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Research Article

Dynamics in a Lotka-Volterra Predator-Prey Model with Time-Varying Delays

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A Lotka-Volterra predator-prey model with time-varying delays is investigated. By using the differential inequality theory, some sufficient conditions which ensure the permanence and global asymptotic stability of the system are established. The paper ends with some interesting numerical simulations that illustrate our analytical predictions.

1. Introduction

In 1992, Berryman [1] pointed out that the dynamical relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Dynamical behavior of predator-prey models has been studied by a lot of papers. It is well known that the investigation on predator-prey models not only focuses on the discussion of stability, periodic oscillatory, bifurcation, and chaos [2–26], but also involves many other dynamical behaviors such as permanence. In many applications, the nature of permanence is of great interest. Recently, Chen [27] investigated the permanence of a discrete n-species food-chain system with delays. Fan and Li [28] gave a theoretical study on permanence of a delayed ratio-dependent predator-prey model with Holling type functional response. Chen [29] focused on the permanence and global attractivity of Lotka-Volterra competition system with feedback control. Zhao and Jiang [30] analyzed the permanence and extinction for nonautonomous Lotka-Volterra system. Teng et al. [31] addressed the permanence criteria for delayed discrete nonautonomous-species Kolmogorov systems. For more research on the permanence behavior of predator-prey models, one can see [32–40].

In 2010, Lv et al. [41] investigated the existence and global attractivity of periodic solution to the following Lotka-Volterra predator-prey system:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)\left[r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) \right.\\
&\qquad \left. - a_{12}(t)x_2(t-\tau_{12}(t)) - a_{13}(t)x_3(t-\tau_{13}(t))\right],\\
\frac{dx_2(t)}{dt} &= x_2(t)\left[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}(t)) \right.\\
&\qquad \left. - a_{22}(t)x_2(t-\tau_{22}(t)) - a_{23}(t)x_3(t-\tau_{23}(t))\right],\\
\frac{dx_3(t)}{dt} &= x_3(t)\left[-r_3(t) + a_{31}(t)x_1(t-\tau_{31}(t)) \right.\\
&\qquad \left. - a_{32}(t)x_2(t-\tau_{32}(t)) - a_{33}(t)x_3(t-\tau_{33}(t))\right],
\end{aligned} \tag{1}$$

where $x_1(t)$ denotes the density of prey species at time t, $x_2(t)$ and $x_3(t)$ stand for the density of predator species at time

t, $r_i, a_{ij} \in C(\mathbb{R}, [0, \infty))$ and $\tau_{ij} \in C(\mathbb{R}, \mathbb{R})$. Using Krasnoselskii's fixed point theorem and constructing Lyapunov function, Lv et al. obtained a set of easily verifiable sufficient conditions which guarantee the permanence and global attractivity of system (1).

For the viewpoint of biology, we shall consider (1) together with the initial conditions $x_i(0) \geq 0$ $(i = 1, 2, 3)$. The principle object of this paper is to explore the dynamics of system (1), applying the differential inequality theory to study the permanence of system (1). Using the method of Lyapunov function, we investigated the globally asymptotically stability of system (1).

The remainder of the paper is organized as follows: in Section 2, basic definitions and Lemmas are given, and some sufficient conditions for the permanence of the Lotka-Volterra predator-prey model in consideration are established. A series of sufficient conditions for the global stability of the Lotka-Volterra predator-prey model in consideration are included in Section 3. In Section 4, we give an example which shows the feasibility of the main results. Conclusions are presented in Section 5.

2. Permanence

For convenience in the following discussing, we always use the notations:

$$f^l = \inf_{t\in\mathbb{R}} f(t), \qquad f^u = \sup_{t\in\mathbb{R}} f(t), \tag{2}$$

where $f(t)$ is a continuous function. In order to obtain the main result of this paper, we shall first state the definition of permanence and several lemmas which will be useful in the proving the main result.

Definition 1 (see [41]). We say that system (1) is permanence if there are positive constants M and m such that for each positive solution $(x_1(t), x_2(t), x_3(t))$ of system (1) satisfies

$$m \leq \lim_{t\to+\infty} \inf x_i(t) \leq \lim_{t\to+\infty} \sup x_i(t) \leq M \quad (i = 1, 2, 3). \tag{3}$$

Lemma 2 (see [42]). *If $a > 0$, $b > 0$, and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\lim_{t\to+\infty} \inf x(t) \geq \frac{b}{a}. \tag{4}$$

If $a > 0$, $b > 0$, and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\lim_{t\to+\infty} \sup x(t) \leq \frac{b}{a}. \tag{5}$$

Now we state our permanence result for system (1).

Theorem 3. *Let M_1, M_2, M_3, and m_1 be defined by* (11), (18), (24), *and* (30), *respectively. Suppose that the following conditions:*

(H1) $a^u_{22} M_1 > r^l_2$, $a^u_{31} M_1 > r^l_3$,

(H2) $r^l_1 > a^u_{12} M_2 + a^u_{13} M_3$, $r^u_2 > a^u_{23} M_3$, $a^l_{31} m_1 > r^u_3 + a^u_{32} M_2$

hold, and then system (1) *is permanent; that is, there exist positive constants m_i, M_i $(i = 1, 2, 3)$ which are independent of the solution of system* (1)*, such that, for any positive solution $(x_1(t), x_2(t), x_3(t))$ of system* (1) *with the initial condition $x_i(0) \geq 0$ $(i = 1, 2, 3)$, one has*

$$m_i \leq \lim_{t\to+\infty} \inf x_i(t) \leq \lim_{t\to+\infty} \sup x_i(t) \leq M_i. \tag{6}$$

Proof. It is easy to see that system (1) with the initial value condition $(x_1(0), x_2(0), x_3(0))$ has positive solution $(x_1(t), x_2(t), x_3(t))$ passing through $(x_1(0), x_2(0), x_3(0))$. Let $(x_1(t), x_2(t), x_3(t))$ be any positive solution of system (1) with the initial condition $(x_1(0), x_2(0), x_3(0))$. It follows from the first equation of system (1) that

$$\frac{dx_1(t)}{dt} \leq r_1(t) x_1(t) \leq r^u_1 x_1(t). \tag{7}$$

Integrating both sides of (7) from $t - \tau_{11}(t)$ to t, we get

$$\ln\left[\frac{x_1(t)}{x_1(t - \tau_{11}(t))}\right] \leq \int_{t-\tau_{11}(t)}^{t} r^u_1 ds \leq r^u_1 \tau^u_{11}, \tag{8}$$

which leads to

$$x_1(t - \tau_{11}(t)) \geq x_1(t) \exp\{-r^u_1 \tau^u_{11}\}. \tag{9}$$

Substituting (9) into the first equation of system (1), it follows that

$$\frac{dx_1(t)}{dt} \leq x_1(t) \left[r^u_1 - a^l_{11} \exp\{-r^u_1 \tau^u_{11}\} x_1(t)\right]. \tag{10}$$

It follows from (10) and Lemma 2 that

$$\lim_{t\to+\infty} \sup x_1(t) \leq \frac{r^u_1}{a^l_{11}} \exp\{r^u_1 \tau^u_{11}\} := M_1. \tag{11}$$

For any positive constant $\varepsilon > 0$, it follows from (11) that there exists a $T_1 > 0$ such that, for all $t > T_1$,

$$x_1(t) \leq M_1 + \varepsilon. \tag{12}$$

For $t \geq T_1 + \tau^u_{21}$, from (12) and the second equation of system (1), we have

$$\begin{aligned} \frac{dx_2(t)}{dt} &\leq x_1(t) \left[-r_2(t) + a_{21}(t) x_1(t - \tau_{21}(t))\right] \\ &\leq x_1(t) \left[-r^l_2 + a^u_{21}(M_1 + \varepsilon)\right]. \end{aligned} \tag{13}$$

Integrating both sides of (13) from $t - \tau_{22}(t)$ to t, we get

$$\begin{aligned} \ln\left[\frac{x_2(t)}{x_2(t - \tau_{22}(t))}\right] &\leq \int_{t-\tau_{22}(t)}^{t} \left[-r^l_2 + a^u_{21}(M_1 + \varepsilon)\right] ds \\ &\leq \left[-r^l_2 + a^u_{21}(M_1 + \varepsilon)\right] \tau^u_{22}, \end{aligned} \tag{14}$$

which leads to

$$x_2(t-\tau_{22}(t)) \geq x_2(t)\exp\left\{\left[r_2^l - a_{21}^u(M_1+\varepsilon)\right]\tau_{22}^u\right\}. \quad (15)$$

Substituting (15) into the second equation of system (1), it follows that

$$\frac{dx_2(t)}{dt} \leq x_2(t)\left\{-r_2^l + a_{22}^u(M_1+\varepsilon) - a_{22}^l \exp\left\{\left[r_2^l - a_{21}^u(M_1+\varepsilon)\right]\tau_{22}^u\right\} x_2(t)\right\}. \quad (16)$$

Thus, as a direct corollary of Lemma 2, according to (16), one has

$$\lim_{t\to+\infty}\sup x_2(t) \leq \frac{-r_2^l + a_{22}^u(M_1+\varepsilon)}{a_{22}^l \exp\left\{\left[r_2^l - a_{21}^u(M_1+\varepsilon)\right]\tau_{22}^u\right\}}. \quad (17)$$

Setting $\varepsilon \to 0$, it follows that

$$\lim_{t\to+\infty}\sup x_2(t) \leq \frac{-r_2^l + a_{22}^u M_1}{a_{22}^l \exp\left\{\left(r_2^l - a_{21}^u M_1\right)\tau_{22}^u\right\}} := M_2. \quad (18)$$

For $t \geq T_1 + \tau_{31}^u$, from (12) and the third equation of system (1), we have

$$\begin{aligned}\frac{dx_3(t)}{dt} &\leq x_3(t)\left[-r_3(t) + a_{31}(t)x_1(t-\tau_{31}(t))\right] \\ &\leq x_3(t)\left[-r_3^l + a_{31}^u(M_1+\varepsilon)\right].\end{aligned} \quad (19)$$

Integrating both sides of (19) from $t-\tau_{33}(t)$ to t, we get

$$\begin{aligned}\ln\left[\frac{x_3(t)}{x_3(t-\tau_{33}(t))}\right] &\leq \int_{t-\tau_{33}(t)}^{t}\left[-r_3^l + a_{31}^u(M_1+\varepsilon)\right]ds \\ &\leq \left[-r_3^l + a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u,\end{aligned} \quad (20)$$

which leads to

$$x_3(t-\tau_{33}(t)) \geq x_3(t)\exp\left\{\left[r_3^l - a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\}. \quad (21)$$

Substituting (21) into the third equation of system (1), it follows that

$$\frac{dx_3(t)}{dt} \leq x_3(t)\left\{-r_3^l + a_{31}^u(M_1+\varepsilon) - a_{33}^l \exp\left\{\left[r_3^l - a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\} x_3(t)\right\}. \quad (22)$$

Thus, as a direct corollary of Lemma 2, according to (22), one has

$$\lim_{t\to+\infty}\sup x_3(t) \leq \frac{-r_3^l + a_{31}^u(M_1+\varepsilon)}{a_{33}^l \exp\left\{\left[r_3^l - a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\}}. \quad (23)$$

Setting $\varepsilon \to 0$, it follows that

$$\lim_{t\to+\infty}\sup x_3(t) \leq \frac{-r_3^l + a_{31}^u M_1}{a_{33}^l \exp\left\{\left(r_3^l - a_{31}^u M_1\right)\tau_{33}^u\right\}} := M_3. \quad (24)$$

For $t \geq T_1 + \max\{\tau_{21}^u, \tau_{31}^u, \tau_{11}^u, \tau_{12}^u, \tau_{13}^u\}$, it follows from the first equation of system (1) that

$$\frac{dx_1(t)}{dt} \geq x_1(t)\left[r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]. \quad (25)$$

Integrating both sides of (25) from $t-\tau_{11}(t)$ to t, one has

$$\begin{aligned}&\ln\left[\frac{x_1(t)}{x_1(t-\tau_{11}(t))}\right] \\ &\geq \int_{t-\tau_{11}(t)}^{t}\left[r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]ds \\ &\geq \left[r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u,\end{aligned} \quad (26)$$

which leads to

$$\begin{aligned}&x_1(t-\tau_{11}(t)) \\ &\quad\leq x_1(t)\exp\left\{-\left[r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}.\end{aligned} \quad (27)$$

Substituting (27) into the first equation of system (1), it follows that

$$\begin{aligned}&\frac{dx_1(t)}{dt} \\ &\geq x_1(t)\left\{r_1^l - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right. \\ &\quad - a_{11}^u\exp\left\{-\left[r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]\right. \\ &\quad \left.\left.\times\, \tau_{22}^u\right\} x_1(t)\right\}.\end{aligned} \quad (28)$$

According to Lemma 2, it follows from (28) that

$$\begin{aligned}&\lim_{t\to+\infty}\inf x_1(t) \\ &\geq \left(r_1^l - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right) \\ &\quad\times\left(a_{11}^u\exp\left\{\left[-r_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon) - a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\right)^{-1}.\end{aligned} \quad (29)$$

Setting $\varepsilon \to 0$ in (29), we can get

$$\begin{aligned}&\lim_{t\to+\infty}\inf x_1(t) \\ &\geq \frac{r_1^l - a_{12}^u M_2 - a_{13}^u M_3}{a_{11}^u\exp\left\{-\left(r_1^l - a_{11}^u M_1 - a_{12}^u M_2 - a_{13}^u M_3\right)\tau_{22}^u\right\}} := m_1.\end{aligned} \quad (30)$$

For $t \geq T_1 + \max\{\tau_{21}^u, \tau_{22}^u, \tau_{23}^u, \tau_{31}^u, \tau_{11}^u, \tau_{12}^u, \tau_{13}^u\}$, from the second equation of system (1), we have

$$\frac{dx_2(t)}{dt} \geq x_2(t)\left[-r_2^u + a_{21}^l(m_1-\varepsilon) - a_{22}^u(M_2+\varepsilon) - a_{23}^u(M_3+\varepsilon)\right]. \tag{31}$$

Integrating both sides of (31) from $t-\tau_{22}(t)$ to t leads to

$$\begin{aligned}\ln\left[\frac{x_2(t)}{x_2(t-\tau_{22}(t))}\right] &\geq \int_{t-\tau_{22}(t)}^{t}\left[-r_2^u + a_{21}^l(m_1-\varepsilon) - a_{22}^u(M_2+\varepsilon) - a_{23}^u(M_3+\varepsilon)\right]ds \\ &\geq \left[-r_2^u + a_{21}^l(m_1-\varepsilon) - a_{22}^u(M_2+\varepsilon) - a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u,\end{aligned} \tag{32}$$

which leads to

$$x_2(t-\tau_{22}(t)) \leq x_2(t)\exp\left\{\left[r_2^u - a_{21}^l(m_1-\varepsilon) + a_{22}^u(M_2+\varepsilon) + a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}. \tag{33}$$

Substituting (33) into the second equation of system (1), it follows that

$$\frac{dx_2(t)}{dt} \geq x_2(t)\left\{r_2^u - a_{22}^u \times \exp\left\{\left[r_2^u - a_{21}^l(m_1-\varepsilon) + a_{22}^u(M_2+\varepsilon) + a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}x_2(t) - a_{23}^u(M_3+\varepsilon)\right\}. \tag{34}$$

By Lemma 2 and (34), we can get

$$\begin{aligned}\lim_{t\to+\infty}\inf x_2(t) &\geq \left(r_2^u - a_{23}^u(M_3+\varepsilon)\right) \\ &\quad\times\left(a_{22}^u\exp\left\{\left[r_2^u - a_{21}^l(m_1-\varepsilon) + a_{22}^u(M_2+\varepsilon) + a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\right)^{-1}.\end{aligned} \tag{35}$$

Setting $\varepsilon \to 0$ in the above inequality, it follows that

$$\lim_{t\to+\infty}\inf x_2(t) \geq \frac{r_2^u - a_{23}^u M_3}{a_{22}^u\exp\{(r_2^u - a_{21}^l m_1 + a_{22}^u M_2 + a_{23}^u M_3)\tau_{22}^u\}} := m_2. \tag{36}$$

For $t \geq T_1 + \max\{\tau_{31}^u, \tau_{32}^u, \tau_{33}^u, \tau_{21}^u, \tau_{22}^u, \tau_{23}^u, \tau_{31}^u, \tau_{11}^u, \tau_{12}^u, \tau_{13}^u\}$, it follows from the third equation of system (1) that

$$\begin{aligned}\frac{dx_3(t)}{dt} &= x_3(t)\left[-r_3(t) + a_{31}(t)x_1(t-\tau_{31}(t)) - a_{32}(t)x_2(t-\tau_{32}(t)) - a_{33}(t)x_3(t-\tau_{33}(t))\right], \\ &\geq x_3(t)\left[-r_3^u + a_{31}^l(m_1-\varepsilon) - a_{32}^u(M_2+\varepsilon) - a_{33}^u(M_3+\varepsilon)\right].\end{aligned} \tag{37}$$

Integrating both sides of (37) from $t-\tau_{33}(t)$ to t, we get

$$\begin{aligned}\ln\left[\frac{x_3(t)}{x_3(t-\tau_{33}(t))}\right] &\geq \int_{t-\tau_{33}(t)}^{t}\left[-r_3^u + a_{31}^l(m_1-\varepsilon) - a_{32}^u(M_2+\varepsilon) - a_{33}^u(M_3+\varepsilon)\right]ds \\ &\geq \left[-r_3^u + a_{31}^l(m_1-\varepsilon) - a_{32}^u(M_2+\varepsilon) - a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u.\end{aligned} \tag{38}$$

Hence

$$x_3(t-\tau_{33}(t)) \leq x_3(t)\exp\left\{\left[r_3^u - a_{31}^l(m_1-\varepsilon) + a_{32}^u(M_2+\varepsilon) + a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}. \tag{39}$$

Substituting (39) into the third equation of system (1), we derive

$$\frac{dx_3(t)}{dt} \geq x_3(t)\left\{-r_3^u + a_{31}^l(m_1-\varepsilon) - a_{32}(M_2+\varepsilon) - a_{33}\exp\left\{\left[r_3^u - a_{31}^l(m_1-\varepsilon) + a_{32}^u(M_2+\varepsilon) + a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}x_3(t)\right\}. \tag{40}$$

In view of Lemma 2 and (40), one has

$$\begin{aligned}\lim_{t\to+\infty}\inf x_3(t) &\geq \left(-r_3^u + a_{31}^l(m_1-\varepsilon) - a_{32}^u(M_2+\varepsilon)\right) \\ &\quad\times\left(a_{33}\exp\left\{\left[r_3^u - a_{31}^l(m_1-\varepsilon) + a_{32}^u(M_2+\varepsilon) + a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\right)^{-1}.\end{aligned} \tag{41}$$

Setting $\varepsilon \to 0$ in (41) leads to

$$\lim_{t\to+\infty} \inf x_3(t) \geq \frac{-r_3^u + a_{31}^l m_1 - a_{32}^u M_2}{a_{33}^u \exp\left\{\left(r_3^u - a_{31}^l m_1 + a_{32}^u M_2 + a_{33}^u M_3\right)\tau_{33}^u\right\}} := m_3. \tag{42}$$

Equations (11), (18), (24), (30), (36), and (42) show that system (1) is permanent. The proof of Theorem 3 is complete. □

3. Global Asymptotically Stability of Positive Solutions

In this section, we formulate the global asymptotically stability of positive solutions of system (1).

Definition 4. A bounded positive solution $(x_1^*(t), x_2^*(t), x_3^*(t))^T$ of system (1) is said to be globally asymptotically stable if, for any other positive bounded solution $(x_1(t), x_2(t), x_3(t))^T$ of system (1), the following equality holds:

$$\lim_{t\to+\infty}\left[\sum_{i=1}^{3}\left|x_i(t) - x_i^*(t)\right|\right] = 0. \tag{43}$$

Definition 5 (see [24]). Let $\widetilde{h}$ be a real number and f be a nonnegative function defined on $[\widetilde{h}, +\infty)$ such that f is integrable on $[\widetilde{h}, +\infty)$ and is uniformly continuous on $[\widetilde{h}, +\infty)$, then $\lim_{t\to+\infty} f(t) = 0$.

Theorem 6. *In addition to (H1)-(H2), assume further that*

(H3) $\lim_{t\to\infty} \inf A_i(t) > 0$,

where A_i $(i = 1, 2, 3)$ are defined by (48), (49), *and* (50), *respectively. Then system* (1) *has a unique positive solution $(x_1^*(t), x_2^*(t), x_3^*(t))^T$ which is global attractivity.*

Proof. According to the conclusion of Theorem 3, there exists $T > 0$ and positive constants m_i, M_i $(i = 1, 2, 3)$ such that

$$m_i < x_i^*(t) \leq M_i \quad i = 1, 2, 3, \ t > T. \tag{44}$$

Define

$$V(t) = \sum_{i=1}^{3}\left|\ln x_i^*(t) - \ln x_i(t)\right|. \tag{45}$$

Calculating the upper-right derivative of $V(t)$ along the solution of (1), it follows for $t \geq T$ that

$$\begin{aligned}
D^+V(t) &= \sum_{i=1}^{3}\left(\frac{x_i^{*\prime}(t)}{x_i^*(t)} - \frac{x_i^{\prime}(t)}{x_i(t)}\right)\operatorname{sgn}\left(x_i^*(t) - x_i(t)\right) \\
&= \operatorname{sgn}\left(x_1^*(t) - x_1(t)\right) \\
&\quad \times \sum_{i=1}^{3} - a_{1i}(t)\left[x_i^*\left(t - \tau_{1i}(t)\right) - x_i\left(t - \tau_{1i}(t)\right)\right] \\
&\quad + \operatorname{sgn}\left(x_2^*(t) - x_2(t)\right) \\
&\quad \times \sum_{i=1}^{3} - a_{2i}(t)\left[x_i^*\left(t - \tau_{2i}(t)\right) - x_i\left(t - \tau_{2i}(t)\right)\right] \\
&\quad + \operatorname{sgn}\left(x_3^*(t) - x_3(t)\right) \\
&\quad \times \sum_{i=1}^{3} - a_{3i}(t)\left[x_i^*\left(t - \tau_{3i}(t)\right) - x_i\left(t - \tau_{3i}(t)\right)\right] \\
&\leq -a_{11}(t)\left|x_1^*\left(t - \tau_{11}(t)\right) - x_1\left(t - \tau_{11}(t)\right)\right| \\
&\quad + a_{12}(t)\left|x_2^*\left(t - \tau_{12}(t)\right) - x_2\left(t - \tau_{12}(t)\right)\right| \\
&\quad + a_{13}(t)\left|x_3^*\left(t - \tau_{13}(t)\right) - x_3\left(t - \tau_{13}(t)\right)\right| \\
&\quad + a_{21}(t)\left|x_1^*\left(t - \tau_{21}(t)\right) - x_1\left(t - \tau_{21}(t)\right)\right| \\
&\quad - a_{22}(t)\left|x_2^*\left(t - \tau_{22}(t)\right) - x_2\left(t - \tau_{22}(t)\right)\right| \\
&\quad + a_{23}(t)\left|x_3^*\left(t - \tau_{23}(t)\right) - x_3\left(t - \tau_{23}(t)\right)\right| \\
&\quad + a_{31}(t)\left|x_1^*\left(t - \tau_{31}(t)\right) - x_1\left(t - \tau_{31}(t)\right)\right| \\
&\quad + a_{32}(t)\left|x_2^*\left(t - \tau_{32}(t)\right) - x_2\left(t - \tau_{32}(t)\right)\right| \\
&\quad - a_{33}(t)\left|x_3^*\left(t - \tau_{33}(t)\right) - x_3\left(t - \tau_{33}(t)\right)\right|.
\end{aligned} \tag{46}$$

It follows that

$$\begin{aligned}
&D^+V(t) \\
&\leq -a_{11}(t)\left|x_1^*\left(t - \tau_{11}(t)\right) - x_1\left(t - \tau_{11}(t)\right)\right| \\
&\quad + a_{12}(t)\left|x_2^*\left(t - \tau_{12}(t)\right) - x_2\left(t - \tau_{12}(t)\right)\right| \\
&\quad + a_{13}(t)\left|x_3^*\left(t - \tau_{13}(t)\right) - x_3\left(t - \tau_{13}(t)\right)\right| \\
&\quad + a_{21}(t)\left|x_1^*\left(t - \tau_{21}(t)\right) - x_1\left(t - \tau_{21}(t)\right)\right| \\
&\quad - a_{22}(t)\left|x_2^*\left(t - \tau_{22}(t)\right) - x_2\left(t - \tau_{22}(t)\right)\right| \\
&\quad + a_{23}(t)\left|x_3^*\left(t - \tau_{23}(t)\right) - x_3\left(t - \tau_{23}(t)\right)\right| \\
&\quad + a_{31}(t)\left|x_1^*\left(t - \tau_{31}(t)\right) - x_1\left(t - \tau_{31}(t)\right)\right| \\
&\quad + a_{32}(t)\left|x_2^*\left(t - \tau_{32}(t)\right) - x_2\left(t - \tau_{32}(t)\right)\right| \\
&\quad - a_{33}(t)\left|x_3^*\left(t - \tau_{33}(t)\right) - x_3\left(t - \tau_{33}(t)\right)\right| \\
&\leq -a_{11}(t)\Big\{\exp\Big\{\Big[r_1^l - a_{11}^u\left(M_1 + \varepsilon\right) - a_{12}^u\left(M_2 + \varepsilon\right) \\
&\qquad\qquad -a_{13}^u\left(M_3 + \varepsilon\right)\Big]\tau_{22}^u\Big\} + \exp\left\{-r^u\tau_{11}^u\right\}\Big\} \\
&\quad \times \left|x_1^*(t) - x_1(t)\right| \\
&\quad + 2a_{12}(t)\exp\Big\{\Big[-r_2^u + a_{21}^l\left(m_1 - \varepsilon\right) - a_{22}^u\left(M_2 + \varepsilon\right) \\
&\qquad\qquad - a_{23}^u\left(M_3 + \varepsilon\right)\Big]\tau_{22}^u\Big\} \\
&\quad \times \left|x_2^*(t) - x_2(t)\right|
\end{aligned}$$

$$
\begin{aligned}
&+2a_{13}(t)\exp\left\{\left[-r_3^u+a_{31}^l(m_1-\varepsilon)-a_{32}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\\
&\times\left|x_3^*(t)-x_3(t)\right|\\
&+2a_{21}(t)\exp\left\{-\left[r_1^l-a_{11}^u(M_1+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{12}^u(M_2+\varepsilon)-a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\times\left|x_1^*(t)-x_1(t)\right|\\
&-a_{22}(t)\left\{\exp\left\{\left[-r_2^u+a_{21}^l(m_1-\varepsilon)-a_{22}^u(M_2+\varepsilon)\right.\right.\right.\\
&\qquad\left.\left.-a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\qquad\left.+\exp\left\{\left[r_2^l-a_{21}^u(M_1+\varepsilon)\right]\tau_{22}^u\right\}\right\}\\
&\times\left|x_2^*(t)-x_2(t)\right|\\
&+2a_{23}(t)\exp\left\{\left[r_3^u-a_{31}^l(m_1-\varepsilon)\right.\right.\\
&\qquad\left.\left.+a_{32}^u(M_2+\varepsilon)+a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\\
&\times\left|x_3^*(t)-x_3(t)\right|\\
&+2a_{31}(t)\exp\left\{-\left[r_1^l-a_{11}^u(M_1+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{12}^u(M_2+\varepsilon)-a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\times\left|x_1^*(t)-x_1(t)\right|\\
&+2a_{32}(t)\exp\left\{\left[-r_2^u+a_{21}^l(m_1-\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{22}^u(M_2+\varepsilon)-a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\times\left|x_2^*(t)-x_2(t)\right|\\
&-a_{33}(t)\left\{\exp\left\{\left[-r_3^u+a_{31}^l(m_1-\varepsilon)\right.\right.\right.\\
&\qquad\left.\left.-a_{32}^u(M_2+\varepsilon)-a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\\
&\qquad\left.+\exp\left\{\left[r_3^l-a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\}\right\}\\
&\times\left|x_3^*(t)-x_3(t)\right|\\
&\le\left[-A_1(t)\left|x_1^*(t)-x_1(t)\right|+A_2(t)\left|x_2^*(t)-x_2(t)\right|\right.\\
&\qquad\left.+A_3\left|x_3^*(t)-x_3(t)\right|\right],
\end{aligned}
\tag{47}
$$

where ε is defined by Theorem 3 and

$$
\begin{aligned}
&A_1(t)\\
&=a_{11}(t)\left\{\exp\left\{\left[r_1^l-a_{11}^u(M_1+\varepsilon)-a_{12}^u(M_2+\varepsilon)\right.\right.\right.\\
&\qquad\left.\left.\left.-a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}+\exp\left\{-r^u\tau_{11}^u\right\}\right\}\\
&\quad-2a_{21}(t)\exp\left\{-\left[r_1^l-a_{11}^u(M_1+\varepsilon)-a_{12}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\quad-2a_{31}(t)\exp\left\{\tau_{22}^u-\left[r_1^l-a_{11}^u(M_1+\varepsilon)-a_{12}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{13}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\},
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
&A_2(t)\\
&=a_{22}(t)\left\{\exp\left\{\left[-r_2^u+a_{21}^l(m_1-\varepsilon)-a_{22}^u(M_2+\varepsilon)\right.\right.\right.\\
&\qquad\left.\left.-a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\qquad\left.+\exp\left\{\left[r_2^l-a_{21}^u(M_1+\varepsilon)\right]\tau_{22}^u\right\}\right\}\\
&\quad-2a_{12}(t)\exp\left\{\left[-r_2^u+a_{21}^l(m_1-\varepsilon)-a_{22}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\}\\
&\quad-2a_{32}(t)\exp\left\{\left[-r_2^u+a_{21}^l(m_1-\varepsilon)-a_{22}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{23}^u(M_3+\varepsilon)\right]\tau_{22}^u\right\},
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
&A_3(t)\\
&=a_{33}(t)\left\{\exp\left\{\left[-r_3^u+a_{31}^l(m_1-\varepsilon)\right.\right.\right.\\
&\qquad+\exp\left\{\left[r_3^l-a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\}\\
&\qquad\left.\left.-a_{32}^u(M_2+\varepsilon)-a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\\
&\qquad\left.+\exp\left\{\left[r_3^l-a_{31}^u(M_1+\varepsilon)\right]\tau_{33}^u\right\}\right.\\
&\quad-2a_{13}(t)\exp\left\{\left[-r_3^u+a_{31}^l(m_1-\varepsilon)-a_{32}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.-a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}\\
&\quad-2a_{23}(t)\exp\left\{\left[r_3^u-a_{31}^l(m_1-\varepsilon)+a_{32}^u(M_2+\varepsilon)\right.\right.\\
&\qquad\left.\left.+a_{33}^u(M_3+\varepsilon)\right]\tau_{33}^u\right\}.
\end{aligned}
\tag{50}
$$

By hypothesis (H3), there exist constants α_i $(i=1,2,3)$ and $T^*>T$ such that

$$A_i(t)\ge\alpha_i>0,\quad (i=1,2,3)\ \text{ for } t\ge T^*. \tag{51}$$

Integrating both sides of (51) on interval $[T^*,t]$ yields

$$V(t)+\sum_{i=1}^{3}\int_{T^*}^{t}A_i(t)\left|x_i^*(t)-x_i(t)\right|ds\le V(T^*). \tag{52}$$

It follows from (51) and (52) that

$$\sum_{i=1}^{3}\int_{T^*}^{t}A_i(t)\left|x_i^*(t)-x_i(t)\right|ds\le V(T^*)<\infty,\quad \text{for } t\ge T^*. \tag{53}$$

Since $x_i^*(t)$ $(i=1,2,3)$ are bounded for $t\ge T^*$, so $|x_i^*(t)-x_i(t)|$ $(i,j=1,2,3)$ are uniformly continuous on $[T^*,\infty)$. By Barbalat's Lemma [24], we have

$$\lim_{t\to\infty}\left|x_i^*(t)-x_i(t)\right|=0,\quad (i=1,2,3). \tag{54}$$

By Theorems 7.4 and 8.2 in [43], we know that the positive solution $(x_1^*(t),x_2^*(t),x_3^*(t))^T$ of (1) is uniformly asymptotically stable. The proof of Theorem 6 is complete. □

4. Numerical Example

To illustrate the theoretical results, we present some numerical simulations. Let us consider the following discrete system:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)\Big[5-\frac{\cos\pi t}{2} - \left(4+\frac{\cos\pi t}{5}\right)x_1\left(t-\left(1-\frac{\sin\pi t}{4}\right)\right) \\
&\quad - \left(\frac{1+\sin\pi t}{4}\right)x_2\left(t-\left(\frac{0.5-\sin\pi t}{4}\right)\right) \\
&\quad - \left(\frac{1+\cos\pi t}{3}\right)x_3\left(t-\left(\frac{0.9-\cos\pi t}{4}\right)\right)\Big], \\
\frac{dx_2(t)}{dt} &= x_2(t)\Big[-\left(\frac{48-\cos\pi t}{12}\right) + \left(2-\frac{\cos\pi t}{4}\right)x_1\left(t-\left(\frac{0.7-\cos\pi t}{5}\right)\right) \\
&\quad - \left(4-\frac{\cos\pi t}{12}\right)x_2\left(t-\left(\frac{1+\sin\pi t}{4}\right)\right) \\
&\quad - \left(1+\frac{\sin\pi t}{4}\right)x_3\left(t-\left(\frac{0.2-\sin\pi t}{12}\right)\right)\Big], \\
\frac{dx_3(t)}{dt} &= x_3(t)\Big[-\left(\frac{1-\cos\pi t}{4}\right) + \left(8+\frac{\sin\pi t}{4}\right)x_1\left(t-\left(\frac{0.8-\sin\pi t}{5}\right)\right) \\
&\quad - \left(\frac{0.6-\sin\pi t}{8}\right)x_2\left(t-\left(\frac{0.6-\cos\pi t}{12}\right)\right) \\
&\quad - \left(20+\frac{\sin\pi t}{4}\right)x_3\left(t-\left(0.5+\frac{\sin\pi t}{2}\right)\right)\Big].
\end{aligned} \tag{55}$$

Here

$$\begin{aligned}
r_1(t) &= 5-\frac{\cos\pi t}{2}, & r_2(t) &= \frac{48-\cos\pi t}{12}, \\
r_3(t) &= \frac{2-\cos\pi t}{4}, & a_{11}(t) &= 4+\frac{\cos\pi t}{5}, \\
a_{12}(t) &= \frac{1+\sin\pi t}{4}, & a_{13}(t) &= \frac{1+\cos\pi t}{3}, \\
a_{21}(t) &= 2-\frac{\cos\pi t}{4}, & a_{22}(t) &= 4-\frac{\cos\pi t}{12}, \\
a_{23}(t) &= 1+\frac{\sin\pi t}{4}, & a_{31}(t) &= 8+\frac{\sin\pi t}{4}, \\
a_{32}(t) &= \frac{0.6-\sin\pi t}{8}, & a_{33}(t) &= 20+\frac{\sin\pi t}{4}, \\
\tau_{11}(t) &= 1-\frac{\sin\pi t}{4}, & \tau_{12}(t) &= \frac{0.5-\sin\pi t}{4}, \\
\tau_{13}(t) &= \frac{0.9-\cos\pi t}{4}, & \tau_{21}(t) &= \frac{0.7-\cos\pi t}{5}, \\
\tau_{22}(t) &= \frac{1+\sin\pi t}{4}, & \tau_{23}(t) &= \frac{0.2-\sin\pi t}{12}, \\
\tau_{31}(t) &= \frac{0.8-\sin\pi t}{5}, & \tau_{32}(t) &= \frac{0.6-\cos\pi t}{12}, \\
\tau_{33}(t) &= 0.5+\frac{\sin\pi t}{2}.
\end{aligned} \tag{56}$$

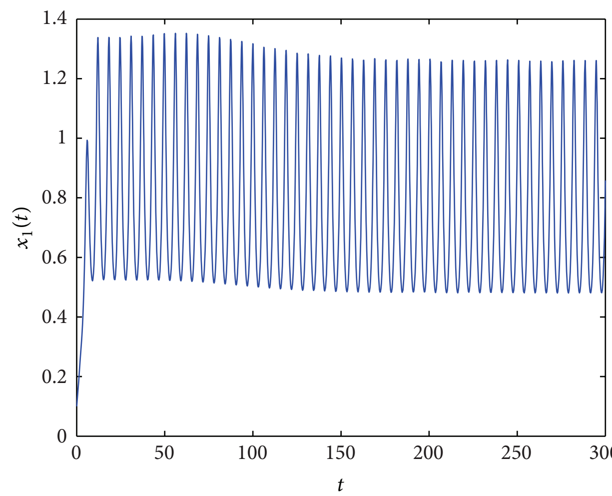

Figure 1: The dynamical behavior of the first component of the solution $(x_1(t), x_2(t), x_3(t))$.

All the coefficients $r_i(t)$ $(i = 1,2,3)$, $a_{ij}(t)$ $(i,j = 1,2,3)$, $\tau_{ij}(t)$ $(i,j = 1,2,3)$ are functions with respect to t, and it is easy to see that

$$\begin{aligned}
a_{22}^u &= \frac{49}{12}, & a_{31}^u &= \frac{33}{4}, & r_2^l &= \frac{47}{12}, \\
r_3^l &= \frac{1}{4}, & a_{12}^u &= \frac{1}{2}, & a_{13}^u &= \frac{2}{3}, \\
r_2^u &= \frac{49}{12}, & a_{23}^u &= \frac{5}{4}, & a_{31}^l &= \frac{31}{4}, \\
r_3^u &= \frac{3}{4}, & a_{32}^u &= 0.2, & r_1^u &= 5.5, \\
a_{11}^l &= 3.8, & \tau_{11}^u &= 1.25, & a_{22}^l &= \frac{47}{12}, \\
a_{21}^u &= 2.25, & \tau_{22}^u &= 0.5, & a_{33}^l &= 19.75.
\end{aligned} \tag{57}$$

Then $M_1 = 1.2451$, $M_2 = 0.7395$, $M_3 = 2.1093$, $m_1 = 0.6422$. Thus it is easy to see that all the conditions of Theorem 6 are satisfied. Thus system (55) is permanent which is shown in Figures 1, 2, and 3.

5. Conclusions

In this paper, we have investigated the dynamical behavior of a Lotka-Volterra predator-prey model with time-varying

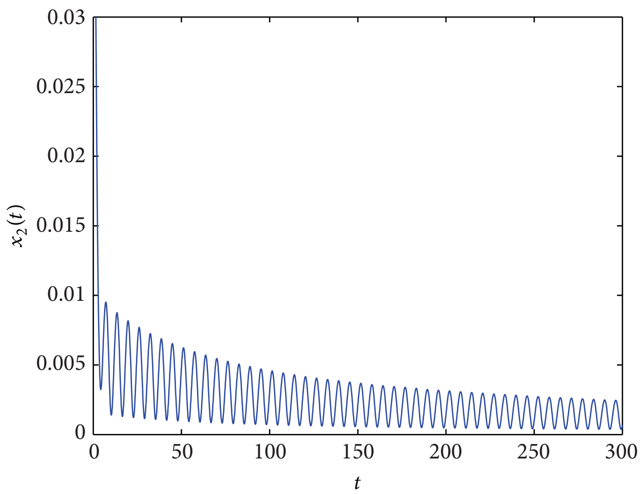

FIGURE 2: The dynamical behavior of the second component of the solution $(x_1(t), x_2(t), x_3(t))$.

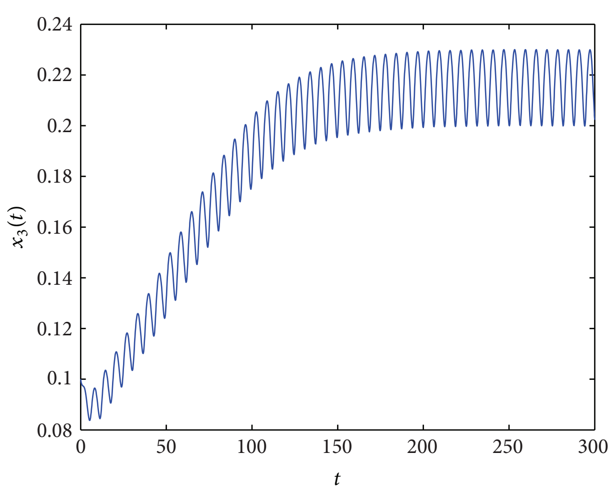

FIGURE 3: The dynamical behavior of the third component of the solution $(x_1(t), x_2(t), x_3(t))$.

delays. Sufficient conditions which ensure the permanence of the system are derived. Moreover, we also deal with the global stability of the system. It is shown that delay has influence on the permanence and the global stability of system. Thus delay is an important factor to decide the permanence and global stability of the system. Numerical simulations show the feasibility of our main results.

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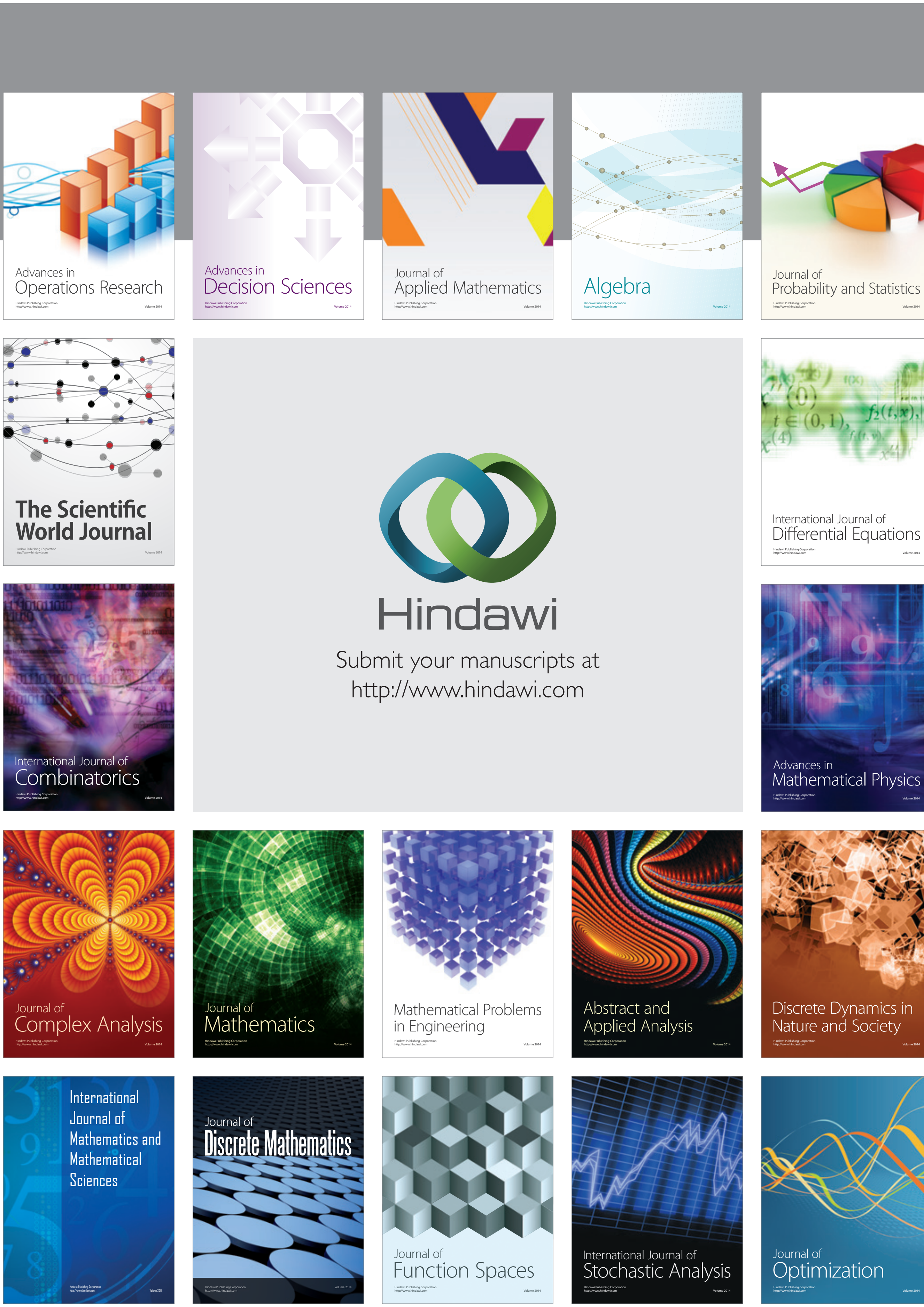
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