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Research Article

Periodic Solution for Impulsive Cellar Neural Networks with Time-Varying Delays in the Leakage Terms

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This paper is concerned with impulsive cellular neural networks with time-varying delays in leakage terms. Without assuming bounded and monotone conditions on activation functions, we establish sufficient conditions on existence and exponential stability of periodic solutions by using Lyapunov functional method and differential inequality techniques. Our results are complement to some recent ones.

1. Introduction

It is well known that impulsive differential equations are mathematical apparatus for simulation of process and phenomena observed in control theory, physics, chemistry, population dynamics, biotechnologies, industrial robotics, economics, and so forth [1–3]. Thus, many neural networks with impulses have been studied extensively, and a great deal of literature is focused on the existence and stability of an equilibrium point [4–7]. In [8–10], the authors discussed the existence and global exponential stability of periodic solution of a class of cellular neural networks (CNNs) with impulse. Recently, Wang et al. [11] considered the following CNNs with impulses and leakage delays:

$$\begin{aligned} x_i'(t) &= -a_i x_i(t-\tau_i) + \sum_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} \beta_{ij}(t) f_j(x_j(t-\sigma_{ij})) + I_i(t), \qquad (1) \\ &\qquad t \geq 0, \quad t \neq t_k, \\ \Delta x_i(t_k) &= x_i(t_k^+) - x_i(t_k^-) = d_{ik} x_i(t_k), \end{aligned}$$

where $\Delta x_i(t_k)$ are the impulses at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$; $a_i > 0$ and $\tau_i > 0$ are constants, and $\alpha_{ij}(t)$, $I_i(t)$, and $\beta_{ij}(t)$ are continuous periodic functions with period T. Suppose that the following conditions are satisfied.

(A_1) There exist constants L_j^f, $j = 1, 2, \ldots n$, such that, for any $\alpha, \beta \in R$,

$$0 < \frac{f_j(\alpha) - f_j(\beta)}{\alpha - \beta} < L_j^f, \quad \alpha \neq \beta, \ j = 1, 2, \ldots n. \qquad (2)$$

(A_2) $f_i(0) = 0$ and for $i = 1, 2, \ldots, n$, there exists a constant $0 < M_i < +\infty$, such that

$$|f_i(\alpha)| \leq M_i, \quad \alpha \in R. \qquad (3)$$

By using the continuation theorem of coincidence degree theory and a suitable degenerate Lyapunov-Krasovskii functional together with model transformation technique, some results were obtained in [11] to guarantee that all solutions of system (1) converge exponentially to a periodic function. However, to the best of our knowledge, few authors have considered the existence and stability of periodic solutions of system (1) without the assumptions (A_1) and (A_2). Thus, it is worthwhile to continue to investigate the convergence behavior of system (1) in this case. In view of the fact that the coefficients and delays in neural networks are usually time varying in the real world, motivated by the above discussions, in this paper, we will consider the problem on periodic solution of the following impulsive CNNs with time-varying delays in the leakage terms:

$$\begin{aligned} x_i'(t) &= -c_i(t)\,x_i(t-\eta_i(t)) + \sum_{j=1}^{n} a_{ij}(t)\,f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)\,g_j(x_j(t-\tau_{ij}(t))) + I_i(t), \\ &\qquad t \ge 0, \quad t \ne t_k, \\ \Delta x_i(t_k) &= x_i(t_k^+) - x_i(t_k^-) = d_{ik} x_i(t_k), \end{aligned} \tag{4}$$

in which n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t, and $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at the time t. $a_{ij}(t)$ and $b_{ij}(t)$ are the connection weights at the time t, $\eta_i(t)$ and $\tau_{ij}(t)$ denote the transmission delays, $I_i(t)$ denotes the external bias on the ith unit at the time t, f_j and g_j are activation functions of signal transmission, $\Delta x_i(t_k)$ are the impulses at moments t_k, and $0 \le t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$, and $i, j = 1, 2, \ldots, n$. It is obvious that when $f = g$ and $\eta_i(t)$ is a constant function, (1) is a special case of (4).

The main purpose of this paper is to give the conditions for the existence and exponential stability of the periodic solutions for system (4). By applying Lyapunov functional method and differential inequality techniques, without assuming (A_1) and (A_2), we derive some new sufficient conditions ensuring the existence, uniqueness, and exponential stability of the periodic solution for system (4), which are new and complement previously known results. Moreover, an example is also provided to illustrate the effectiveness of our results.

Throughout this paper, we assume that the following conditions hold.

(H_1) For $i, j = 1, 2, \ldots, n$, I_i, a_{ij}, $b_{ij} : R \to R$ and c_i, η_i, $\tau_{ij} : R \to R^+$ are continuous periodic functions with period $T > 0$, and $t - \eta_i(t) \ge 0$ for all $t \ge 0$. In addition, there exist constants c_i^+, η_i^+, I_i^+, τ_i, a_{ij}^+, b_{ij}^+, and τ_{ij}^+ such that

$$\begin{gathered} c_i^+ = \sup_{t\in R} c_i(t), \qquad \eta_i^+ = \sup_{t\in R} \eta_i(t), \\ I_i^+ = \sup_{t\in R} |I_i(t)|, \qquad a_{ij}^+ = \sup_{t\in R} |a_{ij}(t)|, \\ b_{ij}^+ = \sup_{t\in R} |b_{ij}(t)|, \qquad \tau_{ij}^+ = \sup_{t\in R} \tau_{ij}(t), \\ \tau_i = \max\left\{\eta_i^+, \max_{j=1,2,\ldots,n} \tau_{ji}^+\right\}. \end{gathered} \tag{5}$$

(H_2) The sequence of times t_k $(k \in N)$ satisfies $t_k < t_{k+1}$ and $\lim_{k\to+\infty} t_k = +\infty$, and d_{ik} satisfies $-2 \le d_{ik} \le 0$ for $i \in \{1, 2, \ldots, n\}$ and $k \in Z^+$, where Z^+ denotes the set of all positive integers.

(H_3) There exists a $q \in Z^+$ such that

$$d_{i(k+q)} = d_{ik}, \quad t_{k+q} = t_k + T, \quad (k \in Z^+). \tag{6}$$

(H_4) For each $j \in \{1, 2, \ldots, n\}$, there exist nonnegative constants L_j^f and L_j^g such that, for all $u, v \in R$,

$$\begin{gathered} g_j(0) = f_j(0) = 0, \quad |g_j(u) - g_j(v)| \le L_j^g |u - v|, \\ |f_j(u) - f_j(v)| \le L_j^f |u - v|. \end{gathered} \tag{7}$$

(H_5) For all $t > 0$ and $i \in \{1, 2, \ldots, n\}$, there exist constants $\xi_i > 0$ and $\eta > 0$ such that

$$\begin{aligned} -\eta &> -\left[c_i(t) - c_i(t)\,\eta_i(t)\,c_i^+\right]\xi_i \\ &\quad + \sum_{j=1}^{n} \left(|a_{ij}(t)| + c_i(t)\,\eta_i(t)\,a_{ij}^+\right) L_j^f \xi_j \\ &\quad + \sum_{j=1}^{n} \left(|b_{ij}(t)| + c_i(t)\,\eta_i(t)\,b_{ij}^+\right) L_j^g \xi_j. \end{aligned} \tag{8}$$

For convenience, let $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, in which "$T$" denotes the transposition. We define $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$ and $\|x\| = \max_{1\le i\le n}\{|x_i|\}$. As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \mapsto (x_1(t), x_2(t), \ldots, x_n(t))^T$, we assume that $(x_1(t), x_2(t), \ldots, x_n(t))^T \equiv (x_1(t-0), x_2(t-0), \ldots, x_n(t-0))^T$. It is clearly that, in general, the derivative $x_i'(t_k)$ does not exist. On the other hand, according to system (4), there exists the limit $x_i'(t_k \mp 0)$. In view of the above convention, we assume that $x_i'(t_k) \equiv x_i'(t_k - 0)$.

The initial conditions associated with (4) are assumed to be of the form

$$x_i(s) = \phi_i(s), \quad s \in [-\tau_i, 0], \ i = 1, 2, \ldots, n, \tag{9}$$

where $\phi_i(\cdot)$ denotes a real-valued continuous function defined on $[-\tau_i, 0]$.

2. Preliminary Results

The following lemmas will be used to prove our main results in Section 3.

Lemma 1. *Let (H_1)–(H_5) hold. Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is a solution of system (1) with the initial conditions*

$$x_i(s) = \varphi_i(s), \quad |\varphi_i(s)| < \xi_i \frac{\gamma}{\eta}, \quad s \in [-\tau_i, 0], \tag{10}$$

where $\gamma = 1 + \max_{i=1,2,\ldots,n}\{[c_i^+\eta_i^+ + 1]I_i^+\}$, $i = 1, 2, \ldots, n$. Then

$$|x_i(t)| < \xi_i \frac{\gamma}{\eta}, \quad \forall t \ge 0, \ i = 1, 2, \ldots, n. \tag{11}$$

Proof. Assume that (11) does not hold. From (H_2), we have

$$|x_i(t_k^+)| = |(1 + d_{ik})|\,|x_i(t_k)| \le |x_i(t_k)|. \tag{12}$$

So, if $|x_i(t_k^+)| > \gamma$, then $|x_i(t_k)| > \gamma$. Thus, we may assume that there exist $i \in \{1, 2, \ldots, n\}$ and $t_* \in (t_k, t_{k+1})$ such that

$$\begin{gathered} \left|x_i(t_*)\right| = \xi_i \frac{\gamma}{\eta}, \qquad \left|x_j(t)\right| < \xi_j \frac{\gamma}{\eta} \\ \forall t \in [-\tau_i, t_*), \quad j = 1, 2, \ldots, n. \end{gathered} \tag{13}$$

According to (4), we get

$$\begin{aligned} x_i'(t) &= -c_i(t) x_i(t - \eta_i(t)) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) + I_i(t) \\ &= -c_i(t) x_i(t) + c_i(t) [x_i(t) - x_i(t - \eta_i(t))] \\ &\quad + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) + I_i(t) \\ &= -c_i(t) x_i(t) + c_i(t) \int_{t-\eta_i(t)}^{t} x_i'(s)\, ds \\ &\quad + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) + I_i(t), \\ &\qquad t > 0, \quad t \neq t_k, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{14}$$

Calculating the upper left derivative of $|x_i(t)|$, together with (13), (14), (H_5), and

$$\gamma > [c_i^+ \eta_i^+ + 1] I_i^+, \tag{15}$$

we obtain

$$\begin{aligned} 0 &\leq D^- \left|x_i(t_*)\right| \\ &\leq -c_i(t_*) \left|x_i(t_*)\right| + c_i(t_*) \int_{t_*-\eta_i(t_*)}^{t_*} \left|x_i'(s)\right| ds \\ &\quad + \sum_{j=1}^{n} \left|a_{ij}(t_*) f_j(x_j(t_*))\right| \\ &\quad + \sum_{j=1}^{n} \left|b_{ij}(t_*)\right| \left|g_j(x_j(t_* - \tau_{ij}(t_*)))\right| + \left|I_i(t_*)\right| \\ &= -c_i(t_*) \left|x_i(t_*)\right| + c_i(t_*) \\ &\quad \times \int_{t_*-\eta_i(t_*)}^{t_*} \Bigg| - c_i(s) x_i(s - \eta_i(s)) \\ &\qquad + \sum_{j=1}^{n} a_{ij}(s) f_j(x_j(s)) \\ &\qquad + \sum_{j=1}^{n} b_{ij}(s) g_j(x_j(s - \tau_{ij}(s))) \\ &\qquad + I_i(s) \Bigg| ds \\ &\quad + \sum_{j=1}^{n} \left|a_{ij}(t_*) f_j(x_j(t_*))\right| \\ &\quad + \sum_{j=1}^{n} \left|b_{ij}(t_*)\right| \left|g_j(x_j(t_* - \tau_{ij}(t_*)))\right| + \left|I_i(t_*)\right| \\ &\leq -[c_i(t_*) - c_i(t_*) \eta_i(t_*) c_i^+] \left|x_i(t_*)\right| \\ &\quad + \sum_{j=1}^{n} \left(\left|a_{ij}(t_*)\right| + c_i(t_*) \eta_i(t_*) a_{ij}^+\right) L_j^f \xi_j \frac{\gamma}{\eta} \\ &\quad + \sum_{j=1}^{n} \left(\left|b_{ij}(t_*)\right| + c_i(t_*) \eta_i(t_*) b_{ij}^+\right) L_j^g \xi_j \frac{\gamma}{\eta} \\ &\quad + [c_i^+ \eta_i^+ + 1] I_i^+ \\ &= \Bigg\{ - [c_i(t_*) - c_i(t_*) \eta_i(t_*) c_i^+] \xi_i \\ &\qquad + \sum_{j=1}^{n} \left(\left|a_{ij}(t_*)\right| + c_i(t_*) \eta_i(t_*) a_{ij}^+\right) L_j^f \xi_j \\ &\qquad + \sum_{j=1}^{n} \left(\left|b_{ij}(t_*)\right| + c_i(t_*) \eta_i(t_*) b_{ij}^+\right) L_j^g \xi_j \Bigg\} \frac{\gamma}{\eta} \\ &\quad + [c_i^+ \eta_i^+ + 1] I_i^+ \\ &< -\eta \frac{\gamma}{\eta} + [c_i^+ \eta_i^+ + 1] I_i^+ \\ &< 0. \end{aligned} \tag{16}$$

It is a contradiction and shows that (11) holds. The proof is now completed. □

Remark 2. After the conditions (H_1)–(H_5), the solution of system (4) always exists (see [1, 2]). In view of the boundedness of this solution, from the theory of impulsive differential equations in [1], it follows that the solution of system (4) with initial conditions (10) can be defined on $[0, +\infty)$.

Lemma 3. *Suppose that (H_1)–(H_5) are true. Let $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ be the solution of system (4) with* *initial value* $\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \ldots, \varphi_n^*(t))^T$, *and let* $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ *be the solution of system* (4) *with initial value* $\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T$. *Then, there exists a positive constant* λ *such that*

$$x_i(t) - x_i^*(t) = O\left(e^{-\lambda t}\right), \quad i = 1, 2, \ldots, n. \tag{17}$$

Proof. Let $y(t) = x(t) - x^*(t)$. Then, for $i \in \{1, 2, \ldots, n\}$, it is followed by

$$\begin{aligned} y_i'(t) &= -c_i(t)\left(x_i\left(t - \eta_i(t)\right) - x_i^*\left(t - \eta_i(t)\right)\right) \\ &\quad + \sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(x_j(t)\right) - f_j\left(x_j^*(t)\right)\right] \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)\left[g_j\left(x_j\left(t - \tau_{ij}(t)\right)\right) - g_j\left(x_j^*\left(t - \tau_{ij}(t)\right)\right)\right], \\ &\qquad t \geq 0, \quad t \neq t_k, \\ y_i(t_k^+) &= \left(1 + d_{ik}\right) y_i(t_k), \quad k = 1, 2, \ldots. \end{aligned} \tag{18}$$

Define continuous functions $\Gamma_i(\omega)$ by setting

$$\begin{aligned} \Gamma_i(\omega) &= -\left[c_i(t) e^{\omega \eta_i(t)} - \omega - c_i(t) e^{\omega \eta_i(t)} \eta_i(t)\left(\omega + c_i^+ e^{\omega \eta_i^+}\right)\right] \xi_i \\ &\quad + \sum_{j=1}^{n}\left(\left|a_{ij}(t)\right| + a_{ij}^+ c_i(t) e^{\omega \eta_i(t)} \eta_i(t)\right) L_j^f \xi_j \\ &\quad + \sum_{j=1}^{n}\left(\left|b_{ij}(t)\right| e^{\omega \tau_{ij}(t)} + b_{ij}^+ c_i(t) e^{\omega \eta_i(t)} \eta_i(t) e^{\omega \tau_{ij}^+}\right) L_j^g \xi_j, \\ &\qquad \omega \geq 0, \quad t \geq 0, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{19}$$

Then

$$\begin{aligned} \Gamma_i(0) &= -\left[c_i(t) - c_i(t) \eta_i(t) c_i^+\right] \xi_i \\ &\quad + \sum_{j=1}^{n}\left(\left|a_{ij}(t)\right| + a_{ij}^+ c_i(t) \eta_i(t)\right) L_j^f \xi_j \\ &\quad + \sum_{j=1}^{n}\left(\left|b_{ij}(t)\right| + b_{ij}^+ c_i(t) \eta_i(t)\right) L_j^g \xi_j \\ &< -\eta, \quad t \geq 0, \ i = 1, 2, \ldots, n, \end{aligned} \tag{20}$$

which, together with the continuity of $\Gamma_i(\omega)$, implies that we can choose two positive constants λ and $\overline{\eta}$ such that

$$\begin{aligned} -\overline{\eta} &> \Gamma_i(\lambda) \\ &= -\left[c_i(t) e^{\lambda \eta_i(t)} - \lambda - c_i(t) e^{\lambda \eta_i(t)} \eta_i(t)\left(\lambda + c_i^+ e^{\lambda \eta_i^+}\right)\right] \xi_i \\ &\quad + \sum_{j=1}^{n}\left(\left|a_{ij}(t)\right| + a_{ij}^+ c_i(t) e^{\lambda \eta_i(t)} \eta_i(t)\right) L_j^f \xi_j \\ &\quad + \sum_{j=1}^{n}\left(\left|b_{ij}(t)\right| e^{\lambda \tau_{ij}(t)} + b_{ij}^+ c_i(t) e^{\lambda \eta_i(t)} \times \eta_i(t) e^{\lambda \tau_{ij}^+}\right) L_j^g \xi_j, \quad t \geq 0, \ i = 1, 2, \ldots, n. \end{aligned} \tag{21}$$

Let

$$Y_i(t) = y_i(t) e^{\lambda t}, \quad i = 1, 2, \ldots, n. \tag{22}$$

Then

$$\begin{aligned} Y_i'(t) &= \lambda Y_i(t) - c_i(t) e^{\lambda t} y_i\left(t - \eta_i(t)\right) \\ &\quad + e^{\lambda t}\left\{\sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(x_j(t)\right) - f_j\left(x_j^*(t)\right)\right] + \sum_{j=1}^{n} b_{ij}(t)\left[g_j\left(x_j\left(t - \tau_{ij}(t)\right)\right) - g_j\left(x_j^*\left(t - \tau_{ij}(t)\right)\right)\right]\right\} \\ &= \lambda Y_i(t) - c_i(t) e^{\lambda \eta_i(t)} Y_i(t) \\ &\quad + c_i(t) e^{\lambda \eta_i(t)}\left[Y_i(t) - Y_i\left(t - \eta_i(t)\right)\right] \\ &\quad + e^{\lambda t}\left\{\sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(x_j(t)\right) - f_j\left(x_j^*(t)\right)\right] + \sum_{j=1}^{n} b_{ij}(t)\left[g_j\left(x_j\left(t - \tau_{ij}(t)\right)\right) - g_j\left(x_j^*\left(t - \tau_{ij}(t)\right)\right)\right]\right\} \end{aligned}$$

$$
\begin{aligned}
&= \lambda Y_i(t) - c_i(t) e^{\lambda \eta_i(t)} Y_i(t) \\
&\quad + c_i(t) e^{\lambda \eta_i(t)} \int_{t-\eta_i(t)}^{t} Y_i'(s)\, ds \\
&\quad + e^{\lambda t} \Bigg\{ \sum_{j=1}^{n} a_{ij}(t) \left[f_j(x_j(t)) - f_j(x_j^*(t)) \right] \\
&\qquad + \sum_{j=1}^{n} b_{ij}(t) \left[g_j(x_j(t-\tau_{ij}(t))) \right. \\
&\qquad\qquad \left. - g_j(x_j^*(t-\tau_{ij}(t))) \right] \Bigg\} \\
&= \lambda Y_i(t) - c_i(t) e^{\lambda \eta_i(t)} Y_i(t) + c_i(t) e^{\lambda \eta_i(t)} \\
&\quad \times \int_{t-\eta_i(t)}^{t} \Bigg\{ \lambda Y_i(s) - c_i(s) e^{\lambda s} y_i(s-\eta_i(s)) \\
&\qquad + e^{\lambda s} \sum_{j=1}^{n} a_{ij}(s) \left[f_j(x_j(s)) - f_j(x_j^*(s)) \right] \\
&\qquad + e^{\lambda s} \sum_{j=1}^{n} b_{ij}(s) \left[g_j(x_j(s-\tau_{ij}(s))) \right. \\
&\qquad\qquad \left. - g_j(x_j^*(s-\tau_{ij}(s))) \right] \Bigg\} ds \\
&\quad + e^{\lambda t} \Bigg\{ \sum_{j=1}^{n} a_{ij}(t) \left[f_j(x_j(t)) - f_j(x_j^*(t)) \right] \\
&\qquad + \sum_{j=1}^{n} b_{ij}(t) \left[g_j(x_j(t-\tau_{ij}(t))) \right. \\
&\qquad\qquad \left. - g_j(x_j^*(t-\tau_{ij}(t))) \right] \Bigg\}, \\
&\qquad t \neq t_k, \quad i = 1, 2, \ldots, n,
\end{aligned} \tag{23}
$$

$$
|Y_i(t_k^+)| = |1 + d_{ik}| \, |Y_i(t_k)|, \quad i = 1, 2, \ldots, n. \tag{24}
$$

We define a positive constant M as follows:

$$
M = \max_{1 \le i \le n} \left\{ \sup_{s \in [-\tau_i, 0]} |Y_i(s)| \right\}. \tag{25}
$$

Let K be a positive number such that

$$
|Y_i(t)| \le M < K\xi_i \quad \forall t \in [-\tau_i, 0], \; i = 1, 2, \ldots, n. \tag{26}
$$

We claim that

$$
|Y_i(t)| < K\xi_i, \quad \forall t > 0, \; i = 1, 2, \ldots, n. \tag{27}
$$

Obviously, (27) holds for $t = 0$. We first prove that (27) is true for $0 < t \le t_1$. Otherwise, there exist $i \in \{1, 2, \ldots, n\}$ and $\rho \in (0, t_1]$ such that one of the following two cases must occur;

$$
\begin{aligned}
&(1) \quad Y_i(\rho) = K\xi_i, \qquad |Y_j(t)| < K\xi_j \\
&\qquad \forall t \in [0, \rho), \quad j = 1, 2, \ldots, n,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
&(2) \quad Y_i(\rho) = -K\xi_i, \qquad |Y_j(t)| < K\xi_j \\
&\qquad \forall t \in [0, \rho), \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{29}
$$

Now, we distinguish two cases to finish the proof.

Case (i). If (28) holds. Then, from (21), (23), and (H_1)–(H_5), we have

$$
\begin{aligned}
0 &\le Y_i'(\rho) \\
&= \lambda Y_i(\rho) - c_i(\rho) e^{\lambda \eta_i(\rho)} Y_i(\rho) + c_i(\rho) e^{\lambda \eta_i(\rho)} \\
&\quad \times \int_{\rho-\eta_i(\rho)}^{\rho} \Bigg\{ \lambda Y_i(s) - c_i(s) e^{\lambda s} y_i(s-\eta_i(s)) \\
&\qquad + e^{\lambda s} \sum_{j=1}^{n} a_{ij}(s) \left[f_j(x_j(s)) - f_j(x_j^*(s)) \right] \\
&\qquad + e^{\lambda s} \sum_{j=1}^{n} b_{ij}(s) \left[g_j(x_j(s-\tau_{ij}(s))) \right. \\
&\qquad\qquad \left. - g_j(x_j^*(s-\tau_{ij}(s))) \right] \Bigg\} ds
\end{aligned}
$$

$$
\begin{aligned}
&+e^{\lambda\rho}\left\{\sum_{j=1}^{n}a_{ij}(\rho)\left[f_j\left(x_j(\rho)\right)-f_j\left(x_j^*(\rho)\right)\right]\right.\\
&\qquad+\sum_{j=1}^{n}b_{ij}(\rho)\left[g_j\left(x_j\left(\rho-\tau_{ij}(\rho)\right)\right)\right.\\
&\qquad\qquad\left.\left.-g_j\left(x_j^*\left(\rho-\tau_{ij}(\rho)\right)\right)\right]\right\}\\
&\le\lambda Y_i(\rho)-c_i(\rho)e^{\lambda\eta_i(\rho)}Y_i(\rho)+c_i(\rho)e^{\lambda\eta_i(\rho)}\\
&\quad\times\int_{\rho-\eta_i(\rho)}^{\rho}\left\{\lambda Y_i(s)+c_i^+e^{\lambda\eta_i(s)}\left|Y_i\left(s-\eta_i(s)\right)\right|\right.\\
&\qquad+\sum_{j=1}^{n}a_{ij}^+L_j^f\left|Y_j(s)\right|\\
&\qquad\left.+\sum_{j=1}^{n}b_{ij}^+L_j^ge^{\lambda\tau_{ij}(s)}\left|Y_j\left(s-\tau_{ij}(s)\right)\right|\right\}ds\\
&\quad+\sum_{j=1}^{n}\left|a_{ij}(\rho)\right|L_j^f\left|Y_j(\rho)\right|\\
&\quad+\sum_{j=1}^{n}\left|b_{ij}(\rho)\right|L_j^ge^{\lambda\tau_{ij}(\rho)}\left|Y_j\left(\rho-\tau_{ij}(\rho)\right)\right|\\
&\le-\left[c_i(\rho)e^{\lambda\eta_i(\rho)}-\lambda-c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right.\\
&\qquad\left.\times\left(\lambda+c_i^+e^{\lambda\eta_i^+}\right)\right]K\xi_i\\
&\quad+\sum_{j=1}^{n}\left(\left|a_{ij}(\rho)\right|+a_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right)L_j^fK\xi_j\\
&\quad+\sum_{j=1}^{n}\left(\left|b_{ij}(\rho)\right|e^{\lambda\tau_{ij}(\rho)}+b_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\right.\\
&\qquad\left.\times\eta_i(\rho)e^{\lambda\tau_{ij}^+}\right)L_j^gK\xi_j\\
&=\left\{-\left[c_i(\rho)e^{\lambda\eta_i(\rho)}-\lambda-c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right.\right.\\
&\qquad\left.\times\left(\lambda+c_i^+e^{\lambda\eta_i^+}\right)\right]\xi_i\\
&\qquad+\sum_{j=1}^{n}\left(\left|a_{ij}(\rho)\right|+a_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right)L_j^f\xi_j\\
&\qquad+\sum_{j=1}^{n}\left(\left|b_{ij}(\rho)\right|e^{\lambda\tau_{ij}(\rho)}+b_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\right.\\
&\qquad\qquad\left.\left.\times\eta_i(\rho)e^{\lambda\tau_{ij}^+}\right)L_j^g\xi_j\right\}K\\
&<-\overline{\eta}K<0.
\end{aligned}
\tag{30}
$$

Case (ii). If (29) holds. Then, from (21), (23), and (H_1)–(H_5), we get

$$
\begin{aligned}
0&\ge Y_i'(\rho)\\
&=\lambda Y_i(\rho)-c_i(\rho)e^{\lambda\eta_i(\rho)}Y_i(\rho)+c_i(\rho)e^{\lambda\eta_i(\rho)}\\
&\quad\times\int_{\rho-\eta_i(\rho)}^{\rho}\left\{\lambda Y_i(s)-c_i(s)e^{\lambda s}y_i\left(s-\eta_i(s)\right)\right.\\
&\qquad+e^{\lambda s}\sum_{j=1}^{n}a_{ij}(s)\left[f_j\left(x_j(s)\right)-f_j\left(x_j^*(s)\right)\right]\\
&\qquad+e^{\lambda s}\sum_{j=1}^{n}b_{ij}(s)\left[g_j\left(x_j\left(s-\tau_{ij}(s)\right)\right)\right.\\
&\qquad\qquad\left.\left.-g_j\left(x_j^*\left(s-\tau_{ij}(s)\right)\right)\right]\right\}ds\\
&\quad+e^{\lambda\rho}\left\{\sum_{j=1}^{n}a_{ij}(\rho)\left[f_j\left(x_j(\rho)\right)-f_j\left(x_j^*(\rho)\right)\right]\right.\\
&\qquad+\sum_{j=1}^{n}b_{ij}(\rho)\left[g_j\left(x_j\left(\rho-\tau_{ij}(\rho)\right)\right)\right.\\
&\qquad\qquad\left.\left.-g_j\left(x_j^*\left(\rho-\tau_{ij}(\rho)\right)\right)\right]\right\}\\
&\ge\lambda Y_i(\rho)-c_i(\rho)e^{\lambda\eta_i(\rho)}Y_i(\rho)+c_i(\rho)e^{\lambda\eta_i(\rho)}\\
&\quad\times\int_{\rho-\eta_i(\rho)}^{\rho}\left\{\lambda Y_i(s)-c_i^+e^{\lambda\eta_i(s)}\left|Y_i\left(s-\eta_i(s)\right)\right|\right.\\
&\qquad-\sum_{j=1}^{n}a_{ij}^+L_j^f\left|Y_j(s)\right|\\
&\qquad\left.-\sum_{j=1}^{n}b_{ij}^+L_j^ge^{\lambda\tau_{ij}(s)}\left|Y_j\left(s-\tau_{ij}(s)\right)\right|\right\}ds\\
&\quad-\sum_{j=1}^{n}\left|a_{ij}(\rho)\right|L_j^f\left|Y_j(\rho)\right|\\
&\quad-\sum_{j=1}^{n}\left|b_{ij}(\rho)\right|L_j^ge^{\lambda\tau_{ij}(\rho)}\left|Y_j\left(\rho-\tau_{ij}(\rho)\right)\right|\\
&\ge-\left[c_i(\rho)e^{\lambda\eta_i(\rho)}-\lambda-c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right.\\
&\qquad\left.\times\left(\lambda+c_i^+e^{\lambda\eta_i^+}\right)\right]\left(-K\xi_i\right)\\
&\quad+\sum_{j=1}^{n}\left(\left|a_{ij}(\rho)\right|+a_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\eta_i(\rho)\right)L_j^f\left(-K\xi_j\right)\\
&\quad+\sum_{j=1}^{n}\left(\left|b_{ij}(\rho)\right|e^{\lambda\tau_{ij}(\rho)}+b_{ij}^+c_i(\rho)e^{\lambda\eta_i(\rho)}\right.\\
&\qquad\left.\times\eta_i(\rho)e^{\lambda\tau_{ij}^+}\right)L_j^g\left(-K\xi_j\right)
\end{aligned}
$$

$$= \Bigg\{ - \Big[c_i(\rho) e^{\lambda\eta_i(\rho)} - \lambda - c_i(\rho) e^{\lambda\eta_i(\rho)} \times \eta_i(\rho) \left(\lambda + c_i^+ e^{\lambda\eta_i^+} \right) \Big] \xi_i + \sum_{j=1}^{n} \left(\left| a_{ij}(\rho) \right| + a_{ij}^+ c_i(\rho) e^{\lambda\eta_i(\rho)} \eta_i(\rho) \right) L_j^f \xi_j + \sum_{j=1}^{n} \Big(\left| b_{ij}(\rho) \right| e^{\lambda\tau_{ij}(\rho)} + b_{ij}^+ c_i(\rho) e^{\lambda\eta_i(\rho)} \times \eta_i(\rho) e^{\lambda\tau_{ij}^+} \Big) L_j^g \xi_j \Bigg\} (-K) > \overline{\eta} K > 0. \tag{31}$$

Therefore, (27) holds for $t \in [0, t_1]$. From (24) and (27), we know that

$$\begin{aligned} \left| Y_i(t_1) \right| &= \left| y_i(t_1) \right| e^{\lambda t_1} < K\xi_i, \quad i = 1, 2, \ldots, n, \\ \left| Y_i(t_1^+) \right| &= \left| 1 + d_{i1} \right| \left| Y_i(t_1) \right| \le \left| Y_i(t_1) \right| < K\xi_i, \\ & \qquad i = 1, 2, \ldots, n. \end{aligned} \tag{32}$$

Thus, for $t \in [t_1, t_2]$, we may repeat the above procedure and obtain

$$\left| Y_i(t) \right| = \left| y_i(t) \right| e^{\lambda t} < K\xi_i, \quad \forall t \in [t_1, t_2], \; i = 1, 2, \ldots, n. \tag{33}$$

Further, we have

$$\left| Y_i(t) \right| = \left| y_i(t) \right| e^{\lambda t} < K\xi_i, \quad \forall t > 0, \; i = 1, 2, \ldots, n. \tag{34}$$

That is,

$$\left| x_i(t) - x_i^*(t) \right| \le K\xi_i e^{-\lambda t}, \quad \forall t > 0, \; i = 1, 2, \ldots, n. \tag{35}$$

□

Remark 4. If $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ is the T-periodic solution of system (4), it follows from Lemma 3 that $x^*(t)$ is globally exponentially stable.

3. Main Results

In this section, we will study existence and exponential stability for periodic solutions of system (4).

Theorem 5. *Suppose that all conditions in Lemma 3 are satisfied. Then system (4) has exactly one T-periodic solution $x^*(t)$. Moreover, $x^*(t)$ is globally exponentially stable.*

Proof. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be a solution of system (4) with initial conditions (10). By Remark 2, the solution $x(t)$ can be defined for all $t \in [0, +\infty)$. By hypothesis (H_1), we have, for any natural number h,

$$\begin{aligned} \left[x_i(t + (h+1)T) \right]' &= -c_i(t) x_i(t + (h+1)T - \eta_i(t)) \\ &\quad + \sum_{j=1}^{n} a_{ij}(t) f_j\left(x_j(t + (h+1)T) \right) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t) g_j\left(x_j\left(t + (h+1)T - \tau_{ij}(t) \right) \right) \\ &\quad + I_i(t), \quad t \ne t_k, \; i = 1, 2, \ldots, n. \end{aligned} \tag{36}$$

Further, by hypothesis of (H_3), we obtain

$$\begin{aligned} x_i\left((t_k + (h+1)T)^+ \right) &= x_i\left(t_{k+(h+1)q}^+ \right) \\ &= \left(1 + d_{i(k+(h+1)q)} \right) x_i\left(t_{k+(h+1)q} \right) \\ &= \left(1 + d_{ik} \right) x_i\left(t_k + (h+1)T \right), \quad k = 1, 2, \ldots. \end{aligned} \tag{37}$$

Thus, for any natural number h, we obtain that $x(t + (h+1)T)$ is a solution of system (4) for all $t + (h+1)T \ge 0$. Hence, $x(t+T)$ is also a solution of (4) with initial values

$$x_i(s + T), \quad s \in [-\tau_i, 0], \; i = 1, 2, \ldots, n. \tag{38}$$

Then, by the proof of Lemma 3, there exists a constant $K > 0$ such that for any natural number h,

$$\begin{aligned} \left| x_i(t + (h+1)T) - x_i(t + hT) \right| &= \left| x_i(t + hT + T) - x_i(t + hT) \right| \\ &\le K\xi_i e^{-\lambda(t + hT)} \\ &= K\xi_i e^{-\lambda t} \left(\frac{1}{e^{\lambda T}} \right)^h, \quad t + hT \ge 0, \\ & \qquad t \ne t_k, \quad i = 1, 2, \ldots, n, \\ \left| x_i\left((t_k + (h+1)T)^+ \right) - x_i\left((t_k + hT)^+ \right) \right| &= \left(1 + d_{ik} \right) \left| x_i(t_k + (h+1)T) - x_i(t_k + hT) \right| \\ &\le K\xi_i e^{-\lambda(t_k + hT)} \\ &= K\xi_i e^{-\lambda t_k} \left(\frac{1}{e^{\lambda T}} \right)^h, \quad \forall k \in Z^+, \; i = 1, 2, \ldots, n. \end{aligned} \tag{39}$$

Moreover, for any natural number m, we can obtain

$$\begin{aligned}
& x_i(t+(m+1)T) \\
&\quad = x_i(t) + \sum_{h=0}^{m} \left[x_i(t+(h+1)T) - x_i(t+hT) \right], \\
&\qquad\qquad t+hT \geq 0, \quad t \neq t_k, \quad i = 1,2,\ldots,n, \\
& \left(x_i\left((t_k + (m+1)T)^+ \right) \right) \\
&\quad = x_i(t) + \sum_{h=0}^{m} \left[x_i\left((t_k + (h+1)T)^+ \right) \right. \\
&\qquad\qquad \left. - \left(x_i\left((t_k + hT)^+ \right) \right) \right], \\
&\qquad\qquad \forall k \in Z^+, \quad i = 1,2,\ldots,n.
\end{aligned} \tag{40}$$

Combining (39) with (40), we know that $x(t+mT)$ will converge uniformly to a piecewise continuous function $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ on any compact set of R.

Now we are in the position of proving that $x^*(t)$ is a T-periodic solution of system (4). It is easily known that $x^*(t)$ is T-periodic since

$$\begin{aligned}
x_i^*(t+T) &= \lim_{m\to+\infty} x_i(t+T+mT) \\
&= \lim_{m+1\to+\infty} x_i(t+(m+1)T) \\
&= x_i^*(t), \quad t \neq t_k, \\
x_i^*\left((t_k+T)^+ \right) &= \lim_{m\to+\infty} x_i\left((t_k+T+mT)^+ \right) \\
&= x_i^*(t_k^+), \quad k = 1,2,\ldots,
\end{aligned} \tag{41}$$

where $i = 1,2,\ldots,n$. Noting that the right side of (4) is piecewise continuous, together with (36) and (37), we know that $\{x_i'(t+(m+1)T)\}$ converges uniformly to a piecewise continuous function on any compact set of $R \setminus \{t_1, t_2, \ldots\}$. Therefore, letting $m \to +\infty$ on both sides of (36) and (37), we get

$$\begin{aligned}
x_i^{*\prime}(t) &= -c_i(t) x_i^*(t-\eta_i(t)) \\
&\quad + \sum_{j=1}^{n} a_{ij}(t) f_j\left(x_j^*(t) \right) \\
&\quad + \sum_{j=1}^{n} b_{ij}(t) g_j\left(x_j^*\left(t - \tau_{ij}(t) \right) \right) \\
&\quad + I_i(t), \quad t \neq t_k, \ i = 1,2,\ldots,n, \\
x_i^*(t_k^+) &= (1+d_{ik}) x_i^*(t_k), \\
k &= 1,2,\ldots, \qquad i = 1,2,\ldots,n.
\end{aligned} \tag{42}$$

Thus, $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ is a T-periodic solution of system (4).

Finally, by Lemma 3, we can prove that $x^*(t)$ is globally exponentially stable. This completes the proof. □

4. An Example

In this section, we give an example to demonstrate the results obtained in the previous sections.

Example 6. Consider the following impulsive cellar neural network consisting of two neurons with time-varying delays in the leakage terms, which is described by

$$\begin{aligned}
x_1'(t) &= -3(|\sin \pi t| + 1) x_1\left(t - \frac{\sin^2 \pi t}{1000} \right) \\
&\quad + \frac{1}{16} \cos^2 \pi t f_1(x_1(t)) \\
&\quad + \frac{1}{16} \sin^2 \pi t f_2(x_2(t)) \\
&\quad + \frac{1}{16} \sin^2 \pi t g_1\left(x_1\left(t - \cos^2 \pi t \right) \right) \\
&\quad + \frac{1}{16} \sin^2 \pi t g_2\left(x_2\left(t - 2\sin^2 \pi t \right) \right) \\
&\quad + 100 \cos \pi t \quad t \neq 2k-1, \\
x_2'(t) &= -3(|\cos \pi t| + 1) x_1\left(t - \frac{\sin^4 \pi t}{1000} \right) \\
&\quad + \frac{1}{16} \cos^3 \pi t f_1(x_1(t)) \\
&\quad + \frac{1}{16} \sin^3 \pi t f_2(x_2(t)) \\
&\quad + \frac{1}{16} \sin^3 \pi t g_1\left(x_1\left(t - \cos^2 \pi t \right) \right) \\
&\quad + \frac{1}{16} \sin^3 \pi t g_2\left(x_2\left(t - 2\sin^2 \pi t \right) \right) \\
&\quad + 100 \sin \pi t \quad t \neq 2k-1, \\
x_i(t_k^+) &= (1+d_{ik}) x_i(t_k), \\
d_{i(2s)} &= -2, \qquad d_{i(2s-1)} = -1, \\
t_k &= k, \qquad i = 1,2, \qquad k, s = 1,2,\ldots.
\end{aligned} \tag{43}$$

Here, it is assumed that the activation functions are

$$\begin{aligned}
g_1(x) &= g_2(x) = x + 2\sin x, \\
f_1(x) &= f_2(x) = x + 3\sin x.
\end{aligned} \tag{44}$$

Noting that

$$\begin{gathered}
\eta_1(t) = \frac{\sin^2 \pi t}{1000}, \qquad \eta_2(t) = \frac{\sin^4 \pi t}{1000}, \\
c_1(t) = 3(|\sin \pi t| + 1), \qquad c_2(t) = 3(|\cos \pi t| + 1),
\end{gathered}$$

$$\begin{aligned} a_{11}(t) &= \frac{1}{16}\cos^2\pi t, & a_{12}(t) &= \frac{1}{16}\sin^2\pi t, \\ b_{11}(t) &= \frac{1}{16}\sin^2\pi t, & b_{12}(t) &= \frac{1}{16}\sin^2\pi t, \\ a_{21}(t) &= \frac{1}{16}\cos^3\pi t, & a_{22}(t) &= \frac{1}{16}\sin^3\pi t, \\ b_{21}(t) &= \frac{1}{16}\sin^3\pi t, & b_{22}(t) &= \frac{1}{16}\sin^3\pi t, \end{aligned}$$
$$\tau_{11}(t) = \tau_{21}(t) = \cos^2\pi t,$$
$$\tau_{12}(t) = \tau_{22}(t) = 2\sin^2\pi t, \tag{45}$$

then we obtain

$$\begin{aligned} &-\left[c_i(t) - c_i(t)\eta_i(t)c_i^+\right]\xi_i \\ &\quad + \sum_{j=1}^{2}\left(\left|a_{ij}(t)\right| + c_i(t)\eta_i(t)a_{ij}^+\right)L_j^f\xi_j \\ &\quad + \sum_{j=1}^{2}\left(\left|b_{ij}(t)\right| + c_i(t)\eta_i(t)b_{ij}^+\right)L_j^g\xi_j \\ &< -\left(3 - 6\times\frac{1}{1000}\times 6\right) \\ &\quad + 2\left(\frac{1}{16} + 6\times\frac{1}{1000}\times\frac{1}{16}\right)\times 3 \\ &\quad + 2\left(\frac{1}{16} + 6\times\frac{1}{1000}\times\frac{1}{16}\right)\times 4 \\ &< -1, \quad \xi_i = 1, \ i = 1, 2. \end{aligned} \tag{46}$$

This yields that system (43) satisfies (H_1)–(H_5). Hence, from Theorem 5, system (43) has exactly one 2-periodic solution. Moreover, the 2-periodic solution is globally exponentially stable.

Remark 7. Since $g_1(x) = g_2(x) = x + 2\sin x$, $f_1(x) = f_2(x) = x + 3\sin x$ and CNNs (43) is a very simple form of CNNs with time-varying delays in the leakage terms, it is clear that the conditions (A_1) and (A_2) are not satisfied. Therefore, all the results in [11–19] and the references therein cannot be applicable to system (43) to obtain the existence and exponential stability of the 2-periodic solutions.

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