Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 572863, 8 pages
http://dx.doi.org/10.1155/2014/572863

Research Article

New Rational Homoclinic and Rogue Waves for Davey-Stewartson Equation

Changfu Liu,[1] Chuanjian Wang,[2] Zhengde Dai,[3] and Jun Liu[4]

[1] *School of Mathematics, Wenshan University, Wenshan 663000, China*
[2] *School of Science, Kunming University of Science and Technology, Kunming 650031, China*
[3] *School of Mathematics and Statistics, Yunnan University, Kunming 650091, China*
[4] *Department of Mathematics and Information Science, Qujing Normal University, Qujing 655000, China*

Correspondence should be addressed to Changfu Liu; chfuliu@163.com

Received 4 February 2014; Revised 20 March 2014; Accepted 25 March 2014; Published 16 April 2014

Academic Editor: Weiguo Rui

A new method, homoclinic breather limit method (HBLM), for seeking rogue wave solution of nonlinear evolution equation is proposed. A new family of homoclinic breather wave solution, and rational homoclinic solution (homoclinic rogue wave) for DSI and DSII equations are obtained using the extended homoclinic test method and homoclinic breather limit method (HBLM), respectively. Moreover, rogue wave solution is exhibited as period of periodic wave in homoclinic breather wave approaches to infinite. This result shows that rogue wave can be generated by extreme behavior of homoclinic breather wave for higher dimensional nonlinear wave fields.

1. Introduction

In recent years, rogue waves, as a special type of solitary waves, has been triggered much interest in various physical branches, although there is no exact definition up to now. Rogue waves is a kind of waves that seems abnormal which is first observed in the deep ocean, it has been the subject of intensive research in oceanography [1, 2], optical fibres [3–5], superfluids [6], Bose-Einstein condensates, financial markets, and related fields [7–10]. A possible mechanism for the formation of rogue waves is associated with modulation instability [11–14].The mysteriousness of rogue wave events mainly lies in the phenomenon which appears out of nowhere and disappears without trace. As is known, there are some methods to seek rogue wave such as Darboux dressing technique, Hirota bilinear method. Based on Hirota bilinear equation of nonlinear evolution equation, for Schrödinger type complex systems, there are some effective techniques such as the Peregrine breather method (PB) [11], whose representation is mathematically a ratio of two polynomials, Ma solitons [4] (MS) and Akhmediev breather methods (ABs) [3]. The main difference between these methods is the test function to Hirota bilinear equation. The test functions of PB, MS, and ABs are $E_1(x,t) = e^{i\phi(x,t)}(1-((G(x,t)+iH(x,t))/D(x,t)))$; $E_2(x,t) = e^{i\phi(x,t)}((a_1\cosh(px)+a_2\cos(kt)+ia_3\sin(kt))/(b_1\cosh(px)+b_2\cos(kt)))$; and $E_3(x,t) = e^{i\phi(x,t)}((a_1\cosh(\alpha t)+a_2\cos(kx)+ia_3\sinh(\alpha t))/(b_1\cosh(\alpha t)+b_2\cos(kx)))$, respectively. Here $\phi(x,t)$ is real function and $G(x,t), H(x,t)$, and $D(x,t)$ are polynomials of (x,t), and $E_i(x,t)$, $i = 2,3$, may generate the rogue wave similar to E_1 as $k \to 0$. The above three methods have been successfully applied to complex system such as Hirota equation, Sasa-Satsuma equation, Davey-Stewartson equation, coupled Gross-Pitaevskii equation, coupled NLS Maxwell-Bloch equation, and coupled Schrödinger-Boussinesq equation [11–18].

In this work, we propose a homoclinic (heteroclinic) breather limit method for seeking rogue wave solution. We take $E(x,t) = e^{i\phi(x,t)}((e^{-p(x-\alpha t)}+a_1\cos(p_1(x-\beta t))+a_2e^{p(x-\alpha t)})/(e^{-p(x-\alpha t)}+a_3\cos(p_1(x-\beta t))+a_4e^{p(x-\alpha t)}))$ as a test function to Hirots bilinear equation. $E(x,t)$ can generate one or two rogue waves as $p_1 \to 0$. It is obvious that the E is different from E_2 and E_3, comparing with E_2, E_3, and E has more complicated structure. Now we consider the application of HBLM to Davey-Stewartson equation.

Davey-Stewartson (DS) equation is written as [19]

$$iu_t = -u_{xx} - \frac{1}{\alpha_0^2}u_{yy} - \frac{2\epsilon}{\alpha_0^2}|u|^2 u - \frac{2}{\alpha_0^2}uv,$$
$$v_{yy} - \alpha_0^2 v_{xx} - 2\alpha_0^2\epsilon\left(|u|^2\right)_{xx} = 0, \tag{1}$$

where $u : R_x \times R_y \times R_t^+ \rightarrow C$, $v : R_x \times R_y \times R_t^+ \rightarrow R$, and ϵ and α_0 are constants. DS equation was introduced in a paper by Davey and Stewartson (1974) to describe the evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow. The function $u(x, y, t)$ stands for the complex amplitude, and $v(x, y, t)$ describes the perturbation of the real velocity. DS equation is called the DSI as $\epsilon = 1$, $\alpha_0 = \pm 1$ and DSII as $\epsilon = 1$, $\alpha_0 = \pm i$. There are known results due to local well-posed, global existence and blow-up of some solutions, exact periodic soliton solutions, solitoff and dromion solutions [20–29]. Recently, homoclinic and heteroclinic tube solutions were obtained [29–33].

We consider DSI equation:

$$iu_t + u_{xx} + u_{yy} = -2|u|^2 u - 2uv,$$
$$v_{xx} - v_{yy} = -2\left(|u|^2\right)_{xx}, \tag{2}$$

and DSII equation:

$$iU_t + U_{xx} - U_{yy} = 2|U|^2 U + 2UV,$$
$$V_{xx} + V_{yy} = -2\left(|U|^2\right)_{xx}. \tag{3}$$

2. Homoclinic Breather and Rogue Wave Solution of DSI

Making transformation $u = (a/\sqrt{2})\exp(ia^2 t)Q$, $v = -\varphi/2$ and substituting it into (2), we can get

$$iQ_t + Q_{xx} + Q_{yy} = -a^2\left(|Q|^2 - 1\right)Q + Q\varphi,$$
$$\varphi_{xx} - \varphi_{yy} = 2a^2\left(|Q|^2\right)_{xx}, \tag{4}$$

where $Q = Q(x, y, t)$ is a complex function and φ is a real. By the dependent variable transformation

$$Q = \frac{G}{F}, \qquad \varphi = -4(\ln F)_{xx}, \tag{5}$$

with G being a complex and F being a real, then (4) can be converted into the form

$$iG_t F - iF_t G + G_{xx}F - 2G_x F_x + GF_{xx} + G_{yy}F$$
$$- 2G_y F_y + GF_{yy} - \left(a^2 + B\right)GF = 0, \tag{6}$$
$$2\left(F_{yy}F - F_y^2 - F_{xx}F + F_x^2\right) - BF^2 - a^2 GG^* = 0,$$

where B is an integration constant and an asterisk denotes the complex conjugation.

By means of the extended homoclinic test approach [33], we take the test function as follows:

$$G = e^{-p(x+y/2+\alpha t)} + a_1\cos\left(p_1\left(x + 2y - \alpha t\right)\right)$$
$$+ a_2 e^{p(x+y/2+\alpha t)},$$
$$F = e^{-p(x+y/2+\alpha t)} + a_3\cos\left(p_1\left(x + 2y - \alpha t\right)\right) \tag{7}$$
$$+ a_4 e^{p(x+y/2+\alpha t)},$$

where all of a_3, a_4, p, p_1, β, β_1, and α are real and a_1, a_2 are complex. Substituting (7) into (6) and equating the coefficients of all powers of $e^{jp(x+y/2+\alpha t)}\cos(p_1(x + 2y - \alpha t))$, $e^{jp(x+y/2+\alpha t)}\sin(p_1(x+2y-\alpha t))$ and $e^{\pm 2(p(x+y/2+\alpha t))}$ ($j = 0, \pm 1$) to zero, we can obtain a set of algebraic equations for p, p_1, β, β_1, α, and a_j, $j = 1, 2, 3, 4$, with

$$B = -a^2, \qquad \left(4pp_1 - ip_1\alpha\right)a_1 + \left(4pp_1 + ip_1\alpha\right)a_3 = 0,$$
$$\left(-ip_1\alpha - 4pp_1\right)a_4 a_1 + \left(ip_1\alpha - 4pp_1\right)a_3 a_2 = 0,$$
$$\left(ip\alpha + \frac{5p^2}{4} - 5p_1^2\right)a_1 + \left(\frac{5p^2}{4} - 5p_1^2 - ip\alpha\right)a_3 = 0,$$
$$\left(\frac{5p^2}{4} - ip\alpha - 5p_1^2\right)a_4 a_1 + \left(ip\alpha + \frac{5p^2}{4} - 5p_1^2\right)a_3 a_2 = 0,$$
$$-10a_1 a_3 p_1^2 + \left(5p^2 + 2ip\alpha\right)a_2 + \left(5p^2 - 2ip\alpha\right)a_4 = 0,$$
$$a^2\left(a_3^2 - a_1 a_1^*\right) = 0, \qquad a^2\left(a_4^2 - a_2 a_2^*\right) = 0,$$
$$\left(2a^2 - \frac{3p^2}{2} - 6p_1^2\right)a_3 - a^2\left(a_1 + a_1^*\right) = 0,$$
$$\left(2a^2 - \frac{3p^2}{2} - 6p_1^2\right)a_3 a_4 - a^2\left(a_1 a_2^* + a_1^* a_2\right) = 0,$$
$$\left(2a^2 - 6p^2\right)a_4 - 6a_3^2 p_1^2 - a^2\left(a_2 + a_2^*\right) = 0. \tag{8}$$

Solving these equations, we obtain the relations between the parameters as

$$B = -a^2, \qquad p_1^2 = \frac{21p^2}{20},$$
$$p^2 = \frac{320a^2 - 39\alpha^2}{624}, \qquad a_1 = \frac{(i\alpha + 4p)a_3}{i\alpha - 4p}, \tag{9}$$
$$a_2 = \frac{(i\alpha + 4p)^2 a_4}{(i\alpha - 4p)^2}, \qquad a_3^2 = \frac{4\left(21\alpha^2 - 80p^2\right)a_4}{21\left(\alpha^2 + 16p^2\right)}.$$

From $p^2 \geq 0$ and $a_3^2 \geq 0$ in (9), we have $(800a^2/507) \leq \alpha^2 \leq (320a^2/39)$. Substituting (9) into (7) and then (5) and taking $a_4 > 0$, we obtain the solution for DSI equation as

$$\begin{aligned} u &= \frac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \frac{2\cosh[\xi + \gamma + i\theta] + (a_3/\sqrt{a_4})\cos(\eta)}{2\cosh(\xi + \gamma) + (a_3/\sqrt{a_4})\cos(\eta)}, \\ v &= 2\left(2a_3\sqrt{a_4}\left(p^2 - p_1^2\right)\cos(\eta)\cosh(\xi + \gamma) \right. \\ &\quad + 4a_3 p p_1 \sqrt{a_4}\sin(\eta)\sinh(\xi + \gamma) \\ &\quad \left. + 4a_4 p^2 - a_3^2 p_1^2\right) \\ &\quad \times \left(\left[a_3\cos(\eta) + 2\sqrt{a_4}\cosh(\xi + \gamma)\right]^2\right)^{-1}, \end{aligned} \tag{10}$$

where $\xi = p(x + y/2 + \alpha t)$, $\eta = p_1(x + 2y - \alpha t)$, $\gamma = \ln\sqrt{a_4}$, $e^{i\theta} = (i\alpha + 4p)/(i\alpha - 4p)$, and p, p_1, α, a_3, and a_4, are given by (9). Note that if $(u(x, y, t), v(x, y, t))$ is the solution of DSI equation, then $(u(x, -y, t), v(x, -y, t))$ is the solution as well. So, we also obtain solution of DSI equation:

$$\begin{aligned} u_1 &= \frac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \frac{2\cosh[\xi_1 + \gamma + i\theta] + (a_3/\sqrt{a_4})\cos(\eta_1)}{2\cosh(\xi_1 + \gamma) + (a_3/\sqrt{a_4})\cos(\eta_1)}, \\ v_1 &= 2\left(2a_3\sqrt{a_4}\left(p^2 - p_1^2\right)\cos(\eta_1)\cosh(\xi_1 + \gamma) \right. \\ &\quad + 4a_3 p p_1 \sqrt{a_4}\sin(\eta_1)\sinh(\xi_1 + \gamma) \\ &\quad \left. + 4a_4 p^2 - a_3^2 p_1^2\right) \\ &\quad \times \left(\left[a_3\cos(\eta_1) + 2\sqrt{a_4}\cosh(\xi_1 + \gamma)\right]^2\right)^{-1}, \end{aligned} \tag{11}$$

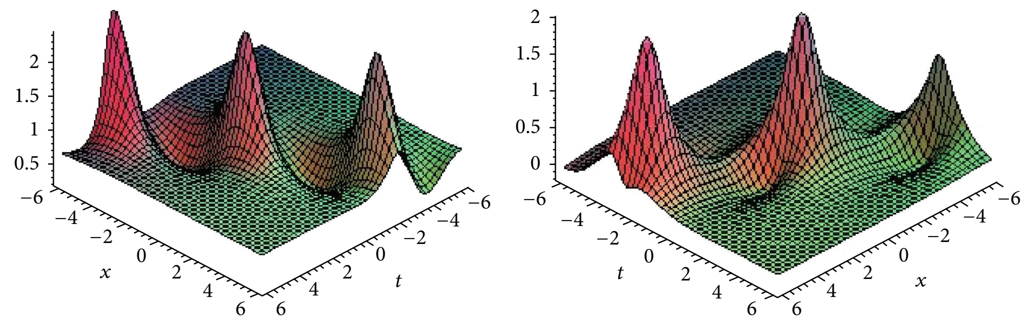
where $\xi_1 = p(x - y/2 + \alpha t)$, $\eta_1 = p_1(x - 2y - \alpha t)$, and $\gamma = \ln\sqrt{a_4}$. Solution (11) is the homoclinic solution of DSI equation. Indeed, we have

$$\begin{aligned} (u_1, v_1) &\longrightarrow \left(\frac{a}{\sqrt{2}}\exp\left(i\left(a^2 t + 2\theta\right)\right), 0\right), \\ &\qquad \text{as } t \longrightarrow +\infty; \\ (u_1, v_1) &\longrightarrow \left(\frac{a}{\sqrt{2}}\exp\left(ia^2 t\right), 0\right), \quad \text{as } t \longrightarrow -\infty, \end{aligned} \tag{12}$$

where 2θ is a phase shift and $(a\exp(ia^2 t), 0)$ is a fixed circle of DSI [30]. Note that solution (11) contains not only a periodic wave $\cos(p_1(x - 2y - \alpha t))$, so its amplitude periodically oscillates with the evolution of time (the breather effect), but also a solitary wave $1/\cosh(p(x - y/2 + \alpha t) + \gamma)$, which shows that interaction between a solitary wave and a periodic wave with the same velocity α and opposite propagation direction can form a new family of homoclinic solution. This is a new phenomenon of evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow (Figure 1).

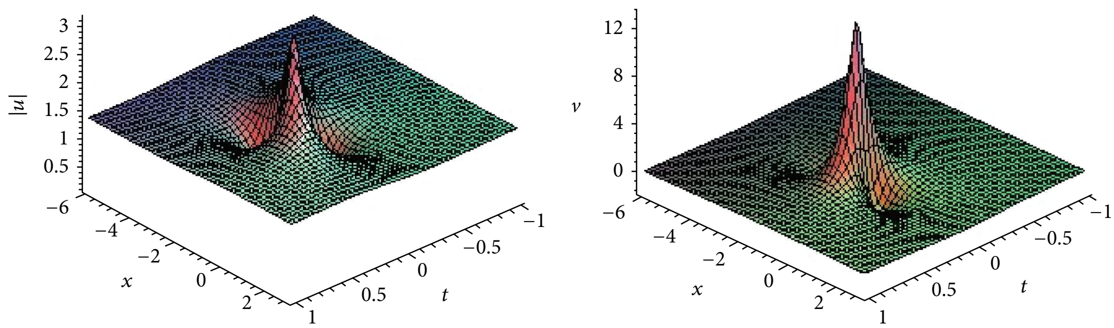
In the above two cases, set $a_3 = -(2/21)\sqrt{(441r^2 - 1680p^2)/(16p^2 + r^2)}$ and $a_4 = 1$, when $\alpha = (8/39)\sqrt{195}a$. Let $p \to 0$, and we can obtain two rogue wave solutions for DSI as follows (Figures 2 and 3):

$$\begin{aligned} u_1 &= \frac{a}{\sqrt{2}}\exp\left(ia^2 t\right) \\ &\quad \times \left(\left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3 \right.\right. \\ &\qquad + \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left. -18560\right) \\ &\quad \times \left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3 \right. \\ &\qquad + \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left. + 8320\right)^{-1} \\ &\quad + i\left(-6720\alpha^2 t + (-3360y - 6720x)\alpha\right) \\ &\quad \times \left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3 \right. \\ &\qquad + \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left.\left. + 8320\right)^{-1}\right), \\ v_1 &= -84\alpha^2\left(-35259\alpha^4 t^2 \right. \\ &\qquad + (50736yt - 1722xt)\alpha^3 \\ &\qquad + \left(35301x^2 + 89544xy + 36939y^2\right)\alpha^2 \\ &\qquad \left. - 341120\right) \\ &\quad \times \left(\left(861\alpha^4 t^2 + (-1344yt - 42xt)\alpha^3 \right.\right. \\ &\qquad + \left(2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left.\left. + 8320\right)^2\right)^{-1}, \\ u_2 &= \frac{a}{\sqrt{2}}\exp\left(ia^2 t\right) \\ &\quad \times \left(\left(861\alpha^4 t^2 + (1344yt - 42xt)\alpha^3 \right.\right. \\ &\qquad + \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left. -18560\right) \\ &\quad \times \left(861\alpha^4 t^2 + (1344yt - 42xt)\alpha^3 \right. \\ &\qquad + \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\ &\qquad \left. + 8320\right)^{-1} \\ &\quad + i\left(-6720\alpha^2 t + (3360y - 6720x)\alpha\right) \end{aligned} \tag{13}$$



(a) (b)

FIGURE 1: (a) Homoclinic breather wave of $|u_1|$ in DSI. (b) Homoclinic breather wave of v_1 in DSI.



(a) (b)

FIGURE 2: (a) Homoclinic rogue wave $|u_1|$ in solution (13). (b) Homoclinic rogue wave v_1 in solution (13).

$$\begin{aligned}
&\times \Big(861\alpha^4 t^2 + (1344yt - 42xt)\,\alpha^3 \\
&\quad + \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\
&\quad +8320\Big)^{-1}\Big), \\
v_2 &= -84\alpha^2 \big(-35259\alpha^4 t^2 \\
&\quad + (-50736yt - 1722xt)\,\alpha^3 \\
&\quad + \left(35301x^2 - 89544xy + 36939y^2\right)\alpha^2 \\
&\quad - 341120\big) \\
&\times \Big(\big(861\alpha^4 t^2 + (-1344yt - 42xt)\,\alpha^3 \\
&\quad + \left(-2184xy + 861x^2 + 1869y^2\right)\alpha^2 \\
&\quad + 8320\big)^2\Big)^{-1},
\end{aligned} \tag{14}$$

where $\alpha = (8/39)\sqrt{195}a$.

Both (u_1, v_1) and (u_2, v_2) are rational homoclinic (rogue) wave solutions of DSI equation. In fact, we have $(u_j, v_j) \to ((a/\sqrt{2})\exp(ia^2 t), 0)$ as $t \to \infty$, $j = 1, 2$.

3. Homoclinic Breather and Rogue Wave Solution of DSII

As we know, the $(ae^{-2a^2 it}, 0)$ is hyperbolic fixed cycle of DSII equation when the period of y is larger than the period of x [30]. Similar to the argument in [30], we can analyze the linear stability of fixed cycle $(ae^{-2|a|^2 it}, 0)$. Similar to the dealing with process of (2), by means of transformation of functions

$$U = \frac{G}{F}, \qquad V = -2(\ln F)_{xx}, \tag{15}$$

Equation (3) can be converted into the bilinear form

$$\begin{aligned}
\left(iD_t + D_x^2 - D_y^2\right) G \cdot F &= \lambda G \cdot F, \\
\left(D_x^2 + D_y^2 + \lambda\right) F \cdot F &= 2GG^*,
\end{aligned} \tag{16}$$

where G is a complex function and F is a real. Now, we take the following ansatz:

$$\begin{aligned}
G &= ae^{-2a^2 it}\left[e^{-p_2(mx+ny+\alpha t)} + b_1 \cos p_1 (kx + ly - \alpha t)\right. \\
&\qquad \left. + b_2 e^{p_2(mx+ny+\alpha t)}\right], \\
F &= e^{-p_2(mx+ny+\alpha t)} + b_3 \cos p_1 (kx + ly - \alpha t) \\
&\qquad + b_4 e^{p_2(mx+ny+\alpha t)},
\end{aligned} \tag{17}$$

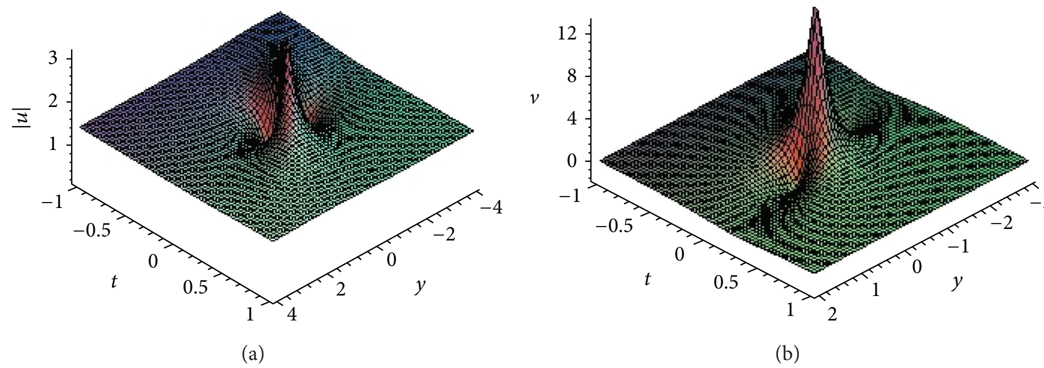


FIGURE 3: (a) Behaviour of $|u_2|$ in solution (14). (b) Behaviour of v_2 in solution (14).

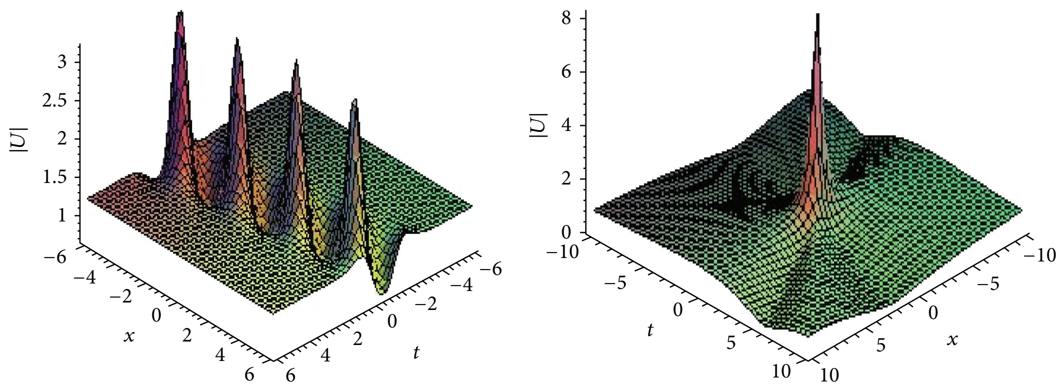
where p_1, p_2, m, n, k, l, α, b_3, and b_4 are real and b_1, b_2 are complex. Substituting (17) into (16), then we obtain

$$\begin{aligned}
\lambda &= 2a^2,\\
p_2^2 &= \Big(8\left(k^2-l^2\right)(km-nl)^2a^2 + \left(l^3n-\left(km+m^2\right)l^2 + k^2nl+n^2k^2-k^3m\right)\alpha^2\Big)\\
&\quad\times\Big(4\left(k^3m-k^2nl-n^2k^2 + l^2km+m^2l^2-l^3n\right)\times(km-nl)^2\Big)^{-1},\\
p_1^2 &= \frac{\left(m^2-n^2-2nl+2km\right)p_2^2}{k^2-l^2},\\
b_1 &= \frac{b_3\left(i\alpha+2p_2mk-2p_2nl\right)}{i\alpha-2p_2mk+2p_2nl},\\
b_2 &= \frac{b_4(i\alpha+2p_2mk-2p_2nl)^2}{(i\alpha-2p_2mk+2p_2nl)^2},\\
b_3^2 &= -4b_4\times\Big(\left(m^2-n^2\right)(mk-nl)^2p_2^2 - \alpha^2\left(2mk-n^2-2nl+m^2\right)\Big)\\
&\quad\times\Big(\left(2mk-n^2-2nl+m^2\right)\times\left(\left(4m^2k^2-8nlmk+4n^2l^2\right)p_2^2+\alpha^2\right)\Big)^{-1},\\
&nl+km = 0.
\end{aligned}\tag{18}$$

So we can obtain breather solution of DSII equation as follows:

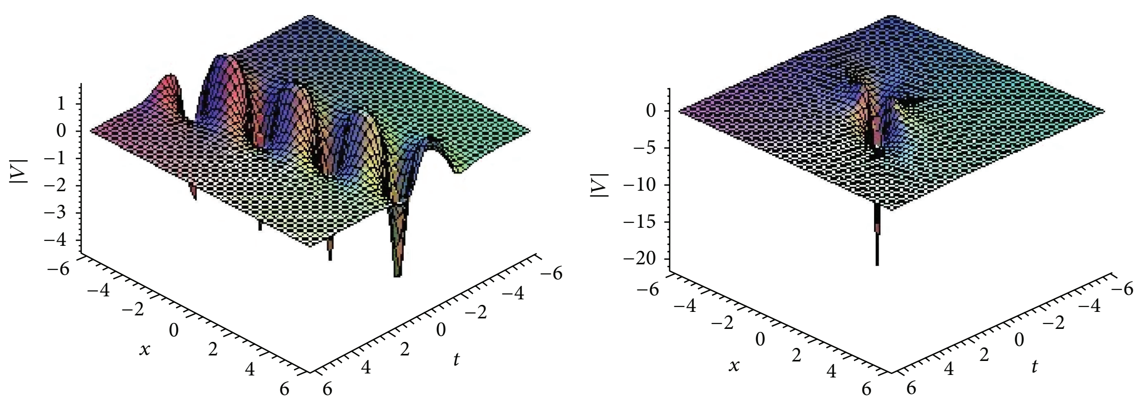
$$\begin{aligned}
U &= ae^{-i(2a^2t-\theta)}\times\Big(2\sqrt{b_4}\cosh\left(p_2\left(mx+ny+\alpha t\right)+i\theta+\phi\right) + b_3\cos\left(p_1\left(kx+ly-\alpha t\right)\right)\Big)\\
&\quad\times\Big(2\sqrt{b_4}\cosh\left(p_2\left(mx+ny+\alpha t\right)+\phi\right) + b_3\cos\left(p_1\left(kx+ly-\alpha t\right)\right)\Big)^{-1},\\
V &= -2\Big(2\sqrt{b_4}\times\cosh\left(p_2\left(mx+ny+\alpha t\right)+\phi\right)p_2^2m^2 - b_3\cos\left(p_1\left(kx+ly-\alpha t\right)\right)p_1^2k^2\Big)\\
&\quad\times\Big(2\sqrt{b_4}\cosh\left(p_2\left(mx+ny+\alpha t\right)+\phi\right) + b_3\cos\left(p_1\left(kx+ly-\alpha t\right)\right)\Big)^{-1}\\
&\quad+2\Big(\Big(2\sqrt{b_4}\sinh\left(p_2\left(mx+ny+\alpha t\right)+\phi\right)p_2m - b_3\sin\left(p_1\left(kx+ly-\alpha t\right)\right)p_1k\Big)^2\Big)\\
&\quad\times\Big(\Big(2\sqrt{b_4}\cosh\left(p_2\left(mx+ny+\alpha t\right)+\phi\right) + b_3\cos\left(p_1\left(kx+ly-\alpha t\right)\right)\Big)^2\Big)^{-1},
\end{aligned}\tag{19}$$

where $\theta = \arctan b_1$ and $\phi = \ln(\sqrt{b_4})$.



(a) (b)

FIGURE 4: (a) Bright breather structure $|U|$ in DSII. (b) Bright rogue wave $|U|$ in DSII.



(a) (b)

FIGURE 5: (a) Dark breather structure V in DSII. (b) Dark rogue wave V in DSII.

The rogue wave of the DSII system is derived when the period of periodic wave goes to infinite. Indeed, by letting $p_1 \to 0$ in solution (19), solution (19) becomes rogue waves:

$$\begin{aligned} U &= ae^{2ia^2t} \\ &\quad \times \left(-(mx+ny+\alpha t)^2 + 2(kx+ly-\alpha t)^2 \right. \\ &\qquad \left. +16i(mx+ny+\alpha t)\delta_1\delta_2 + \delta_4 \right) \\ &\quad \times \left(2(mx+ny+\alpha t)^2\delta_1 \right. \\ &\qquad \left. + 2(kx+ly-\alpha t)^2 + \delta_3 \right)^{-1}, \\ V &= -2\left(4m^2\delta_1 + 4k^2\right) \\ &\quad \times \left(2(mx+ny+\alpha t)^2\delta_1 \right. \\ &\qquad \left. + 2(kx+ly-\alpha t)^2 + \delta_3 \right)^{-1} \\ &\quad + 2\left(4(mx+ny+\alpha t)\delta_1 m \right. \\ &\qquad \left. + 4(kx+ly-\alpha t)k \right)^2 \\ &\quad \times \left(\left(2(mx+ny+\alpha t)^2\delta_1 \right.\right. \\ &\qquad \left.\left. +2(kx+ly-\alpha t)^2 + \delta_3 \right)^2 \right)^{-1}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \delta_1 &= \frac{k^2-l^2}{m^2-n^2-2ln+2mk}, \\ \delta_2 &= \frac{mk-ln}{\alpha}, \\ \delta_3 &= 16\,\frac{\left(k^2-l^2\right)\left(mk+m^2-n^2-ln\right)(mk-ln)^2}{\left(m^2-n^2-2ln+2mk\right)^2\alpha^2}, \end{aligned}$$

$$\delta_4 = 32 \frac{\left(k^2 - l^2\right)\left(5mk + 3m^2 - 5ln - 3n^2\right)(mk - ln)^2}{\left(m^2 - n^2 - 2ln + 2mk\right)^2 \alpha^2},$$

$$m = \frac{1312\alpha^2}{9\left(41\alpha^2 + 512a^2\right)},$$

$$n = -\frac{1640\alpha^2}{9\left(41\alpha^2 + 512a^2\right)}. \tag{21}$$

Figure 4(a): The dynamical evolution of bright breather $|U(x,t)|$ in solution (19) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 4(b): The dynamical evolution of bright rogue wave $|U(x,t)|$ in solution (20) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 5(a): The dynamical evolution of dark breather $V(x,t)$ in solution (19) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

Figure 5(b): The dynamical evolution of dark rogue wave $V(x,t)$ in solution (20) is plotted with parameters $a = \alpha = k = 2$, $b_4 = 1$, and $l = 1.6$.

It is easy to verify that both solution (19) and (20) are solutions of DSII. Similar to DSI, we can show that solution (20) is a rational homoclinic rogue waves.

4. Conclusion

In summary, based on Hirota bilinear form, applying homoclinic breather limit method to DSI and DSII equations, we obtain a new kind of homoclinic solutions with locally oscillatory structure and rational homoclinic rogue wave solutions. We also investigate and exhibit the different homoclinic rogue wave structures of solutions. These results show the complexity and variety of dynamical behavior of the DS system. Following these ideas in this work, the problem needed to further study is other types of nonlinear evolution equations whether have this kind of rational homoclinic solutions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by Chinese Natural Science Foundation Grant nos. 11261049 and 11361048.

References

[1] M. Hopkin, "Sea snapshots will map frequency of freak waves," *Nature*, vol. 430, no. 6999, p. 492, 2004.

[2] C. Kharif, E. Pelinovsky, and A. Slunyaev, *Rogue Waves in the Ocean: Observation, Theories and Modeling*, Springer, New York, NY, USA, 2009.

[3] N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, "Rogue waves and rational solutions of the nonlinear Schrödinger equation," *Physical Review E*, vol. 80, no. 2, Article ID 026601, 9 pages, 2009.

[4] Y. C. Ma, "The perturbed plane-wave solutions of the cubic Schrödinger equation," *Studies in Applied Mathematics*, vol. 60, no. 1, pp. 43–58, 1979.

[5] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," *Nature*, vol. 450, no. 7172, pp. 1054–1057, 2007.

[6] Y. V. Bludov, V. V. Konotop, and N. Akhmediev, "Rogue waves as spatial energy concentrators in arrays of nonlinear waveguides," *Optics Letters*, vol. 34, no. 19, pp. 3015–3017, 2009.

[7] A. N. Ganshin, V. B. Efimov, G. V. Kolmakov, L. P. Mezhov-Deglin, and P. V. E. McClintock, "Observation of an inverse energy cascade in developed acoustic turbulence in superfluid helium," *Physical Review Letters*, vol. 101, no. 6, Article ID 065303, 4 pages, 2008.

[8] Y. V. Bludov, V. V. Konotop, and N. Akhmediev, "Matter rogue waves," *Physical Review A: Atomic, Molecular, and Optical Physics*, vol. 80, no. 3, Article ID 033610, 5 pages, 2009.

[9] A. Montina, U. Bortolozzo, S. Residori, and F. T. Arecchi, "Non-gaussian statistics and extreme waves in a nonlinear optical cavity," *Physical Review Letters*, vol. 103, no. 17, Article ID 173901, 4 pages, 2009.

[10] Z. Y. Yan, "Financial rogue waves," *Communications in Theoretical Physics*, vol. 54, no. 5, pp. 947–949, 2010.

[11] D. H. Peregrine, "Water waves, nonlinear Schrödinger equations and their solutions," *Australian Mathematical Society B: Applied Mathematics*, vol. 25, no. 1, pp. 16–43, 1983.

[12] Y. Tao and J. He, "Multisolitons, breathers, and rogue waves for the Hirota equation generated by the Darboux transformation," *Physical Review E*, vol. 85, no. 2, Article ID 026601, 7 pages, 2012.

[13] U. Bandelow and N. Akhmediev, "Persistence of rogue waves in extended nonlinear Schrödinger equations: Integrable Sasa-Satsuma case," *Physics Letters A: General, Atomic and Solid State Physics*, vol. 376, no. 18, pp. 1558–1561, 2012.

[14] Y. Ohta and J. Yang, "Dynamics of rogue waves in the Davey-Stewartson II equation," *Journal of Physics A: Mathematical and Theoretical*, vol. 46, no. 10, Article ID 105202, 19 pages, 2013.

[15] L. C. Zhao and J. Liu, "Rogue-wave solutions of a three-component coupled nonlinear Schrödinger equation," *Physical Review E*, vol. 87, no. 1, Article ID 013201, 8 pages, 2013.

[16] W. P. Zhong, "Rogue wave solutions of the generalized one-dimensional Gross-Pitaevskii equation," *Journal of Nonlinear Optical Physics & Materials*, vol. 21, no. 2, Article ID 1250026, 9 pages, 2012.

[17] C. Li, J. He, and K. Porseizan, "Rogue waves of the Hirota and the Maxwell-Bloch equations," *Physical Review E*, vol. 87, no. 1, Article ID 012913, 13 pages, 2013.

[18] G. Mu and Z. Qin, "Rogue waves for the coupled Schrödinger-Boussinesq equation and the coupled Higgs equation," *Journal of the Physical Society of Japan*, vol. 81, no. 8, Article ID 084001, 6 pages, 2012.

[19] A. Davey and K. Stewartson, "On three-dimensional packets of surface waves," *Proceedings of the Royal Society A: Mathematical, Physical & Engineering Sciences*, vol. 338, pp. 101–110, 1974.

[20] V. A. Arkadiev, A. K. Pogrebkov, and M. C. Polivanov, "Inverse scattering transform method and soliton solutions for Davey-Stewartson II equation," *Physica D: Nonlinear Phenomena*, vol. 36, no. 1-2, pp. 189–197, 1989.

[21] M. Boiti, J. J. Leon, L. Martina, and F. Pempinelli, "Scattering of localized solitons in the plane," *Physics Letters A*, vol. 132, no. 8-9, pp. 432–439, 1988.

[22] M. Boiti, J. P. Leon, and F. Pempinelli, "Multidimensional solitons and their spectral transforms," *Journal of Mathematical Physics*, vol. 31, no. 11, pp. 2612–2618, 1990.

[23] K. W. Chow, "A class of doubly periodic waves for nonlinear evolution equations," *Wave Motion*, vol. 35, no. 1, pp. 71–90, 2002.

[24] A. S. Fokas and P. M. Santini, "Coherent structures in multidimensions," *Physical Review Letters*, vol. 63, no. 13, pp. 1329–1333, 1989.

[25] C. R. Gilson, "Resonant behaviour in the Davey-Stewartson equation," *Physics Letters. A*, vol. 161, no. 5, pp. 423–428, 1992.

[26] J. Hietarinta and R. Hirota, "Multidromion solutions to the Davey-Stewartson equation," *Physics Letters A*, vol. 145, no. 5, pp. 237–244, 1990.

[27] J. Satsuma and M. J. Ablowitz, "Two-dimensional lumps in nonlinear dispersive systems," *Journal of Mathematical Physics*, vol. 20, no. 7, pp. 1496–1503, 1979.

[28] M. Tajiri and T. Arai, "Periodic soliton solutions to the Davey-Stewartson equation," *Proceedings of Institute of Mathematics of NAS of Ukraine*, vol. 30, part 1, pp. 210–217, 2000.

[29] Y. Watanabe and M. Tajiri, "Periodic soliton resonance: solutions to the Davey-Stewartson I equation," *Journal of the Physical Society of Japan*, vol. 67, no. 3, pp. 705–708, 1998.

[30] Z. Dai and J. Huang, "Homoclinic tubes for the Davey-Stewartson II equation with periodic boundary conditions," *Chinese Journal of Physics*, vol. 43, no. 2, pp. 349–356, 2005.

[31] J. Huang and Z. Dai, "Homoclinic solutions for Davey-Stewartson equation," *Chaos, Solitons & Fractals*, vol. 35, no. 5, pp. 996–1002, 2008.

[32] Z. D. Dai, A. J. Zhu, S. L. Li, and D. L. Li, "Homoclinic tubes for the Davey-Stewartson II equation with periodic boundary conditions," *Chinese Journal of Physics*, vol. 45, no. 1, pp. 62–71, 2007.

[33] L. B. Santos, M. Rojas-Medar, and G. Ruiz-Garzón, "Existence of weakly efficient solutions in vector optimization," *Acta Mathematica Sinica*, vol. 24, no. 4, pp. 599–606, 2008.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
Algebra
Journal of
Probability and Statistics
The Scientific
World Journal
International Journal of
Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of
Combinatorics
Advances in
Mathematical Physics
Journal of
Complex Analysis
Journal of
Mathematics
Mathematical Problems
in Engineering
Abstract and
Applied Analysis
Discrete Dynamics in
Nature and Society
International
Journal of
Mathematics and
Mathematical
Sciences
Journal of
Discrete Mathematics
Journal of
Function Spaces
International Journal of
Stochastic Analysis
Journal of
Optimization