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Research Article

Oscillations for Nonlinear Neutral Delay Differential Equations with Variable Coefficients

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A class of nonlinear neutral delay differential equations is considered. Some new oscillation criteria of all solutions are derived. The obtained results generalize and extend some of well known previous results in the literature.

1. Introduction and Preliminaries

Consider the nonlinear neutral delay differential equation of the form

$$(a(t)x(t) - p(t)x(t-\tau))' + q(t)\prod_{i=1}^{n}|x(t-\sigma_i)|^{\alpha_i}\operatorname{sign} x(t-\sigma_i) = 0, \tag{E}$$

where

$$a(t), p(t), q(t) \in C([t_0,\infty), \mathbb{R}^+), \qquad \tau, \sigma_i > 0, \qquad \alpha_i \ge 0, \qquad \sum_{i=1}^{n}\alpha_i = 1, \qquad i = 1, 2, \ldots, n. \tag{1}$$

Let $m = \max\{\tau, \sigma_i, 1 \le i \le n\}$. By a solution of (E) we mean a function $x \in C[[\tilde{t} - m, \infty), \mathbb{R}]$ for some $\tilde{t} \ge t_0$ such that $a(t)x(t) - p(t)x(t-\tau)$ is continuously differentiable for $t \ge \tilde{t}$ and such that (E) is satisfied for $t \ge \tilde{t}$. Let $\tilde{t} \ge t_0$ be a given initial point and let $\Phi \in C[[\tilde{t} - m, \tilde{t}], \mathbb{R}]$ be a given initial function. Then, one can show by using the method of steps that (E) has a unique solution on $[\tilde{t}, \infty)$ satisfying the initial function

$$x(t) = \Phi(t), \quad \tilde{t} - m \le t \le \tilde{t}. \tag{2}$$

As usual, a solution of (E) is said to be oscillatory if it has arbitrarily large zeros and nonoscillatory if it is either eventually positive or eventually negative. Equation (E) is said to be oscillatory if all its solutions are oscillatory.

In the sequel, unless otherwise specified, when we write a functional inequality, we assume that it holds for all sufficiently large t.

In the case where $n = 1$, $a(t) \equiv 1$, and p and q are constants, Karpuz and Öcalan [1] improved the result of Ladas and Sficas [2] holding $0 \le p \le 1$, $q \ge 0$, and $q(\tau - \sigma) > (1/e)(1-p)$ conditions for oscillation. Also, the case including continuous functions as coefficients

$$(x(t) - p(t)x(t-\tau))' + q(t)x(t-\sigma) = 0, \quad t \ge t_0, \tag{3}$$

has been studied by many authors; see, for example, Kubiaczyk and Saker [3], Karpuz and Öcalan [1], Ahmed at al. [4], Chuanxi et al. [5], and Yu et al. [6]. In particular, Chen et al. [7] succeeded in getting some oscillation theorems for (3) which involve joint behaviour of p and q using the condition

$$p(t-\sigma)q(t) \le q(t-\tau). \tag{4}$$

Some further results on the oscillation for neutral delay differential equations can be found in the excellent paper of Saker and Kubiaczyk [8] and the recent paper of Ahmed et al. [9]. See also Shen and Debnath [10] and Wang [11].

Li [12] extended results of Chen et al. [7] for (E) in the case when $a(t) \equiv 1$ and introduced some new oscillation criteria under the hypothesis

$$\int_{t_0}^{\infty} q(s) \exp\left\{\frac{1}{\tau}\int_s^{\infty} uq(u)\,du\right\} ds = \infty. \quad (5)$$

Kubiaczyk et al. [13] have given some several sufficient conditions for oscillation of all solutions depending on the functions p and q when $p(t) - 1$ is allowed to oscillate, while Zhou [14] has established some new sufficient conditions for oscillation depending on an additional constant λ.

Here, in this paper, we continue in this direction of finding some sufficient conditions for (E) to oscillate in the case when $\int_{t_0}^{\infty} q(s)ds < \infty$. To this end, let us site the next two results which will enable us to complete the proofs of our main results.

Lemma 1 (see [8]). *Assume that there exists $t^* \geq t_0 > 0$ such that*

$$\frac{p(t^* + n^*\tau)}{a(t^* + (n^* - 1)\tau)} \leq 1 \quad \text{for } n^* = 0, 1, 2, \ldots. \quad (6)$$

Let $x(t)$ be an eventually positive solution of (E). *Let*

$$z(t) = a(t)x(t) - p(t)x(t - \tau). \quad (7)$$

Then

$$z(t) > 0, \qquad z'(t) \leq 0. \quad (8)$$

Theorem 2 (see [15]). *Assume that $p(t) \equiv 1$; then every solution of* (3) *oscillates if*

$$\int_{t_0}^{\infty} sq(s)\int_s^{\infty} q(u)\,du\,ds = \infty. \quad (9)$$

2. Main Results

Theorem 3. *Assume that condition* (6) *holds and either*

$$p(t) + \tau q(t) > 0, \quad (10)$$

or

$$\tau > 0,$$
$$q(s) \text{ does not identically equal zero for } s \in [t, t + \tau]. \quad (11)$$

Then all solutions of (E) *oscillate if and only if the corresponding differential inequality*

$$\begin{aligned}&(a(t)x(t) - p(t)x(t-\tau))' \\ &\quad + q(t)\prod_{i=1}^{n}|x(t-\sigma_i)|^{\alpha_i}\operatorname{sign} x(t-\sigma_i) \leq 0\end{aligned} \quad (12)$$

has no eventually positive solution.

Proof. The sufficiency is obvious. To prove necessity, assume that $x(t)$ is an eventually positive solution of (12). We plan to show that (E) has a nonoscillatory solution. Set $z(t)$ as in (7). Then, from (E) we have

$$z'(t) = -q(t)\prod_{i=1}^{n}|x(t-\sigma_i)|^{\alpha_i}\operatorname{sign} x(t-\sigma_i). \quad (13)$$

Integrating the last equation from t to ∞, and using Lemma 1, we have

$$z(t) \geq \int_t^{\infty} q(s)\prod_{i=1}^{n}x(t-\sigma_i)^{\alpha_i}ds. \quad (14)$$

That is,

$$a(t)x(t) \geq p(t)x(t-\tau) + \int_t^{\infty} q(s)\prod_{i=1}^{n}x(t-\sigma_i)^{\alpha_i}ds, \quad (15)$$

which leads to

$$x(t) \geq \frac{1}{a(t)}\left(p(t)x(t-\tau) + \int_t^{\infty} q(s)\prod_{i=1}^{n}x(t-\sigma_i)^{\alpha_i}ds\right). \quad (16)$$

Let $T > t_0$ be fixed so that (16) holds for all $t \geq T$. Set $T_0 = \max\{\tau, \sigma_i, 1 \leq i \leq n\}$ and consider the set of functions

$$X = \{u \in C([T - T_0, \infty), \mathbb{R}^+); 0 \leq u(t) \leq 1, t \geq T - T_0\}. \quad (17)$$

Define a mapping F on X as

$$(Fu)(t) = \begin{cases} \dfrac{1}{a(t)x(t)}\left(p(t)u(t-\tau)x(t-\tau) + \displaystyle\int_t^{\infty} q(s)\prod_{i=1}^{n}x(s-\sigma_i)^{\alpha_i}u(s-\sigma_i)^{\alpha_i}ds\right), & t \geq T, \\ \dfrac{t - T + T_0}{T_0}(Fu)(T) + \left(1 - \dfrac{t - T + T_0}{T_0}\right), & T - T_0 \leq t \leq T. \end{cases} \quad (18)$$

It is easy to see, by using (16), that F maps X into itself. Moreover, for any $u \in X$ we have $(Fu)(t) > 0$ for $T - T_0 \leq t \leq T$.

Next, define the sequence $u_i(t)$ in X as follows:

$$\begin{aligned} u_0(t) &= 1, \quad t \geq T - T_0, \\ u_{j+1}(t) &= (Fu_j)(t), \quad j = 0, 1, 2, \ldots. \end{aligned} \quad (19)$$

Therefore, by using (16) and a simple induction, we can easily see that

$$0 \leq u_{j+1}(t) \leq u_j(t) \leq 1, \quad t \geq T - T_0, \ j = 0, 1, 2, \ldots. \quad (20)$$

Set

$$u(t) = \lim_{j\to\infty} u_j(t); \quad t \ge T - T_0. \tag{21}$$

Then from Lebesgue's Dominated Convergence Theorem, it follows that $u(t)$ satisfies

$$u(t) = \frac{1}{a(t)x(t)} \times \left(p(t)u(t-\tau)x(t-\tau) + \int_t^\infty q(s)\prod_{i=1}^n x(s-\sigma_i)^{\alpha_i}u(s-\sigma_i)^{\alpha_i}ds \right), \quad t \ge T,$$
$$u(t) = \frac{t-T+T_0}{T_0}(Fu)(T) + \left(1 - \frac{t-T+T_0}{T_0}\right), \quad T - T_0 \le t \le T. \tag{22}$$

Again set

$$\omega(t) = u(t)a(t)x(t). \tag{23}$$

Then

$$\omega(t) > 0, \quad T - T_0 \le t < T \tag{24}$$

and satisfies, for $t \ge T$,

$$\omega(t) = \left[\frac{1}{a(t)x(t)} \times \left(p(t)u(t-\tau)x(t-\tau) + \int_t^\infty q(s)\prod_{i=1}^n x(s-\sigma_i)^{\alpha_i}u(s-\sigma_i)^{\alpha_i}ds \right)\right] \times a(t)x(t). \tag{25}$$

This implies that

$$\omega(t) = \overline{p}(t)\omega(t-\tau) + \int_t^\infty \overline{q}(s)\prod_{i=1}^n \omega(s-\sigma_i)^{\alpha_i}ds, \quad t \ge T, \tag{26}$$

where

$$\overline{q}(s) = \frac{q(s)}{\prod_{i=1}^n a(s-\sigma_i)^{\alpha_i}}, \qquad \overline{p}(t) = \frac{p(t)}{a(t-\tau)}. \tag{27}$$

Clearly, $\omega(t)$ is continuous on $t \ge T - T_0$. To show that $\omega(t)$ is positive for all $t \ge T - T_0$, assume that there exists $t^* \ge T - T_0$ such that $\omega(t) > 0$ for $T - T_0 \le t < t^*$ and $\omega(t^*) = 0$. Then $t^* \ge T$ and by (26) we obtain

$$0 = \omega(t^*) = \overline{p}(t^*)\omega(t^*-\tau) + \int_{t^*}^\infty \overline{q}(s)\prod_{i=1}^n \omega(s-\sigma_i)^{\alpha_i}ds, \quad t \ge T. \tag{28}$$

Then

$$\overline{p}(t^*) \equiv 0, \qquad \overline{q}(s)\prod_{i=1}^n \omega^{\alpha_i}(t-\sigma_i) \equiv 0, \quad \forall t \ge t^*. \tag{29}$$

This is a contradiction with (10) or (11). Therefore, $\omega(t^*)$ is positive on $[T-T_0, \infty)$. Furthermore, it is easy to see that $\omega(t)$ is a positive solution of (E), which implies that the inequality (12) having no eventually positive solution is a necessary condition for the oscillation of all solutions of (E). The proof is complete. □

Remark 4. Theorem 3 is an extent of Theorem 2.1 due to Lalli and Zhang [16], Theorem 1 due to Chen et al. [7], and Theorem 1 due to Li [12].

Now we give an application of Theorem 3.

Theorem 5. *Consider* (E) *with* $n = 1$. *Suppose that condition* (6) *holds with*

$$\int_{t_0}^\infty s\overline{q}(s)\int_{t_0}^s \overline{q}(u)\,du\,ds = \infty, \tag{30}$$

$$p(t-\sigma)\overline{q}(t) \ge q(t-\tau), \tag{31}$$

where

$$\overline{q}(t) = \frac{q(t)}{a(t-\sigma)}. \tag{32}$$

Then all solutions of (E) *are oscillatory.*

Proof. Suppose that (E) has an eventually positive solution $x(t)$. Set $z(t)$ as in (7). Then, by Lemma 6, we have

$$z(t) > 0, \qquad z'(t) \le 0. \tag{33}$$

From (7), we have

$$x(t) = \frac{1}{a(t)}\left(z(t) + p(t)x(t-\tau)\right), \tag{34}$$

$$x(t-\sigma) = \frac{1}{a(t-\sigma)}\left(z(t-\sigma) + p(t-\sigma)x(t-\sigma-\tau)\right). \tag{35}$$

Hence, from (E), (31), and (35), respectively, we have

$$\begin{aligned} z'(t) &= -q(t)x(t-\sigma) \\ &= -q(t)\left(\frac{1}{a(t-\sigma)}\left(z(t-\sigma) + p(t-\sigma)x(t-\sigma-\tau)\right)\right) \\ &= -\frac{q(t)}{a(t-\sigma)}z(t-\sigma) - q(t-\tau)x(t-\sigma-\tau). \end{aligned} \tag{36}$$

That is,

$$z'(t) \le -\frac{q(t)}{a(t-\sigma)} z(t-\sigma) + z'(t-\tau) \quad (37)$$

or

$$z'(t) - z'(t-\tau) + \overline{q}(t) z(t-\sigma) \le 0, \quad (38)$$

where

$$\overline{q}(t) = \frac{q(t)}{a(t-\sigma)}. \quad (39)$$

In view of Theorem 3, we have that the equation

$$z'(t) - z'(t-\tau) + \overline{q}(t) z(t-\sigma) = 0 \quad (40)$$

has an eventually positive solution. On the other hand, in view of Theorem 2, condition (30) implies that (40) cannot have an eventually positive solution. This is a contradiction. The proof is complete. □

Lemma 6. *Suppose that*

$$0 < a(t) \le 1; \quad (41)$$

$$p(t) \ge 1, \quad (42)$$

$$\int_{t_0}^{\infty} q(s) \exp\left(\frac{1}{\tau}\int_{t_0}^{s} uq(u)\,du\right) ds = \infty. \quad (43)$$

Let $x(t)$ be an eventually positive solution of (E) *and $z(t)$ defined by* (7). *Then*

$$z(t) < 0, \qquad z'(t) \le 0. \quad (44)$$

Proof. From (E) and (7), we have

$$z'(t) = -q(t) \prod_{i=1}^{n} x(t-\sigma_i)^{\alpha_i} \le 0. \quad (45)$$

Therefore, if (44) does not hold, then we have eventually that $z(t) > 0$; that is,

$$a(t) x(t) \ge p(t) x(t-\tau) \quad (46)$$

which together with (41) and (42) yields

$$x(t) \ge x(t-\tau). \quad (47)$$

Let $t_1 \ge t_0$ be such that

$$x(t-\tau) > 0, \quad \text{for } t_1 \ge t_0, \quad (48)$$

and also such that (16) holds for $t_1 \ge t_0$. Define

$$\kappa = \min\{x(t) : t \in [t_1 - \tau, t_1]\}. \quad (49)$$

Then, $x(t) \ge \kappa$ for $t \ge t_1$. Set $\sigma^* = \max\{\tau, \sigma_1, \dots, \sigma_n\}$, and we have

$$x(t) \ge \kappa \quad \text{for } t \ge t_1 + \sigma^* = t_2. \quad (50)$$

For convenience, we denote

$$N(t) = \left[\frac{t - t_2}{\tau}\right], \quad (51)$$

where $[(t-t_2)/\tau]$ is the greatest integer parts of $(t-t_2)/\tau$. Then from (7), (41), and (42), we obtain

$$x(t) \ge a(t) x(t) = z(t) + p(t) x(t-\tau) \ge z(t) + x(t-\tau). \quad (52)$$

Thus, we have

$$\begin{aligned} x(t) &\ge z(t) + x(t-\tau) \\ &\ge z(t) + z(t-\tau) + \cdots + z(t-(N(t)-1)\tau) \\ &\quad + x(t - N(t)\tau), \quad t \ge t_2. \end{aligned} \quad (53)$$

But $z(t)$ is decreasing, and $x(t - N(t)\tau) \ge \kappa$ for $t \ge t_1$. Therefore, from (53), we get

$$x(t) \ge N(t) z(t) + \kappa, \quad t \ge t_1. \quad (54)$$

Substituting in (E), we obtain

$$z'(t) + q(t) \prod_{i=1}^{n} [N(t-\sigma_i) z(t-\sigma_i) + \kappa]^{\alpha_i} \le 0, \quad t \ge t_3 \ge t_2. \quad (55)$$

By Holder's inequality, we have

$$\begin{aligned} &\prod_{i=1}^{n} [N(t-\sigma_i) z(t-\sigma_i) + \kappa]^{\alpha_i} \\ &\quad \ge \prod_{i=1}^{n} N^{\alpha_i}(t-\sigma_i) \prod_{i=1}^{n} z^{\alpha_i}(t-\sigma_i) + \kappa. \end{aligned} \quad (56)$$

Then, from (55), we get

$$z'(t) + q(t) \prod_{i=1}^{n} N^{\alpha_i}(t-\sigma_i) z(t) + q(t)\kappa \le 0, \quad t \ge t_3. \quad (57)$$

Hence,

$$\begin{aligned} &\left[z(t) \exp\left(\int_{t_3}^{t} q(s) \prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)\,ds\right)\right]' \\ &\quad + \kappa q(t) \exp\left(\int_{t_3}^{t} q(s) \prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)\,ds\right) \le 0. \end{aligned} \quad (58)$$

Integrating (58) from t_3 to $t \ge t_3$, we have

$$\begin{aligned} &z(t) \exp\left(\int_{t_3}^{t} q(s) \prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)\,ds\right) - y(t_3) + \kappa \\ &\quad \times \int_{t_3}^{t} q(s) \exp\left(\int_{t_3}^{s} q(u) \prod_{i=1}^{n} N^{\alpha_i}(u-\sigma_i)\,du\right) ds \le 0. \end{aligned} \quad (59)$$

Also we have

$$\int_{t_3}^{\infty} q(s)\,ds < \infty. \tag{60}$$

Now, by noting that

$$\left(\frac{\prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)}{t}\right) \longrightarrow \frac{1}{\tau} \quad \text{as } t \longrightarrow \infty, \tag{61}$$

we see that

$$\int_{t_3}^{\infty} q(s)\left(\frac{s}{\tau} - \prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)\right) ds < \infty. \tag{62}$$

Thus, we have eventually that

$$\lim_{s\to\infty} \frac{\exp\left(\int_{t_3}^{s} q(u)\prod_{i=1}^{n} N^{\alpha_i}(s-\sigma_i)\,du\right)}{\exp\left((1/\tau)\int_{t_3}^{s} uq(u)\,du\right)} \tag{63}$$

exists. Then, from condition (43), we have

$$\int_{t_3}^{\infty} q(s)\exp\left(\int_{t_3}^{s} q(u)\prod_{i=1}^{n} N^{\alpha_i}(u-\sigma_i)\,du\right) ds = \infty. \tag{64}$$

Letting $t \to \infty$ in (59), we obtain a contradiction with (64). The proof is complete. □

Remark 7. Lemma 6 extends Lemma 2 in Li [12] where $a(t) \equiv 1$.

Theorem 8. *Suppose that condition* (6) *holds with*

$$\int_{t_0}^{\infty} \overline{q}(s)\exp\left(\frac{1}{\tau}\int_{t_0}^{s} u\overline{q}(u)\,du\right) ds = \infty, \tag{65}$$

$$\prod_{i=1}^{n} \overline{p}_1^{\alpha_i}(t-\sigma_i)\,q(t) \ge q(t-\tau), \tag{66}$$

where $\overline{q}$ is defined as in Theorem 3 and

$$\prod_{i=1}^{n} \overline{p}_1^{\alpha_i}(t-\sigma_i) = \frac{\prod_{i=1}^{n} p^{\alpha_i}(t-\sigma_i)}{\prod_{i=1}^{n} a^{\alpha_i}(t-\sigma_i)}. \tag{67}$$

Then, all solutions of (E) *are oscillatory.*

Proof. Suppose that (E) has an eventually positive solution $x(t)$. Set $z(t)$ as in (7). Then by Lemma 6 we have

$$z(t) > 0, \qquad z'(t) \le 0. \tag{68}$$

From (7), we have

$$x(t-\sigma_i) = \frac{1}{a(t-\sigma_i)}\left(z(t-\sigma_i) + p(t-\sigma_i)\,x(t-\sigma_i-\tau)\right); \quad i = 1,2,\ldots,n. \tag{69}$$

Hence, from (E) and (69), we have

$$\begin{aligned} z'(t) &= -q(t)\prod_{i=1}^{n} x^{\alpha_i}(t-\sigma_i) \\ &= -q(t)\prod_{i=1}^{n}\left(\frac{1}{a(t-\sigma_i)} \times \left[z(t-\sigma_i) + p(t-\sigma_i)\,x(t-\sigma_i-\tau)\right]\right)^{\alpha_i} \\ &= -q(t)\prod_{i=1}^{n}\left(\left[\frac{z(t-\sigma_i)}{a(t-\sigma_i)} + \frac{p(t-\sigma_i)}{a(t-\sigma_i)}x(t-\sigma_i-\tau)\right]\right)^{\alpha_i}. \end{aligned} \tag{70}$$

Applying Holder's inequality, we obtain

$$\begin{aligned} z'(t) \le{}& -q(t)\prod_{i=1}^{n}\left(\frac{z(t-\sigma_i)}{a(t-\sigma_i)}\right)^{\alpha_i} \\ & - q(t)\prod_{i=1}^{n}\left(\frac{p(t-\sigma_i)}{a(t-\sigma_i)}x(t-\sigma_i-\tau)\right)^{\alpha_i} \end{aligned} \tag{71}$$

which implies that

$$\begin{aligned} z'(t) &\le -\frac{q(t)}{\prod_{i=1}^{n} a^{\alpha_i}(t-\sigma_i)}\prod_{i=1}^{n} z^{\alpha_i}(t-\sigma_i) - q(t) \times \prod_{i=1}^{n}\left[\frac{p(t-\sigma_i)}{a(t-\sigma_i)}\right]^{\alpha_i}\prod_{i=1}^{n} x^{\alpha_i}(t-\sigma_i-\tau) \\ &\le -\frac{q(t)}{\prod_{i=1}^{n} a^{\alpha_i}(t-\sigma_i)}\prod_{i=1}^{n} z^{\alpha_i}(t-\sigma_i) + z'(t-\tau), \end{aligned} \tag{72}$$

where we have used condition (66) to obtain the last inequality. This implies that $z(t)$ is a positive solution of the inequality

$$z'(t) - z'(t-\tau) + \frac{q(t)}{\prod_{i=1}^{n} a^{\alpha_i}(t-\sigma_i)}\prod_{i=1}^{n} z^{\alpha_i}(t-\sigma_i) \le 0; \tag{73}$$

that is,

$$z'(t) - z'(t-\tau) + \overline{q}(t)\prod_{i=1}^{n} z^{\alpha_i}(t-\sigma_i) \le 0, \tag{74}$$

where

$$\overline{q}(t) = \frac{q(t)}{\prod_{i=1}^{n} a^{\alpha_i}(t-\sigma_i)}. \tag{75}$$

As we see, (74) satisfies all conditions of Lemma 6; hence, $Z(t) = z(t) - z(t-\tau) > 0$ eventually. On the other hand, since (74) satisfies all conditions of Lemma 6, then $Z(t) = z(t) - z(t-\tau) < 0$ eventually, which is a contradiction. The proof is complete. □

Remark 9. Theorem 8 improves and extends Theorem 2 in Chen et al. [7] and Theorem 4 in Li [12], where $a(t) \equiv 1$. See also Theorem 3.4 in Yu et al. [6].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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