overline{\tau_i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_1\dot{x}(s)\,ds\\
&\qquad - \int_{t-d_{1i}}^{t} d_{1i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_2\dot{x}(s)\,ds\\
&\qquad - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_3\dot{x}(s)\,ds\\
&\qquad - \int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_4\dot{x}(s)\,ds,
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma V_7(x_t,i)\\
&\quad= \dot{x}^T(t)\left[\frac{\overline{\tau_i}^4}{4}W_1 + \frac{d_{1i}^4}{4}W_2 + \varrho_{3i}^2 W_3 + \varrho_{4i}^2 W_4\right]\dot{x}(t)\\
&\qquad - 2\varepsilon V_7(x_t,i)\\
&\qquad - \int_{-\overline{\tau_i}}^{0}\int_{t+\theta}^{t} \frac{\overline{\tau_i}^2}{2}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_1\dot{x}(s)\,ds\,d\theta\\
&\qquad - \int_{-d_{1i}}^{0}\int_{t+\theta}^{t} \frac{d_{1i}^2}{2}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_2\dot{x}(s)\,ds\,d\theta\\
&\qquad - \int_{-d_{mi}}^{-d_{1i}}\int_{t+\theta}^{t} \varrho_{3i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_3\dot{x}(s)\,ds\,d\theta\\
&\qquad - \int_{-d_{2i}}^{-d_{mi}}\int_{t+\theta}^{t} \varrho_{4i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_4\dot{x}(s)\,ds\,d\theta.
\end{aligned} \tag{57}
$$

Define

$$
\begin{aligned}
\xi(t) = \operatorname{col}\Big\{ & x(t)\quad \dot{x}(t)\quad x(t-d_i(t))\quad \dot{x}(t-d_i(t))\quad x(t-d_{1i})\quad x(t-d_{mi})\quad x(t-d_{2i})\quad \dot{x}(t-d_{1i})\quad \dot{x}(t-d_{mi})\quad \dot{x}(t-d_{2i})\\
& \int_{t-d_{1i}}^{t} x(s)\,ds\quad \int_{t-d_i(t)}^{t-d_{1i}} x(s)\,ds\quad \int_{t-d_{2i}}^{t-d_i(t)} x(s)\,ds\quad \int_{t-d_{mi}}^{t-d_{1i}} x(s)\,ds\quad \int_{t-d_{2i}}^{t-d_{mi}} x(s)\,ds\quad x(t-\overline{\tau_i})\quad \dot{x}(t-\overline{\tau_i})\\
& x(t-\tau_i(t))\quad \dot{x}(t-\tau_i(t))\quad \int_{t-\tau_i(t)}^{t} x(s)\,ds\quad \int_{t-\overline{\tau_i}}^{t} x(s)\,ds\quad f_1\quad f_2\quad f_3\Big\}.
\end{aligned} \tag{58}
$$

Then, there exist matrices $J = \operatorname{col}\{J_k, (k = 1, 2, \ldots, 24)\}$ with appropriate dimensions, such that the following equality holds according to (13):

$$
\begin{aligned}
2\xi^T(t)J[&-\dot{x}(t) + A_i x(t) + B_i x(t-d_i(t))\\
&+ C_i\dot{x}(t-\tau_i(t)) + D_i f_1 + E_i f_2 + F_i f_3] = 0.
\end{aligned} \tag{59}
$$

Due to $\sum_{j=1}^{N}\pi_{ij} = 0$, the following zero equations hold for arbitrary matrices $X_i = X_i^T$, $Y_{1i} = Y_{1i}^T$, $Y_{2i} = Y_{2i}^T$, $Z_{1i} = Z_{1i}^T$, $Z_{2i} = Z_{2i}^T$, $i \in S$; that is,

$$
\begin{aligned}
&-x^T(t)\left[\sum_{j\in\mathcal{S}_k^i}\pi_{ij}X_i + \sum_{j\in\mathcal{S}_{uk}^i}\pi_{ij}X_i\right]x(t) = 0,\\
&-\int_{t-\overline{\tau_j}}^{t}\exp\{2\varepsilon(s-t)\}x^T(s)\\
&\qquad\times\left[\sum_{j\in\mathcal{S}_k^i}\pi_{ij}Y_{1i} + \sum_{j\in\mathcal{S}_{uk}^i}\pi_{ij}Y_{1i}\right]x(s)\,ds = 0,\\
&-\int_{t-\overline{\tau_j}}^{t}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)\\
&\qquad\times\left[\sum_{j\in\mathcal{S}_k^i}\pi_{ij}Y_{2i} + \sum_{j\in\mathcal{S}_{uk}^i}\pi_{ij}Y_{2i}\right]\dot{x}(s)\,ds = 0,\\
&-\int_{t-d_{2j}}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}x^T(s)\\
&\qquad\times\left[\sum_{j\in\mathcal{S}_k^i}\pi_{ij}Z_{1i} + \sum_{j\in\mathcal{S}_{uk}^i}\pi_{ij}Z_{1i}\right]x(s)\,ds = 0,\\
&-\int_{t-d_{2j}}^{t-d_{1i}}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)\\
&\qquad\times\left[\sum_{j\in\mathcal{S}_k^i}\pi_{ij}Z_{2i} + \sum_{j\in\mathcal{S}_{uk}^i}\pi_{ij}Z_{2i}\right]\dot{x}(s)\,ds = 0.
\end{aligned} \tag{60}
$$

In view of (9), the following inequalities hold for any scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and $\varepsilon_3 > 0$:

$$
\varepsilon_1\left[\alpha^2 x^T(t)x(t) - f_1^T(x(t),t)f_1(x(t),t)\right] \ge 0,
$$

$$\begin{aligned}&\varepsilon_2\left[\beta^2 x^T(t-d_i(t))x(t-d_i(t))\right.\\&\qquad\left.-f_2^T(x(t-d_i(t)),t)f_2(x(t-d_i(t)),t)\right]\geq 0,\\&\varepsilon_3\left[\gamma^2\dot{x}^T(t-\tau_i(t))\dot{x}(t-\tau_i(t))\right.\\&\qquad\left.-f_3^T(\dot{x}(t-\tau_i(t)),t)f_3(\dot{x}(t-\tau_i(t)),t)\right]\geq 0.\end{aligned}\tag{61}$$

From (46) and (61), we have

$$\begin{aligned}&\Gamma V(x(t),i,t)\\&\quad\leq\sum_{k=1}^{7}\Gamma V_k(x_t,i)\\&\qquad+\varepsilon_1\left[\alpha^2 x^T(t)x(t)-f_1^T f_1\right]\\&\qquad+\varepsilon_2\left[\beta^2 x^T(t-d_i(t))x(t-d_i(t))-f_2^T f_2\right]\\&\qquad+\varepsilon_3\left[\gamma^2\dot{x}^T(t-\tau_i(t))\dot{x}(t-\tau_i(t))-f_3^T f_3\right].\end{aligned}\tag{62}$$

Since it is easy to see $-(1-\dot{\tau}_i(t))\exp\{-2\varepsilon\tau_i(t)\}\leq-(1-\nu_i)\exp\{-2\varepsilon\overline{\tau}_i\}$, then from (8) we also obtain that

$$-\left(1-\dot{d}_i(t)\right)\exp\{-2\varepsilon d_i(t)\}\leq\varpi(\varepsilon,\mu_i),\tag{63}$$

where $\varpi(\varepsilon,\mu_i)$ is defined in Theorem 10. Notice (a) of Lemma 7, then

$$\begin{aligned}&-\int_{t-d_{1i}}^{t}d_{1i}\exp\{2\varepsilon(s-t)\}x^T(s)U_2x(s)\,ds\\&\qquad\leq-\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\}-1}\xi^T(t)e_{11}U_2e_{11}^T\xi(t),\\&-\int_{t-d_{1i}}^{t}d_{1i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_2\dot{x}(s)\,ds\\&\qquad\leq-\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\}-1}\xi^T(t)(e_1-e_5)V_2\left(e_1^T-e_5^T\right)\xi(t).\end{aligned}\tag{64}$$

Notice (b) of Lemma 7, then

$$\begin{aligned}&-\int_{-\overline{\tau}_i}^{0}\int_{t+\theta}^{t}\frac{\overline{\tau}_i^2}{2}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_1\dot{x}(s)\,ds\,d\theta\\&\qquad\leq-\exp\{-2\varepsilon\overline{\tau}_i\}\xi^T(t)(\overline{\tau}_ie_1-e_{21})\\&\qquad\quad\times W_1\left(\overline{\tau}_ie_1^T-e_{21}^T\right)\xi(t),\\&-\int_{-d_{1i}}^{0}\int_{t+\theta}^{t}\frac{d_{1i}^2}{2}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_2\dot{x}(s)\,ds\,d\theta\\&\qquad\leq-\exp\{-2\varepsilon d_{1i}\}\xi^T(t)(d_{1i}e_1-e_{11})\\&\qquad\quad\times W_2\left(d_{1i}e_1^T-e_{11}^T\right)\xi(t),\\&-\int_{-d_{mi}}^{-d_{1i}}\int_{t+\theta}^{t}\varrho_{3i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_3\dot{x}(s)\,ds\,d\theta\\&\qquad\leq-\exp\{-2\varepsilon d_{mi}\}\xi^T(t)(\varrho_{1i}e_1-e_{14})\\&\qquad\quad\times W_3\left(\varrho_{1i}e_1^T-e_{14}^T\right)\xi(t),\\&-\int_{-d_{2i}}^{-d_{mi}}\int_{t+\theta}^{t}\varrho_{4i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)W_4\dot{x}(s)\,ds\,d\theta\\&\qquad\leq-\exp\{-2\varepsilon d_{2i}\}\xi^T(t)(\varrho_{2i}e_1-e_{15})\\&\qquad\quad\times W_4\left(\varrho_{2i}e_1^T-e_{15}^T\right)\xi(t).\end{aligned}\tag{65}$$

For $d_i(t)\in[d_{1i},d_{mi}]$, the following is held from (a) of Lemma 7:

$$\begin{aligned}&-\int_{t-d_{mi}}^{t-d_{1i}}\varrho_{1i}\exp\{2\varepsilon(s-t)\}\dot{x}^T(s)V_3\dot{x}(s)\,ds\\&\quad=-\left\{\int_{t-d_i(t)}^{t-d_{1i}}+\int_{t-d_{mi}}^{t-d_i(t)}\right\}\varrho_{1i}\exp\{2\varepsilon(s-t)\}\\&\qquad\times\dot{x}^T(s)V_3\dot{x}(s)\,ds\\&\quad\leq-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_i(t)\}-\exp\{2\varepsilon d_{1i}\}}\\&\qquad\times\left[\int_{t-d_i(t)}^{t-d_{1i}}\dot{x}^T(s)\,ds\right]V_3\left[\int_{t-d_i(t)}^{t-d_{1i}}\dot{x}(s)\,ds\right]\\&\qquad-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d(t)\}}\\&\qquad\times\left[\int_{t-d_{mi}}^{t-d_i(t)}\dot{x}^T(s)\,ds\right]V_3\left[\int_{t-d_{mi}}^{t-d_i(t)}\dot{x}(s)\,ds\right]\\&\quad=-\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}\xi^T(t)\\&\qquad\times\left[\frac{1}{\lambda_{1i}(t)}(e_5-e_3)V_3\left(e_5^T-e_3^T\right)\right.\\&\qquad\quad\left.+\frac{1}{\lambda_{2i}(t)}(e_3-e_6)V_3\left(e_3^T-e_6^T\right)\right]\xi(t),\end{aligned}\tag{66}$$

where

$$\begin{aligned}\lambda_{1i}(t)&=\frac{\exp\{2\varepsilon d_i(t)\}-\exp\{2\varepsilon d_{1i}\}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}},\\\lambda_{2i}(t)&=\frac{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_i(t)\}}{\exp\{2\varepsilon d_{mi}\}-\exp\{2\varepsilon d_{1i}\}}.\end{aligned}\tag{67}$$

By Lemma 8, there exists matrix N_2 with appropriate dimensions such that

$$\begin{aligned}
&-\int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_3 \dot{x}(s)\, ds \\
&\quad \le -\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\} - \exp\{2\varepsilon d_{1i}\}} \xi^T(t) \\
&\quad \times \begin{bmatrix} e_5 - e_3 & e_3 - e_6 \end{bmatrix} \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} \begin{bmatrix} e_5^T - e_3^T \\ e_3^T - e_6^T \end{bmatrix} \xi(t), \\
&\begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0.
\end{aligned} \tag{68}$$

Similarly, considering $-\int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} x^T(s) U_3 x(s) ds$ and following the same procedure, there exists matrix N_1 with appropriate dimensions such that

$$\begin{aligned}
&-\int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} x^T(s) U_3 x(s)\, ds \\
&\quad \le -\frac{2\varepsilon\varrho_{1i}}{\exp\{2\varepsilon d_{mi}\} - \exp\{2\varepsilon d_{1i}\}} \xi^T(t) \\
&\quad \times \begin{bmatrix} e_{12} & e_{14} - e_{12} \end{bmatrix} \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} \begin{bmatrix} e_{12}^T \\ e_{14}^T - e_{12}^T \end{bmatrix} \xi(t), \\
&\begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0.
\end{aligned} \tag{69}$$

For $\tau_i(t) \in [0, \overline{\tau_i}]$, with the same matrix inequalities technique, we obtain the following:

$$\begin{aligned}
&-\int_{t-\overline{\tau_i}}^{t} \overline{\tau_i} \exp\{2\varepsilon(s-t)\} x^T(s) U_1 x(s)\, ds \\
&\quad \le -\frac{2\varepsilon\overline{\tau_i}}{\exp\{2\varepsilon\overline{\tau_i}\} - 1} \xi^T(t) \begin{bmatrix} e_{20} & e_{21} - e_{20} \end{bmatrix} \\
&\quad \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} e_{20}^T \\ e_{21}^T - e_{20}^T \end{bmatrix} \xi(t), \\
&\begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} > 0, \\
&-\int_{t-\overline{\tau_i}}^{t} \overline{\tau_i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_1 \dot{x}(s)\, ds \\
&\quad \le -\frac{2\varepsilon\overline{\tau_i}}{\exp\{2\varepsilon\overline{\tau_i}\} - 1} \xi^T(t) \begin{bmatrix} e_1 - e_{18} & e_{18} - e_{16} \end{bmatrix} \\
&\quad \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} e_1^T - e_{18}^T \\ e_{18}^T - e_{16}^T \end{bmatrix} \xi(t), \\
&\begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} > 0.
\end{aligned} \tag{70}$$

Consider $-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} x^T(s) U_4 x(s) ds$ and $-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_4 \dot{x}(s) ds$, which are directly estimated by (a) of Lemma 7; that is,

$$\begin{aligned}
&-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} x^T(s) U_4 x(s)\, ds \\
&\quad \le -\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} \xi^T(t) e_{15} U_4 e_{15}^T \xi(t), \\
&-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_4 \dot{x}(s)\, ds \\
&\quad \le -\frac{2\varepsilon\varrho_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} \xi^T(t) \\
&\quad \times (e_6 - e_7) V_4 \left(e_6^T - e_7^T\right) \xi(t).
\end{aligned} \tag{71}$$

Substituting (47), (54)–(60), and (63)–(71) into (62) we obtain

$$\begin{aligned}
&\Gamma V(x(t), i, t) + 2\varepsilon V(x(t), i, t) \\
&\quad \le \xi^T(t) \left(\Omega_{i0}^{\varepsilon} + \Omega_{i1}^{\varepsilon}\right) \xi(t) + \mho,
\end{aligned} \tag{72}$$

where

$$\begin{aligned}
\mho = &\int_{t-\overline{\tau_j}}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) \Bigg\{ \sum_{j\ne i} \pi_{ij} Q_{1j} - \sum_{j\in\mathcal{S}_k^i} \pi_{ij} Y_{1i} \\
&\quad - \sum_{j\in\mathcal{S}_{uk}^i} \pi_{ij} Y_{1i} \Bigg\} x(s)\, ds \\
&+ \int_{t-\overline{\tau_j}}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) \Bigg\{ \sum_{j\ne i} \pi_{ij} Q_{2j} - \sum_{j\in\mathcal{S}_k^i} \pi_{ij} Y_{2i} \\
&\quad - \sum_{j\in\mathcal{S}_{uk}^i} \pi_{ij} Y_{2i} \Bigg\} \dot{x}(s)\, ds \\
&+ \int_{t-d_{2j}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} x^T(s) \Bigg\{ \sum_{j\ne i} \pi_{ij} R_{1j} - \sum_{j\in\mathcal{S}_k^i} \pi_{ij} Z_{1i} \\
&\quad - \sum_{j\in\mathcal{S}_{uk}^i} \pi_{ij} Z_{1i} \Bigg\} x(s)\, ds \\
&+ \int_{t-d_{2j}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) \Bigg\{ \sum_{j\ne i} \pi_{ij} T_{1j} - \sum_{j\in\mathcal{S}_k^i} \pi_{ij} Z_{2i} \\
&\quad - \sum_{j\in\mathcal{S}_{uk}^i} \pi_{ij} Z_{2i} \Bigg\} x(s)\, ds \\
&+ x^T(t) \Bigg\{ \sum_{j\in\mathcal{S}_{uk}^i} \pi_{ij} \left(P_j - X_i\right) \Bigg\} x(t).
\end{aligned} \tag{73}$$

On the other hand, for $d_i(t) \in [d_{mi}, d_{2i}]$, the integral terms

$$\begin{aligned}&-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} x^T(s) U_4 x(s)\, ds,\\&-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_4 \dot{x}(s)\, ds\end{aligned} \tag{74}$$

are disposed and estimated by Lemma 8, and

$$\begin{aligned}&-\int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} x^T(s) U_3 x(s)\, ds,\\&-\int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) V_3 \dot{x}(s)\, ds\end{aligned} \tag{75}$$

are directly estimated by (a) of Lemma 7. Therefore,

$$\begin{aligned}&\Gamma V(x(t), i, t) + 2\varepsilon V(x(t), i, t)\\&\quad \le \xi^T(t)\left(\Omega_{i0}^{\varepsilon} + \Omega_{i2}^{\varepsilon}\right)\xi(t) + \mho.\end{aligned} \tag{76}$$

With (72) and (76), the following inequality (77) is held for $d_i(t) \in [d_{1i}, d_{2i}]$ if (26)–(34) are satisfied

$$\Gamma V(x(t), i, t) + 2\varepsilon V(x(t), i, t) < 0. \tag{77}$$

From the stochastic Lyapunov functional (38) and (77), it is held that

$$\begin{aligned}V(x(t), i, t) &\ge \min_{i\in S}\{\lambda_{\min}(P_i)\}\|x(t)\|^2\\&= \underline{\lambda}\|x(t)\|^2,\\V(x(t), i, t) &< \exp\{-2\varepsilon(t-t_0)\}\\&\quad \times V\left(x_{t_0}, r_{t_0}, t_0\right).\end{aligned} \tag{78}$$

Moreover, we have

$$V\left(x_{t_0}, r_{t_0}, t_0\right) = \sum_{k=1}^{6} V_k\left(x_{t_0}, r_{t_0}\right) \le \overline{\lambda}\left\|x_{t_0}\right\|_{\varsigma}^2. \tag{79}$$

Then from (78) and (79), it is readily seen that

$$\|x(t)\| \le \sqrt{\frac{\overline{\lambda}}{\underline{\lambda}}} \exp\{-\varepsilon(t-t_0)\}\left\|x_{t_0}\right\|_{\varsigma}, \tag{80}$$

where $\kappa = \sqrt{\overline{\lambda}/\underline{\lambda}} \ge 1$.

Therefore, by Definition 4, the system (13) is exponentially stable with a decay rate ε. This completes the proof. □

Remark 12. It is noted that the integral intervals in (79) are enlarged as follows:

$$\begin{aligned}&\sum_{k=1}^{6} V_k\left(x_{t_0}, r_{t_0}\right)\\&\quad \le \int_{t_0-\varsigma}^{t_0} \exp\{2\varepsilon(s-t_0)\} x^T(s)\\&\quad\quad \times \left\{Q_{1i} + Q_3 + R_{1i} + \sum_{k=2}^{4} R_k\right\} x(s)\, ds\\&\quad\quad + \int_{t_0-\varsigma}^{t_0} \exp\{2\varepsilon(s-t_0)\} \dot{x}^T(s)\\&\quad\quad \times \left\{Q_{2i} + Q_4 + T_{1i} + \sum_{l=2}^{4} T_l\right\} \dot{x}(s)\, ds\\&\quad\quad + \int_{-\varsigma}^{0}\int_{t_0+\theta}^{t_0} \varsigma \exp\{2\varepsilon(s-t_0)\} x^T(s)\\&\quad\quad \times \left(\sum_{m=1}^{4} U_m\right) x(s)\, ds\, d\theta\\&\quad\quad + \int_{-\varsigma}^{0}\int_{t_0+\theta}^{t_0} \varsigma \exp\{2\varepsilon(s-t_0)\} \dot{x}^T(s)\\&\quad\quad \times \left(\sum_{n=1}^{4} V_n\right) \dot{x}(s)\, ds\, d\theta\\&\quad\quad + \int_{-\varsigma}^{0}\int_{\theta}^{0}\int_{t_0+\lambda}^{t_0} \frac{\varsigma^2}{2} \exp\{2\varepsilon(s-t_0)\} \dot{x}^T(s)\\&\quad\quad\quad \times \left(\sum_{s=1}^{4} W_s\right) \dot{x}(s)\, ds\, d\lambda\, d\theta\\&\quad\quad + x^T(t_0) P_i x(t_0)\end{aligned} \tag{81}$$

(79) can be obtained by letting $\overline{\lambda}$ be defined as previously mentioned.

Remark 13. In Theorem 10, the factors

$$\begin{aligned}&\frac{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_i(t)\}}{2\varepsilon},\\&\frac{\exp\{2\varepsilon d_i(t)\} - \exp\{2\varepsilon d_{1i}\}}{2\varepsilon}\end{aligned} \tag{82}$$

may be enlarged as $(\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{1i}\})/2\varepsilon$. This will lead conservative results due to the fact that $d_i(t)$ cannot achieve d_{1i} and d_{2i} at the same time. While we apply Lemma 7 to these terms, the method by using reciprocally convex lemma [38] can achieve less conservative results. Moreover, for $\varepsilon > 0$, the factor $(\dot{d}_i(t) - 1)\exp\{-2\varepsilon d_i(t)\}$ that appeared in the derivative of Lyapunov functional may be directly enlarged as $\mu_i \exp\{-2\varepsilon d_{1i}\} - \exp\{-2\varepsilon d_{2i}\}$. In this paper, we enlarge it as $\varpi(\varepsilon, \mu_i)$ to reduce the conservativeness of the obtained criteria. In the literature [36, 45, 46], this factor is enlarged as $(\mu_i - 1)\exp\{-2\varepsilon d_{2i}\}$, which only holds for $\mu_i < 1$.

Remark 14. The information on the lower bound of the delay is sufficiently used in the Lyapunov functional by introducing the terms such as $\int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\}x^T(s)R_3x(s)ds$, $\int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\}x^T(s)R_4x(s)ds$, and $\int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\}x^T(s)R_{1i}x(s)ds$, which is equivalent to the improved bounding technique.

Remark 15. It should be also mentioned that the result obtained in Theorem 10 is delay-range-dependent and decay rate-dependent stability condition for (13), which is less conservative than the previous ones and will be verified in Section 4. Although the large number of introduced free weighting matrices may increase the complexity of computation, utilizing the technique of free weighting matrices would reduce the conservativeness. In addition, the given results can be extended to more general systems with neutral delay $\tau_i(t)$. That is, $\tau_{1i} \le \tau_i(t) \le \tau_{2i}$. The corresponding results can be obtained by using the similar methods.

In Theorem 10, it is assumed that $\varepsilon \neq 0$. For $\varepsilon = 0$, by L'Hospital rule, the following asymptotic stability criterion can be obtained.

Corollary 16. *For given scalars π_i^m, π_i^M, α, β, γ, $\overline{\tau}_i$, ν_i, d_{1i}, d_{2i}, μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the systems described by (13) with partially known transition rates are asymptotically stable if $\|C_i\| + \gamma < 1$ and there exist symmetric positive matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_{1i} > 0$, $T_{1i} > 0$, $(i \in S)$, $Q_j > 0$, $(j = 3, 4)$, $R_k > 0$, $T_l > 0$, $(k, l = 2, 3, 4)$, $U_m > 0$, $V_n > 0$, $W_s > 0$, $(m, n, s = 1, 2, 3, 4)$ and matrices M_1, M_2, N_1, N_2, N_3, N_4 for any scalars ε_1, ε_2, ε_3, any symmetric matrices X_i, Y_{1i}, Y_{2i}, Z_{1i}, Z_{2i}, $(i \in S)$ and any matrices J_k, $(k = 1, 2, \ldots, 24)$ with appropriate dimensions, such that (26)–(32) and the following linear matrix inequalities hold:*

$$\begin{aligned}
&\text{(i)} \quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\
&\text{(ii)} \quad \Omega_{i0} + \Omega_{i1} < 0, \\
&\text{(iii)} \quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\
&\text{(iv)} \quad \Omega_{i0} + \Omega_{i2} < 0,
\end{aligned} \tag{83}$$

where

$$\begin{aligned}
\Omega_{i0} = {} & \sum_{m=1}^{24} e_m \widehat{\Upsilon}_m e_m^T + \mathscr{L}(\Xi) \\
& - (e_1 - e_5) V_2 \left(e_1^T - e_5^T\right) \\
& - \left(\overline{\tau}_i e_1 - e_{21}\right) W_1 \left(\overline{\tau}_i e_1^T - e_{21}^T\right) \\
& - \left(d_{1i} e_1 - e_{11}\right) W_2 \left(d_{1i} e_1^T - e_{11}^T\right) \\
& - \left(\varrho_{1i} e_1 - e_{14}\right) W_3 \left(\varrho_{1i} e_1^T - e_{14}^T\right) \\
& - \left(\varrho_{2i} e_1 - e_{15}\right) W_4 \left(\varrho_{2i} e_1^T - e_{15}^T\right) \\
& - \begin{bmatrix} e_{20} & e_{21} - e_{20} \end{bmatrix} \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} e_{20}^T \\ e_{21}^T - e_{20}^T \end{bmatrix} \\
& - \begin{bmatrix} e_1 - e_{18} & e_{18} - e_{16} \end{bmatrix} \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} e_1^T - e_{18}^T \\ e_{18}^T - e_{16}^T \end{bmatrix}, \\
\widehat{\Upsilon}_1 = {} & A_i^T P_i + P_i A_i + \sum_{j \in \mathcal{S}_k^i} \pi_{ij} \left(P_j - X_i\right) \\
& + Q_{1i} + Q_3 + R_2 + \overline{\tau}_i^2 U_1 + d_{1i}^2 U_2 \\
& + \varrho_{1i}^2 U_3 + \varrho_{2i}^2 U_4 + \varepsilon_1 \alpha^2 I + J_1 A_i + A_i^T J_1^T, \\
\widehat{\Upsilon}_3 = {} & -\left(1 - \mu_i\right) R_{1i} + \varepsilon_2 \beta^2 I + J_3 B_i + B_i^T J_3^T, \\
& \widehat{\Upsilon}_4 = -\left(1 - \mu_i\right) T_{1i}, \\
& \widehat{\Upsilon}_5 = R_{1i} + R_3 - R_2, \qquad \widehat{\Upsilon}_6 = R_4 - R_3, \\
& \widehat{\Upsilon}_7 = -R_4, \qquad \widehat{\Upsilon}_8 = T_{1i} + T_3 - T_2, \\
& \widehat{\Upsilon}_9 = T_4 - T_3, \qquad \widehat{\Upsilon}_{10} = -T_4, \qquad \widehat{\Upsilon}_{11} = -U_2, \\
& \widehat{\Upsilon}_{16} = -Q_3, \qquad \widehat{\Upsilon}_{17} = -Q_4, \qquad \widehat{\Upsilon}_{18} = -\left(1 - \nu_i\right) Q_{1i}, \\
\widehat{\Upsilon}_{19} = {} & -\left(1 - \nu_i\right) Q_{2i} + \varepsilon_3 \gamma^2 I + J_{19} C_i + C_i^T J_{19}^T.
\end{aligned} \tag{84}$$

Other $\widehat{\Upsilon}_m = \Upsilon_m$, $(m = 2, 12, 13, 14, 15, 20, \ldots, 24)$, which have been defined in Theorem 10,

$$\begin{aligned}
\Omega_{i1} = {} & -e_{15} U_4 e_{15}^T - (e_6 - e_7) V_4 \left(e_6^T - e_7^T\right) \\
& - \begin{bmatrix} e_{12} & e_{14} - e_{12} \end{bmatrix} \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} \begin{bmatrix} e_{12}^T \\ e_{14}^T - e_{12}^T \end{bmatrix} \\
& - \begin{bmatrix} e_5 - e_3 & e_3 - e_6 \end{bmatrix} \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} \begin{bmatrix} e_5^T - e_3^T \\ e_3^T - e_6^T \end{bmatrix}, \\
\Omega_{i2} = {} & -e_{14} U_3 e_{14}^T - (e_5 - e_6) V_3 \left(e_5^T - e_6^T\right) \\
& - \begin{bmatrix} e_{13} & e_{15} - e_{13} \end{bmatrix} \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} \begin{bmatrix} e_{13}^T \\ e_{15}^T - e_{13}^T \end{bmatrix} \\
& - \begin{bmatrix} e_3 - e_7 & e_6 - e_3 \end{bmatrix} \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} \begin{bmatrix} e_3^T - e_7^T \\ e_6^T - e_3^T \end{bmatrix}
\end{aligned} \tag{85}$$

and the remaining notations are the same as Theorem 10.

Further, the information on the transition rates may be completely unknown in some circumstances, which viewed the systems as switched systems with arbitrary switching. The following corollary is therefore given to guarantee the exponential stability for this case.

Corollary 17. *For given scalars π_i^m, π_i^M, α, β, γ, ε, $\overline{\tau}_i$, ν_i, d_{1i}, d_{2i}, μ_i, and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the* *systems described by* (13) *with completely unknown transition rates are exponentially stable with decay rate* ε *and* $\kappa = \sqrt{\overline{\lambda}/\underline{\lambda}}$ *if* $\|C_i\| + \gamma < 1$ *and there exist symmetric positive matrices* $P > 0$, $Q_j > 0$, $R_k > 0$, $T_l > 0$, $U_m > 0$, $V_n > 0$, $W_s > 0$, $(j, k, l, m, n, s = 1, 2, 3, 4)$ *and matrices* M_1, M_2, N_1, N_2, N_3, N_4 *for any scalars* ε_1, ε_2, ε_3, *any symmetric matrices* Y_1, Y_2, Z_1, Z_2, *and any matrices* J_k, $(k = 1, 2, \ldots, 24)$ *with appropriate dimensions, such that the following linear matrix inequalities hold:*

$$\begin{aligned} &-\sum_{j\in\mathcal{S}^i_{uk}, j\neq i} \pi_i^M (Q_1 - Y_1) - \pi_i^m Y_1 \le 0; \\ &-\sum_{j\in\mathcal{S}^i_{uk}, j\neq i} \pi_i^M (Q_2 - Y_2) - \pi_i^m Y_2 \le 0, \end{aligned} \tag{86}$$

$$\begin{aligned} &-\sum_{j\in\mathcal{S}^i_{uk}, j\neq i} \pi_i^M (R_1 - Z_1) - \pi_i^m Z_1 \le 0; \\ &-\sum_{j\in\mathcal{S}^i_{uk}, j\neq i} \pi_i^M (T_1 - Z_2) - \pi_i^m Z_2 \le 0, \end{aligned} \tag{87}$$

$$\begin{gathered} \text{(i)}\quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\ \text{(ii)}\quad \widetilde{\Omega}^{\varepsilon}_{i0} + \widetilde{\Omega}^{\varepsilon}_{i1} < 0, \end{gathered} \tag{88}$$

$$\begin{gathered} \text{(iii)}\quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\ \text{(iv)}\quad \widetilde{\Omega}^{\varepsilon}_{i0} + \widetilde{\Omega}^{\varepsilon}_{i2} < 0, \end{gathered} \tag{89}$$

where

$$\begin{aligned} \widetilde{\Omega}^{\varepsilon}_{i0} = {} & \sum_{m=1}^{24} e_m \widetilde{\Upsilon}_m e_m^T + \mathcal{L}(\Xi) \\ & - \frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} (e_1 - e_5) V_2 \left(e_1^T - e_5^T\right) \\ & - \exp\{-2\varepsilon\overline{\tau}_i\} (\overline{\tau}_i e_1 - e_{21}) W_1 \left(\overline{\tau}_i e_1^T - e_{21}^T\right) \\ & - \exp\{-2\varepsilon d_{1i}\} (d_{1i} e_1 - e_{11}) W_2 \left(d_{1i} e_1^T - e_{11}^T\right) \\ & - \exp\{-2\varepsilon d_{mi}\} (\varrho_{1i} e_1 - e_{14}) W_3 \left(\varrho_{1i} e_1^T - e_{14}^T\right) \\ & - \exp\{-2\varepsilon d_{2i}\} (\varrho_{2i} e_1 - e_{15}) W_4 \left(\varrho_{2i} e_1^T - e_{15}^T\right) \\ & - \frac{2\varepsilon\overline{\tau}_i}{\exp\{2\varepsilon\overline{\tau}_i\} - 1} \begin{bmatrix} e_{20} & e_{21} - e_{20} \end{bmatrix} \\ & \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} e_{20}^T \\ e_{21}^T - e_{20}^T \end{bmatrix} \\ & - \frac{2\varepsilon\overline{\tau}_i}{\exp\{2\varepsilon\overline{\tau}_i\} - 1} \begin{bmatrix} e_1 - e_{18} & e_{18} - e_{16} \end{bmatrix} \\ & \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} e_1^T - e_{18}^T \\ e_{18}^T - e_{16}^T \end{bmatrix}, \\ \widetilde{\Upsilon}_1 = {} & A_i^T P + P A_i + 2\varepsilon P + Q_1 + Q_3 \\ & + R_2 + \overline{\tau}_i^2 U_1 + d_{1i}^2 U_2 + \varrho_{1i}^2 U_3 \\ & + \varrho_{2i}^2 U_4 + \varepsilon_1 \alpha^2 I + J_1 A_i + A_i^T J_1^T, \\ \widetilde{\Upsilon}_2 = {} & Q_2 + Q_4 + T_2 + \overline{\tau}_i^2 V_1 + d_{1i}^2 V_2 \\ & + \varrho_{1i}^2 V_3 + \varrho_{2i}^2 V_4 + \frac{\overline{\tau}_i^4}{4} W_1 + \frac{d_{1i}^4}{4} W_2 \\ & + \varrho_{3i}^2 W_3 + \varrho_{4i}^2 W_4 - J_2 - J_2^T, \\ \widetilde{\Upsilon}_3 = {} & \varpi(\varepsilon, \mu_i) R_1 + \varepsilon_2 \beta^2 I + J_3 B_i + B_i^T J_3^T, \\ \widetilde{\Upsilon}_4 = {} & \varpi(\varepsilon, \mu_i) T_1, \\ \widetilde{\Upsilon}_5 = {} & \exp\{-2\varepsilon d_{1i}\} (R_1 + R_3 - R_2), \\ \widetilde{\Upsilon}_6 = {} & \exp\{-2\varepsilon d_{mi}\} (R_4 - R_3), \\ \widetilde{\Upsilon}_7 = {} & -\exp\{-2\varepsilon d_{2i}\} R_4, \\ \widetilde{\Upsilon}_8 = {} & \exp\{-2\varepsilon d_{1i}\} (T_1 + T_3 - T_2), \\ \widetilde{\Upsilon}_9 = {} & \exp\{-2\varepsilon d_{mi}\} (T_4 - T_3), \\ \widetilde{\Upsilon}_{10} = {} & -\exp\{-2\varepsilon d_{2i}\} T_4, \\ \widetilde{\Upsilon}_{11} = {} & -\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} U_2, \\ \widetilde{\Upsilon}_{16} = {} & -\exp\{-2\varepsilon\overline{\tau}_i\} Q_3, \\ \widetilde{\Upsilon}_{17} = {} & -\exp\{-2\varepsilon\overline{\tau}_i\} Q_4, \\ \widetilde{\Upsilon}_{18} = {} & -(1 - \nu_i) \exp\{-2\varepsilon\overline{\tau}_i\} Q_1, \\ \widetilde{\Upsilon}_{19} = {} & -(1 - \nu_i) \exp\{-2\varepsilon\overline{\tau}_i\} Q_2 + \varepsilon_3 \gamma^2 I + J_{19} C_i + C_i^T J_{19}^T, \\ \widetilde{\Upsilon}_{22} = {} & -\varepsilon_1 I + J_{22} D_i + D_i^T J_{22}^T, \\ \widetilde{\Upsilon}_{23} = {} & -\varepsilon_2 I + J_{23} E_i + E_i^T J_{23}^T, \\ \widetilde{\Upsilon}_{24} = {} & -\varepsilon_3 I + J_{24} F_i + F_i^T J_{24}^T, \\ \widetilde{\Upsilon}_m = {} & 0, \quad (m = 12, 13, 14, 15, 20, 21), \end{aligned} \tag{90}$$

where

$$\begin{aligned} & \widetilde{\Omega}^{\varepsilon}_{i1} = \Omega^{\varepsilon}_{i1}, \qquad \widetilde{\Omega}^{\varepsilon}_{i2} = \Omega^{\varepsilon}_{i2}, \\ \overline{\lambda} = {} & \lambda_{\max}(P) + \frac{1 - \exp\{-2\varepsilon\varsigma\}}{2\varepsilon} \\ & \times \left\{ \sum_{j=1}^{4} \lambda_{\max}(Q_j) + \sum_{k=1}^{4} \lambda_{\max}(R_k) + \sum_{l=1}^{4} \lambda_{\max}(T_l) \right\} \\ & + \frac{\exp\{-2\varepsilon\varsigma\} + 2\varepsilon\varsigma - 1}{4\varepsilon^2} \\ & \times \left\{ \sum_{m=1}^{4} \lambda_{\max}(U_m) + \sum_{n=1}^{4} \lambda_{\max}(V_n) \right\} \\ & + \frac{1 - \exp\{-2\varepsilon\varsigma\} + 2\varepsilon^2\varsigma^2 - 2\varepsilon\varsigma}{8\varepsilon^3} \left\{ \sum_{s=1}^{4} \lambda_{\max}(W_s) \right\} \end{aligned} \tag{91}$$

and the remaining notations are the same as Theorem 10.

Proof. Since the elements of transition rates are entirely unknown, we choose the Lyapunov functional as follows:

$$V(x(t), i, t) = x^T(t) P x(t) + \sum_{k=2}^{7} V_k(x_t, i), \quad (92)$$

where

$$\begin{aligned} V_2(x_t, i) &= \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_1 x(s)\, ds \\ &+ \int_{t-\tau_i(t)}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_2 \dot{x}(s)\, ds \\ &+ \int_{t-\overline{\tau}_i}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) Q_3 x(s)\, ds \\ &+ \int_{t-\overline{\tau}_i}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) Q_4 \dot{x}(s)\, ds, \end{aligned} \quad (93)$$

$$\begin{aligned} V_3(x_t, i) &= \int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} x^T(s) R_1 x(s)\, ds \\ &+ \int_{t-d_{1i}}^{t} \exp\{2\varepsilon(s-t)\} x^T(s) R_2 x(s)\, ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} x^T(s) R_3 x(s)\, ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} x^T(s) R_4 x(s)\, ds, \end{aligned} \quad (94)$$

$$\begin{aligned} V_4(x_t, i) &= \int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_1 \dot{x}(s)\, ds \\ &+ \int_{t-d_{1i}}^{t} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_2 \dot{x}(s)\, ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_3 \dot{x}(s)\, ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} \dot{x}^T(s) T_4 \dot{x}(s)\, ds \end{aligned} \quad (95)$$

and $V_5(x_t, i)$, $V_6(x_t, i)$, $V_7(x_t, i)$ are the same as (43), (44), and (45).

Then, we follow a similar line as in proof of Theorem 10 and obtain the result. □

3.2. Extension to the Uncertain Case. In this subsection, the uncertain neutral Markovian jump systems described by (6) with partially unknown transition rates are considered. The delay-range-dependent and rate-dependent exponential stability conditions are presented in the following theorems and corollaries.

Theorem 18. *For given scalars π_i^m, π_i^M, α, β, γ, ε, $\overline{\tau}_i$, ν_i, d_{1i}, d_{2i}, μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the systems described by* (6) *with partially known transition rates are exponentially stable with decay rate ε and $\kappa = \sqrt{\overline{\lambda}/\underline{\lambda}}$ if $\|C_i\| + \gamma < 1$ and there exist scalars $\delta_1 > 0$, $\delta_2 > 0$, symmetric positive matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_{1i} > 0$, $T_{1i} > 0$, $(i \in S)$, $Q_j > 0$, $(j = 3, 4)$, $R_k > 0$, $T_l > 0$, $(k, l = 2, 3, 4)$, $U_m > 0$, $V_n > 0$, $W_s > 0$, $(m, n, s = 1, 2, 3, 4)$ and matrices M_1, M_2, N_1, N_2, N_3, N_4 for any scalars ε_1, ε_2, ε_3, any symmetric matrices X_i, Y_{1i}, Y_{2i}, Z_{1i}, Z_{2i}, $(i \in S)$, and any matrices J_k, $(k = 1, 2, \ldots, 24)$ with appropriate dimensions, such that* (26)-(32) *and the following inequalities hold:*

$$\begin{aligned} &\text{(i)} \quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\ &\text{(ii)} \quad \begin{bmatrix} \delta_1 \Omega_{i0}^{\varepsilon} + \delta_1 \Omega_{i1}^{\varepsilon} + \delta_1^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \end{aligned} \quad (96)$$

$$\begin{aligned} &\text{(iii)} \quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\ &\text{(iv)} \quad \begin{bmatrix} \delta_2 \Omega_{i0}^{\varepsilon} + \delta_2 \Omega_{i2}^{\varepsilon} + \delta_2^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \end{aligned} \quad (97)$$

where

$$\begin{aligned} \mathbb{M} &= e_1 N_{Ai}^T + e_3 N_{Bi}^T + e_{22} N_{Di}^T + e_{23} N_{Ei}^T + e_{24} N_{Fi}^T, \\ \mathbb{N} &= L_i^T P_i^T e_1 + \sum_{m=1}^{24} L_i^T J_m^T e_m \end{aligned} \quad (98)$$

and other notations are the same as Theorem 10.

Proof. Defining $\mathbb{E} = \mathrm{col}\{e_m, (m = 1, 2, \ldots, 24)\}$ and $\mathbb{J} = \mathrm{col}\{P_i + J_1, J_m, (m = 2, \ldots, 24)\}$, we replace A_i, B_i, D_i, E_i, and F_i with $A_i + \Delta A_i(t)$, $B_i + \Delta B_i(t)$, $D_i + \Delta D_i(t)$, $E_i + \Delta E_i(t)$, and $F_i + \Delta F_i(t)$ on the basis of Theorem 10. That is,

$$\begin{aligned} &\text{(i)} \quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\ &\text{(ii)} \quad \Omega_{i0}^{\varepsilon} + \Omega_{i1}^{\varepsilon} + \mathscr{L}\left[e_1 \Delta A_i^T(t) \mathbb{J}^T \mathbb{E} + e_3 \Delta B_i^T(t) \mathbb{J}^T \mathbb{E} \right. \\ &\qquad + e_{22} \Delta D_i^T(t) \mathbb{J}^T \mathbb{E} + e_{23} \Delta E_i^T(t) \mathbb{J}^T \mathbb{E} \\ &\qquad \left. + e_{24} \Delta F_i^T(t) \mathbb{J}^T \mathbb{E}\right] < 0, \end{aligned} \quad (99)$$

$$\begin{aligned} &\text{(iii)} \quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\ &\text{(iv)} \quad \Omega_{i0}^{\varepsilon} + \Omega_{i2}^{\varepsilon} + \mathscr{L}\left[e_1 \Delta A_i^T(t) \mathbb{J}^T \mathbb{E} + e_3 \Delta B_i^T(t) \mathbb{J}^T \mathbb{E} \right. \\ &\qquad + e_{22} \Delta D_i^T(t) \mathbb{J}^T \mathbb{E} + e_{23} \Delta E_i^T(t) \mathbb{J}^T \mathbb{E} \\ &\qquad \left. + e_{24} \Delta F_i^T(t) \mathbb{J}^T \mathbb{E}\right] < 0. \end{aligned} \quad (100)$$

Considering (ii) of (99) and combining the uncertainties condition (11), we have

$$\Omega_{i0}^{\varepsilon} + \Omega_{i1}^{\varepsilon} + \mathscr{L} \times \left\{ \begin{bmatrix} e_1 & e_3 & e_{22} & e_{23} & e_{24} \end{bmatrix} \begin{bmatrix} N_{Ai}^T \\ N_{Bi}^T \\ N_{Di}^T \\ N_{Ei}^T \\ N_{Fi}^T \end{bmatrix} H_i^T(t) L_i^T \mathbb{J}^T \mathbb{E} \right\} < 0. \tag{101}$$

By the definition of $\mathscr{L}$, we obtain

$$\Omega_{i0}^{\varepsilon} + \Omega_{i1}^{\varepsilon} + \mathbb{M} H_i^T(t) \mathbb{N} + \mathbb{N}^T H_i(t) \mathbb{M}^T < 0. \tag{102}$$

According to (12), by Lemmas 9 and 6, with (102) we obtain (ii) of (96). Following the same procedure, (iv) of (100) is considered and (iv) of (97) can be obtained. Finally, following the proof of Theorem 10, the systems described by (6) are exponentially stable with a decay rate ε. This completes the proof. □

Considering the uncertain case, the following corollaries are given, for $\varepsilon = 0$ and entirely unknown transition rates, respectively.

Corollary 19. *For given scalars π_i^m, π_i^M, α, β, γ, $\overline{\tau}_i$, ν_i, d_{1i}, d_{2i}, μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the systems described by* (6) *with partially known transition rates are asymptotically stable if $\|C_i\| + \gamma < 1$ and there exist scalars $\delta_1 > 0$, $\delta_2 > 0$, symmetric positive matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_{1i} > 0$, $T_{1i} > 0$, $(i \in S)$, $Q_j > 0$, $(j = 3, 4)$, $R_k > 0$, $T_l > 0$, $(k, l = 2, 3, 4)$, $U_m > 0$, $V_n > 0$, and $W_s > 0$, $(m, n, s = 1, 2, 3, 4)$ and matrices M_1, M_2, N_1, N_2, N_3, N_4 for any scalars ε_1, ε_2, ε_3, any symmetric matrices X_i, Y_{1i}, Y_{2i}, Z_{1i}, Z_{2i}, $(i \in S)$ and any matrices J_k, $(k = 1, 2, \ldots, 24)$ with appropriate dimensions, such that* (26)–(32)*, and the following inequalities hold:*

$$\begin{aligned} &\text{(i)} \quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\ &\text{(ii)} \quad \begin{bmatrix} \delta_1 \Omega_{i0} + \delta_1 \Omega_{i1} + \delta_1^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \\ &\text{(iii)} \quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\ &\text{(iv)} \quad \begin{bmatrix} \delta_2 \Omega_{i0} + \delta_2 \Omega_{i2} + \delta_2^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \end{aligned} \tag{103}$$

where

$$\begin{aligned} \mathbb{M} &= e_1 N_{Ai}^T + e_3 N_{Bi}^T + e_{22} N_{Di}^T + e_{23} N_{Ei}^T + e_{24} N_{Fi}^T, \\ \mathbb{N} &= L_i^T P_i^T e_1 + \sum_{m=1}^{24} L_i^T J_m^T e_m \end{aligned} \tag{104}$$

and other notations are the same as Corollary 16.

Corollary 20. *For given scalars π_i^m, π_i^M, α, β, γ, ε, $\overline{\tau}_i$, ν_i, d_{1i}, d_{2i}, μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the systems described by* (6) *with completely unknown transition rates are exponentially stable with decay rate ε and $\kappa = \sqrt{\overline{\lambda}/\underline{\lambda}}$ if $\|C_i\| + \gamma < 1$ and there exist scalars $\delta_1 > 0$, $\delta_2 > 0$, symmetric positive matrices $P > 0$, $Q_j > 0$, $R_k > 0$, $T_l > 0$, $U_m > 0$, $V_n > 0$, $W_s > 0$, $(j, k, l, m, n, s = 1, 2, 3, 4)$ and matrices M_1, M_2, N_1, N_2, N_3, N_4 for any scalars ε_1, ε_2, ε_3, any symmetric matrices Y_1, Y_2, Z_1, Z_2, and any matrices J_k, $(k = 1, 2, \ldots, 24)$ with appropriate dimensions, such that* (86), (87)*, and the following inequalities hold:*

$$\begin{aligned} &\text{(i)} \quad \begin{bmatrix} U_3 & N_1 \\ N_1^T & U_3 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_3 & N_2 \\ N_2^T & V_3 \end{bmatrix} > 0, \\ &\text{(ii)} \quad \begin{bmatrix} \delta_1 \widetilde{\Omega}_{i0}^{\varepsilon} + \delta_1 \widetilde{\Omega}_{i1}^{\varepsilon} + \delta_1^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \\ &\text{(iii)} \quad \begin{bmatrix} U_4 & N_3 \\ N_3^T & U_4 \end{bmatrix} > 0, \qquad \begin{bmatrix} V_4 & N_4 \\ N_4^T & V_4 \end{bmatrix} > 0, \\ &\text{(iv)} \quad \begin{bmatrix} \delta_2 \widetilde{\Omega}_{i0}^{\varepsilon} + \delta_2 \widetilde{\Omega}_{i2}^{\varepsilon} + \delta_2^2 \mathbb{M}^T \mathbb{M} & \mathbb{N}^T \\ * & -I \end{bmatrix} < 0, \end{aligned} \tag{105}$$

where

$$\begin{aligned} \mathbb{M} &= e_1 N_{Ai}^T + e_3 N_{Bi}^T + e_{22} N_{Di}^T + e_{23} N_{Ei}^T + e_{24} N_{Fi}^T, \\ \mathbb{N} &= L_i^T P_i^T e_1 + \sum_{m=1}^{24} L_i^T J_m^T e_m \end{aligned} \tag{106}$$

and other notations are the same as Corollary 17.

4. Numerical Examples

In this section, numerical examples are given to demonstrate that the proposed theoretical results in this paper are effectiveness. In particular, when $i \in S = \{1\}$, $d_i(t)$, d_{1i}, d_{mi}, and d_{2i} are expressed into $d(t)$, d_1, d_m, and d_2.

Example 1. We consider an interval mode-dependent time-varying delay system in the form of (6) with three modes and nonlinear perturbations:

$$\begin{aligned} \|f_1(x(t), t)\| &\le 0.1 \|x(t)\|, \\ \|f_2(x(t - d(t)), t)\| &\le 0.2 \|x(t - d(t))\|, \\ \|f_3(\dot{x}(t - \tau(t)), t)\| &\le 0.1 \|\dot{x}(t - \tau(t))\|; \end{aligned} \tag{107}$$

that is, $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.1$. The parametric matrices of the system are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.75 & -0.75 \\ 1.50 & -1.50 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.15 & -0.09 \\ 1.50 & -0.10 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -0.30 & -0.15 \\ 0.50 & -0.50 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.11 & 0.23 \\ -0.52 & -0.37 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -0.59 & 0.02 \\ -0.06 & -0.61 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0.51 & 0.24 \\ 0.02 & -0.44 \end{bmatrix}, \end{aligned}$$

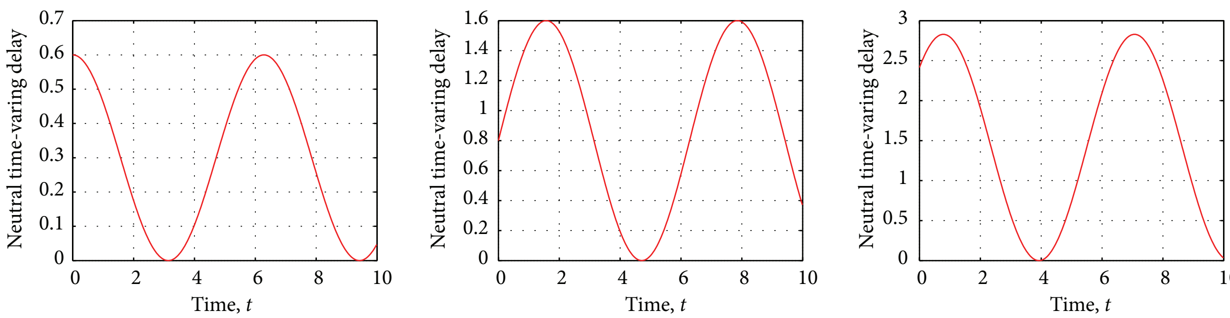

FIGURE 1: Neutral time-varying delay $\tau_i(t)$ at Mode 1, Mode 2, and Mode 3.

$$C_1 = \begin{bmatrix} 0.30 & 0 \\ 0 & 0.20 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.20 \end{bmatrix},$$
$$C_3 = \begin{bmatrix} 0.20 & 0 \\ 0 & 0.40 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} -0.70 & -0.34 \\ 0.50 & -0.65 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} -0.67 & -0.50 \\ 0.39 & 0.23 \end{bmatrix}, \qquad D_3 = \begin{bmatrix} -0.23 & 0.16 \\ 0.02 & -0.57 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} -0.30 & -0.44 \\ 0.50 & -0.65 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} -0.40 & -0.35 \\ 0.39 & 0.28 \end{bmatrix},$$
$$E_3 = \begin{bmatrix} -0.24 & 0.19 \\ 0.12 & -0.38 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} -0.50 & -0.36 \\ 0.26 & -0.75 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -0.48 & -0.33 \\ 0.27 & 0.22 \end{bmatrix}, \qquad F_3 = \begin{bmatrix} -0.45 & 0.23 \\ 0.12 & -0.35 \end{bmatrix},$$
$$L_1 = L_2 = L_3 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$
$$N_{A1} = N_{A2} = N_{A3} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$N_{B1} = N_{B2} = N_{B3} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$
$$N_{D1} = N_{D2} = N_{D3} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix},$$
$$N_{E1} = N_{E2} = N_{E3} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$
$$N_{F1} = N_{F2} = N_{F3} = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}. \tag{108}$$

The partially known transition rates matrix is described by

$$\begin{bmatrix} -0.8 & ? & ? \\ ? & -0.9 & ? \\ 0.7 & 0.4 & -1.1 \end{bmatrix}, \tag{109}$$

where $\pi_i^m = -1.1$, $\pi_i^M = -0.8$.

The interval mode-dependent time-varying neutral delays are assumed to be

$$\tau_1(t) = 0.3(1 + \cos t), \qquad \tau_2(t) = 0.8(1 + \sin t),$$
$$\tau_3(t) = \sqrt{2} + \sin t + \cos t. \tag{110}$$

The interval mode-dependent time-varying retarded delays are assumed to be

$$d_1(t) = 0.4\left(1 + \sin^2(3t)\right), \qquad d_2(t) = 0.5\left(1 + \cos^2(4t)\right),$$
$$d_3(t) = 0.6\sin^4 t. \tag{111}$$

They are governed by the Markov process $\{r_t, t \geq 0\}$ and shown in Figures 1 and 2. It can be readily obtain that

$$\overline{\tau_1} = 0.6, \qquad \overline{\tau_2} = 1.6, \qquad \overline{\tau_3} = 2\sqrt{2};$$
$$\nu_1 = 0.3, \qquad \nu_2 = 0.8, \qquad \nu_3 = \sqrt{2};$$
$$d_{11} = 0.4, \qquad d_{21} = 0.8; \qquad d_{12} = 0.5, \qquad d_{22} = 1;$$
$$d_{13} = 0, \qquad d_{23} = 0.6;$$
$$\mu_1 = 1.2, \qquad \mu_2 = 2, \qquad \mu_3 = \frac{9\sqrt{3}}{20}. \tag{112}$$

Providing that decay rate $\varepsilon = 0.5$, we choose qualified values of d_{m1}, d_{m2}, d_{m3}, for example, $d_{m1} = 0.5$, $d_{m2} = 0.7$, $d_{m3} = 0.3$, and then we solve (26)–(32), (96), and (97) in Theorem 18. Thus, it is facile to establish the exponential stability of this uncertain neutral system described by (6), which shows that the approach presented in this paper is effectiveness.

Example 2. Consider the nominal system (13) with four operation modes $S = \{1, 2, 3, 4\}$, $\alpha = \beta = \gamma = 0$, and the following parameters:

$$A_1 = \begin{bmatrix} -1.15 & -0.75 \\ 1.50 & -1.50 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -1.20 & 0.12 \\ 0.24 & -0.25 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.15 & -0.06 \\ 0.50 & -0.50 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2.15 & -0.49 \\ 1.50 & -2.10 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} -1.45 & -0.96 \\ 0.47 & -1.57 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.25 & -0.35 \\ 0.05 & -0.65 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -1.30 & -0.15 \\ 1.50 & -1.80 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0.58 & -0.68 \\ -0.13 & 0.96 \end{bmatrix},$$

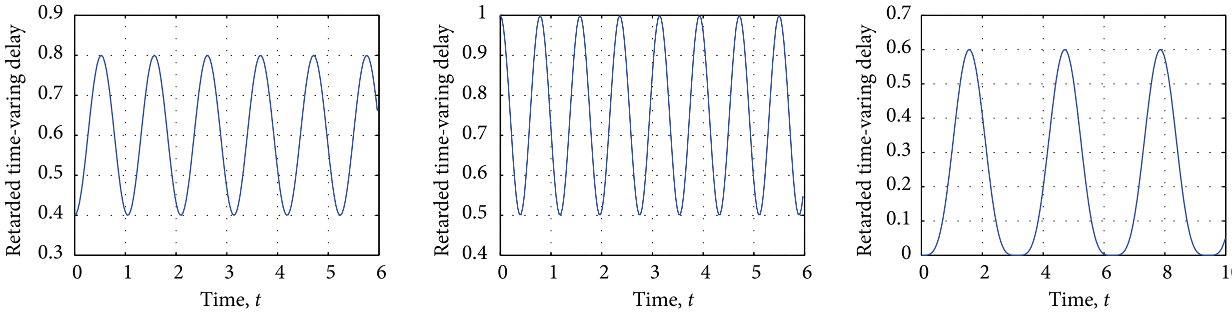

Figure 2: Retarded time-varying delay $d_i(t)$ at Mode 1, Mode 2, and Mode 3.

Table 1: Maximum upper bound of d_2 with Π_1 and different parameter μ.

Methods	μ	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25
Xiong et al. [40]	d_2	0.4489	0.3317	0.2242	0.1814	0.1486	0.1220	0.0983	0.0781
Corollary 16	d_2	0.4771	0.3664	0.2583	0.2165	0.1897	0.1473	0.1144	0.0956

$$C_3 = \begin{bmatrix} 0.33 & 0.07 \\ 0.19 & -0.36 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} -1.90 & -0.34 \\ 1.50 & -1.65 \end{bmatrix},$$
$$B_4 = \begin{bmatrix} -0.67 & -1.50 \\ 1.39 & 1.23 \end{bmatrix}, \qquad C_4 = \begin{bmatrix} -0.23 & 0.16 \\ 0.02 & -0.57 \end{bmatrix}. \tag{113}$$

The partially known transition rate matrix is considered as the following two cases:

$$\Pi_1 = \begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ 0.6 & ? & -1.5 & ? \\ 0.4 & ? & ? & ? \end{bmatrix},$$
$$\Pi_2 = \begin{bmatrix} -1.3 & 0.2 & 0.4 & 0.7 \\ ? & ? & 0.3 & 0.3 \\ 0.6 & ? & -1.5 & ? \\ 0.4 & ? & ? & ? \end{bmatrix}, \tag{114}$$

where $\pi_i^m = -1.8$ and $\pi_i^M = -1.3$.

Providing that $\varepsilon = 0$ and $\tau_i(t) = d_i(t) = d(t)$, $i \in S = \{1,2,3,4\}$, we have $d_1 = 0, \overline{\tau} = d_2, \nu = \mu$. Set $d_m = d_2/2$ and employ Corollary 16; the maximum upper bound of the time delay d_2 which satisfies LMIs (26)–(32), and (83), can be calculated by solving a quasi-convex optimization problem. This neutral Markovian jump system with partially unknown transition rates was also considered in [40]. The results on the maximum upper bound of d_2 are compared in Tables 1 and 2.

From Tables 1 and 2, we consider the previous system with Π_1, Π_2 and obtain the maximum upper bound of delay $d_2 = 0.4771$, $d_2 = 12.4394$, respectively, in this paper by setting $\mu = 0.18$, while the maximum upper bound of delay $d_2 = 0.4489$, $d_2 = 12.0750$, respectively, for [40]. The results are also given by setting $\mu = 0.19$, $\mu = 0.20$, $\mu = 0.21$, $\mu = 0.22$, $\mu = 0.23$, $\mu = 0.24$, and $\mu = 0.25$, and it is found that the maximum upper bound of delay in this paper is larger than that of [40]. So it can be seen that our proposed method is less conservative than the result in [40]. Besides, according to the data in Tables 1 and 2, we know that the maximum upper bound of delay to guarantee stability is dependent on transition rates knowledge.

Example 3. To show the reduced conservativeness of the exponential stability condition in Theorem 10, consider the time-delay system in the form of (13) with $C_i = 0$ and $\alpha = \beta = \gamma = 0$, $d_i(t) = d$ and

$$A_i = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix}, \quad i \in S = \{1\}. \tag{115}$$

For given d, the maximum exponential decay rate ε, which satisfies the LMIs (32), (33), and (34) in Theorem 10, can be calculated by solving a quasi-convex optimization problem. The results are presented in Table 3.

From Table 3, we know that the maximum exponential decay rate $\varepsilon = 0.9569$ in this paper by setting $d = 0.8$, while the maximum exponential decay rate $\varepsilon = 0.7344$ for [41], $\varepsilon = 0.9367$ for [42], and $\varepsilon = 0.9366$ for [43]. The results are also given by setting $d = 1.0$, $d = 1.2$, $d = 1.4$, $d = 1.6$, $d = 1.8$, and $d = 2.0$, and it is found that the maximum exponential decay rate in this paper is larger than those in [41–43]. So it can be demonstrated that Theorem 10 in this paper yields less conservative results than [41–43].

Consider the previous system again, but with parametric matrices and parameter uncertainties as follows:

$$A_i = \begin{bmatrix} -4 & 1 \\ 0 & -4 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0.1 & 0 \\ 4 & 0.1 \end{bmatrix}, \quad i \in S = \{1\} \tag{116}$$

and the uncertain matrices $\Delta A_i(t)$ and $\Delta B_i(t)$ satisfy

$$L_i = 0.2I, \quad N_{Ai} = N_{Bi} = I, \quad \|H_i(t)\| < 1, \quad i \in S = \{1\}. \tag{117}$$

Table 2: Maximum upper bound of d_2 with Π_2 and different parameter μ.

Methods	μ	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25
Xiong et al. [40]	d_2	12.0750	0.9924	0.5147	0.3749	0.2742	0.2139	0.1737	0.1433
Corollary 16	d_2	12.4394	1.2587	0.6263	0.4037	0.3003	0.2485	0.2018	0.1796

Table 3: Maximum upper bound of ε with different parameter d.

Methods	d	0.8	1	1.2	1.4	1.6	1.8	2.0
Mondié and Kharitonov [41]	ε	0.7344	0.6715	0.6145	0.5642	0.5202	0.4818	0.4481
Liu [42]	ε	0.9367	0.5903	0.3400	0.1813	0.0752	0.0014	0
Xu et al. [43]	ε	0.9366	0.9192	0.8991	0.8115	0.6990	0.6148	0.5494
Theorem 10	ε	0.9569	0.9407	0.9218	0.9024	0.8154	0.7394	0.6933

Table 4: Maximum upper bound of ε with different parameter d.

Methods	d	0.3	0.5	0.7	0.9	1.1	1.3	1.5
Mondié and Kharitonov [41]	ε	0.6255	0.4760	0.3825	0.3191	0.2735	0.2392	0.2125
Xu et al. [43]	ε	1.0108	0.8366	0.7103	0.6156	0.5425	0.4845	0.4375
Theorem 18	ε	1.0419	0.9207	0.8035	0.7048	0.6234	0.5876	0.5043

For given d, the maximum exponential decay rate ε, which satisfies the LMIs in (32), (96), and (97) in Theorem 18, can be calculated by solving a quasi-convex optimization problem. The results are presented in Table 4 and it also can be seen that the delay-range-dependent and rate-dependent exponential stability conditions in this paper are less conservative than previous results in [41, 43].

Example 4. Consider the system (6) with $\tau_i(t) = d_i(t) = d(t)$ and the parameters are listed in the following:

$$A_i = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad B_i = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad C_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$D_i = E_i = F_i = I, \quad i \in S = \{1\},$$
$$\alpha = 0.05, \qquad \beta = 0.1, \qquad \gamma = 0,$$
$$d_1 = 0, \qquad d_2 = \overline{\tau}, \qquad \mu = \nu. \tag{118}$$

For given ε and μ, choose $d_m = d_2/2$ and utilize Theorem 10, the maximum upper bound of d_2, which satisfies the LMIs in (32), (33), and (34), can be obtained by solving a quasi-convex optimization problem. The results are presented in Tables 5, 6, and 7.

From Tables 5, 6, and 7, we consider $\mu = 0$, $\mu = 0.5$, and $\mu = 0.9$ and obtain the maximum upper bound of delay $d_2 = 1.5167$, $d_2 = 1.0643$, and $d_2 = 0.7136$, respectively, in this paper by setting $\varepsilon = 0.1$, while the maximum upper bound of delay $d_2 = 1.2999$, $d_2 = 0.9442$, and $d_2 = 0.5471$, respectively, for [44], the maximum upper bound of delay $d_2 = 1.4008$, $d_2 = 1.0120$, and $d_2 = 0.6438$, respectively, for [13]. The results are also given by setting $\varepsilon = 0.3$, $\varepsilon = 0.5$, $\varepsilon = 0.7$, and $\varepsilon = 0.9$, and it is found that the maximum upper bound of delay in this paper is larger than those in [13, 44]. According to the comparative result, it can be seen that our proposed method is less conservative than those in [13, 44].

Example 5. Partial element equivalent circuit (PEEC) model can be represented as a stochastic jump system as in (6) with the abrupt variation in structures and parameters [47]. Consider the practical PEEC system described by (6) with $\alpha = \beta = \gamma = 0$ and completely unknown transition rates

$$\Pi = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}, \quad i \in S = \{1, 2\}, \tag{119}$$

where $\pi_i^m = -1.0$ and $\pi_i^M = -0.6$.

The parametric matrices of the system are given as follows:

$$A_1 = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1 & 0 \\ -0.9 & -1.2 \end{bmatrix},$$
$$C_1 = 0.5I, \qquad C_2 = 0.3I, \tag{120}$$
$$L_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0 \\ -0.3 \end{bmatrix},$$
$$N_{A1} = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \qquad N_{A2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix},$$
$$N_{B1} = \begin{bmatrix} -0.3 & 0.3 \end{bmatrix}, \qquad N_{B2} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}.$$

Given the decay rate $\varepsilon = 0.4$, the bound of mode-dependent time-varying neutral delay $0 \le \tau_1(t) \le 0.5$, $0 \le \tau_2(t) \le 0.6$, and the bound of mode-dependent time-varying retarded delay $0.1 \le d_1(t) \le 0.2$, $0.3 \le d_2(t) \le 0.4$. Without loss of generality, we choose $d_{m1} = 0.12$ and $d_{m2} = 0.33$. Since the information on the delay derivative is not available, by setting $Q_1 = Q_2 = 0$, $R_1 = 0$, $T_1 = 0$ in the Lyapunov

Table 5: Maximum upper bound of d_2 with different ε and parameter $\mu = 0$.

Methods	ε	0.1	0.3	0.5	0.7	0.9
Chen et al. [44]	d_2	1.2999	0.8781	0.6917	0.5792	0.5015
Qiu and Cui [13]	d_2	1.4008	1.0199	0.8457	0.7395	0.6667
Theorem 10	d_2	1.5167	1.1365	1.0674	0.9677	0.7558

Table 6: Maximum upper bound of d_2 with different ε and parameter $\mu = 0.5$.

Methods	ε	0.1	0.3	0.5	0.7	0.9
Chen et al. [44]	d_2	0.9442	0.7275	0.6096	0.5321	0.4761
Qiu and Cui [13]	d_2	1.0120	0.8324	0.7311	0.6941	0.6063
Theorem 10	d_2	1.0643	0.9156	0.7985	0.7368	0.6849

Table 7: Maximum upper bound of d_2 with different ε and parameter $\mu = 0.9$.

Methods	ε	0.1	0.3	0.5	0.7	0.9
Chen et al. [44]	d_2	0.5471	0.5015	0.4650	0.4350	0.4089
Qiu and Cui [13]	d_2	0.6438	0.5789	0.5214	0.4954	0.4231
Theorem 10	d_2	0.7136	0.6344	0.5987	0.5473	0.4963

functional (92), we follow the same procedure as in the proof of Theorem 10 and obtain the corresponding result. Thus, the exponential stability of the practical PEEC system can be readily established, which shows the effectiveness of the approach presented in this paper.

5. Conclusions

The problem of exponential stability for neutral Markovian jumping systems with interval mode-dependent time-varying delays, nonlinear perturbations, and partially unknown transition rates is investigated in this paper. A novel augmented stochastic Lyapunov-Krasovskii functional is constructed, which contains some triple-integral terms and sufficiently takes advantage of the delay bound. Then, less conservative delay-range-dependent and rate-dependent exponential stability criteria are obtained by novel technique of matrix inequalities and free weighting matrices. These theoretical results are successfully verified through some numerical examples. Finally, the main contributions of this paper can be summarized as follows: (1) the constructed stochastic Lyapunov functional contains some triple-integral terms which are very effective in the reduction of conservativeness and has not appeared in the context of neutral Markovian jump systems with partially known transition rates and nonlinear perturbations before; (2) the bound of the delay is fully utilized in this paper; that is, improved bounding technique is used to reduce the conservativeness; (3) the reciprocally convex lemma is used to derive the delay-range-dependent and rate-dependent stability conditions, which can well reduce the conservativeness of the investigated systems; and (4) the proposed results are applicable to the partially known transition rates and expressed in a new representation, which are proved to be less conservative than some existing ones.

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References

[1] V. B. Kolmanovskii and A. Myshkis, *Applied Theory of Functional Differential Equations*, Kluwer Academic Publishers, Dodrecht, The Netherlands, 1992.

[2] S.-I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*, vol. 269 of *Lecture Notes in Control and Information Sciences*, Springer, London, UK, 2001.

[3] P. L. Liu, "Further results on the exponential stability criteria for time delay singular systems with delay-dependence," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 6, pp. 4015–4024, 2012.

[4] P. L. Liu, "Improved delay-dependent robust exponential stabilization criteria for uncertain time-varying delay singular systems," *International Journal of Innovative Computing, Information and Control*, vol. 9, no. 1, pp. 165–178, 2013.

[5] C.-H. Lien and J.-D. Chen, "Discrete-delay-independent and discrete-delay-dependent criteria for a class of neutral systems," *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME*, vol. 125, no. 1, pp. 33–41, 2003.

[6] J. Tian and S. Zhong, "Improved delay-dependent stability criterion for neural networks with time-varying delay," *Applied Mathematics and Computation*, vol. 217, no. 24, pp. 10278–10288, 2011.

[7] J. Tian and X. Zhou, "Improved asymptotic stability criteria for neural networks with interval time-varying delay," *Expert Systems with Applications*, vol. 37, no. 12, pp. 7521–7525, 2010.

[8] J. Tian and X. Xie, "New asymptotic stability criteria for neural networks with time-varying delay," *Physics Letters A*, vol. 374, no. 7, pp. 938–943, 2010.

[9] M. N. A. Parlakçı, "Delay-dependent robust stability criteria for uncertain neutral systems with mixed time-varying discrete and neutral delays," *Asian Journal of Control*, vol. 9, no. 4, pp. 411–421, 2007.

[10] O. M. Kwon and J. H. Park, "Delay-range-dependent stabilization of uncertain dynamic systems with interval time-varying delays," *Applied Mathematics and Computation*, vol. 208, no. 1, pp. 58–68, 2009.

[11] S. Xu and J. Lam, "A survey of linear matrix inequality techniques in stability analysis of delay systems," *International Journal of Systems Science*, vol. 39, no. 12, pp. 1095–1113, 2008.

[12] C.-C. Shen and S.-M. Zhong, "New delay-dependent robust stability criterion for uncertain neutral systems with time-varying delay and nonlinear uncertainties," *Chaos, Solitons & Fractals*, vol. 40, no. 5, pp. 2277–2285, 2009.

[13] F. Qiu and B.-T. Cui, "Improved exponential stability criteria for uncertain neutral system with nonlinear parameter perturbations," *International Journal of Automation and Computing*, vol. 7, no. 4, pp. 413–418, 2010.

[14] G. Zhang, C. Han, Y. Guan, and L. Wu, "Exponential stability analysis and stabilization of discrete-time nonlinear switched systems with time delays," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 3A, pp. 1973–1986, 2012.

[15] H. R. Karimi, M. Zapateiro, and N. Luo, "Stability analysis and control synthesis of neutral systems with time-varying delays and nonlinear uncertainties," *Chaos, Solitons & Fractals*, vol. 42, no. 1, pp. 595–603, 2009.

[16] J. H. Park, "Novel robust stability criterion for a class of neutral systems with mixed delays and nonlinear perturbations," *Applied Mathematics and Computation*, vol. 161, no. 2, pp. 413–421, 2005.

[17] J. Zhang, P. Shi, and J. Qiu, "Robust stability criteria for uncertain neutral system with time delay and nonlinear uncertainties," *Chaos, Solitons & Fractals*, vol. 38, no. 1, pp. 160–167, 2008.

[18] Y. He, Q.-G. Wang, L. Xie, and C. Lin, "Further improvement of free-weighting matrices technique for systems with time-varying delay," *IEEE Transactions on Automatic Control*, vol. 52, no. 2, pp. 293–299, 2007.

[19] A. Tanikawa, "On new smoothing algorithms for discrete-time linear stochastic systems with unknown disturbances," *International Journal of Innovative Computing, Information and Control*, vol. 4, no. 1, pp. 15–24, 2008.

[20] B. Chen, H. Li, P. Shi, C. Lin, and Q. Zhou, "Delay-dependent stability analysis and controller synthesis for Markovian jump systems with state and input delays," *Information Sciences*, vol. 179, no. 16, pp. 2851–2860, 2009.

[21] J. Liu, Z. Gu, and S. Hu, "H_∞ filtering for Markovian jump systems with time-varying delays," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 3, pp. 1299–1310, 2011.

[22] J. Xiong and J. Lam, "Stabilization of discrete-time Markovian jump linear systems via time-delayed controllers," *Automatica*, vol. 42, no. 5, pp. 747–753, 2006.

[23] J. Wu, T. Chen, and L. Wang, "Delay-dependent robust stability and H_∞ control for jump linear systems with delays," *Systems & Control Letters*, vol. 55, no. 11, pp. 939–948, 2006.

[24] P. Shi, Y. Xia, G. P. Liu, and D. Rees, "On designing of sliding-mode control for stochastic jump systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 97–103, 2006.

[25] S. Xu, J. Lam, and X. Mao, "Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays," *IEEE Transactions on Circuits and Systems I*, vol. 54, no. 9, pp. 2070–2077, 2007.

[26] J. Tian, Y. Li, J. Zhao, and S. Zhong, "Delay-dependent stochastic stability criteria for Markovian jumping neural networks with mode-dependent time-varying delays and partially known transition rates," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5769–5781, 2012.

[27] L. L. Xiong, X. B. Zhou, J. Qiu, and J. Lei, "Stability analysis for Markovian jump neutral systems with mixed delays and partially known transition rates," *Abstract and Applied Analysis*, vol. 2012, Article ID 450168, 22 pages, 2012.

[28] L. Zhang and J. Lam, "Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1695–1701, 2010.

[29] Q. Ma, S. Xu, and Y. Zou, "Stability and synchronization for Markovian jump neural networks with partly unknown transition probabilities," *Neurocomputing*, vol. 74, no. 17, pp. 3404–3411, 2011.

[30] X. Luan, F. Liu, and P. Shi, "Finite-time filtering for non-linear stochastic systems with partially known transition jump rates," *IET Control Theory & Applications*, vol. 4, no. 5, pp. 735–745, 2010.

[31] Y. Zhang, Y. He, M. Wu, and J. Zhang, "Stabilization for Markovian jump systems with partial information on transition probability based on free-connection weighting matrices," *Automatica*, vol. 47, no. 1, pp. 79–84, 2011.

[32] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional-Differential Equations*, vol. 99 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 1993.

[33] X.-M. Sun, J. Zhao, and D. J. Hill, "Stability and L_2-gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.

[34] X. Mao, "Stability of stochastic differential equations with Markovian switching," *Stochastic Processes and Their Applications*, vol. 79, no. 1, pp. 45–67, 1999.

[35] H. B. Ji, *Algebra Foundation of Control Theory*, University of Science and Technology of China Press, Hefei, China, 2008.

[36] Y. G. Sun, L. Wang, and G. Xie, "Exponential stability of switched systems with interval time-varying delay," *IET Control Theory & Applications*, vol. 3, no. 8, pp. 1033–1040, 2009.

[37] J. Sun, G. P. Liu, and J. Chen, "Delay-dependent stability and stabilization of neutral time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 19, no. 12, pp. 1364–1375, 2009.

[38] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[39] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *International Journal of Control*, vol. 63, no. 4, pp. 741–750, 1996.

[40] L. Xiong, J. Tian, and X. Liu, "Stability analysis for neutral Markovian jump systems with partially unknown transition

probabilities," *Journal of the Franklin Institute*, vol. 349, no. 6, pp. 2193–2214, 2012.

[41] S. Mondié and V. L. Kharitonov, "Exponential estimates for retarded time-delay systems: an LMI approach," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 268–273, 2005.

[42] P.-L. Liu, "Exponential stability for linear time-delay systems with delay dependence," *Journal of the Franklin Institute*, vol. 340, no. 6-7, pp. 481–488, 2003.

[43] S. Xu, J. Lam, and M. Zhong, "New exponential estimates for time-delay systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 9, pp. 1501–1505, 2006.

[44] Y. Chen, A. Xue, R. Lu, and S. Zhou, "On robustly exponential stability of uncertain neutral systems with time-varying delays and nonlinear perturbations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 68, no. 8, pp. 2464–2470, 2008.

[45] D. Wang, W. Wang, and P. Shi, "Exponential H_∞ filtering for switched linear systems with interval time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 19, no. 5, pp. 532–551, 2009.

[46] D. Wang, W. Wang, P. Shi, and X.-M. Sun, "Controller failure analysis for systems with interval time-varying delay: a switched method," *Circuits, Systems, and Signal Processing*, vol. 28, no. 3, pp. 389–407, 2009.

[47] S. He and F. Liu, "Exponential stability for uncertain neutral systems with Markov jumps," *Journal of Control Theory and Applications*, vol. 7, no. 1, pp. 35–40, 2009.