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*Research Article*

# Stability Criteria for Singular Stochastic Hybrid Systems with Mode-Dependent Time-Varying Delay

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This paper provides a delay-dependent criterion for a class of singular stochastic hybrid systems with mode-dependent time-varying delay. In order to reduce conservatism, a new Lyapunov-Krasovskii functional is constructed by decomposing the delay interval into multiple subintervals. Based on the new functional, a stability criterion is derived in terms of strict linear matrix inequality (LMI), which guarantees that the considered system is regular, impulse-free, and mean-square exponentially stable. Numerical examples are presented to illustrate the effectiveness of proposed method.

## 1. Introduction

Over the past years, a lot of attention has been devoted to singular systems, which are also referred to as descriptor systems, generalized state-space systems, differential-algebraic systems, or semistate systems, because they can better describe physical systems than regular ones. Since the introduction of the first model for a jump system by Krasovskii and Lidskiĭ [1], the Markovian jump systems have become more popular in the area of control and operation research due to the fact that they are more powerful and appropriate to model varieties of systems mainly those with abrupt changes in their structure [2–6]. Consequently, singular hybrid systems or say singular systems with Markovian switching have been extensively studied in recent years. For more details on this class of systems, we refer the readers to see [7–10] and the references therein.

As is well known, environmental noise exists and cannot be neglected in many dynamical systems [11]. The systems with noise are called stochastic system. Up to date, some results have been reported on stochastic systems with Markovian switching [12–16]. It should be pointed out that the problems of stability and control for singular stochastic systems with Markovian switching or say singular stochastic hybrid systems are more complicated than that for regular ones, because the singular systems usually have three types of modes, namely, finite-dynamic modes, impulsive modes, and nondynamic modes, while the latter two do not appear in state-space systems. That is the reason why only a few results have been reported on this class of systems in the literature. In [17], the stochastic stability and stochastic stabilization for a class of nonlinear singular stochastic hybrid systems were investigated. The problem of sliding mode control for singular stochastic hybrid systems was investigated in [18]. In [19], under an assumption, authors study the problem of mean-square stability for singular stochastic systems with Markovian switching. However, no time delay was considered in [17–19], which often causes instability and poor performance of the systems [20]. In addition, the stability conditions in [17–19] are difficult to calculate since the equality constraints are used, which are often fragile and usually not met perfectly. To the author's best knowledge, the problem of delay-dependent mean-square exponential stability for singular stochastic hybrid systems with mode-dependent time-varying delay has not been fully investigated, which remains important and challenging.

This paper will focus on the derivation of delay-dependent mean-square exponential stability for a class of singular stochastic hybrid systems with mode-dependent time-varying delay. Inspired by the discretized Lyapunov functional method proposed by Gu [21], a new Lyapunov-Krasovskii functional is constructed to reduce conservatism. Based on the functional, a delay-dependent stability criterion is proposed, which guarantees that the system is regular,

impulse-free, and mean-square exponentially stable. The criterion is formulated in terms of strict LMI, which can be easily solved by standard software. Finally, two examples are given to illustrate the effectiveness of proposed method.

*Notation.* $\mathbf{R}^n$ denotes the $n$-dimensional Euclidean space; $\mathbf{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $C_{n,d2} = C([-d_2, 0], \mathbf{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d_2, 0]$ into $\mathbf{R}^n$ with norm $\|\varphi(t)\|_{d2} = \sup_{-d2\le s\le 0}\|\varphi(s)\|$. $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ is a probability space, $\Omega$ is the sample space, $\mathbf{F}$ is the $\sigma$-algebra of subsets of the sample space, and $\mathbf{P}$ is the probability measure on $\mathbf{F}$. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathbf{P}$. The superscript "$*$" denotes the term that is induced by symmetry.

## 2. Problem Formulation and Preliminaries

Let $\{r_t, t \ge 0\}$ be a continuous-time Markov process with a right continuous trajectory taking values in a finite set $\mathbf{S} = \{1, 2, \ldots, s\}$ with transition probability matrix $\Lambda = \{\pi_{ij}\}$ given by

$$\mathbf{P}\left[r_{t+\Delta t} = j \mid r_t = i\right] = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } j \ne i \\ 1 + \pi_{ij}\Delta + o(\Delta) & \text{if } j = i, \end{cases} \tag{1}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, $\pi_{ij} > 0$, $j \ne i$, and $\pi_{ii} = -\sum_{j\ne i}\pi_{ij}$ for each $i \in \mathbf{S}$.

Fix a probability space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ and consider the singular stochastic system with Markovian switching as follows:

$$\begin{aligned} Edx(t) &= \left[A(r_t)x(t) + A_d(r_t)(x - d(t, r_t))\right]dt \\ &\quad + \left[C(r_t)x(t) + D(r_t)(x - d(t, r_t))\right]d\varpi(t) \\ x(t) &= \phi(t), \quad \forall t \in [-d, t], \end{aligned} \tag{2}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $\varpi(t)$ is a scalar Brownian motion defined on a probability space. $E \in \mathbf{R}^{n\times n}$ may be singular; we assume that rank $E = r \le n$. $A(r_t)$, $A_d(r_t)$, $C(r_t)$, and $D(r_t)$ are real constant matrices of appropriate dimensions.

For notational simplicity, in the sequel, for each possible $r_t \in i$, $i \in \mathbf{S}$, a matrix $M(r_t)$ will be denoted by $M_i$; for example, $A(r_t)$ is denoted by $A_i$, $A_d(r_t)$ is denoted by $A_{di}$, and so on.

The mode-dependent time-varying delay $d_i(t)$ is a time-varying continuous function that satisfies

$$0 \le d_i(t) \le d_i, \qquad \dot{d}_i(t) \le \mu_i < 1, \quad \forall i \in \mathbf{S}, \tag{3}$$

where $d_i$ and $\mu_i$ are constants; $\phi(t) \in [-d, 0]$ is the initial condition of continuous state with $d = \max\{d_i, i \in \mathbf{S}\}$.

The following preliminary assumption is made for system (2).

*Assumption 1.* For singular stochastic hybrid system (2), the following condition is satisfied:

$$\operatorname{rank}(E) = \operatorname{rank}\left[E \;\; C_i \;\; D_i\right], \quad i \in \mathbf{S}. \tag{4}$$

The objective of this paper is to present a criterion for mean-square exponential stability of system (2). It should be pointed out that, for simplicity only, we do not consider uncertainties and disturbance input in our models. The proposed method can also be easily extended to systems with multiple and distributed delays.

The following definition and lemma will be used.

*Definition 2* (see [22]). (1) System (2) is said to be regular and impulse-free for any time delay $d_i(t)$ satisfying (3), if the pairs $(E, A_i)$ and $(E, A_i + A_{di})$ are regular and impulse-free for each $i \in \mathbf{S}$.

(2) System (2) is said to be mean-square exponentially stable, if there exist scalars $\alpha > 0$ and $\beta > 0$ such that $E\{\|x(t)\|^2\} \le \alpha e^{-\beta t}\|\phi(t)\|^2_{d_2}$, $t > 0$.

(3) System (2) is said to be mean-square exponentially admissible, if it is regular, impulse-free, and mean-square exponentially stable.

**Lemma 3** (see [23]). *Suppose that a positive continuous function $f(t)$ satisfies*

$$\mathbf{E}\{f(t)\} \le \lambda_1 \mathbf{E}\left\{\sup_{t-d\le s\le t} f(t)\right\} + \lambda_2 e^{-\varepsilon t}, \tag{5}$$

*where $\varepsilon > 0$, $0 < \lambda_1 < 1$, $0 < \lambda_1 e^{\varepsilon\alpha} < 1$, $\lambda_2 > 0$, and $\alpha > 0$; then*

$$\mathbf{E}\{f(t)\} \le e^{-\varepsilon t}\|f(s)\|_d + \frac{\lambda_2 e^{-\varepsilon t}}{1 - \lambda_1 e^{\varepsilon\alpha}}. \tag{6}$$

**Lemma 4** (see [24]). *For any vectors $x, y \in \mathbf{R}^n$, scalar $\gamma > 0$, and matrix $P > 0$ with appropriate dimension, the following inequality is always satisfied:*

$$2x^T y \le \gamma x^T P^{-1} x + \frac{1}{\gamma} y^T P y. \tag{7}$$

## 3. Stability of Singular Stochastic Hybrid Systems

In this section, we study the mean-square exponential stability of singular hybrid system (2).

**Theorem 5.** *For any delays $d_i(t)$ satisfying (3), the singular stochastic hybrid system (2) is mean-square exponentially admissible for $\delta > 0$, if there exist symmetric matrices $P_i > 0$, $Q_{ki} > 0$, $U_k > 0$, $V_k > 0$, $Q_k > 0$, $W_i > 0$, and $W > 0$ and matrices $S$, $M_{kl}$, $k = 1, 2, \ldots N$, $l = 0, 1, \ldots N + 1$, with appropriate dimensions such that for each $i \in \mathbf{S}$,*

$$\Theta_i = \begin{bmatrix} \Omega_i & \widehat{\Xi}_{i12} & \widehat{\Xi}_{i13} & \widehat{\Xi}_{i14} & \widehat{\Xi}_{i15} \\ * & -\delta\sum_{k=1}^{N} U_k & 0 & 0 & 0 \\ * & * & -N_i & 0 & 0 \\ * & * & * & \widehat{\Xi}_{i44} & 0 \\ * & * & * & * & \widehat{\Xi}_{i55} \end{bmatrix} < 0, \tag{8}$$

$$\sum_{j=1}^{s}\pi_{ij}Q_{kj} \le Q_k, \quad k = 1, 2, \ldots N, \tag{9}$$

$$\sum_{j=1}^{s}\pi_{ij}W_j \le W, \tag{10}$$

*where*

$$
\begin{gathered}
M_k = \left[M_{k0}^T \;\; M_{k1}^T \;\; \cdots \;\; M_{k(N-1)}^T \;\; M_{kN}^T \;\; M_{k(N+1)}^T\right]^T, \\
N_i = E^T P_i E + \delta \sum_{k=1}^{N} V_k, \\
\widehat{\Xi}_{i12} = \left[\delta \sum_{k=1}^{N} U_k A_i \;\; 0 \;\; \cdots \;\; 0 \;\; \delta \sum_{k=1}^{N} U_k A_{di}\right]^T, \\
\widehat{\Xi}_{i13} = \left[N_i^T C_i \;\; 0 \;\; \cdots \;\; 0 \;\; N_i^T D_i\right]^T, \\
\widehat{\Xi}_{i14} = \widehat{\Xi}_{i15} = \left[M_1 \;\; M_2 \;\; \cdots \;\; M_N\right], \\
\widehat{\Xi}_{i44} = -\operatorname{diag}\left(V_1, V_2, \ldots, V_N\right), \\
\widehat{\Xi}_{i55} = -\operatorname{diag}\left(U_1, U_2, \ldots, U_N\right),
\end{gathered}
\tag{11}
$$

$$
\Omega_{i1} = \begin{bmatrix}
\Xi_{i11} & -M_{10}E + M_{20}E & -M_{20}E + M_{30}E & \cdots & \Xi_{i1N} & -M_{N0} & \Xi_{i1(N+2)} \\
* & \Xi_{i22} & -M_{21}E + M_{31}E & \cdots & \Xi_{i2N} & -M_{N1} & \Xi_{i2(N+2)} \\
* & * & \Xi_{i33} & \cdots & \Xi_{3N} & -M_{NN} & \Xi_{i3(N+2)} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & \cdots & \Xi_{NN} & -M_{N(N-1)} & \Xi_{iN(N+2)} \\
* & * & * & \cdots & * & \Xi_{(N+1)(N+1)} & \Xi_{(N+1)(N+2)} \\
* & * & * & \cdots & * & * & \Xi_{(N+2)(N+2)}
\end{bmatrix}
$$

*with*

$$
\begin{aligned}
\Xi_{i11} = {} & A_i^T\left(P_i E + RS^T\right) + \left(E^T P_i + SR^T\right) A_i \\
& + \sum_{j=1}^{s} \pi_{ij} E^T P_j E \\
& + Q_{1i} + W_i + \delta \sum_{k=1}^{N} Q_k + dW + M_{10}E + E^T M_{10}^T, \\
\Xi_{i1N} = {} & -M_{(N-1)0}E + M_{N0}E, \\
\Xi_{i1(N+2)} = {} & \left(E^T P_i + SR^T\right) A_{di} + E^T M_{1(N+1)}^T, \\
\Xi_{i22} = {} & Q_{2i} - Q_{1i} - M_{11}E - E^T M_{11}^T + M_{21}E + E^T M_{21}^T, \\
\Xi_{i2N} = {} & -M_{(N-1)1}E + M_{N1}E, \\
\Xi_{i2(N+2)} = {} & -E^T M_{1(N+1)}^T + E^T M_{2(N+1)}^T, \\
\Xi_{i33} = {} & Q_{3i} - Q_{2i} - M_{22}E - E^T M_{22}^T + M_{32}E + E^T M_{32}^T, \\
\Xi_{3N} = {} & -M_{(N-1)2}E + M_{N2}E, \\
\Xi_{NN} = {} & Q_{Ni} - Q_{(N-1)i} - M_{(N-1)(N-1)}E - E^T M_{(N-1)(N-1)}^T \\
& + M_{N(N-1)}E + E^T M_{N(N-1)}^T, \\
\Xi_{iN(N+2)} = {} & -E^T M_{(N-1)(N+1)}^T + E^T M_{N(N+1)}^T, \\
\Xi_{(N+1)(N+1)} = {} & -Q_{Ni} - M_{NN}E - E^T M_{NN}^T, \\
\Xi_{i(N+1)(N+2)} = {} & -E^T M_{N(N+1)}^T + E^T M_{(N+1)(N+1)}^T, \\
\Xi_{(N+2)(N+2)} = {} & -\left(1 - \mu_i\right) W_i,
\end{aligned}
\tag{12}
$$

*and* $R \in \mathbf{R}^{n\times(n-r)}$ *is any matrix with full rank and satisfying* $E^T R = 0$.

*Proof.* We first show that the system (2) is regular and impulse-free.

Since rank $E = r \le n$, there exist nonsingular matrices $G$ and $H$ such that

$$
\widehat{E} = GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{13}
$$

Then, $R$ can be parameterized as $R = G^T \left[\begin{smallmatrix} 0 \\ \Phi \end{smallmatrix}\right]$, where $\Phi \in \mathbf{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

Define

$$
\begin{gathered}
\widehat{A}_i = GA_iH = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \\
G^{-T} P_i G^{-1} = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix}, \qquad H^T S = \begin{bmatrix} S_{11} \\ S_{21} \end{bmatrix}.
\end{gathered}
\tag{14}
$$

Premultiplying and postmultiplying $\Xi_{i11} < 0$ by $H^T$ and $H$, respectively, we have

$$
A_{i22}^T \Phi S_{21}^T + S_{21} \Phi^T A_{i22} < 0 \tag{15}
$$

and thus $A_{i22}$ is nonsingular. Otherwise, supposing $A_{i22}$ is singular, there must exist a nonzero vector $\zeta \in \mathbf{R}^{n-r}$

which ensures that $A_{i22}\zeta = 0$. Then, we can conclude that $\zeta^T(A_{i22}^T\Phi S_{21}^T + S_{21}\Phi^T A_{i22})\zeta = 0$, and this contradicts (15). So $A_{i22}$ is nonsingular, and thus the pair $(\widehat{E}, \widehat{A}_i)$ is regular and impulse-free for each $i \in \mathbf{S}$. Since $\det(sE - A_i) = \det(s\widehat{E} - \widehat{A}_i)$, we can easily see that the pair $(E, A_i)$ is regular and impulse-free. On the other hand, premultiplying and postmultiplying $\Omega_{i1} < 0$ by $[I \;\; I \;\; \cdots \;\; I]$ and $[I \;\; I \;\; \cdots \;\; I]^T$, respectively, we have

$$\begin{aligned}&(A_i + A_{di})^T \left(P_i E + RS^T\right) \\ &\quad + \left(E^T P_i + SR^T\right)(A_i + A_{di}) \\ &\quad + \sum_{j=1}^{s} \pi_{ij} E^T P_j E < 0.\end{aligned} \tag{16}$$

This implies that the pair $(E, A_i + A_{di})$ is regular and impulse-free for each $i \in \mathbf{S}$ according to Theorem 10.1 of [25]. Then, by Definition 2, system (2) is regular and impulse part.

In the following, we will prove the mean-square exponential stability of system (2). Define a new process $\{(x_t, r_t), t \geq 0\}$, by $x_t(s) = x(t+s)$, $-2d \leq s \leq 0$; then $\{(x_t, r_t), t \geq 0\}$ is a Markov process with initial state $(\phi(\cdot), r_0)$. Choose a Lyapunov function candidate as follows:

$$\begin{aligned}V(x_t, r_t, t) &= x^T(t)\left(E^T P(r_t) + SR^T\right) Ex(t) \\ &\quad + \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} x^T(\alpha) Q_k(r_t) x(\alpha)\, d\alpha \\ &\quad + \int_{t-d_i(t)}^{t} x^T(\alpha) W(r_t) x(\alpha)\, d\alpha \\ &\quad + \sum_{k=1}^{N} \int_{-k\delta}^{-(k-1)\delta} \int_{t+\beta}^{t} f^T(\alpha) U_k f(\alpha)\, d\alpha\, d\beta \\ &\quad + \sum_{k=1}^{N} \int_{-k\delta}^{-(k-1)\delta} \int_{t+\beta}^{t} g^T(\alpha) V_k g(\alpha)\, d\alpha\, d\beta \\ &\quad + \sum_{k=1}^{N} \int_{-k\delta}^{-(k-1)\delta} \int_{t+\beta}^{t} x^T(\alpha) Q_k x(\alpha)\, d\alpha\, d\beta \\ &\quad + \int_{-d}^{0} \int_{t+\beta}^{t} x^T(\alpha) W x(\alpha)\, d\alpha\, d\beta,\end{aligned} \tag{17}$$

where $\delta = \tau/N$ and $N$ is the number of divisions of the interval $[-d, 0]$,

$$\begin{aligned}f(t) &= \left[A(r_t) x(t) + A_d(r_t)(t - d_i(t))\right], \\ g(t) &= \left[C(r_t) x(t) + D(r_t)(t - d_i(t))\right].\end{aligned} \tag{18}$$

By Itô's Lemma, we have

$$\begin{aligned}dV(x_t, r_t, t) &= LV(x_t, r_t, t)\, dt \\ &\quad + 2x^T(t) E^T P(r_t) Eg(t)\, d\varpi(t),\end{aligned} \tag{19}$$

where

$$\begin{aligned}&LV(x_t, i, t) \\ &\leq 2x^T(t)\left(E^T P_i + SR^T\right) f(t) \\ &\quad + x^T(t)\left[\sum_{j=1}^{s} \pi_{ij} E^T P_j E\right] x(t) \\ &\quad + g^T(t) E^T P_i E g(t) \\ &\quad + \sum_{k=1}^{N} x^T(t - (k-1)\delta) Q_{ki} x(t - (k-1)\delta) \\ &\quad - \sum_{k=1}^{N} x^T(t - k\delta) Q_{ki} x(t - k\delta) \\ &\quad + \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} x^T(\alpha) \left(\sum_{j=1}^{s} \pi_{ij} Q_{kj}\right) x(\alpha)\, d\alpha \\ &\qquad + x^T(t) W_i x(t) \\ &\qquad - (1 - \mu_i) x^T(t - d_i(t)) W_i x(t - d_i(t)) \\ &\quad + \sum_{j=1}^{s} \pi_{ij} \int_{t-d_j(t)}^{t} x^T(\alpha) W_j x(\alpha)\, d\alpha \\ &\quad + \delta \sum_{k=1}^{N} f^T(t) U_k f(t) \\ &\quad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} f^T(\alpha) U_k f(\alpha)\, d\alpha \\ &\quad + \delta \sum_{k=1}^{N} g^T(t) V_k g(t) \\ &\quad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} g^T(\alpha) V_k g(\alpha)\, d\alpha \\ &\quad + \delta \sum_{k=1}^{N} x^T(t) Q_k x(t) \\ &\quad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} x^T(\alpha) Q_k x(\alpha)\, d\alpha \\ &\qquad + dx^T(t) W x(t) \\ &\qquad - \int_{t-d}^{t} x^T(\alpha) W x(\alpha)\, d\alpha.\end{aligned} \tag{20}$$

From (19), for any appropriately dimensioned matrix $M_k$, $k = 1, 2, \ldots N$, we have

$$\begin{aligned}&2\sum_{k=1}^{N} \zeta^T(t) M_k \\ &\quad \times \Bigg[Ex(t - (k-1)\delta) - Ex(t - k\delta) \\ &\qquad - \int_{t-k\delta}^{t-(k-1)\delta} f(\alpha)\, d\alpha - \int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\Bigg] = 0,\end{aligned} \tag{21}$$

where

$$\zeta(t) = \left[x^T(t) \;\; x^T(t-\delta) \;\; x^T(t-2\delta) \;\; \cdots \;\; x^T(t-(N-1)\delta) \;\; x^T(t-N\delta) \;\; x^T(t-d_i(t))\right]^T. \tag{22}$$

From Lemma 4, we obtain

$$\begin{aligned} 2\zeta^T(t) M_k \int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha) &\le \zeta^T(t) M_k V_k^{-1} M_k^T \zeta(t) \\ &+ \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right)^T V_k \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right). \end{aligned} \tag{23}$$

Thus, it follows from (20)–(23) that

$$\begin{aligned} &LV(x_t, i, t) \\ &\quad \le 2x^T(t)\left(E^T P_i + SR^T\right) f(t) \\ &\qquad + x^T(t)\left[\sum_{j=1}^{s} \pi_{ij} E^T P_j E\right] x(t) + g^T(t) E^T P_i E g(t) \\ &\qquad + \sum_{k=1}^{N} x^T(t-(k-1)\delta) Q_{ki} x(t-(k-1)\delta) \\ &\qquad - \sum_{k=1}^{N} x^T(t-k\delta) Q_{ki} x(t-k\delta) \\ &\qquad\quad + x^T(t) W_i x(t) \\ &\qquad\quad - (1-\mu_i) x^T(t-d_i(t)) W_i x(t-d_i(t)) \\ &\qquad + \delta \sum_{k=1}^{N} f^T(t) U_k f(t) + \delta \sum_{k=1}^{N} g^T(t) V_k g(t) \\ &\qquad + \delta \sum_{k=1}^{N} x^T(t) Q_k x(t) + d x^T(t) W x(t) \\ &\qquad + 2\sum_{k=1}^{N} \zeta^T(t) M_k \left[Ex(t-(k-1)\delta) - Ex(t-k\delta)\right] \\ &\qquad - 2\sum_{k=1}^{N} \zeta^T(t) M_k \int_{t-k\delta}^{t-(k-1)\delta} f(\alpha)\, d\alpha \\ &\qquad + \sum_{k=1}^{N} \zeta^T(t) M_k V_k^{-1} M_k^T \zeta(t) \\ &\qquad + \delta \sum_{k=1}^{N} \zeta^T(t) M_k U_k^{-1} M_k^T \zeta(t) \\ &\qquad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} g^T(\alpha) V_k g(\alpha)\, d\alpha \\ &\qquad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} \left[\zeta^T(t) M_k + f^T(\alpha) U_k\right] \\ &\qquad\qquad \times U_k^{-1}\left[\zeta^T(t) M_k + f^T(\alpha) U_k\right]^T d\alpha \\ &\qquad + \sum_{k=1}^{N} \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right)^T \\ &\qquad\qquad \times V_k \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right) \\ &\qquad + \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} x^T(\alpha) \left(\sum_{j=1}^{s} \pi_{ij} Q_{kj}\right) x(\alpha)\, d\alpha \\ &\qquad + \sum_{j=1}^{s} \pi_{ij} \int_{t-d_j(t)}^{t} x^T(\alpha) W_j x(\alpha)\, d\alpha \\ &\qquad - \sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} x^T(\alpha) Q_k x(\alpha)\, d\alpha \\ &\qquad\qquad - \int_{t-d}^{t} x^T(\alpha) W x(\alpha)\, d\alpha. \end{aligned} \tag{24}$$

Noting $\pi_{ij} > 0$ for $j \ne i$ and $\pi_{ii} < 0$, then we have

$$\begin{aligned} &\sum_{j=1}^{s} \pi_{ij} \int_{t-d_j(t)}^{t} x^T(\alpha) W_j x(\alpha)\, d\alpha \\ &\qquad \le \int_{t-d}^{t} x^T(\alpha) \left(\sum_{j=1, j\ne i}^{s} \pi_{ij} W_j\right) x(\alpha)\, d\alpha. \end{aligned} \tag{25}$$

Note

$$\begin{aligned} &\mathbf{E}\left\{\sum_{k=1}^{N} \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right)^T \right. \\ &\qquad \left. \times V_k \left(\int_{t-k\delta}^{t-(k-1)\delta} g(\alpha)\, d\varpi(\alpha)\right)\right\} \\ &\quad = \mathbf{E}\left\{\sum_{k=1}^{N} \int_{t-k\delta}^{t-(k-1)\delta} g^T(\alpha) V_k g(\alpha)\, d\alpha\right\}. \end{aligned} \tag{26}$$

From (9)-(10) and (24)–(26), we have

$$\mathbf{E}LV(x_t,i,t) \le \mathbf{E}\left\{\zeta^T(t)\left[\Omega_{i1}+\Omega_{i2}\right]\zeta(t)\right\}, \tag{27}$$

where

$$\begin{aligned}\Omega_{i2} = {} & \delta\sum_{k=1}^{N}\Gamma_{1i}^T U_k \Gamma_{1i} + \Gamma_{2i}^T E^T P_i E \Gamma_{2i} \\ & + \delta\sum_{k=1}^{N}\Gamma_{2i}^T V_k \Gamma_{2i} \\ & + \sum_{k=1}^{N} M_k V_k^{-1} M_k^T + \delta\sum_{k=1}^{N} M_k U_k^{-1} M_k^T\end{aligned} \tag{28}$$

with

$$\begin{aligned}\Gamma_{1i} &= \left[A_i \;\; 0 \;\; 0 \;\; \cdots \;\; 0 \;\; A_{di}\right], \\ \Gamma_{2i} &= \left[C_i \;\; 0 \;\; 0 \;\; \cdots \;\; 0 \;\; D_i\right].\end{aligned} \tag{29}$$

From (8) and using Schur complement lemma, it is easy to see that there exists a scalar $\rho > 0$ such that for each $i \in \mathbf{S}$,

$$\mathbf{E}LV(x_t,i,t) \le -\rho\mathbf{E}\|x(t)\|^2. \tag{30}$$

Since $A_{i22}$ is nonsingular for each $i \in \mathbf{S}$, we set $\overline{G} = \begin{bmatrix} I_r & A_{i12}A_{i22}^{-1} \\ 0 & A_{i22}^{-1}\end{bmatrix} G$.

It is easy to see that

$$\overline{G}EH = \begin{bmatrix} I_r & 0 \\ 0 & 0\end{bmatrix}, \qquad \overline{G}A_iH = \begin{bmatrix} \widehat{A}_{i11} & 0 \\ \widehat{A}_{i21} & I\end{bmatrix}, \tag{31}$$

where $\widehat{A}_{i11} = A_{i11} - A_{i12}A_{i22}^{-1}A_{i21}$ and $\widehat{A}_{i21} = A_{i22}^{-1}A_{i21}$. Define

$$\begin{gathered}\overline{P}_i = P_iE + RS^T, \qquad \overline{G}A_{di}H = \begin{bmatrix} A_{id1} & A_{id2} \\ A_{id3} & A_{id4}\end{bmatrix}, \\ \overline{G}^{-T}\overline{P}_iH = \begin{bmatrix} \widehat{P}_{i11} & \widehat{P}_{i12} \\ \widehat{P}_{i21} & \widehat{P}_{i22}\end{bmatrix}, \qquad H^TW_iH = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22}\end{bmatrix}, \\ H^{-1}C_iH = \begin{bmatrix} C_{i11} & C_{i12} \\ 0 & 0\end{bmatrix}, \qquad H^{-1}D_iH = \begin{bmatrix} D_{i11} & D_{i12} \\ 0 & 0\end{bmatrix}, \\ \zeta(t) = \begin{bmatrix}\zeta_1(t) \\ \zeta_2(t)\end{bmatrix} = H^{-1}x(t).\end{gathered} \tag{32}$$

From $E^T\overline{P}_i = \overline{P}_i^T E$, we can deduce that $\widehat{P}_{i11} > 0$ and $\widehat{P}_{i12} = 0$ for each $i \in \mathbf{S}$.

Then, for each $i \in \mathbf{S}$, system (2) is equivalent to

$$\begin{aligned} d\zeta_1(t) = {} & \left[\widehat{A}_{i11}\zeta_1(t) + A_{id1}\zeta_1(t-d_i(t)) \right. \\ & \left. + A_{id2}\zeta_2(t-d_i(t))\right]dt \\ & + \left[C_{i11}\zeta_1(t) + C_{i12}\zeta_2(t) \right. \\ & \left. + D_{i11}\zeta_1(t-d_i(t)) \right. \\ & \left. + D_{i12}\zeta_2(t-d_i(t))\right]d\varpi(t), \\ -\zeta_2(t) = {} & \widehat{A}_{i21}\zeta_1(t) + A_{id3}\zeta_1(t-d_i(t)) \\ & + A_{id4}\zeta_2(t-d_i(t)), \\ \zeta(t) = {} & \psi(t) = H^{-1}\phi(t), \quad t \in [-d_2, 0].\end{aligned} \tag{33}$$

To prove the mean-square exponential stability, we define a new function as

$$\overline{V}(x_t,r_t,t) = e^{\varepsilon t}V(x_t,r_t,t), \tag{34}$$

where $\varepsilon > 0$. By Dynkin's formula [26], we get that for each $i \in \mathbf{S}$,

$$\begin{aligned}\mathbf{E}\left\{\overline{V}(x_t,r_t,t)\right\} \le {} & \mathbf{E}\left\{\overline{V}(x_0,r_0,0)\right\} \\ & + \mathbf{E}\left\{\int_0^t e^{\varepsilon s}\left[\varepsilon V(x_t,r_t,t) - \rho\|x(s)\|^2\right]ds\right\}.\end{aligned} \tag{35}$$

By using the similar method of [27], it can be seen from (17), (34), and (35) that, if $\varepsilon$ is chosen small enough, a constant $\kappa > 0$ can be found such that

$$\begin{aligned}& \min_{i\in\mathbf{S}}\left\{\lambda_{\min}\left(\widehat{P}_{i11}\right)\right\}\mathbf{E}\left\{\|\zeta_1(t)\|^2\right\} \\ & \le \mathbf{E}\left\{x^T(t)E^T\overline{P}_ix(t)\right\} \\ & \le \mathbf{E}\left\{V(x_t,r_t,t)\right\} \le \kappa e^{-\varepsilon t}\|\phi(t)\|_d^2.\end{aligned} \tag{36}$$

Hence, for any $t > 0$

$$\mathbf{E}\left\{\|\zeta_1(t)\|^2\right\} \le \eta e^{-\varepsilon t}\|\phi(t)\|_d^2, \tag{37}$$

where $\eta = \left(\min_{i\in\mathbf{S}}\{\lambda_{\min}(\widehat{P}_{i11})\}\right)^{-1}\kappa$. Define

$$e(t) = \widehat{A}_{i21}\zeta_1(t) + A_{id3}\zeta_1(t-d_i(t)). \tag{38}$$

Then, from (37), there exists a constant $m > 0$ such that when $t > 0$

$$\mathbf{E}\left\{\|e(t)\|^2\right\} \le me^{-\varepsilon t}\|\phi(t)\|_d^2. \tag{39}$$

Define a function as follows:

$$J(t) = \zeta_2^T(t)W_{22}\zeta_2(t) - \zeta_2^T(t-d_i(t))W_{22}\zeta_2(t-d_i(t)). \tag{40}$$

By premultiplying the second equation of (33) with $\zeta_2^T(t)\widehat{P}_{i22}^T$, we get

$$\begin{aligned}&\zeta_2^T(t)\widehat{P}_{i22}^T\zeta_2(t)+\zeta_2^T(t)\widehat{P}_{i22}^T A_{id4}\zeta_2(t-d_i(t))\\&\quad+\zeta_2^T(t)\widehat{P}_{i22}^T e(t)=0.\end{aligned}\tag{41}$$

Adding (41) to (40) yields that

$$\begin{aligned}J(t)&=\zeta_2^T(t)\left(\widehat{P}_{i22}^T+\widehat{P}_{i22}+W_{22}\right)\zeta_2(t)\\&\quad+2\zeta_2^T(t)\widehat{P}_{i22}^T A_{id4}\zeta_2(t-d_i(t))\\&\quad-\zeta_2^T(t-d_i(t))W_{22}\zeta_2(t-d_i(t))+2\zeta_2^T(t)\widehat{P}_{i22}^T e(t)\\&\le\begin{bmatrix}\zeta_2(t)\\ \zeta_2(t-d_i(t))\end{bmatrix}^T\\&\quad\times\begin{bmatrix}\widehat{P}_{i22}^T+\widehat{P}_{i22}+W_{22} & \widehat{P}_{i22}^T A_{id4}\\ * & -W_{22}\end{bmatrix}\begin{bmatrix}\zeta_2(t)\\ \zeta_2(t-d_i(t))\end{bmatrix}\\&\quad+\tau_1\zeta_2^T(t)\zeta_2(t)+\tau_1^{-1}e^T(t)\widehat{P}_{i22}\widehat{P}_{i22}^T e(t),\end{aligned}\tag{42}$$

where $\tau_1$ is any positive scalar. Premultiplying and postmultiplying $\Omega_{i1}<0$ by $\mathrm{diag}(H^T\ \cdots\ H^T)$and $\mathrm{diag}(H\ \cdots\ H)$, respectively, we get a constant $\tau_2>0$ such that

$$\begin{bmatrix}\widehat{P}_{i22}^T+\widehat{P}_{i22}+W_{22} & \widehat{P}_{i22}^T A_{id4}\\ * & -W_{22}\end{bmatrix}\le-\begin{bmatrix}\tau_2 I & 0\\ 0 & 0\end{bmatrix}.\tag{43}$$

Since $\tau_1$ can be chosen arbitrarily, $\tau_1$ is chosen small enough such that $\tau_2-\tau_1>0$. Then, we can always find a scalar $\tau_3>1$ such that

$$W_{22}-(\tau_1-\tau_2)I\ge\tau_3 W_{22}.\tag{44}$$

From (40), (42), and (43), we get

$$\begin{aligned}&\zeta_2^T(t)W_{22}\zeta_2(t)-\zeta_2^T(t-d_i(t))W_{22}\zeta_2(t-d_i(t))\\&\quad\le-\tau_2\zeta_2^T(t)\zeta_2(t)+\tau_1\zeta_2^T(t)\zeta_2(t)\\&\quad+\tau_1^{-1}e^T(t)\widehat{P}_{i22}\widehat{P}_{i22}^T e(t).\end{aligned}\tag{45}$$

It is easy to see that the above inequality and (44) imply

$$\begin{aligned}\tau_3\zeta_2^T(t)W_{22}\zeta_2(t)&\le\zeta_2^T(t)\left(W_{22}-(\tau_1-\tau_2)I\right)\zeta_2(t)\\&\le\zeta_2^T(t-d_i(t))W_{22}\zeta_2(t-d_i(t))\\&\quad+\tau_1^{-1}e^T(t)\widehat{P}_{i22}\widehat{P}_{i22}^T e(t).\end{aligned}\tag{46}$$

Then, from above inequality, we have

$$\begin{aligned}\zeta_2^T(t)W_{22}\zeta_2(t)&\le\tau_3^{-1}\zeta_2^T(t-d_i(t))W_{22}\zeta_2(t-d_i(t))\\&\quad+(\tau_3\tau_1)^{-1}e^T(t)\widehat{P}_{i22}\widehat{P}_{i22}^T e(t).\end{aligned}\tag{47}$$

Then, from (39) and (47), we deduce that

$$\mathbf{E}\{f(t)\}\le\tau_3^{-1}\mathbf{E}\left\{\sup_{t-d\le s\le t}f(s)\right\}+\tau_4 e^{-\delta t},\tag{48}$$

where $0<\delta<\min\{\varepsilon,d^{-1}\ln\tau_3\}$, $f(t)=\zeta_2^T(t)W_{22}\zeta_2(t)$, and $\tau_4=(\tau_3\tau_1)^{-1}m\max_{i\in\mathbf{S}}\|W_{22}\|^2\|\phi(t)\|_d^2$.

Therefore, by Lemma 3, the above inequality yields that

$$\begin{aligned}\mathbf{E}\|\zeta_2(t)\|^2&\le\lambda_{\min}^{-1}(W_{22})\lambda_{\max}^{-1}(W_{22})e^{-\delta t}\|\zeta_2(t)\|_d^2\\&\quad+\frac{\lambda_{\min}^{-1}(W_{22})\tau_4 e^{-\delta t}}{1-\tau_3^{-1}e^{\delta d}},\end{aligned}\tag{49}$$

which means that the system (2) is mean-square exponentially 10 stable. This completes the proof. □

*Remark 6.* In Theorem 5, a delay-dependent criterion for the mean-square exponential admissibility of system (2) is derived in terms of strict LMI, which can be easily solved by standard software. In order to reduce conservatism, a new Lyapunov-Krasovskii functional is constructed base on the discretized Lyapunov functional method. The functional in (17) allows to take different weighing matrices on different subintervals, which will yield less conservative delay-dependent stability criterion. The conservatism will be reduced with $N$ increasing and will be illustrated via examples in the next section. The method can also be applied to this class of systems with interval time-varying delays by decomposing the lower bound of the time delay.

*Remark 7.* Specially, if there are no stochastic terms, the system (2) becomes a singular Markovian jump system with mode-dependent time-varying delay:

$$\begin{aligned}E\dot{x}(t)&=A(r_t)x(t)+A_d(r_t)(x-d(t,r_t))\\x(t)&=\phi(t),\quad\forall t\in[-d,t].\end{aligned}\tag{50}$$

Based on the proof of Theorem 5, we have the following corollary.

**Corollary 8.** *For any delays $d_i(t)$ satisfying* (3)*, the singular Markovian jump system* (50) *is mean-square exponentially admissible for $\delta>0$, if there exist matrices $P_i>0$, $Q_{ki}>0$, $U_k>0$, $Q_k>0$, $M_{kl}$, $k=1,2,\ldots N$, $l=0,1,\ldots N+1$, $W_i>0$, and $W>0$ and matrix $S$ with appropriate dimensions such that for each $i\in\mathbf{S}$,*

$$\begin{gathered}\Theta_i=\begin{bmatrix}\Omega_i & \widehat{\Xi}_{i12} & \widehat{\Xi}_{i14}\\ * & -\delta\sum_{k=1}^{N}U_k & 0\\ * & * & \widehat{\Xi}_{i55}\end{bmatrix}<0\\ \sum_{j=1}^{s}\pi_{ij}Q_{kj}\le Q_k,\quad k=1,2,\ldots N\\ \sum_{j=1}^{s}\pi_{ij}W_j\le W.\end{gathered}\tag{51}$$

*Remark 9.* Corollary 8 provides a new exponential stability criterion for continuous-time singular Markovian jump systems with mode-dependent time delays, which has only been studied by Balasubramiam et al. [28]. It should be

pointed out that, even in the case $N = 1$, our criterion still has less conservatism than the results in [28], which will be illustrated via a numerical example. The reduced conservatism of Corollary 8 comes from the matrices $Q_{ki}$ and $W_i$, which are selected to be mode dependent in our paper.

## 4. Numerical Examples

*Example 1.* Consider the system (50) with two modes and the parameters as follows:

$$A_1 = \begin{bmatrix} -1.5 & -2.6 \\ -2.6 & -1.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1.8 & -2.7 \\ -2.7 & -1.8 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -1.2 & 0.7 \\ 0.7 & -1.2 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -1.3 & 0.6 \\ 0.6 & -1.3 \end{bmatrix}, \tag{52}$$
$$\Lambda = \begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix}, \qquad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For given $\mu_1 = 0.4$, $\mu_2 = 0.6$, and $d_1 = d_2 = d$, the comparison results of the maximum upper bounds $d$ of the time delay are given in Table 1. From Table 1, we can see that our results still have less conservatism than the results obtained in [28] even with the case $N = 1$.

*Example 2.* Consider the following singular stochastic hybrid system with mode-dependent time-varying delay:

$$A_1 = \begin{bmatrix} -1.1 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.9 & 0.0 \\ 0.0 & -1.0 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0.3 & 0.1 \\ 0.9 & 0.4 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.8 & 0 \\ -0.5 & -0.6 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0 \end{bmatrix}, \tag{53}$$
$$D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\Lambda = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, \qquad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

By Theorem 5, for given $d_1 = 0.1$ and $\mu_1 = \mu_2 = \mu$, the maximum allowable upper bound of $d_2$ to guarantee the system is mean-square exponentially stable for different $\mu$ is listed in Table 2.

From Table 2, we can see that the conservatism will be reduced with $N$ increasing.

## 5. Conclusions

In this paper, the problem of delay-dependent stability problem for a class of singular hybrid systems with mode-dependent time delay has been investigated. Based on the discretized Lyapunov functional method, a criterion which guarantees the systems is regular, impulse-free, and mean-square exponentially stable has been derived in terms of strict LMI. Two numerical examples have been given to show the effectiveness of proposed method.

Table 1: Maximum allowed delay bound $d$ via different method.

| Balasubramaniam et al. [28] | Corollary 8 $N = 1$ | Corollary 8 $N = 2$ | Corollary 8 $N = 3$ |
|---|---|---|---|
| 1.2741 | 2.1841 | 2.3936 | 2.4285 |

Table 2: Maximum allowed $d_2$ via different $\mu$.

| | $\mu$ | | |
|---|---|---|---|
| | 0.3 | 0.6 | 0.9 |
| Theorem 5, $N = 1$ | 7.836 | 4.397 | 1.037 |
| Theorem 5, $N = 2$ | 8.163 | 4.712 | 1.373 |
| Theorem 5, $N = 3$ | 8.311 | 4.890 | 1.514 |

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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