

## Research Article

# Mathematical Model Based on BP Neural Network Algorithm for the Deflection Identification of Storage Tank and Calibration of Tank Capacity Chart

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The tank capacity chart calibration problem of two oil tanks with deflection was studied, one of which is an elliptical cylinder storage tank with two truncated ends and another is a cylinder storage tank with two spherical crowns. Firstly, the function relation between oil reserve and oil height based on the integral method was precisely deduced, when the storage tank has longitudinal inclination but has no deflection. Secondly, the nonlinear optimization model which has both longitudinal inclination parameter  $\alpha$  and lateral deflection parameter  $\beta$  was constructed, using cut-complement method and approximate treatment method. Then the deflection tank capacity chart calibration with a 10 cm oil level height interval was worked out. Lastly, the tank capacity chart was corrected by BP neural network algorithm and got proportional error of theoretical and experimental measurements ranges from 0% to 0.00015%. Experimental results demonstrated that the proposed method has better performance in terms of tank capacity chart calibration accuracy compared with other existing approaches and has a strongly practical significance.

## 1. Introduction

Tanks for storing oil have been in existence for almost one hundred years. Many of the underground storage tanks are horizontal tanks; they are mainly divided into square, cylindrical, and elliptic cylindrical and their roofs can be divided into flat top, conic top, ball lacunarity, and so on [1, 2]. In this paper, cylindrical storage tank with two spherical crowns was primarily discussed, which is more practical. The oil reserve measurement of storage tank is a challenging problem, especially after a period of time, due to foundation deformation and other reasons; oil storage tanks tend to be vertical or horizontal displacement, resulting in inaccurate tank volume tables.

Although some install the automated measurement system, the measurement accuracy is not high. And the price of the imported high-precision liquid level instrument is too

high. Therefore, on the basis of the situation and the development trend of the current domestic and foreign oil tank liquid level measurement technology, developing a kind of liquid level measurement technology which is suitable for China's national conditions is very important [3]. People usually use flow meter and oil level gauge to measure input or output oil, oil height of the tank, and other data and to get the changeable relation of oil height and oil volume, by means of the precalibration tank capacity table (the corresponding relationship between oil height and oil volume of storage tank), so as to determine whether to add oil or not [4–8].

Since the early 1870s, some scholars have already tried to solve this problem [9, 10]. In particular after 2010s, many researchers have made much study as to how to improve the accuracy of calibration of tank capacity chart and proposed many improved methods. Most of them adopted pure integral and infinitesimal method to handle tank issues [11–13] while

some of them used method of the minimum squares [14, 15]. However, for the methods of error correction, few of the papers use methods to correct the result of the calibration of tank capacity chart [16]. And few papers have been presented on handling tank issues by BP neural network [17]. Nevertheless, many issues and problems about calibration have been addressed and resolved and got a good result when using BP neural network [18–20].

The error between theoretical oil reserve and actual oil reserve results from two main reasons. One of the reasons is the irregular geometry of the actual storage tank and another is the volatilization of the oil, the thickness, and the capillary absorption phenomenon of storage tank, which leads to a certain deviation between the theoretical oil reserve and actual ones. As the rule of this kind deviation is relatively fuzzy, in order to further reduce error and improve the accuracy of the calibration, the BP neural network, a method with self-learning ability, is adopted in this paper to revise the calibration value [21].

Artificial neural networks (ANNS) are powerful tools for prediction of nonlinearities. These mathematical models comprise individual processing units called neurons that resemble neural activity [22]. After the first simple neural network was developed by McCulloch and Pitts in 1943 [23], many types of ANN have been proposed. BP neural network simulates the human nervous system structure and the neural network model with multilayer perceptron is the most mature, widely used model among ANN.

Currently the error back-propagation (BP) neural network is the most widely used, which consists of three layers, namely, input, hidden, and output layers. One artificial neuron is simple, but a lot of artificial neurons can compose complicated neural network which can achieve highly nonlinear mapping relation between the input and output through the interaction of artificial neurons and realize the information processing and storage [24]. Due to its highly parallel structure, high-speed self-learning ability, self-adaptable processing ability, arbitrary function mapping ability, powerful pattern classification, and pattern recognition capabilities for modeling complex nonlinear systems [25], BP neural network algorithm studies show promising results in calibration and is used in this study.

The paper is mainly organized into four sections. Section 2 describes the model establishment and solution of small elliptic storage tank with deflection identification and calibration of tank capacity chart. It is divided into three parts as follows: Section 2.1 mathematical models of the relation between oil reserve and oil height of the small nondeflection elliptic storage tank, Section 2.2 model 2 for tank capacity chart calibration problem of small elliptic storage tank at an inclination angle  $\alpha$  of  $4.1^\circ$ , and Section 2.3 correction model of calibration based upon BP neural network. Section 3 describes the establishment and solution of the model with deflection identification and calibration of tank capacity chart of actual storage tank. It also includes three parts: Section 3.1 model 3 of actual storage tank with longitudinal inclination and lateral deflection, Section 3.2 the determination of the deflection parameter, and Section 3.3 model solving of actual

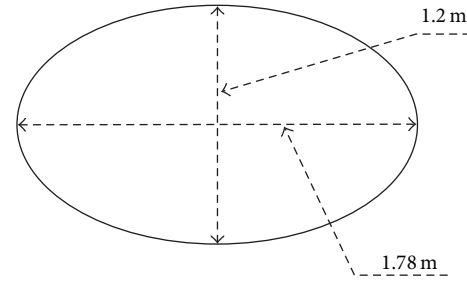


FIGURE 1: Cross-section schematic of small elliptic storage tank.

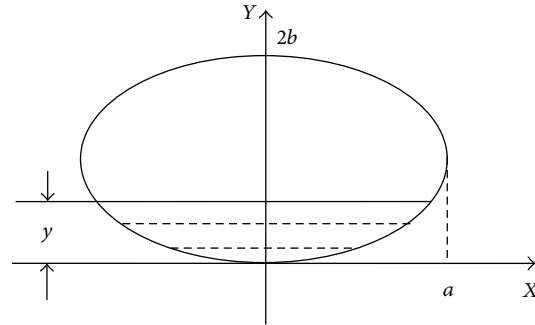


FIGURE 2: Cross-section diagram of the small elliptical tank without deflection.

storage tank and calibration of storage tank chart. Finally, some concluding remarks are drawn in Section 4.

## 2. Model Establishment and Solution of Small Elliptic Storage Tank with Deflection Identification and Calibration of Tank Capacity Chart

**2.1. Mathematical Models of the Relation between Oil Reserve and Oil Height of the Small Nondeflection Elliptic Storage Tank.** According to the cross-section diagram of the small elliptical storage tank as shown in Figure 1, we set up a coordinate system as shown in Figure 2. Make the profile nadir of the storage tank for origin, the high for  $y$  shaft, and the basal level tangent for  $x$  shaft.

The cross-section oil surface area of the small elliptical tank can be calculated according to the application of definite integration:

$$s_1(h_1) = 2 \int_0^{h_1} x dy. \quad (1)$$

According to the elliptic equation  $(x^2/a^2) + (y-b)^2/b^2 = 1$ , substitute  $x = a\sqrt{1 - (y-b)^2/b^2}$  into formula (1) and integral, which can obtain the following:

$$s_1(h_1) = \frac{ab}{2}\pi - a\sqrt{k_h} + \frac{a}{b}h_1\sqrt{k_h} + ab \arcsin\left(-1 + \frac{h_1}{b}\right), \quad (2)$$

where  $k_h$  is  $-h_1(h_1 - 2b)$ .

TABLE 1: Testing data and result of model one.

Site 1					Site 2				
OH/mm	AOR/L	TOR/L	IOR/L	IER	OH/mm	AOR/L	TOR/L	IOR/L	IER
159.02	312.00	322.88	313.46	0.47%	486.89	1512.00	1564.74	1511.08	0.06%
176.14	362.00	374.63	362.90	0.25%	498.95	1562.00	1616.49	1561.20	0.05%
192.59	412.00	426.36	412.41	0.10%	510.97	1612.00	1668.24	1611.33	0.04%
208.50	462.00	478.13	462.03	0.01%	522.95	1662.00	1719.98	1661.46	0.03%
223.93	512.00	529.85	511.67	0.06%	534.90	1712.00	1771.73	1711.59	0.02%
238.97	562.00	581.61	561.39	0.11%	546.82	1762.00	1823.46	1761.71	0.02%
253.66	612.00	633.35	611.16	0.14%	558.72	1812.00	1875.19	1811.84	0.01%
268.04	662.00	685.08	660.95	0.16%	570.61	1862.00	1926.95	1862.00	0.00%
282.16	712.00	736.85	710.81	0.17%	582.48	1912.00	1978.68	1912.12	0.01%
296.03	762.00	788.58	760.67	0.17%	594.35	1962.00	2030.43	1962.27	0.01%
309.69	812.00	840.33	810.58	0.18%	606.22	2012.00	2082.20	2012.44	0.02%
323.15	862.00	892.06	860.49	0.18%	618.09	2062.00	2133.95	2062.59	0.03%
336.44	912.00	943.80	910.44	0.17%	629.96	2112.00	2185.67	2112.71	0.03%
349.57	962.00	995.54	960.41	0.17%	641.85	2162.00	2237.43	2162.86	0.04%
362.56	1012.00	1047.30	1010.41	0.16%	653.75	2212.00	2289.16	2212.99	0.04%
375.42	1062.00	1099.05	1060.43	0.15%	665.67	2262.00	2340.89	2263.10	0.05%
388.16	1112.00	1150.81	1110.47	0.14%	677.63	2312.00	2392.67	2313.27	0.06%
400.79	1162.00	1202.55	1160.51	0.13%	678.54	2315.83	2396.61	2317.09	0.05%
413.32	1212.00	1254.29	1210.56	0.12%	690.53	2365.83	2448.37	2367.23	0.06%
425.76	1262.00	1306.03	1260.62	0.11%	690.82	2367.06	2449.62	2368.45	0.06%
438.12	1312.00	1357.77	1310.69	0.10%	702.85	2417.06	2501.40	2418.60	0.06%
450.40	1362.00	1409.49	1360.75	0.09%	714.91	2467.06	2553.11	2468.69	0.07%
462.62	1412.00	1461.24	1410.85	0.08%	727.03	2517.06	2604.88	2518.82	0.07%
474.78	1462.00	1512.98	1460.95	0.07%	739.19	2567.06	2656.59	2568.89	0.07%

Where OH, AOR, TOR, IOR, and IER are, respectively, oil height, actual oil reserve, theoretical oil reserve, improved oil reserve, and improved error ratio.

We get theoretical oil reserve of small elliptical storage tank according to cylinder volume formula:

$$v_1(h_1) = l \left( \frac{ab}{2} \pi - a \sqrt{s_1} + \frac{a}{b} h_1 \sqrt{s_1} + ab \arcsin \left( -1 + \frac{h_1}{b} \right) \right), \quad (3)$$

where  $s_1 = -h_1(h_1 - 2b)$ ,  $l$  is the length of the storage tank cylinder, and  $h_1$  is oil height of the tank capacity chart.

According to the data provided by the topic A of 2010 National Mathematical Contest in Modeling (Table 1) [26], we substitute the known data into formula (3) and then compare the theory oil reserve with the actual oil reserve. It is obvious that the proportional error is so large, as high as 3.4% ~ 3.5%, that it is necessary to take an error analysis.

**2.1.1. Error Analysis.** With the increase of liquid level, the part of the pipe submerged in the oil is increasing, which makes the theoretical data larger than the actual data. According to the actual situation, the capacity of the pipe in the oil and the probe will take a linear change. Fit the two groups of data, namely, the theoretical and actual oil reserve difference values and the height of the liquid. The results are shown in the Figure 3.

It turns out to be that the above two groups of data meet the linear relationship, and the curve fitting goes to

$R^2 = 0.9967$  from the diagram. It also obtains the capacity of the pipe in the oil and the probe which is named  $\Delta v_1$ :

$$\Delta v_1 = 1.3493h_1 - 12.031. \quad (4)$$

**2.1.2. Model One (Improved Model 1).** The relationship between the capacity and the height of the oil in the tank without deflection can be acquired through formula (3), (4)

$$\bar{v}_1 = l \left( \frac{ab}{2} \pi - a \sqrt{-h_1(h_1 - 2b)} + \frac{a}{b} h_1 \sqrt{-h_1(h_1 - 2b)} + ab \arcsin \left( -1 + \frac{h_1}{b} \right) \right) - \Delta v_1, \quad (5)$$

where  $l$  is the length of storage tank cylinder and  $h_1$  is oil height of the tank capacity chart.

Substituting the oil height into formula (5) can obtain the improved oil reserve and the error ratio is within 0.47%. It means that the precision has been improved by more than 10 times compared with the original model. The specific testing data are shown in Table 1.

**2.2. Model 2 for Tank Capacity Chart Calibration Problem of Small Elliptic Storage Tank at an Inclination Angle  $\alpha$  of  $4.1^\circ$ .** When the inclination angle  $\alpha$  equals  $4.1^\circ$ , take left inclination for example as shown in Figure 4.

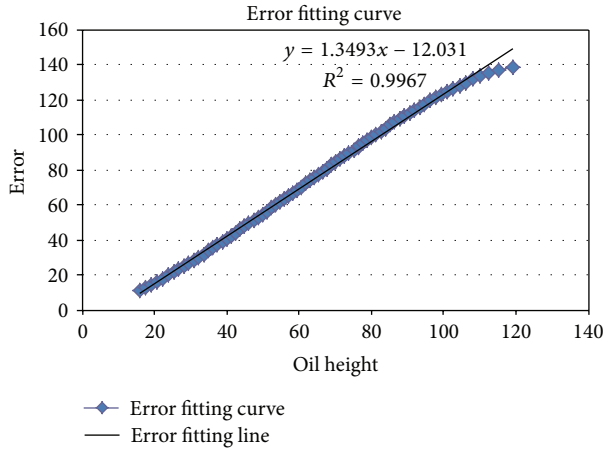


FIGURE 3: The fitting result of the difference between the theoretical and actual oil reserve and the height of the liquid.

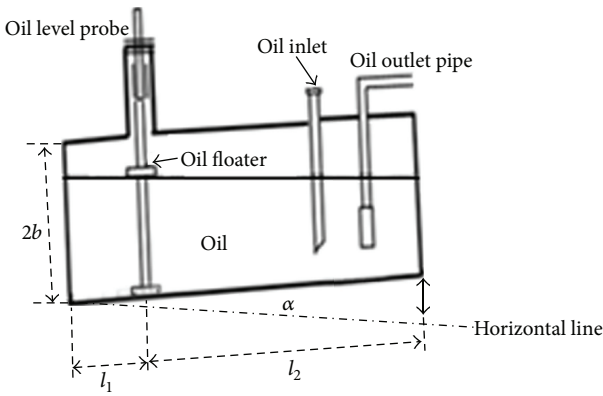


FIGURE 4: Facade schematic of small elliptic storage tank.

Considering the relation of mutative oil reserve and oil height, this problem can be divided into three conditions to discuss as shown in Figure 5.

Make the lower left quarter of the storage tank for origin, the length of storage tank for  $x$  shaft, and the high for  $y$  shaft. Then the coordinate system can be built as shown in Figure 6.

Based upon Figures 5 and 6, (1) there is little oil in storage tank that is; oil level is under line AB. Now,  $0 \leq h_2 < l_2 \tan \alpha$ .

(2) There is moderate oil in storage tank; that is, oil level should be between line CD and AB. Now,  $l_2 \tan \alpha \leq h_2 < 2b - l_1 \tan \alpha$ .

(3) there is much oil in storage tank, that is; oil level should be over line CD. Now,  $2b - l_1 \tan \alpha \leq h_2 \leq 2b$ .

For above three situations, we build model 2 according to the relation of oil height and oil reserve. The detailed solving is below.

First of all, establish equation of the liquid level. Obviously, the slope of this line is  $-\tan \alpha$  and

$$k = \frac{h_2 - b}{l_1} = -\tan \alpha, \quad (6)$$

where  $l_1$  is the length of OC.

The other equation is obtained as follows:

$$b = h_2 + l_1 \tan \alpha. \quad (7)$$

So, the relation between oil height  $y$  and horizontal ordinate  $x$  is defined as follows:

$$y = (l_1 - x) \tan \alpha + h_2. \quad (8)$$

Make differential on both sides of the function at the same time which can obtain the following:

$$dx = -\cot \alpha dy. \quad (9)$$

The theoretical oil reserve of storage tank at longitudinal angle  $\alpha$  of 4.1 degrees can be acquired through the stereoscopic volume formula [27] with known parallel cross-section area,

$$v_2 = \int_0^{245} A(x) dx; \quad (10)$$

that is,

$$v_2 = \int_0^l \left( \frac{ab}{2} \pi - a \sqrt{p_1} + \frac{a}{b} y \sqrt{p_1} + ab \arcsin \left( -1 + \frac{y}{b} \right) \right) dx, \quad (11)$$

where  $p_1 = -y(y - 2b)$ ,  $A(x)$  is the parallel cross-section area when inclined angle with deflection of  $\alpha$  is 4.1 degrees,  $l$  is the length of storage tank cylinder, and the value of  $y$  is  $(l_1 - x) \tan \alpha + h_2$ .

As was discussed above, the relation model between oil height  $h_2$  and oil reserve of storage tank can be obtained, as shown in the following model:

$$V_2 = \begin{cases} 10^{-3} \int_0^{h_2 + l_1 \tan \alpha} \cot \alpha \left[ \frac{ab}{2} \pi - a \sqrt{-y(y - 2b)} + \frac{a}{b} y \sqrt{-y(y - 2b)} + ab \arcsin \left( -1 + \frac{y}{b} \right) \right] dy & 0 \leq h_2 < l_2 \tan \alpha, \\ 10^{-3} \int_{h_2 - l_2 \tan \alpha}^{h_2 + l_1 \tan \alpha} \cot \alpha \left[ \frac{ab}{2} \pi - a \sqrt{-y(y - 2b)} + \frac{a}{b} y \sqrt{-y(y - 2b)} + ab \arcsin \left( -1 + \frac{y}{b} \right) \right] dy & l_2 \tan \alpha \leq h_2 < 2b - l_1 \tan \alpha, \\ v_1(2b) - 10^{-3} \int_0^{h_2 - l_2 \tan \alpha} \cot \alpha \left[ \frac{ab}{2} \pi - a \sqrt{-y(y - 2b)} + \frac{a}{b} y \sqrt{-y(y - 2b)} + ab \arcsin \left( -1 + \frac{y}{b} \right) \right] dy & 2b - l_1 \tan \alpha \leq h_2 \leq 2b. \end{cases} \quad (12)$$

According to the data provided by 2010 National Mathematical Contest in Modeling the title of A [26] (Table 1),

model 2 can be tested by the inclined oil-taking data. What is more, the displayed oil reserve of oil height between

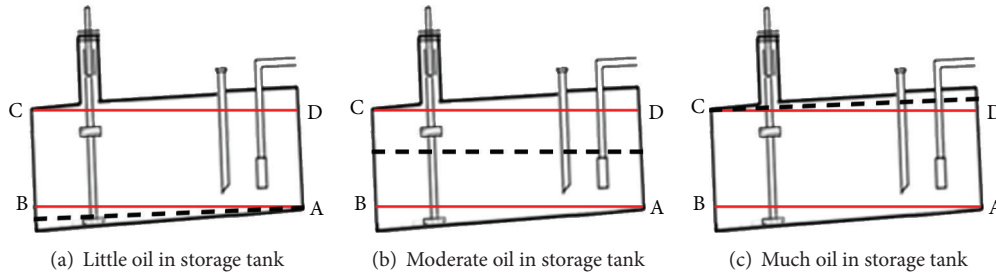


FIGURE 5: Different oil reserve profiles.

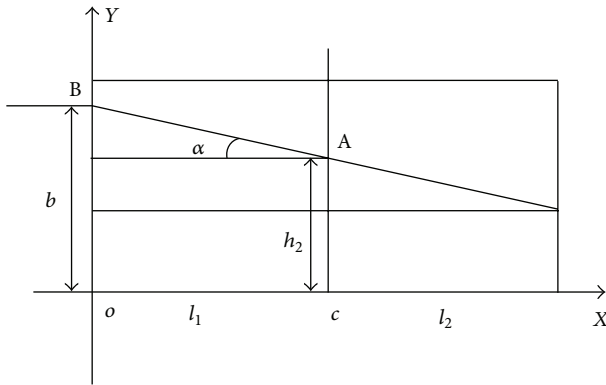


FIGURE 6: Facade schematic of small elliptic storage tank with longitudinal inclination.

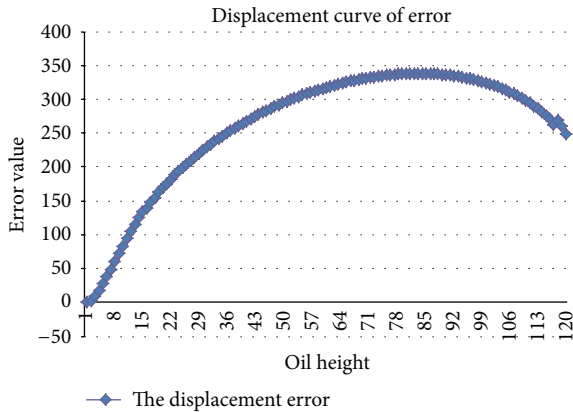


FIGURE 7: Calibrated error curve graph with inclined angle of 4.1 degrees.

0 and 120 cm accordingly and theoretical oil reserve of inclined deflection can be calculated. The chart can be generated as shown in Figure 7 using the calibrated error value and oil height.

The original tank capacity chart can no longer reflect the real oil capacity when the tank inclines. As shown in Figure 7, when the oil height is more than 90 cm, error should be smaller with the increasing oil height.

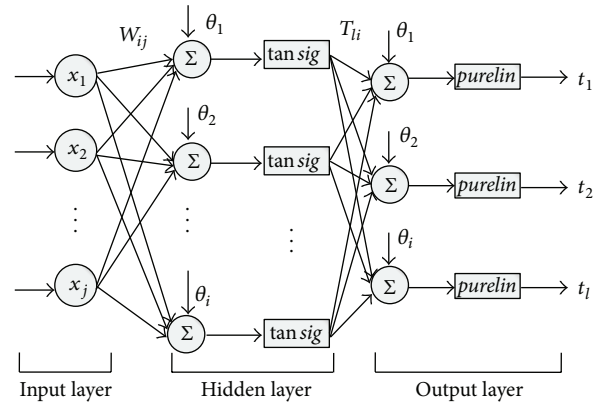


FIGURE 8: The topological structure of the BP network model.

**2.3. Correction Model of Calibration Based upon BP Neural Network.** BP neural network is a nonlinear adaptive dynamic system consisting of many parallel neurons with learning ability, memory ability, calculation ability, and intelligent processing ability [28]. Commonly, a typical BP neural network model is a full-connected neural network including input layer, hidden layer, and output layer [29, 30]. Each layer has multiple neurons, and the nodes between two adjacent layers connect in single direction. It has been proved by Kolmogorov's theorem, a neural network theory theorem, that the fully studied three-layer BP network can approach any function [28].

Some researchers also claim that networks with a single hidden layer can approximate any continuous function to any desired accuracy and are enough for most forecasting problems [31–33].

In this study, a three-layer neural network is applied in calibration of storage tank chart modeling. What's more, the network training is actually an unconstrained nonlinear minimization problem, and the nonlinear model is used in this study. Therefore, it can achieve better effect to process residual correction of this model by BP neural network.

The input node, hidden node, and output node are hypothesized as  $x_j$ ,  $y_i$  and  $O_i$ , respectively. The connection weight between the input node and the hidden node is  $w_{ij}$ , while the connection weight between the hidden node and the output node is  $T_{li}$ . Giving the maximum iterating times and error precision, Figure 8 is the topological structure of the BP neural network model.

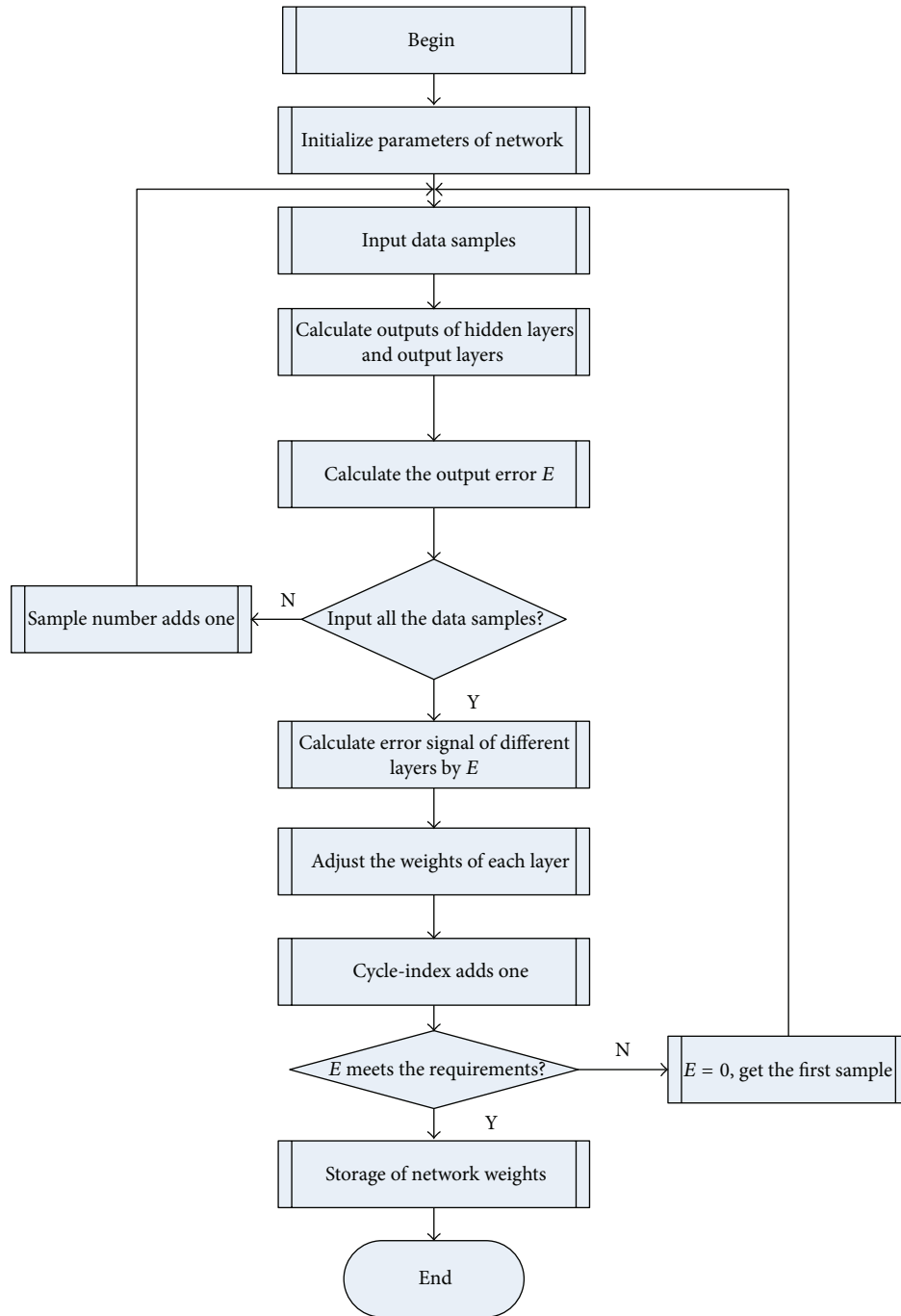


FIGURE 9: Flowchart of back-propagation (BP) neural networks algorithm.

Various steps of the BP training procedure are described in Figure 9.

According to Figures 8 and 9, suppose that the expected output value of the output node is  $t_l$ ; then the BP network model adopts a learning algorithm for training as follows.

Firstly, Initialize by giving random number between  $-1$  and  $1$  to the connection weights  $w_{ij}$ ,  $T_{li}$  and threshold values  $\theta_i$ ,  $\theta_l$ , choosing a mode and giving network to  $x_j$ ,  $t_l$ .

Secondly, the output of the hidden node is  $y_i = f_1(\sum_j w_{ij} x_j - \theta_i)$ .

The output of the output node is  $O_l = f_2(\sum_i T_{li} y_i - \theta_l)$ .

Thirdly, calculate new connection weights and threshold values. The correction value of connection between hidden and output node is defined as is  $T_{li}(k) = T_{li}(k) + \eta \delta_l y_i$ .

The correction of the threshold values is  $\theta_l(k+1) = \theta_l(k) + \eta \delta_l$ .



TABLE 2: The oil reserve of oil-out and analysis of percentage error with BP neural network statistical table.

Site 1					Site 2				
OH/mm	AOR/L	TOR/L	FENN/L	PEC	OH/mm	AOR/L	TOR/L	FENN/L	PEC
1020.65	3464.74	3405.10	59.61	0.00%	715.32	2214.74	2222.10	10.08	0.12%
1007.73	3414.74	3361.80	53.15	0.01%	705.43	2164.74	2180.70	11.76	0.19%
994.32	3364.74	3315.90	48.64	0.01%	693.52	2114.74	2130.60	13.97	0.09%
980.96	3314.74	3269.40	45.20	0.00%	682.5	2064.74	2084.30	16.09	0.17%
967.10	3264.74	3220.10	42.05	0.08%	671.02	2014.74	2036.00	18.31	0.15%
956.01	3214.74	3180.10	39.59	0.15%	658.68	1964.74	1984.00	20.60	0.07%
941.54	3164.74	3127.20	36.34	0.04%	647.74	1914.74	1938.00	22.50	0.04%
929.69	3114.74	3083.20	33.60	0.07%	635.76	1864.74	1887.60	24.34	0.08%
916.44	3064.74	3033.40	30.45	0.03%	624.61	1814.74	1840.70	25.80	0.01%
904.14	3014.74	2986.60	27.46	0.02%	612.53	1764.74	1790.00	27.08	0.10%
891.9	2964.74	2939.50	24.47	0.03%	600.69	1714.74	1740.30	27.99	0.14%
879.23	2914.74	2890.30	21.39	0.10%	589.40	1664.74	1693.10	28.55	0.01%
868.99	2864.74	2850.20	18.96	0.15%	577.00	1614.74	1641.30	28.82	0.14%
855.13	2814.74	2795.50	15.81	0.12%	564.58	1564.74	1589.60	28.76	0.25%
844.02	2764.74	2751.20	13.45	0.00%	554.33	1514.74	1547.10	28.48	0.26%
831.64	2714.74	2701.60	11.07	0.08%	540.76	1464.74	1491.00	27.83	0.11%
820.47	2664.74	2656.50	9.20	0.04%	528.65	1414.74	1441.20	27.02	0.04%
808.16	2614.74	2606.50	7.50	0.03%	517.19	1364.74	1394.20	26.09	0.25%
796.00	2564.74	2556.90	6.24	0.06%	504.87	1314.74	1344.10	24.97	0.33%
785.04	2514.74	2511.90	5.51	0.11%	490.78	1264.74	1287.00	23.55	0.10%
773.07	2464.74	2462.60	5.18	0.12%	478.06	1214.74	1235.90	22.20	0.09%
762.09	2414.74	2417.20	5.30	0.12%	465.97	1164.74	1187.70	20.88	0.18%
750.81	2364.74	2370.30	5.86	0.01%	452.40	1114.74	1134.00	19.39	0.01%
739.42	2314.74	2322.90	6.84	0.06%	439.98	1064.74	1085.30	18.04	0.24%
727.09	2264.74	2271.40	8.32	0.07%	425.83	1014.74	1030.30	16.55	0.10%

Where OH, AOR, TOR, FENN, and PEC are, respectively, oil height, actual oil reserve, theoretical oil reserve, fitting error of neural network, and proportional error with correction.

The correction value of connection between input and hidden node is defined as:  $w_{ij}(k+1) = w_{ij}(k) + \eta' \theta_i' x_j$ .

The correction of the threshold values:  $\theta_i(k+1) = \theta_i(k) + \eta' \delta_i'$ .

Where  $\eta$  and  $\eta'$  reflect learning efficiency,  $\delta_i = (t_i - O_i) \cdot O_i \cdot (1 - O_i)$ ,  $\delta_i' = y_i(1 - y_i) \sum_l \delta_l T_{li}$ .

Lastly, select the next input mode and return to step (2). Keep training until the error precision of the network settings meets the requirements. Then finish the training. Thus the BP neural network model is established.

Calibration correction: take the oil-out level height data of the small longitudinal tilting elliptical tank as input data while take the  $D$ -value between the theoretical oil reserve and the actual measurement of oil as output data. Construct a BP neural network model with single input, single output and hidden layer with three-node by matlab 2010.

Then train the model with inspecting data of the oil-in level height data and the practical measurement oil reserve. The training results are shown in Figures 10 and 11.

As mentioned above, it turns out to be that the accuracy of the results of the correction BP neural network model is very high. Part of the results can be seen in Table 2.

As shown in Table 2, the proportional error of theoretical value and experimental measurement value with BP neural

network correction ranges from 0.00% to 0.38%. Error reduces a lot more than before. Using the correction model, the calibration of tank capacity chart value can be calculated with the internal of oil height for 1 cm after the deflection of storage tank as shown in Table 3.

### 3. Establishment and Solution of the Model with Deflection Identification and Calibration of Tank Capacity Chart of Actual Storage Tank

#### 3.1. Model 3 of Actual Storage Tank with Longitudinal Inclination and Lateral Deflection

3.1.1. Model Establishment of Actual Storage Tank with Longitudinal Inclination. The graph in Figure 12 clearly shows that

$$v_3 = v_C + v_L + v_R, \quad (13)$$

$$y_1 = h + l_1 \tan \alpha,$$

$$y_2 = h - \left( \frac{l}{2} + l_2 \right) \tan \alpha. \quad (14)$$

TABLE 3: Calibration results of tank capacity chart of the small deflected elliptic storage tank after the correction of BP neural network.

Site 1		Site 2		Site 3		Site 4	
OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L
0	1.03	31	615.49	62	1847.65	93	3118.02
1	3.42	32	650.15	63	1888.48	94	3157.49
2	6.55	33	685.36	64	1929.13	95	3196.50
3	10.57	34	721.10	65	1969.62	96	3235.08
4	15.59	35	757.36	66	2009.97	97	3273.22
5	21.67	36	794.10	67	2050.20	98	3310.96
6	28.92	37	831.32	68	2090.35	99	3348.43
7	37.39	38	868.99	69	2130.48	100	3385.85
8	47.17	39	907.08	70	2170.61	101	3423.58
9	58.30	40	945.60	71	2210.80	102	3462.14
10	70.85	41	984.50	72	2251.08	103	3502.02
11	84.86	42	1023.78	73	2291.50	104	3543.18
12	100.40	43	1063.41	74	2332.10	105	3584.59
13	117.51	44	1103.38	75	2372.89	106	3624.60
14	136.25	45	1143.66	76	2413.88	107	3661.99
15	159.60	46	1184.22	77	2455.08	108	3696.57
16	180.85	47	1225.04	78	2496.49	109	3728.74
17	203.42	48	1266.10	79	2538.10	110	3759.03
18	227.17	49	1307.36	80	2579.86	111	3787.81
19	252.01	50	1348.80	81	2621.76	112	3815.32
20	277.86	51	1390.38	82	2663.74	113	3841.66
21	304.66	52	1432.06	83	2705.77	114	3866.82
22	332.36	53	1473.84	84	2747.80	115	3890.82
23	360.90	54	1515.64	85	2789.78	116	3913.55
24	390.25	55	1557.46	86	2831.66	117	3934.90
25	420.38	56	1599.25	87	2873.39	118	3943.58
26	451.23	57	1640.96	88	2914.91	119	3962.31
27	482.79	58	1682.60	89	2956.19	120	3979.40
28	515.02	59	1724.10	90	2997.17		
29	547.90	60	1765.45	91	3037.83		
30	581.40	61	1806.64	92	3078.12		

Where OH is oil height and TOR is theoretical oil reserve.

In the case of no deflection, the formula in references [34] can be cited; namely,

$$v_C = \frac{a}{b} l \left[ q_h + b^2 \arcsin \left( \frac{h}{b} - 1 \right) + \frac{1}{2} \pi b^2 \right], \quad (15)$$

where  $q_h = (h - b) \sqrt{h(2b - h)}$ ,

$$v_S = \frac{\pi a c}{2b^2} \left[ b^2 (h - b) - \frac{1}{3} (h - b)^3 + \frac{2}{3} b^3 \right], \quad (16)$$

where  $c$  reflects sagittal of the storage tank.

Calculate the oil capacity in the tank with longitudinal angel of  $\alpha$  by integration, as shown in Figure 12. As both sides of the storage tank are irregular solid, it is difficult to calculate accurately. But, the angel  $\alpha$  is very tiny according to the fact that both longitudinal angle and lateral deflection angle are small angles, so cut-complement method can be adopted to make an approximate disposal. The extra volume

of left approximately equals the insufficient volume of the right; that is:

$$\Delta v_L - \Delta v_R \longrightarrow 0. \quad (17)$$

Hence, the relation between oil reserve of storage tank and oil height can be defined as follows:

$$v_3 = v_C + v_{S|h=y_1} + v_{S|h=y_2}. \quad (18)$$

The calculations of  $v_C$  are as shown in Figure 13.

The boundary of the cylinder's longisecton is rectangular. As shown in Figure 13, firstly, draw a line perpendicular to the base through the base's midpoint and the line intersects with the metal line. Secondly, draw a parallel line to the base through the above point of intersection. Then the parallel line, metal line, and two boundaries form two triangles named  $v'$  and  $v''$ , both of which are right-angled triangles with equal vertical angles and horizontal right-angle side.



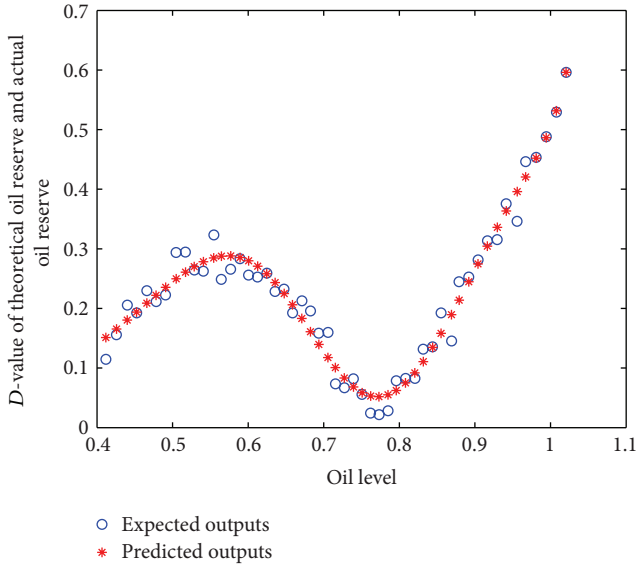


FIGURE 10: The outputs of the network prediction.

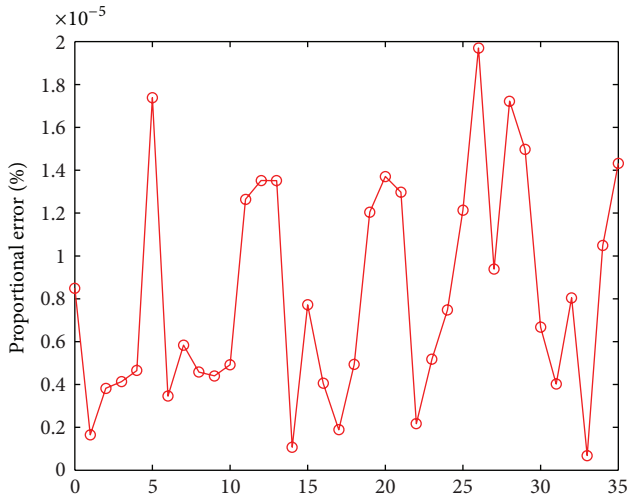


FIGURE 11: The relative error percentage of the BP neural network prediction.

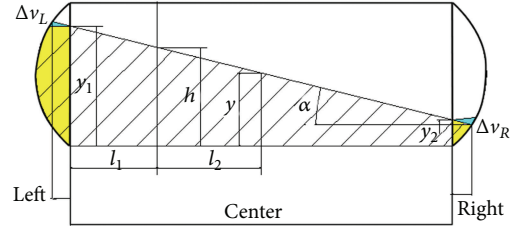
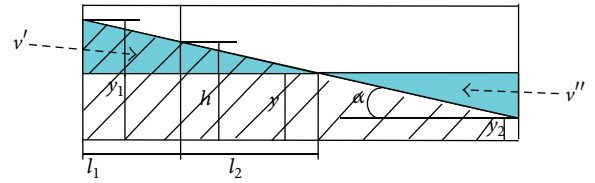
So the two triangles are congruent. Apparently, their areas are also equal. According to the ZuYuan principle, their corresponding volumes in the cylinder are also equal; that is,  $v' = v''$ . Therefore, it can be acquired through the cut-complement method as follows:

$$v_C = \frac{a}{b} l \left[ q_y + b^2 \arcsin \left( \frac{y_C}{b} - 1 \right) + \frac{1}{2} \pi b^2 \right], \quad (19)$$

where  $q_y$  is  $(y_C - b) \sqrt{y_C(2b - y_C)}$ .

Suppose that the metal line's slope equals  $k$ ; then

$$k = \frac{y_C - h_3}{l_2} = -\tan \alpha \Rightarrow y_C = h_3 - l_2 \tan \alpha. \quad (20)$$


 FIGURE 12: Facade schematic of actual elliptic storage tank with angle  $\alpha$  of longitudinal inclination.

 FIGURE 13: Facade schematic of cylinder with inclined angle of  $\alpha$ .

Substituting formula (20) into formula (19),

$$v_C = \frac{a}{b} l \left[ (h_3 - l_2 \tan \alpha - b) \sqrt{q_a} + b^2 \arcsin \left( \frac{h_3 - l_2 \tan \alpha}{b} - 1 \right) + \frac{1}{2} \pi b^2 \right], \quad (21)$$

where  $q_a$  is  $(h_3 - l_2 \tan \alpha)(2b - (h_3 - l_2 \tan \alpha))$ .

The spherical crown's metal line of both ends parallels the undersurface of the cylindrical, after the approximate disposal, which is equivalent to the case of no deflection. From formula (16), the calculation of  $v_s$  can be defined as follows:

$$v_{S|h=y_1} = \frac{\pi a c}{2b^2} \left[ b^2 (y_1 - b) - \frac{1}{3} (y_1 - b)^3 + \frac{2}{3} b^3 \right], \quad (22)$$

$$v_{S|h=y_2} = \frac{\pi a c}{2b^2} \left[ b^2 (y_2 - b) - \frac{1}{3} (y_2 - b)^3 + \frac{2}{3} b^3 \right].$$

Substitute formula (14) into formula (22); thus

$$v_L = \frac{\pi a c}{2b^2} \left[ b^2 t_b - \frac{1}{3} t_b^3 + \frac{2}{3} b^3 \right], \quad (23)$$

where  $t_b = h_3 + l_1 \tan \alpha - b$ .

$$v_R = \frac{\pi a c}{2b^2} \left[ b^2 t_l - \frac{1}{3} t_l^3 + \frac{2}{3} b^3 \right], \quad (24)$$

TABLE 4: The results of calibration of storage tank with deflection of actual storage tank without correction by BP neural network.

Site 1		Site 2		Site 3		Site 4	
OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L
0	3.58	80	11827.88	160	33484.74	240	54967.27
10	140.31	90	14271.04	170	36340.02	250	57230.13
20	885.88	100	16823.93	180	39177.91	260	59331.74
30	2084.66	110	19468.14	190	41983.45	270	61244.02
40	3601.50	120	22186.49	200	44741.33	280	62931.80
50	5372.10	130	24962.73	210	47435.73	290	64346.91
60	7353.89	140	27781.26	220	50050.13	300	65410.85
70	9514.61	150	30626.89	230	52566.96		

Where OH is oil height and TOR is theoretical oil reserve.

where  $t_1$  is  $h_3 - (l/2 + l_2) \tan \alpha - b$ .

$$\begin{aligned}
v_3 &= v_C + v_L + v_R \\
&= \frac{a}{b} l \left[ (h_3 - l_2 \tan \alpha - b) \right. \\
&\quad \times \sqrt{(h_3 - l_2 \tan \alpha) (2b - (h_3 - l_2 \tan \alpha))} \\
&\quad + b^2 \arcsin \left( \frac{h_3 - l_2 \tan \alpha}{b} - 1 \right) + \frac{1}{2} \pi b^2 \Big] \\
&\quad + \frac{\pi a c}{2b^2} \left[ b^2 t_2 - \frac{1}{3} t_2^3 + \frac{2}{3} b^3 \right] \\
&\quad + \frac{\pi a c}{2b^2} \left[ b^2 t_1 - \frac{1}{3} t_1^3 + \frac{2}{3} b^3 \right], \tag{25}
\end{aligned}$$

where  $t_1 = h_3 - (l/2 + l_2) \tan \alpha - b$  and  $t_2 = h_3 + l_1 \tan \alpha - b$ .

**3.1.2. Establishment of the Model with Deflection Identification and Calibration of Tank Capacity Chart of Actual Storage Tank.** As shown in Figure 14

$$\begin{aligned}
h_4 &= r + t, \\
t &= \frac{s}{\cos \beta}, \\
s &= h_3 - r. \tag{26}
\end{aligned}$$

Hence, the  $h$  is defined by

$$h = (h_4 - r) \cos \beta + r. \tag{27}$$

Substitute formula (27) into formula (25). Then, the theoretical model about  $\alpha, \beta, h$  of oil reserve with longitudinal inclination and lateral deflection of storage tank is obtained below:

$$\begin{aligned}
v_4 &= \frac{a}{b} l \left[ (h - l_2 \tan \alpha - b) \right. \\
&\quad \times \sqrt{(h - l_2 \tan \alpha) (2b - (h - l_2 \tan \alpha))} \\
&\quad + b^2 \arcsin \left( \frac{h - l_2 \tan \alpha}{b} - 1 \right) + \frac{1}{2} \pi b^2 \Big]
\end{aligned}$$

$$\begin{aligned}
&+ \frac{\pi a c}{2b^2} \left[ b^2 (h + l_1 \tan \alpha - b) \right. \\
&\quad \left. - \frac{1}{3} (h + l_1 \tan \alpha - b)^3 + \frac{2}{3} b^3 \right] \\
&+ \frac{\pi a c}{2b^2} \left[ b^2 r_a - \frac{1}{3} r_a^3 + \frac{2}{3} b^3 \right], \tag{28}
\end{aligned}$$

where  $h = (h_4 - r) \cos \beta + r$ ,  $l$  is the length of storage tank cylinder,  $r_a$  is  $h - (l/2 + l_2) \tan \alpha - b$ , and  $h$  is the oil height of the calibration of storage tank chart.

**3.2. The Determination of the Deflection Parameter.** As was discussed in Section 3.1.2 and Figure 7 of model 2, the error nearby  $h_4 = 150$  is micro, even without error. For this reason, equations can be established by selecting three contiguous groups of data near  $h_4 = 150$  to ascertain the deflection parameters,  $\alpha$  and  $\beta$ . The equations can be defined by

$$\begin{aligned}
v_4(151.073, \alpha, \beta) - v_4(150.765, \alpha, \beta) &= 86.76, \\
v_4(150.765, \alpha, \beta) - v_4(150.106, \alpha, \beta) &= 187.61. \tag{29}
\end{aligned}$$

Substitute the formula (28) into formula (29). Then, solve them by the software of Mathematica [35] and Matlab [36] using quasi-Newton iterative algorithm and the result can be got as follows:

$$\alpha = 2.3592^0, \quad \beta = 3.80127^0. \tag{30}$$

The angles are very tiny which is realistic.

**3.3. Model Solving of Actual Storage Tank and Calibration of Storage Tank Chart.** According to the data provided by 2010 National Mathematical Contest in Modeling the title of A [26] (Table 2), we substitute  $\alpha, \beta$  and collected oil height of actual storage tank into model 3. The calibration of tank capacity chart value can be calculated with the internal of oil height for 10 cm after the deflection of storage tank, as shown in Table 4.

Error can be controlled under 2%, when testing model 3 by actual collected data of storage tank. But it is still large for the volume of this tank. Similarly, in order to further reduce error and improve the accuracy of the calibration, model 3 also uses the BP neural network which is a method with self-learning ability to revise the calibration value.

TABLE 5: The statistics of oil reserve of actual storage tank with oil-out and the percentage of error.

Site 1					
OH/cm	AOR/L	TOR/L	ATDV	FENN/L	PEC
2014.29	46552.67	45130.62	1422.05	1422.10	0.015%
2003.74	46275.12	44843.34	1431.78	1431.80	0.011%
1995.29	46052.18	44612.73	1439.45	1439.50	0.006%
1989.53	45899.88	44455.27	1444.61	1444.60	0.004%
1985.62	45796.35	44348.27	1448.08	1448.10	0.003%
1979.33	45629.56	44175.95	1453.61	1453.60	0.002%
1972.51	45448.37	43988.83	1459.54	1459.50	0.001%
1969.31	45363.23	43900.94	1462.29	1462.30	0.000%
1961.41	45152.73	43683.70	1469.03	1469.00	0.004%
1957.15	45039.03	43566.41	1472.62	1472.60	0.005%
1951.30	44882.67	43405.18	1477.49	1477.50	0.003%
1943.47	44673.02	43189.07	1483.95	1483.90	0.006%
1938.96	44552.06	43064.44	1487.62	1487.60	0.006%
1934.05	44420.22	42928.64	1491.58	1491.60	0.006%
1925.88	44200.47	42702.39	1498.08	1498.10	0.005%
1921.23	44075.20	42573.46	1501.74	1501.70	0.005%
1910.98	43798.57	42288.87	1509.70	1509.70	0.009%
1899.16	43478.71	41960.06	1518.65	1518.60	0.006%
1889.86	43226.43	41700.88	1525.55	1525.50	0.006%
1884.42	43078.62	41549.09	1529.53	1529.50	0.006%
1876.58	42865.28	41330.10	1535.18	1535.20	0.005%
1873.50	42781.36	41243.99	1537.37	1537.40	0.004%
1862.44	42479.59	40934.45	1545.14	1545.10	0.005%
1851.64	42184.24	40631.69	1552.55	1552.50	0.005%
1841.46	41905.25	40345.88	1559.37	1559.40	0.003%
1836.85	41778.73	40216.32	1562.41	1562.40	0.003%
1828.91	41560.56	39992.99	1567.57	1567.60	0.002%
1826.68	41499.23	39930.22	1569.01	1569.00	0.003%
1820.43	41327.19	39754.21	1572.98	1573.00	0.001%
1815.31	41186.12	39609.91	1576.21	1576.20	0.003%
1811.69	41086.29	39507.84	1578.45	1578.40	0.000%
1807.90	40981.71	39400.92	1580.79	1580.80	0.001%
1801.67	40809.66	39225.07	1584.59	1584.60	0.001%
1798.73	40728.40	39142.04	1586.36	1586.40	0.000%
1790.75	40507.64	38916.53	1591.11	1591.10	0.000%
1784.04	40321.79	38726.76	1595.03	1595.00	0.001
1775.08	40073.31	38473.15	1600.16	1600.20	0.003

Where OH, AOR, TOR, ATDV, FENN, and PEC are, respectively, oil height, actual oil reserve, theoretical oil reserve,  $D$ -value of theoretical and actual oil reserve, fitting error of neural network, and proportional error with correction.

By taking the oil-out level height data of the actual storage tank as input data and the  $D$ -value between the theoretical oil reserve and the actual measurement of oil as output data, the whole network reflects the function mapping relation between input node and output node.

Then, train the network, so it can have a certain ability of association and prediction for this kind of problem.

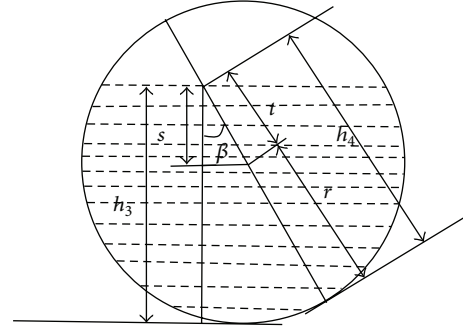


FIGURE 14: Cross-section schematic of lateral deflection.

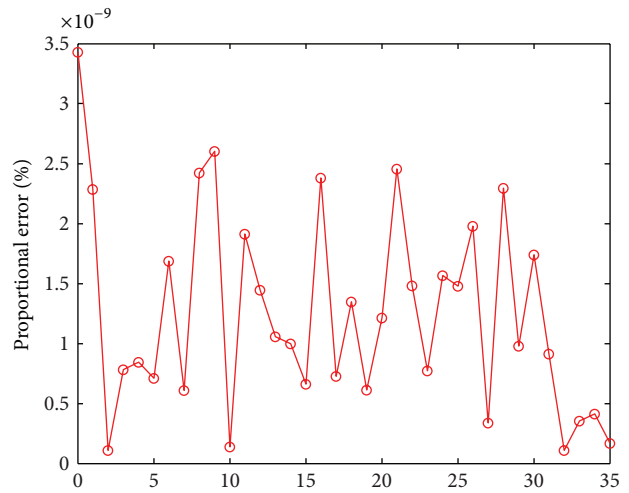


FIGURE 15: The relative error percentage of the BP neural network prediction.

Calibration correction: randomly take 150 groups of the oil-out level height data of the actual storage tank as input data; corresponding, take same groups of the  $D$ -value between the theoretical oil reserve and the actual measurement of oil as output data, while 50 groups of data are taken as testing data. A BP neural network model can be constructed and trained with single input and single output and three-node hidden layer by matlab 2010. The training result is as shown in Figure 15.

As discussed above, it turns out to be that the accuracy of the results of the correction BP neural network model is very high. Part of the results can be seen in Table 5.

As shown in Table 5, the proportional error of theoretical value and experimental measurement value with BP neural network correction ranges from 0.00000% to 0.00015%. Error is micro. Using the correction model, the calibration of tank capacity chart value can be calculated with the internal of oil height for 10 cm after the deflection of storage tank as shown in Table 6.

#### 4. Conclusions

In this paper, using geometrical relationship of the storage tank, the integral models are established from simple to

TABLE 6: The results of calibration of storage tank with deflection of actual storage tank with correction by BP neural network.

Site 1		Site 2		Site 3		Site 4	
OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L	OH/cm	TOR/L
0	264.71	80	13394.67	160	35163.23	240	55909.19
10	705.23	90	15892.60	170	37978.85	250	58008.41
20	1703.45	100	18487.94	180	40764.00	260	59932.14
30	3105.15	110	21163.01	190	43501.00	270	61654.34
40	4783.19	120	23900.97	200	46176.56	280	63142.53
50	6682.45	130	26685.62	210	48772.32	290	64351.76
60	8767.99	140	29501.21	220	51271.70	300	65207.10
70	11012.99	150	32332.21	230	53656.90		

Where OH is oil height and TOR is theoretical oil reserve.

complicated, which makes the models simple and easy to understand. Taking into account many possible oil level conditions and giving the common theoretical relation between the oil reserve and oil height with the known deflection parameters, it has strong universality and is easy to popularize. Cut-complement algorithm is designed to construct this model, according to the special inclined angle. And the nonlinear equations are effectively solved by quasi-Newton iterative method. A novel method is applied to calibrate the storage tank chart which combines the advantages of the polynomial fitting method and BP neural network. Models are tested by the known data and the improved models are got by polynomial fitting method. Based upon fuzzification of system measurement error, BP neural network is proposed to correct results. Quasi-Newton iterative algorithm is used to calculate deflection parameters  $\alpha = 2.3592^\circ$ ,  $\beta = 3.80127^\circ$  by Mathematica, Matlab software. However, when oil in the storage tank is approximately full or there is very little oil in it, it is unable to get the accurate calibration method, so more research efforts should be devoted to validating these issues. Developing better models of solving these problems is the next step we will undertake.

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