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Research Article

A Novel Analytical Technique to Obtain Kink Solutions for Higher Order Nonlinear Fractional Evolution Equations

Qazi Mahmood Ul Hassan, Jamshad Ahmad, and Muhammad Shakeel

Department of Mathematics, Faculty of Sciences, HITEC University Taxila Cantt Pakistan, Taxila, Pakistan

Correspondence should be addressed to Qazi Mahmood Ul Hassan; qazimahmood@yahoo.com

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We use the fractional derivatives in Caputo's sense to construct exact solutions to fractional fifth order nonlinear evolution equations. A generalized fractional complex transform is appropriately used to convert this equation to ordinary differential equation which subsequently resulted in a number of exact solutions.

1. Introduction

The concept of differentiation and integration to noninteger order is not new in any case. The notion of fractional calculus emerged when the ideas of classical calculus were proposed by Leibniz, who mentioned it in a letter to L'Hospital in 1695. The foundation of the earliest, more or less, systematic studies can be traced back to the beginning and middle of the 19th century by Liouville in 1832, Riemann in 1853, and Holmgren in 1864, although Euler in 1730, Lagrange in 1772, and others also made contributions. Recently, it has turned out those differential equations involving derivatives of noninteger [1, 2]. For example, the nonlinear oscillation of earthquakes can be modeled with fractional derivatives [3]. There has been some attempt to solve linear problems with multiple fractional derivatives [3, 4]. Not much work has been done on nonlinear problems and only a few numerical schemes have been proposed for solving nonlinear fractional differential equations [5, 6]. More recently, applications have included classes of nonlinear equation with multiorder fractional derivatives. The generalized fractional complex transform was applied in [7–13] to convert fractional order differential equation to ordinary differential equation. Finally, by using Exp-function method [14–25] we obtain generalized solitary solutions and periodic solutions. Recently the theory of local fractional integrals and derivatives [26–28] is one of useful tools to handle the fractal and continuously nondifferentiable functions. It is to be tinted that that $c = d$ and $p = q$ are the only relations that can be obtained by applying Exp-function method [29] to any nonlinear ordinary differential equation. Most scientific problems and phenomena in different fields of sciences and engineering occur nonlinearly. Except in a limited number of these problems are linear, this method has been effectively and accurately shown to solve a large class of nonlinear problems. The solution procedure of this method, with the aid of Maple, is of utter simplicity and this method can easily extend to other kinds of nonlinear evolution equations. In engineering and science, scientific phenomena give a variety of solutions that are characterized by distinct features. Traveling waves appear in many distinct physical structures in solitary wave theory [30] such as solitons, kinks, peakons, cuspons, compactons, and many others. Solitons are localized traveling waves which are asymptotically zero at large distances. In other words, solitons are localized wave packets with exponential wings or tails. Solitons are generated from robust balance between nonlinearity and dispersion. Solitons exhibit properties typically associated with particles. Kink waves [30, 31] are solitons that rise or descend from one asymptotic state to another and hence another type of traveling waves as in the case of the Burgers hierarchy. Peakons, that are peaked solitary wave solutions, are another type of travelling waves as in the case of Camassa-Holm equation. For peakons, the traveling wave solutions are smooth except for a peak at a corner of its crest. Peakons are the points at which spatial derivative changes sign so that peakons have a finite jump in 1st derivative of the solution. Cuspons are other forms of solitons where solution exhibits cusps at their crests. Unlike peakons where the derivatives at the peak differ only by a sign, the derivatives at the jump of a cuspon diverge. The compactons are solitons with compact spatial support such that each compacton is a soliton confined to a finite core or a soliton without exponential tails or wings. Other types of travelling waves arise in science such as negatons, positons, and complexitons. In this research, we use the Exp-function method along with generalized fractional complex transform to obtain new Kink waves' solutions for [30–32].

2. Preliminaries and Notation [1, 2]

In this section, we give some basic definitions and properties of the fractional calculus theory [1, 2] which will be used further in this work. For the finite derivative in $[a, b]$ we define the following fractional integral and derivatives.

Definition 1. A real function $f(x)$, $x > 0$, is said to be in the space $C\mu$, $\mu \in R$ if there exists a real number ($p > \mu$) such that $f(x) = x^p f_1(x)$, where $f_1(x) = C(0, \infty)$, and it is said to be in the space $C_\mu^m \mu$ if $f^m \in C\mu$, $m \in N$.

Definition 2. The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$ of a function $f \in C\mu$, $\mu \geq -1$, is defined as

$$J^\alpha(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)\, dt, \qquad \alpha > 0, \quad x > 0, \quad J^0(x) = f(x). \tag{1}$$

Properties of the operator J^a can be found in [1]; we mention only the following.

For $f \in C\mu$, $\mu \geq -1$, $\alpha, \beta \geq 0$, and $\gamma \geq -1$

$$\begin{aligned} J^\alpha J^\beta f(x) &= J^{\alpha+B} f(x), \\ J^\alpha J^B f(x) &= J^B J^\alpha f(x), \\ J^\alpha x^\gamma &= \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}. \end{aligned} \tag{2}$$

The Riemann-Liouville derivative has certain disadvantages when trying to model real-world phenomena with fractional differential equations. Therefore, we will introduce a modified fractional differential operator proposed by M. Caputo in his work on the theory of viscoelasticity [2].

Definition 3. For m to be the smallest integer that exceeds α, the Caputo time fractional derivative operator of order $\alpha > 0$ defined as

$$\begin{aligned} {}_a^C D_t^\alpha f(t) &= J^{m-\alpha} D^m f(t) \\ &= \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^m(t)\, dt, \end{aligned} \tag{3}$$

for $m - 1 < \alpha \leq 1m$, $m \in N$, $t > 0$, $f \in C_{-1}^m$.

3. Chain Rule for Fractional Calculus and Fractional Complex Transform

In [7], the authors used the following chain rule $\partial^\alpha u/\partial t^\alpha = (\partial u/\partial s)(\partial^\alpha s/\partial t^\alpha)$ to convert a fractional differential equation with Jumarie's modification of Riemann-Liouville derivative into its classical differential partner. In [10], the authors showed that this chain rule is invalid and showed following relation [8]:

$$D_t^a u = \sigma_t' \frac{du}{d\eta} D_t^a \eta, \qquad D_x^a u = \sigma_x' \frac{du}{d\eta} D_x^a \eta. \tag{4}$$

To determine σ_s, consider a special case as follows:

$$s = t^\alpha, \qquad u = s^m \tag{5}$$

and we have

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\Gamma(1+m\alpha)\, t^{m\alpha-\alpha}}{\Gamma(1+m\alpha-\alpha)} = \sigma \cdot \frac{\partial u}{\partial s} = \sigma m t^{m\alpha-\alpha}. \tag{6}$$

Thus one can calculate σ_s as

$$\sigma_s = \frac{\Gamma(1+m\alpha)}{\Gamma(1+m\alpha-\alpha)}. \tag{7}$$

Other fractional indexes $(\sigma_x', \sigma_y', \sigma_z')$ can determine in similar way. Li and He [3, 7–9] proposed the following fractional complex transform for converting fractional differential equations into ordinary differential equations, so that all analytical methods for advanced calculus can be easily applied to fractional calculus:

$$u(x,t) = u(\eta), \quad \eta = \frac{kx^\beta}{\Gamma(1+\beta)} + \frac{\omega t^\alpha}{\Gamma(1+\alpha)} + \frac{Mx^\gamma}{\Gamma(1+\gamma)}, \tag{8}$$

where k, ω, and M are constants.

4. Exp-Function Method [33–36]

Consider the general nonlinear partial differential equation of fractional order:

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots, D_t^\alpha u, D_x^\alpha u, D_{xx}^\alpha u, \ldots) = 0, \qquad 0 < \alpha \leq 1, \tag{9}$$

where $D_t^\alpha u$, $D_x^\alpha u$, and $D_{xx}^\alpha u$ are the fractional derivative of u with respect to t, x, xx, respectively.

Use

$$u(x,t) = u(\eta), \quad \eta = \frac{kx^\beta}{\Gamma(1+\beta)} + \frac{\omega t^\alpha}{\Gamma(1+\alpha)} + \frac{Mx^\gamma}{\Gamma(1+\gamma)}, \tag{10}$$

where k, ω, and M are constants.

Then (9) becomes

$$Q\left(u, u', u'', u''', u^{iv}\right) = 0, \tag{11}$$

where the prime denotes derivative with respect to η. In accordance with Exp-function method, we assume that the wave solution can be expressed in the following form:

$$u(\eta) = \frac{\sum_{n-c}^{d} a_n \exp[n\eta]}{\sum_{m-p}^{q} b_m \exp[m\eta]}, \tag{12}$$

where p, q, c, and d are positive integers which are known to be further determined and a_n and b_m are unknown constants. Equation (8) can be rewritten as

$$u(\eta) = \frac{a_c \exp(c\eta) + \cdots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \cdots + b_{-q} \exp(-q\eta)}. \tag{13}$$

This equivalent formulation plays an important and fundamental for finding the analytic solution of problems. c and p can be determined by [29].

5. Solution Procedure

Consider the following new fifth order nonlinear (2 + 1)-dimensional evolution equations of fractional order:

$$D_t^{3\alpha} u - \left(D_t^\alpha u\right)_{xxxx} - \left(D_t^\alpha u\right)_{xx} - 4\left(u_x \left(D_t^\alpha u\right)_x\right)_x = 0. \tag{14}$$

Using (8) in (14) then it can be converted to an ordinary differential equation. Consider

$$-\omega^3 \ddot{u} + \omega k^4 u^{(v)} + k^2 \omega \ddot{u} + 4\omega k^3 \dot{u}\ddot{u} = 0, \tag{15}$$

where the prime denotes the derivative with respect to η. The solution of (15) can be expressed in form (13). To determine the value of c and p, by using [26],

$$p = c, \qquad q = d. \tag{16}$$

Case 1. We can freely choose the values of c and d, but we will illustrate that the final solution does not strongly depend upon the choice of values of c and d. For simplicity, we set $p = c = 1$ and $q = d = 1$ (15) reduces to

$$u(\eta) = \frac{a_1 \exp[\eta] + a_0 + a_{-1} \exp[-\eta]}{b_1 \exp[\eta] + a_0 + b_{-1} \exp[-\eta]}. \tag{17}$$

Substituting (17) into (15), we have

$$\begin{aligned} \frac{1}{A} & [c_4 a_1 \exp[4\eta] = c_3 \exp[3\eta] + c_2 \exp[2\eta] + c_1 \exp[\eta] \\ & + c_0 + c_{-1} \exp[-\eta] + c_{-2} \exp[-2\eta] \\ & + c_{-3} \exp[-3\eta] + c_{-4} \exp[-4\eta]] = 0, \end{aligned} \tag{18}$$

where $A = (b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta))^4$ and c_i are constants obtained by Maple software 16. Equating the coefficients of $\exp(n\eta)$ to be zero, we obtain

$$\begin{gathered} c_{-4} = 0, \; c_{-3} = 0, \; c_{-2} = 0, \; c_{-1} = 0, \\ c_0 = 0, \; c_1 = 0, \; c_2 = 0, \; c_3 = 0, \; c_4 = 0. \end{gathered} \tag{19}$$

For solution of (19) we have five solution sets satisfying the given (15).

1st Solution Set. Consider

$$\begin{gathered} \omega = -\sqrt{k^2 + 1}k, \; a_{-1} = \frac{b_{-1}(3kb_0 + a_0)}{b_0}, \\ a_0 = a_0, \; a_1 = 0, \; b_{-1} = b_{-1}, \; b_0 = b_0, \; b_1 = 0. \end{gathered} \tag{20}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of (14) (Figure 1):

$$u(x,t) = \frac{b_{-1}(3kb_0 + a_0) e^{-kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}/b_0 + a_0}{b_{-1} e^{-kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))} + b_0}. \tag{21}$$

2nd Solution Set. Consider

$$\begin{gathered} \omega = \sqrt{4k^2 + 1}k, \; a_{-1} = \frac{b_{-1}(6kb_1 + a_1)}{b_1}, \\ a_0 = a_0, \; a_1 = a_1, \; b_{-1} = b_{-1}, \; b_0 = 0, \; b_1 = b_1. \end{gathered} \tag{22}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of (14) (Figure 2):

$$\begin{aligned} u(x,t) = & \left(b_{-1}(6kb_1 + a_1) e^{-kx+(\sigma\sqrt{4k^2+1}kt^\alpha/\Gamma(1+\alpha))}/b_1 \right. \\ & \left. + a_1 e^{kx-(\sigma\sqrt{4k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right) \\ & \times \left(b_{-1} e^{-kx+(\sigma\sqrt{4k^2+1}kt^\alpha/\Gamma(1+\alpha))} \right. \\ & \left. + b_1 e^{kx-(\sigma\sqrt{4k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right)^{-1}. \end{aligned} \tag{23}$$

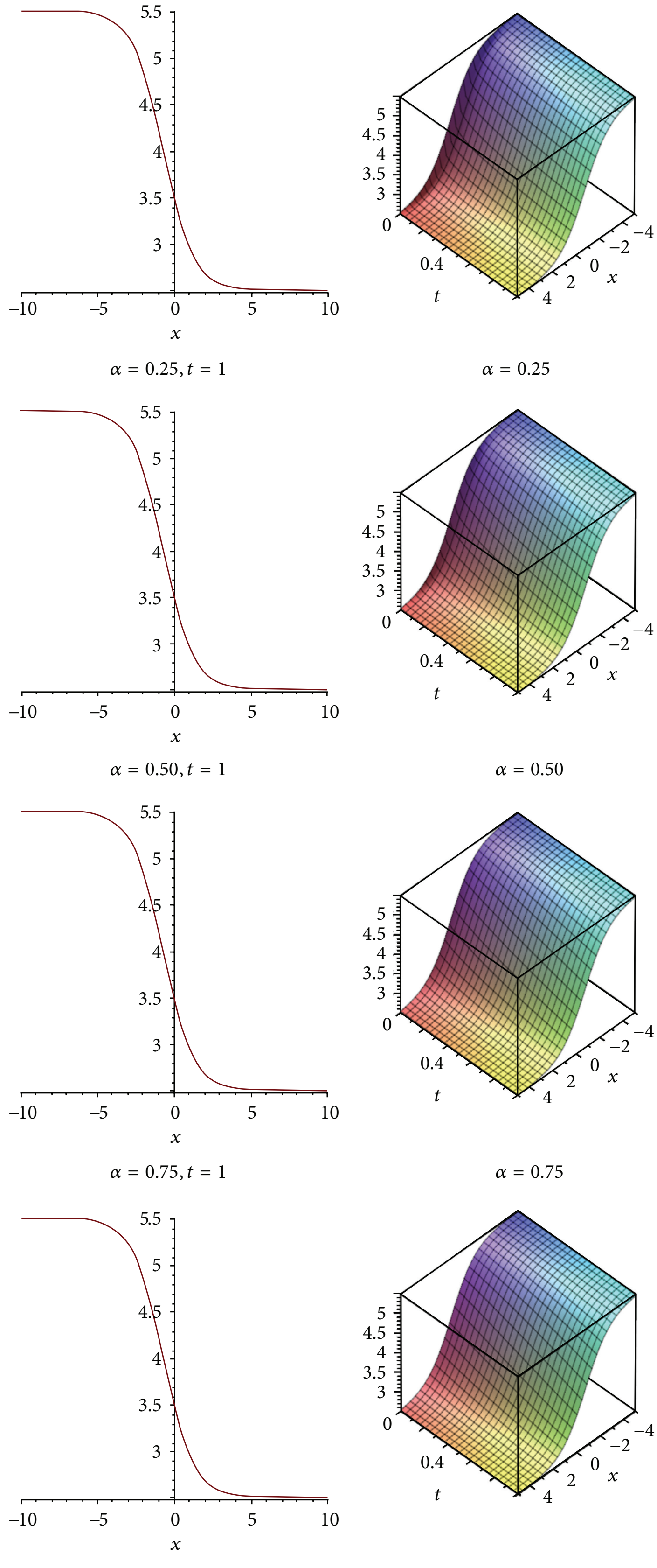


Figure 1: Kink waves' solutions of (14) for 1st solution set.

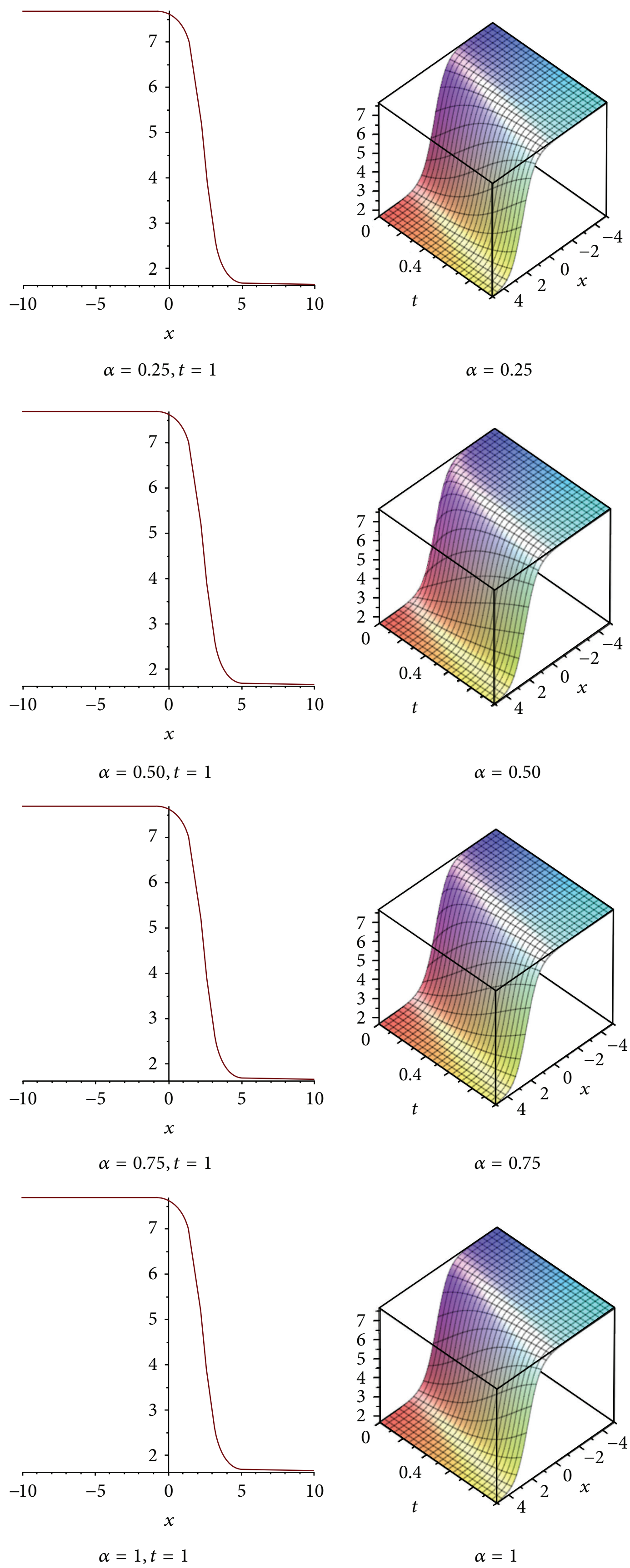


FIGURE 2: Kink waves' solutions of (14) for 2nd solution set.

3rd Solution Set. Consider

$$\omega = \sqrt{k^2+1}k, \quad a_{-1} = \frac{b_{-1}\left(3kb_0+a_0\right)}{b_0},$$
$$a_0 = a_0, \quad a_1 = 0, \quad b_{-1} = b_{-1}, \quad b_0 = b_0, \quad b_1 = 0. \tag{24}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of (14) (Figure 3):

$$u(x,t) = \frac{b_{-1}\left(3kb_0+a_0\right)e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}/b_0 + a_0}{b_{-1}e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}+b_0}. \tag{25}$$

4th Solution Set. Consider

$$\omega = \sqrt{k^2+1}k,$$
$$a_{-1} = \frac{1}{9}\frac{1}{k^2b_1^4}\left(9k^2a_0b_0b_1^3 - 9k^2a_1b_0^2b_1^2 - 3ka_0^2b_1^3 + 9ka_0a_1b_0b_1^2 - 6ka_1^2b_0^2b_1 - a_0^2a_1b_1^2 + 2a_0a_1^2b_0b_1 - a_1^3b_0^2\right),$$
$$b_1 = b_1, \quad b_0 = b_0,$$
$$a_0 = a_0, \quad a_1 = a_1,$$
$$b_{-1} = \frac{1}{9}\frac{3ka_0b_0b_1^2 - 3ka_1b_0^2b_1 - a_0^2b_1^2 + 2a_0a_1b_0b_1 - a_1^2b_0^2}{k^2b_1^3}. \tag{26}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of (14) (Figure 4):

$$\begin{aligned}u(x,t) = {} & \left(\frac{1}{9}\frac{1}{k^2b_1^4}\left(\left(9k^2a_0b_0b_1^3 - 9k^2a_1b_0^2b_1^2 - 3ka_0^2b_1^3 + 9ka_0a_1b_0b_1^2 - 6ka_1^2b_0^2b_1 - a_0^2a_1b_1^2 + 2a_0a_1^2b_0b_1 - a_1^3b_0^2\right)\times e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right) + a_0 + a_1e^{kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right) \\ & \times\left(\frac{1}{9}\left(\left(3ka_0b_0b_1^2 - 3ka_1b_0^2b_1 - a_0^2b_1^2 + 2a_0a_1b_0b_1 - a_1^2b_0^2\right)\times e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right)\left(k^2b_1^3\right)^{-1} + b_0 + b_1e^{kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}\right)^{-1}.\end{aligned} \tag{27}$$

5th Solution Set. Consider

$$\omega = \sqrt{k^2+1}k, \ a_{-1} = \frac{b_{-1}\left(6kb_1+a_1\right)}{b_1},$$
$$a_0 = 0, \ a_1 = a_1, \ b_{-1} = b_{-1}, \ b_0 = 0, \ b_1 = b_1. \tag{28}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of (14) (Figure 5):

$$u(x,t) = \frac{b_{-1}e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))} + a_1e^{kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}}{b_{-1}e^{-kx+(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))} + b_1e^{kx-(\sigma\sqrt{k^2+1}kt^\alpha/\Gamma(1+\alpha))}}. \tag{29}$$

Case 2. If $p = c = 2$ and $q = d = 1$ then trial solution (14) reduces to

$$u(\eta) = \frac{a_2\exp[2\eta] + a_1\exp[\eta] + a_0 + a_{-1}\exp[-\eta]}{b_2\exp[2\eta] + b_1\exp[\eta] + b_0 + b_{-1}\exp[-\eta]}. \tag{30}$$

Proceeding as before, we obtain the following.

1st Solution. Consider

$$a_{-1} = a_{-1}, \quad a_0 = \frac{a_{-1}b_0}{b_{-1}}, \quad a_1 = \frac{a_{-1}b_1}{b_{-1}}, \quad a_2 = \frac{a_{-1}b_2}{b_{-1}},$$
$$b_{-1} = b_{-1}, \quad b_0 = b_0, \quad b_1 = b_1, \quad b_2 = b_2. \tag{31}$$

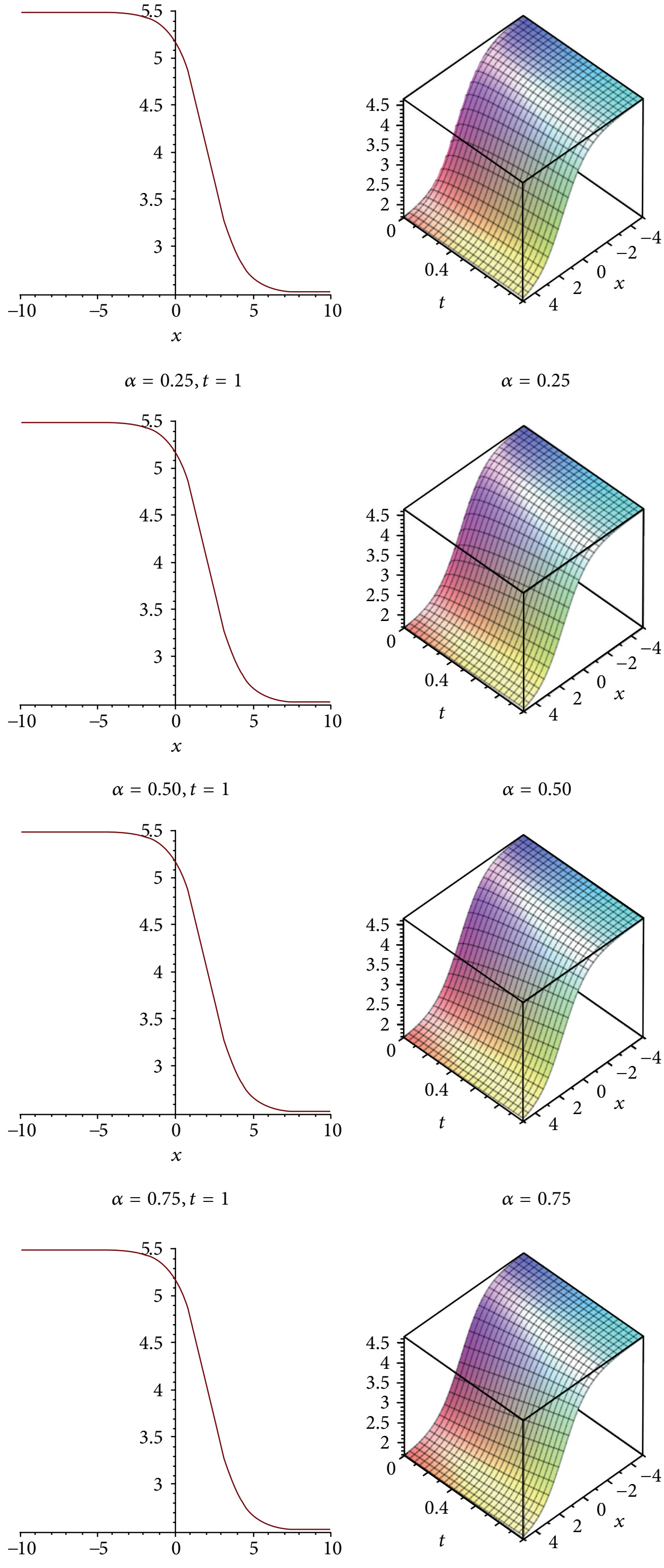
Hence we get the generalized solitary wave solution $u(x,t)$ of (14) (Figure 6):

$$\begin{aligned}u(x,t) = {} & \left(a_{-1}e^{-kx+(\sigma\omega kt^\alpha/\Gamma(1+\alpha))} + \frac{a_{-1}b_0}{b_{-1}} + \frac{a_{-1}b_0}{b_{-1}}e^{kx-(\sigma\omega kt^\alpha/\Gamma(1+\alpha))} + \frac{a_{-1}b_2}{b_{-1}}e^{2kx-2(\sigma\omega kt^\alpha/\Gamma(1+\alpha))}\right) \\ & \times\left(b_{-1}e^{-kx+(\sigma\omega kt^\alpha/\Gamma(1+\alpha))} + b_0 + b_1e^{kx-(\sigma\omega kt^\alpha/\Gamma(1+\alpha))} + b_2e^{2kx-2(\sigma\omega kt^\alpha/\Gamma(1+\alpha))}\right)^{-1}.\end{aligned} \tag{32}$$

In both cases, for different choices of c, p, d, and q we get the same soliton solutions which clearly illustrate that final solution does not strongly depend on these parameters.

6. Conclusions

Exp-function method is applied to construct solitary solutions of the nonlinear new fifth order evolution equations of fractional orders. The reliability of proposed algorithm is fully supported by the computational work, the subsequent results, and graphical representations. It is observed that Exp-function method is very convenient to apply and is very useful for finding solutions of a wide class of nonlinear problems of fractional orders.



$\alpha = 0.25, t = 1$

$\alpha = 0.25$

$\alpha = 0.50, t = 1$

$\alpha = 0.50$

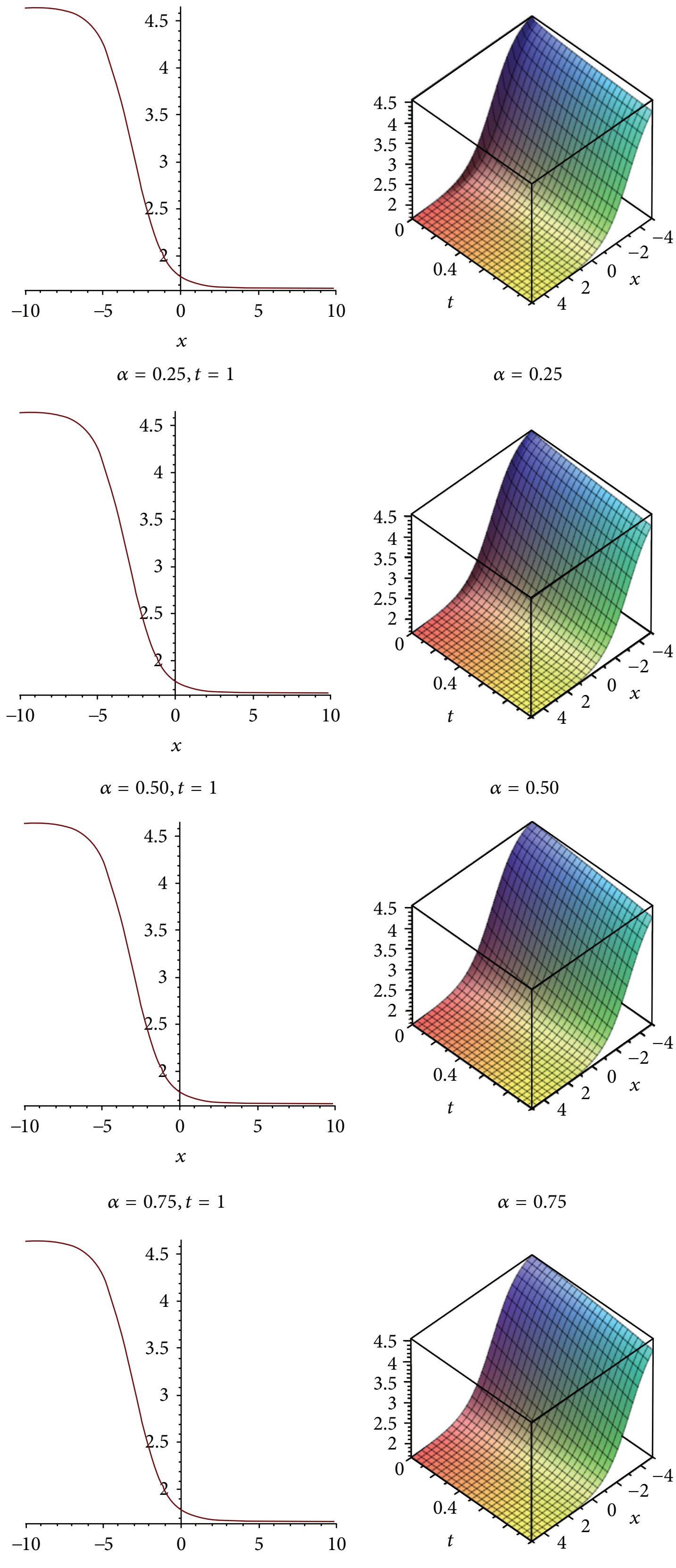
$\alpha = 0.75, t = 1$

$\alpha = 0.75$

$\alpha = 1, t = 1$

$\alpha = 1$

FIGURE 3: Kink waves' solutions of (14) for 3rd solution set.



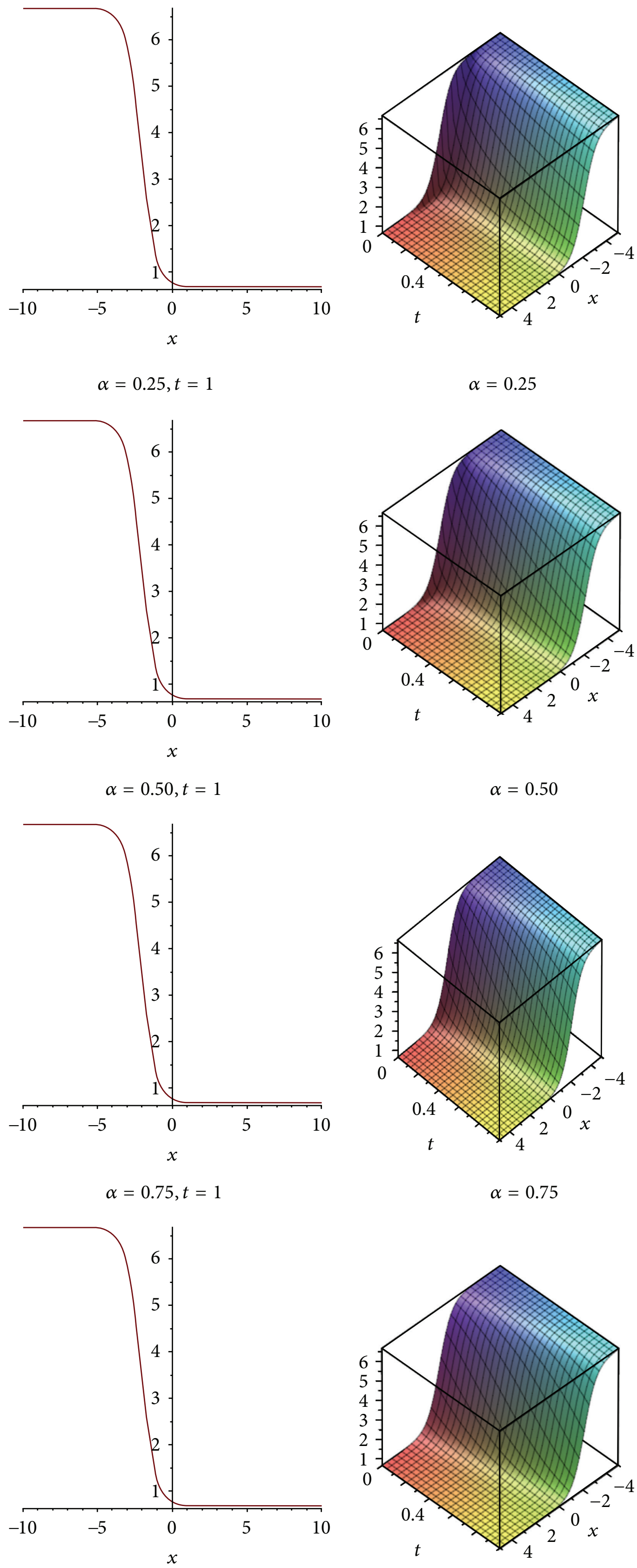
$\alpha = 0.25, t = 1$ $\alpha = 0.25$

$\alpha = 0.50, t = 1$ $\alpha = 0.50$

$\alpha = 0.75, t = 1$ $\alpha = 0.75$

$\alpha = 1, t = 1$ $\alpha = 1$

FIGURE 4: Kink waves' solutions of (14) for 4th solution set.



$\alpha = 0.25, t = 1$ $\alpha = 0.25$

$\alpha = 0.50, t = 1$ $\alpha = 0.50$

$\alpha = 0.75, t = 1$ $\alpha = 0.75$

$\alpha = 1, t = 1$ $\alpha = 1$

FIGURE 5: Kink waves' solutions of (14) for 5th solution set.

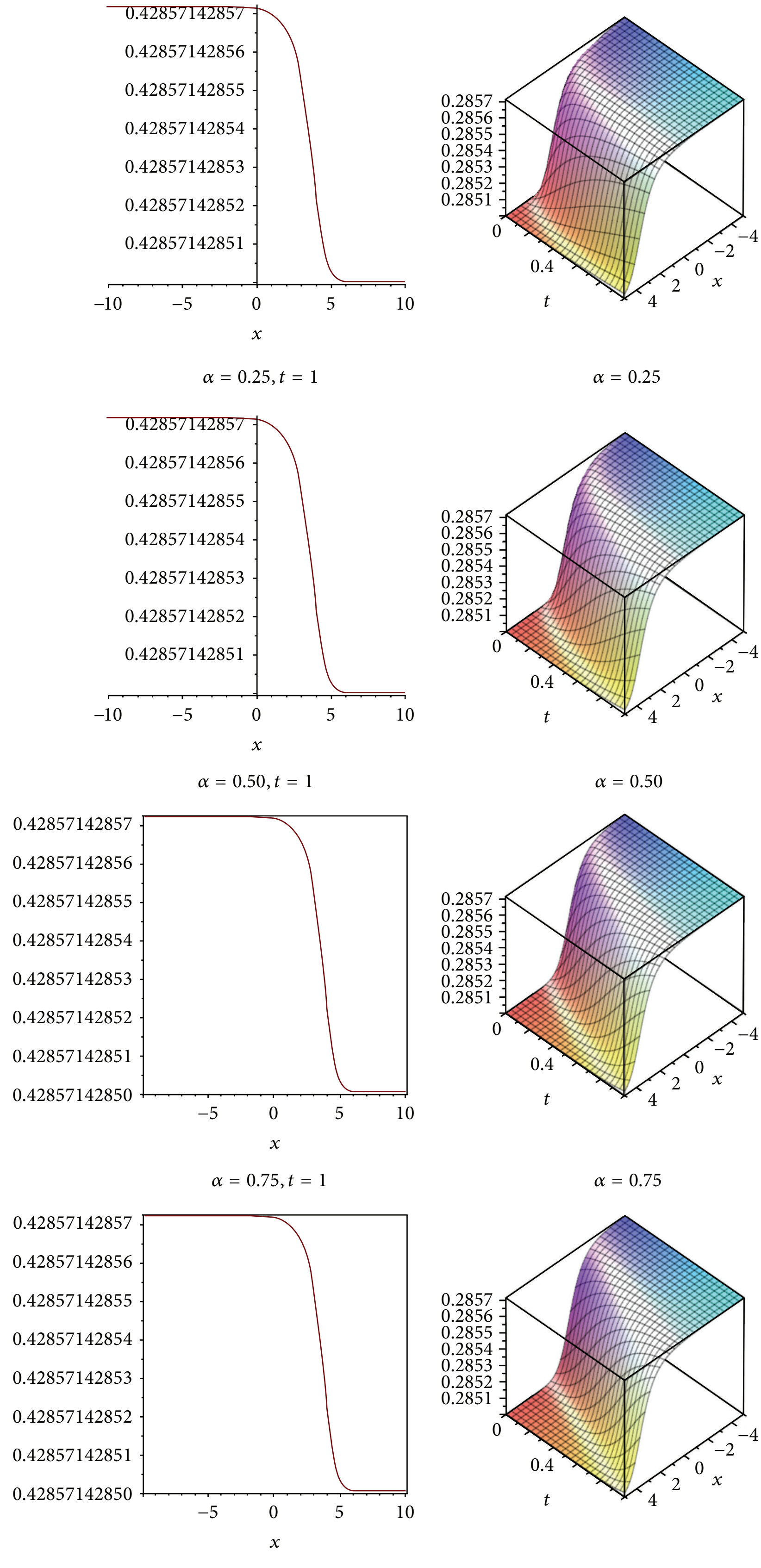


Figure 6: Kink waves' solutions of (14) for 1st solution set of Case 2.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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