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Research Article

Local Fractional Variational Iteration Method for Local Fractional Poisson Equations in Two Independent Variables

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The local fractional Poisson equations in two independent variables that appear in mathematical physics involving the local fractional derivatives are investigated in this paper. The approximate solutions with the nondifferentiable functions are obtained by using the local fractional variational iteration method.

1. Introduction

As it is known the Poisson equation plays an important role in mathematical physics [1, 2]; that is, it describes the electrodynamics and intersecting interface (see, e.g., [3, 4] and the cited references therein). The solution of this equation was discussed by using different methods [5–9]. We notice that recently fractional Poisson equations based on fractional derivatives were analyzed in [10] and the existence and approximations of its solutions can be found in [11]. The Legendre wavelet method was used to find the fractional Poisson equation with Dirichlet boundary conditions [12]. In [13], the Dirichlet problem for the fractional Poisson's equation with Caputo derivatives was reported. Furthermore, the fractional Poisson equation based on the shifted Grünwald estimate was obtained in [14].

The variational iteration method structured in [15–17] was applied to deal with the following type of equations: Helmholtz [18], Burger's and coupled Burger's [19], Klein-Gordon [20], KdV [21], the oscillation [22], Schrodinger [23], reaction-diffusion [24], diffusion equation [25], Bernoulli equation [26], and others. The extended variational iteration method, called the fractional variational iteration method, was developed and applied to handle some fractional differential equations within the modified Riemann-Liouville derivative [27–31]. More recently, the local fractional variational iteration method, initiated in [32], was used to find the nondifferentiable solutions for the heat-conduction [32], Laplace [33], damped and dissipative wave [34], Helmholtz [35] and Fokker-Planck [36] equations, the wave equation on Cantor sets [37], and the fractal heat transfer in silk cocoon hierarchy [38] with local fractional derivative.

We mention that developing a numerical algorithm for local fractional differential equations on Cantor set is not straightforward. Thus, in this paper, we deal with the local fractional Poisson equation in two independent variables, namely,

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} = f(x,y), \quad (x,y) \in [0,+\infty] \times [0,l], \tag{1}$$

where the nondifferentiable functions $u(x,y)$ and $f(x,y)$ are adopted the local fractional differential operators and α denotes the fractal dimension, subject to the initial and boundary conditions

$$\begin{aligned} u(x,0) &= 0, \\ u(x,l) &= 0, \\ u(0,y) &= \varphi(y), \\ \frac{\partial^\alpha}{\partial x^\alpha} u(0,y) &= \phi(y). \end{aligned} \tag{2}$$

We recall that the local fractional Laplace equation presented in [33] is a special case of the local fractional Poisson equation with source term $f(x,y)=0$. Taking all the above thinks into account, the aim of this paper is to find the nondifferentiable solutions for (1) with different conditions by utilizing the local fractional variational iteration algorithm.

The paper has the following organization. In Section 2 the concepts of local fractional complex derivatives and integrals are briefly reviewed. In Section 3 the local fractional variational iteration method is recalled. In Section 4 the nondifferentiable solutions for local fractional Poisson equations are presented. Finally, Section 5 outlines the main conclusions.

2. A Brief Review of the Local Fractional Calculus

Definition 1 (see [32–38]). Let the function $f(x) \in C_\alpha(a,b)$, if it satisfies the condition

$$\left| f(x) - f(x_0) \right| < \varepsilon^\alpha, \tag{3}$$

where $|x - x_0| < \delta$, for $\varepsilon > 0$, $0 < \alpha < 1$, and $\varepsilon \in R$.

Definition 2 (see [32–38]). Let $f(x) \in C_\alpha(a,b)$. The local fractional derivative of $f(x)$ of order α is defined as

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \tag{4}$$

where

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1+\alpha)\left[f(x) - f(x_0)\right]. \tag{5}$$

The formulas of local fractional derivatives of special functions [37] used in the paper are as follows:

$$\begin{aligned} \frac{d^\alpha}{dx^\alpha} \frac{x^{n\alpha}}{\Gamma(1+n\alpha)} &= \frac{x^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)}, \quad n \in N, \\ D_x^{(\alpha)} a g(x) &= a D_x^{(\alpha)} g(x), \\ D_x^{(\alpha)} \left[D_x^{(\alpha)} f(x) \right] &= D_x^{(2\alpha)} f(x), \end{aligned} \tag{6}$$

where $g(x)$ is a local fractional continuous function, a is a constant, and N is a set of positive integers.

Definition 3 (see [32–38]). Let $f(x) \in C_\alpha[a,b]$. The local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined as

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha, \end{aligned} \tag{7}$$

where the partitions of the interval $[a,b]$ are denoted by (t_j, t_{j+1}), $j = 0,\ldots,N-1$, $t_0 = a$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$.

The formulas of local fractional integrals of special functions used in the paper are presented as follows [37]:

$$\begin{aligned} {}_0I_x^{(\alpha)} a g(x) &= a\, {}_0I_x^{(\alpha)} g(x), \\ {}_0I_t^{(\alpha)} \left(\frac{(t-s)^\alpha t^{n\alpha}}{\Gamma(1+\alpha)\Gamma(1+n\alpha)} \right) &= \frac{t^{(n+2)\alpha}}{\Gamma(1+(n+2)\alpha)}, \\ {}_0I_x^{(\alpha)} \frac{x^{(n-1)\alpha}}{\Gamma(1+(n-1)\alpha)} &= \frac{x^{n\alpha}}{\Gamma(1+n\alpha)}, \quad n \in N, \end{aligned} \tag{8}$$

where $g(x)$ is a local fractional continuous function, a is a constant, and N is a set of positive integers.

3. Analysis of the Method

The local fractional variational iteration method structured in [32] was applied to deal with the local fractional differential equations arising in mathematical physics (see, e.g., [33–38]). In this section, we introduce the idea of the local fractional variational iteration method.

Let us consider the local fractional operator equation in the form

$$L_\alpha u + N_\alpha u = g(t), \tag{9}$$

where L_α and N_α are linear and nonlinear local fractional operators, respectively, and $g(t)$ is the source term within the nondifferentiable function.

Local fractional variational iteration algorithm reads as

$$u_{n+1}(t) = u_n(t) + {}_0I_t^{(\alpha)} \left\{ \eta \left[L_\alpha u_n(s) + N_\alpha u_n(s) - g(s) \right] \right\}, \tag{10}$$

where η is a fractal Lagrange multiplier and $L_\alpha = \partial^{n\alpha}/\partial u^{n\alpha}$.

Therefore, a local fractional correction functional was structured as follows:

$$u_{n+1}(t) = u_n(t) + {}_0I_t^{(\alpha)} \left\{ \xi^\alpha \left[L_\alpha u_n(s) + N_\alpha \tilde{u}_n(s) - \tilde{g}(s) \right] \right\}, \tag{11}$$

where $\tilde{u}_n$ is considered as a restricted local fractional variation and η is a fractal Lagrange multiplier. That is, $\delta^\alpha \tilde{u}_n = 0$ [27, 30].

After the fractal Lagrangian multiplier is determined, for $n \geq 0$, the successive approximations u_{n+1} of the solution u

can be readily given by using any selective local fractional function u_0. Consequently, we obtain the solution in the following form:

$$u = \lim_{n\to\infty} u_n. \tag{12}$$

The local fractional variational method was compared with the fractional series expansion and decomposition technologies.

If $L_\alpha = \partial^{2\alpha}/\partial u^{2\alpha}$, then we have the local fractional variational iteration formula [32–34, 36, 37] as follows:

$$u_{n+1}(t) = u_n(t) - {}_0I_t^{(\alpha)} \times \left\{ \frac{(s-t)^\alpha}{\Gamma(1+\alpha)} \left[L_\alpha u_n(s) + N_\alpha u_n(s) - g(s) \right] \right\}. \tag{13}$$

The above formula plays an important role in dealing with the 2α-order local fractional differential equation with either linearity or nonlinearity.

4. The Nondifferentiable Solutions for Local Fractional Poisson Equations

In this section we investigate the nondifferentiable solutions for the local fractional Poisson equations in two independent variables with different initial-boundary conditions.

Example 1. We analyze the local fractional Poisson equation in the following form:

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} = \frac{y^\alpha}{\Gamma(1+\alpha)}, \tag{14}$$

subject to the initial and boundary conditions, namely,

$$u(x,0) = 0, \tag{15}$$

$$u(x,l) = 0, \tag{16}$$

$$u(0,y) = \frac{y^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{17}$$

$$\frac{\partial^\alpha}{\partial x^\alpha} u(0,y) = \sin_\alpha(y^\alpha). \tag{18}$$

In view of (17) and (18), we take the initial value given by

$$u_0(x,y) = \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \frac{x^\alpha}{\Gamma(1+\alpha)}. \tag{19}$$

From (13), the local fractional iteration procedure is given by

$$\begin{aligned} u_{n+1}(x,y) &= u_n(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_n(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_n(s,x)}{\partial y^{2\alpha}} - \frac{y^\alpha}{\Gamma(1+\alpha)} \right) \right\}. \end{aligned} \tag{20}$$

Making use of (19) and (20), we get the first approximation as follows:

$$\begin{aligned} u_1(x,y) &= u_0(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_0(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_0(s,x)}{\partial y^{2\alpha}} - \frac{y^\alpha}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \sum_{i=0}^{1} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{21}$$

The second approximation can be written as

$$\begin{aligned} u_2(x,y) &= u_1(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_1(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_1(s,x)}{\partial y^{2\alpha}} - \frac{y^\alpha}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \sum_{i=0}^{2} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{22}$$

The third approximation reads as

$$\begin{aligned} u_3(x,y) &= u_2(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_2(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_2(s,x)}{\partial y^{2\alpha}} - \frac{y^\alpha}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \sum_{i=0}^{3} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{23}$$

The fourth approximation is as follows:

$$\begin{aligned} u_4(x,y) &= u_3(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_3(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_3(s,x)}{\partial y^{2\alpha}} - \frac{y^\alpha}{\Gamma(1+\alpha)} \right) \right\} \\ &= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \sum_{i=0}^{4} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)} \\ &\quad \vdots \end{aligned} \tag{24}$$

and so on.

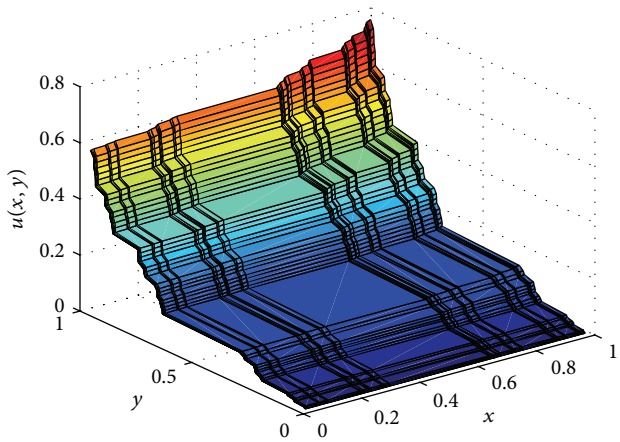

FIGURE 1: Plot of the nondifferentiable solution of (14) with the parameter $\alpha = \ln 2/\ln 3$.

Finally, by direct calculations we obtain

$$u_n(x, y) = \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha) \times \sum_{i=0}^{n}(-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \tag{25}$$

Hence, we report the nondifferentiable solution of (14)

$$\begin{aligned} u(x, y) &= \lim_{n\to\infty} u_n(x, y) \\ &= \frac{y^{3\alpha}}{\Gamma(1+3\alpha)} + \sin_\alpha(y^\alpha)\sin_\alpha(x^\alpha) \end{aligned} \tag{26}$$

and its graph is shown in Figure 1.

Example 2. Next we discuss the local fractional Poisson equations as

$$\frac{\partial^{2\alpha} u(x, y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x, y)}{\partial y^{2\alpha}} = E_\alpha(y^\alpha), \tag{27}$$

with the initial and boundary conditions given as follows:

$$\begin{aligned} u(x, 0) &= 0, \\ u(x, l) &= 0, \\ u(0, y) &= E_\alpha(y^\alpha), \\ \frac{\partial^\alpha}{\partial x^\alpha} u(0, y) &= \cos_\alpha(y^\alpha). \end{aligned} \tag{28}$$

In view of (13), the local fractional iteration procedure becomes

$$\begin{aligned} u_{n+1}(x, y) &= u_n(x, y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u(s, y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u(s, x)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha) \right) \right\}, \end{aligned} \tag{29}$$

where the initial value is given by

$$u_0(x, y) = E_\alpha(y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} \cos_\alpha(y^\alpha). \tag{30}$$

Making use of (29) and (30), the first approximation reads as follows:

$$\begin{aligned} u_1(x, y) &= u_0(x, y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_0(s, y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_0(s, x)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha) \right) \right\} \\ &= E_\alpha(y^\alpha) + \cos_\alpha(y^\alpha) \sum_{i=0}^{1}(-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{31}$$

The expression of the second approximation is as follows:

$$\begin{aligned} u_2(x, y) &= u_1(x, y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_1(s, y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_1(s, x)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha) \right) \right\} \\ &= E_\alpha(y^\alpha) + \cos_\alpha(y^\alpha) \sum_{i=0}^{2}(-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{32}$$

The third approximation becomes

$$\begin{aligned} u_3(x, y) &= u_2(x, y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_2(s, y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_2(s, x)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha) \right) \right\} \\ &= E_\alpha(y^\alpha) + \cos_\alpha(y^\alpha) \sum_{i=0}^{3}(-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}. \end{aligned} \tag{33}$$

The fourth approximation is given by

$$\begin{aligned} u_4(x, y) &= u_3(x, y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_3(s, y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_3(s, x)}{\partial y^{2\alpha}} - E_\alpha(y^\alpha) \right) \right\} \\ &= E_\alpha(y^\alpha) + \cos_\alpha(y^\alpha) \sum_{i=0}^{4}(-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)} \\ &\quad \vdots \end{aligned} \tag{34}$$

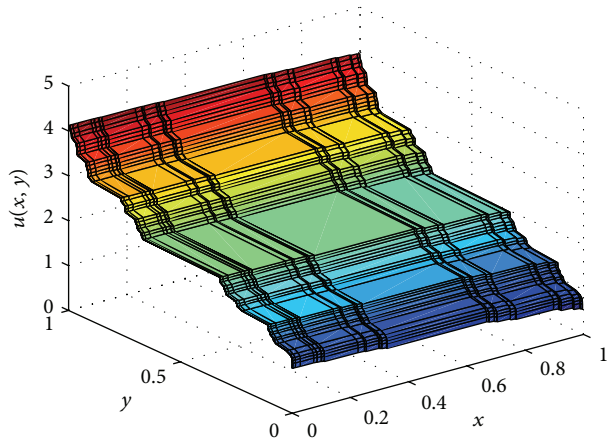

FIGURE 2: Plot of the nondifferentiable solution of (27) with the parameter $\alpha = \ln 2/\ln 3$.

Therefore, we get the nondifferentiable solution of (27)

$$u(x,y) = \lim_{n\to\infty} u_n(x,y) = E_\alpha(y^\alpha) + \cos_\alpha(y^\alpha)\sin_\alpha(x^\alpha), \tag{35}$$

and the corresponding graph is depicted in Figure 2.

Example 3. The next particular case is the local fractional Poisson equations as follows:

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} = \sin_\alpha(x^\alpha), \tag{36}$$

subject to the initial and boundary conditions

$$\begin{gathered} u(x,0) = 0, \\ u(x,l) = 0, \\ u(0,y) = \sin_\alpha(y^\alpha), \\ \frac{\partial^\alpha}{\partial x^\alpha} u(0,y) = E_\alpha(-y^\alpha). \end{gathered} \tag{37}$$

We start with the initial value as follows:

$$u_0(x,t) = \sin_\alpha(y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-y^\alpha). \tag{38}$$

The local fractional iteration procedure leads us to

$$\begin{aligned} u_{n+1}(x,y) &= u_n(x,y) \\ &\quad + {}_0I_x^{(\alpha)} \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \right. \\ &\qquad \left. \times \left(\frac{\partial^{2\alpha} u(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u(s,x)}{\partial y^{2\alpha}} - \sin_\alpha(y^\alpha) \right) \right\}. \end{aligned} \tag{39}$$

In view of (38) and (39), we obtain the following successive approximations:

$$\begin{aligned} u_1(x,y) &= u_0(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_0(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_0(s,x)}{\partial y^{2\alpha}} - \sin_\alpha(y^\alpha) \right) \right\} \\ &= \sin_\alpha(y^\alpha) + E_\alpha(-y^\alpha) \sum_{i=0}^{1} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}, \\ u_2(x,y) &= u_1(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_1(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_1(s,x)}{\partial y^{2\alpha}} - \sin_\alpha(y^\alpha) \right) \right\} \\ &= \sin_\alpha(y^\alpha) + E_\alpha(-y^\alpha) \sum_{i=0}^{2} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}, \\ u_3(x,y) &= u_2(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_2(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_2(s,x)}{\partial y^{2\alpha}} - \sin_\alpha(y^\alpha) \right) \right\} \\ &= \sin_\alpha(y^\alpha) + E_\alpha(-y^\alpha) \sum_{i=0}^{3} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}, \\ u_4(x,y) &= u_3(x,y) + {}_0I_x^{(\alpha)} \\ &\quad \times \left\{ \frac{(s-x)^\alpha}{\Gamma(1+\alpha)} \left(\frac{\partial^{2\alpha} u_3(s,y)}{\partial s^{2\alpha}} + \frac{\partial^{2\alpha} u_3(s,x)}{\partial y^{2\alpha}} - \sin_\alpha(y^\alpha) \right) \right\} \\ &= \sin_\alpha(y^\alpha) + E_\alpha(-y^\alpha) \sum_{i=0}^{4} (-1)^i \frac{x^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)} \\ &\quad \vdots \end{aligned} \tag{40}$$

and so on.

Thus, the nondifferentiable solution of (36) has the form

$$u(x,y) = \lim_{n\to\infty} u_n(x,y) = \sin_\alpha(y^\alpha) + E_\alpha(-y^\alpha)\sin_\alpha(x^\alpha), \tag{41}$$

and its graph is shown in Figure 3.

5. Conclusions

The local fractional operators started to be deeply investigated during the last few years. One of the major problems is

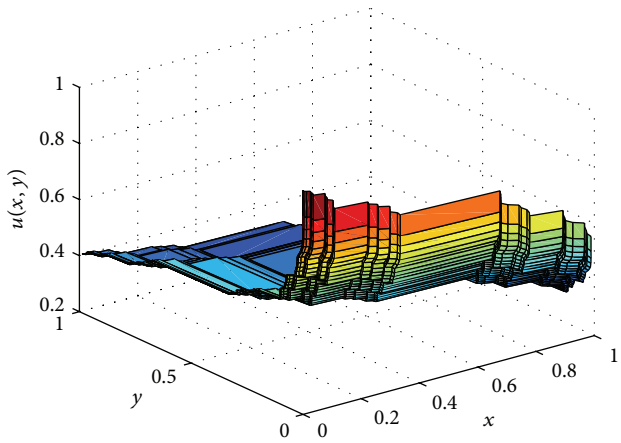

Figure 3: Plot of the nondifferentiable solution of (36) with the parameter $\alpha = \ln 2/\ln 3$.

to find new methods and techniques to solve some given important local fractional partial differential equations on Cantor set. In this line of thought we consider that three local fractional Poisson equations with differential initial and boundary values were solved by using the local fractional variational iteration method. The graphs of the nondifferentiable solutions were also obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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