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Research Article

Improved Robust H_∞ Filtering Approach for Nonlinear Systems

Jun Chen and Haiqiao Sun

Key Laboratory of Advanced Process Control for Light Industry, Ministry of Education, Institute of Automation, Jiangnan University, Wuxi 214122, China

Correspondence should be addressed to Jun Chen; chenjun1860@126.com

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An improved design approach of robust H_∞ filter for a class of nonlinear systems described by the Takagi-Sugeno (T-S) fuzzy model is considered. By introducing a free matrix variable, a new sufficient condition for the existence of robust H_∞ filter is derived. This condition guarantees that the filtering error system is robustly asymptotically stable and a prescribed H_∞ performance is satisfied for all admissible uncertainties. Particularly, the solution of filter parameters which are independent of the Lyapunov matrix can be transformed into a feasibility problem in terms of linear matrix inequalities (LMIs). Finally, a numerical example illustrates that the proposed filter design procedure is effective.

1. Introductions

In recent years, when the external disturbance and the statistical properties of the measurement noise are unknown, using H_∞ filtering approach to estimate the states of a linear system becomes one of the focuses on the estimated theoretical research, and some useful research results [1–4] are obtained. However, how to design an effective filter for nonlinear systems is still a very difficult problem. Over the past two decades, there has been a rapidly growing interest in fuzzy control of nonlinear systems. In particular, the fuzzy model proposed by Takagi and Sugeno [5] receives a great deal of attention. And it indicates that this type of fuzzy model has a good approximation performance for the complex nonlinear systems, so some scholars attempt to apply this fuzzy model to design H_∞ filter for nonlinear systems. Feng et al. [6] were prior scholars to study the filter for nonlinear systems by using T-S fuzzy model and linear matrix inequality (LMI) techniques. For a class of discrete nonlinear dynamic systems, Tseng and Chen [7] and Pan et al. [8] studied a fuzzy H_∞ filtering problem. After that, Tseng [9, 10] and Tian et al. [11] discussed the design problem of robust H_∞ fuzzy filter for a class of continuous nonlinear systems. Moreover, the above-obtained results were extended to the fuzzy H_∞ filter or robust H_∞ filter design for nonlinear systems with time delay [12–15]. In addition, H_∞ filtering approach is also applied to Markovian jump systems [16], nonlinear interconnected systems [17], chaotic systems [18], and networked nonlinear systems [19] for the discrete-time case and stochastic systems [20] and singular systems [21] for the continuous-time case. Nevertheless, in the above-mentioned results, the solving process of filter parameters is related to the Lyapunov matrix, which will more or less bring some conservative to the results. The reason is that most of the existence conditions of filter are sufficient conditions; if the Lyapunov matrix cannot be found, then the filter parameters which maybe exist cannot be constructed. For this reason, de Oliveira et al. [22] proposed a novel filter design method by introducing free matrices to the framework of the quadratic Lyapunov function. By means of decoupling the relations between the Lyapunov matrix and the system matrix, the conservative of the results will be reduced. But due to the restrictions of LMI characteristics, this method can only be applied to the discrete systems [17, 23, 24]. Lately, Apkarian et al. [25] extended this idea to the linear continuous systems with the aid of Projection Theorem. And this idea has been used in other fields [26–28]. Unfortunately, to the best of our knowledge, this idea has not yet been introduced to the design of robust H_∞ filter for the uncertain continuous nonlinear systems.

Taking into account the above-mentioned results, this paper will discuss a new design method of robust H_∞ filter for a class of uncertain nonlinear systems. Firstly, the

T-S fuzzy model is employed to represent the nonlinear systems. Then, on the basis of the bounded real lemma of continuous systems, a new criterion for the existence of the improved robust H_∞ filter is obtained via introducing a free matrix variable. Based on this criterion, the solution of the filter parameters independent of the Lyapunov matrix can be obtained. Combined with the linear matrix inequality techniques, the filter design problem can be transformed into a feasibility problem of a set of linear matrix inequalities. Finally, a simulation example will be given to verify the validity of the proposed method.

2. Problem Formulation

Consider a class of uncertain nonlinear systems described by the following T-S fuzzy models.

Plant Rule i:

$$\begin{aligned}
&\text{IF } v_1(t) \text{ is } M_{i1} \text{ and} \cdots \text{and } v_p(t) \text{ is } M_{ip} \\
&\text{THEN } \dot{x}(t) = \left(A_i + \Delta A_i(t)\right) x(t) + B_i w(t), \\
&\qquad y(t) = \left(C_i + \Delta C_i(t)\right) x(t) + D_i w(t), \\
&\qquad z(t) = L_i x(t), \\
&\qquad x_0 = x(0), \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{1}$$

where $x(t) \in \mathfrak{R}^n$ is the state vector, $y(t) \in \mathfrak{R}^m$ is the measured output, $z(t) \in \mathfrak{R}^l$ is the signal to be estimated, and $w(t) \in L_2^q[0, \infty)$ is the noise signal vector (including process and measurement noises). x_0 is the initial state condition of the system, which is considered to be known and, without loss of generality, assumed to be zero. $v_1(t), \ldots, v_p(t)$ are the premise variables, M_{ij} $(j = 1, 2, \ldots, p)$ is the fuzzy set, and r is the number of IF-THEN rules. A_i, B_i, C_i, D_i, and L_i are known real constant matrices with appropriate dimensions of the ith subsystem, respectively. The uncertain time-varying matrices $\Delta A_i(t)$ and $\Delta C_i(t)$ represent the parameter uncertainties in the system model and are assumed to be norm-bounded of the following forms:

$$\begin{aligned}
\Delta A_i(t) &= E_{ai} F(t) H_{ai}, \\
\Delta C_i(t) &= E_{ci} F(t) H_{ci}, \\
i &= 1, 2, \ldots, r,
\end{aligned} \tag{2}$$

where E_{ai}, E_{ci}, H_{ai}, and H_{ci} are known constant matrices of appropriate dimensions, which reflect the structural information of uncertainty, and $F(t)$ is an uncertainty matrix function with Lebesgue measurable elements and satisfies

$$F^{\mathrm{T}}(t) F(t) \le I. \tag{3}$$

By using the weighted average method for defuzzification, the uncertain fuzzy dynamic model for the system (1) can be inferred as follows:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} h_i(v(t)) \left[\left(A_i + \Delta A_i(t)\right) x(t) + B_i w(t)\right], \\
y(t) &= \sum_{i=1}^{r} h_i(v(t)) \left[\left(C_i + \Delta C_i(t)\right) x(t) + D_i w(t)\right], \\
z(t) &= \sum_{i=1}^{r} h_i(v(t)) L_i x(t),
\end{aligned} \tag{4}$$

where $v(t) = [v_1(t), v_2(t), \ldots, v_p(t)]^{\mathrm{T}}$, and

$$\begin{aligned}
h_i(v(t)) &= \frac{\mu_i(v(t))}{\sum_{i=1}^{r} \mu_i(v(t))}, \\
\mu_i(v(t)) &= \prod_{j=1}^{p} M_{ij}\left(v_j(t)\right), \\
i &= 1, 2, \ldots, r,
\end{aligned} \tag{5}$$

in which $M_{ij}(v_j(t))$ is the grade of membership of $v_j(t)$ in the fuzzy set M_{ij}, while $\mu_i(v(t))$ is the grade of membership of the ith rule.

In general, it is assumed that $\mu_i(v(t)) \ge 0$, $i = 1, 2, \ldots, r$, and $\sum_{i=1}^{r} \mu_i(v(t)) > 0$. Therefore, it is easy to obtain that $h_i(v(t)) \ge 0$, $i = 1, 2, \ldots, r$, and $\sum_{i=1}^{r} h_i(v(t)) = 1$.

Based on the T-S fuzzy models (1), the full-order filter is constructed as follows.

Filter Rule i:

$$\begin{aligned}
&\text{IF } v_1(t) \text{ is } M_{i1} \text{ and} \cdots \text{and } v_p(t) \text{ is } M_{ip} \\
&\text{THEN } \dot{\hat{x}}(t) = A_{fi} \hat{x}(t) + B_{fi} y(t), \\
&\qquad \hat{z}(t) = C_{fi} \hat{x}(t) + D_{fi} y(t), \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{6}$$

where $\hat{x}(t) \in \mathfrak{R}^n$ is the state vector of filter and $\hat{z}(t) \in \mathfrak{R}^l$ is an estimate value of the filter output. The matrices A_{fi}, B_{fi}, C_{fi}, and D_{fi} are filter parameters to be determined. Here, it is assumed that the initial condition of filter is $\hat{x}_0 = 0$. Then the whole fuzzy filter can be expressed as

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(v(t)) \left[A_{fi} \hat{x}(t) + B_{fi} y(t)\right], \\
\hat{z}(t) &= \sum_{i=1}^{r} h_i(v(t)) \left[C_{fi} \hat{x}(t) + D_{fi} y(t)\right].
\end{aligned} \tag{7}$$

Set the state variable as $\tilde{x}(t) = [x^{\mathrm{T}}(t) \hat{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$ and estimated error as $\tilde{z}(t) = z(t) - \hat{z}(t)$. Then the filtering error

dynamic equation inferred from formulas (4) and (7) can be described as follows:

$$\begin{aligned}\dot{\tilde{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(v(t))\, h_j(v(t)) \\ &\quad \times \left[\left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}(t)\right)\tilde{x}(t) + \widetilde{B}_{ij} w(t)\right], \\ \tilde{z}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(v(t))\, h_j(v(t)) \\ &\quad \times \left[\left(\widetilde{C}_{ij} + \Delta\widetilde{C}_{ij}(t)\right)\tilde{x}(t) + \widetilde{D}_{ij} w(t)\right],\end{aligned} \tag{8}$$

where

$$\begin{aligned}\widetilde{A}_{ij} &= \begin{bmatrix} A_j & 0 \\ B_{fi}C_j & A_{fi}\end{bmatrix}, \qquad \widetilde{B}_{ij} = \begin{bmatrix} B_j \\ B_{fi} & D_j\end{bmatrix}, \\ \widetilde{C}_{ij} &= \left[L_j - D_{fi}C_j \quad -C_{fi}\right], \qquad \widetilde{D}_{ij} = -D_{fi}D_j, \\ \Delta\widetilde{A}_{ij}(t) &= \begin{bmatrix} \Delta A_j(t) & 0 \\ B_{fi}\Delta C_j(t) & 0\end{bmatrix}, \\ \Delta\widetilde{C}_{ij}(t) &= \left[-D_{fi}\Delta C_j(t) \quad 0\right].\end{aligned} \tag{9}$$

Let $H_{\tilde{z}w}(s)$ be the transfer function from the disturbance input $w(t)$ to the estimation error $\tilde{z}(t)$. Then the robust H_∞ filter design problem considered in this paper can be described as follows: for a given constant $\gamma > 0$, find a H_∞ full-order filter in the form of (7) so that the filtering error dynamic system (8) is robustly asymptotically stable and the H_∞ norm of the transfer function $H_{\tilde{z}w}(s)$ is less than the given constant γ; that is, $\|H_{\tilde{z}w}(s)\|_\infty < \gamma$ is satisfied. Here, constant γ is called a prescribed H_∞ performance level.

For brevity, the functions $h_i(v(t))$ will be replaced by h_i in the subsequence, and $\Delta A_i(t)$, $\Delta C_i(t)$, $\Delta\widetilde{A}_{ij}(t)$, $\Delta\widetilde{C}_{ij}(t)$ will be replaced by ΔA_i, ΔC_i, $\Delta\widetilde{A}_{ij}$, $\Delta\widetilde{C}_{ij}$.

3. Robust H_∞ Filter Design

According to the bounded real lemma of continuous-time systems, this section firstly gives a sufficient condition for the existence of robust H_∞ filter for the uncertain fuzzy system (4). That is, for a given constant $\gamma > 0$, the filtering error system (8) is robustly asymptotically stable and satisfies $\|H_{\tilde{z}w}(s)\|_\infty < \gamma$, if there exists a symmetric positive definite matrix $P \in \mathfrak{R}^{2n\times 2n}$, such that the following matrix inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} \left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}\right)^{\mathrm{T}} P + P\left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}\right) & P\widetilde{B}_{ij} & \left(\widetilde{C}_{ij} + \Delta\widetilde{C}_{ij}\right)^{\mathrm{T}} \\ \widetilde{B}_{ij}^{\mathrm{T}} P & -\gamma^2 I & \widetilde{D}_{ij}^{\mathrm{T}} \\ \widetilde{C}_{ij} + \Delta\widetilde{C}_{ij} & \widetilde{D}_{ij} & -I \end{bmatrix} < 0. \tag{10}$$

With the aid of the basic idea of [25], an improved robust H_∞ filter design method is obtained by introducing a free matrix variable, in which the Lyapunov function matrix and the filtering error system matrix are separated. Then the filter parameters to be determined can be solved independently of the Lyapunov function matrix. This kind of processing method can reduce the conservatism of the results.

Theorem 1. *For a given constant $\gamma > 0$, the filtering error system (8) is robustly asymptotically stable and satisfies $\|H_{\tilde{z}w}(s)\|_\infty < \gamma$, if there exist a symmetric positive definite matrix $P \in \mathfrak{R}^{2n\times 2n}$ and matrix $V \in \mathfrak{R}^{2n\times 2n}$, such that the following matrix inequality holds:*

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} -\left(V + V^{\mathrm{T}}\right) & V^{\mathrm{T}}\left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}\right) + P & V^{\mathrm{T}}\widetilde{B}_{ij} & 0 & V^{\mathrm{T}} \\ * & -P & 0 & \left(\widetilde{C}_{ij} + \Delta\widetilde{C}_{ij}\right)^{\mathrm{T}} & 0 \\ * & * & -\gamma^2 I & \widetilde{D}_{ij}^{\mathrm{T}} & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P \end{bmatrix} < 0. \tag{11}$$

Proof. Rewrite the matrix inequality (11) in the following form:,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left[\Phi + \widetilde{M}^{\mathrm{T}} V^{\mathrm{T}} \widetilde{N} + \widetilde{N}^{\mathrm{T}} V \widetilde{M}\right] < 0, \tag{12}$$

where

$$\Phi = \begin{bmatrix} 0 & P & 0 & 0 & 0 \\ * & -P & 0 & \left(\widetilde{C}_{ij} + \Delta\widetilde{C}_{ij}\right)^{\mathrm{T}} & 0 \\ * & * & -\gamma^2 I & \widetilde{D}_{ij}^{\mathrm{T}} & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P \end{bmatrix},$$

$$\widetilde{M} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\widetilde{N} = \begin{bmatrix} -I & \widetilde{A}_{ij} + \Delta\widetilde{A}_{ij} & \widetilde{B}_{ij} & 0 & I \end{bmatrix}. \tag{13}$$

□

According to the Projection Theorem [25], inequality (12) is equivalent to the following inequality; that is,

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} P\left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}\right) + \left(\widetilde{A}_{ij} + \Delta\widetilde{A}_{ij}\right)^{\mathrm{T}} P - P & P\widetilde{B}_{ij} & \left(\widetilde{C}_{ij} + \Delta\widetilde{C}_{ij}\right)^{\mathrm{T}} & P \\ \widetilde{B}_{ij}^{\mathrm{T}} P & -\gamma^2 I & \widetilde{D}_{ij}^{\mathrm{T}} & 0 \\ \widetilde{C}_{ij} + \Delta\widetilde{C}_{ij} & \widetilde{D}_{ij} & -I & 0 \\ P & 0 & 0 & -P \end{bmatrix} < 0. \tag{14}$$

Applying the Schur complement, it is easy to know that inequality (10) can be deduced from the above inequality. That is to say, inequality (11) is a sufficient condition of the establishment of inequality (10), which can guarantee the filtering error system (8) is robustly asymptotically stable and satisfies the prescribed H_∞ performance level.

Take into account that inequality (11) is a nonlinear matrix inequality on the matrix variables (P, A_{fi}, B_{fi}, C_{fi}, D_{fi}, $i = 1, 2, \ldots, r$), so it is very difficult to solve these variables directly. In this end, the variable substitution method will be utilized in the following derivation to transform inequality (11) into the form of linear matrix inequalities. Then the parameters of robust H_∞ filter can be easily achieved by applying the MATLAB LMI toolbox.

Lemma 2 (see [29]). *Given matrices Y, H, and E of appropriate dimensions, where Y is symmetric, then the inequality $Y + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$ holds for all F satisfying $F^{\mathrm{T}}F \leq I$, if and only if there exists a constant $\varepsilon > 0$ such that the equality $Y + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0$ holds.*

Let matrices V, V^{-1}, and P be partitioned as follows:

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \qquad V^{-1} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix},$$
$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix}, \tag{15}$$

where $V_{11}, W_{11}, P_{11} \in \mathfrak{R}^{n\times n}$.

Then introduce the following nonsingular matrices:

$$\Pi_1 = \begin{bmatrix} V_{11} & I \\ V_{21} & 0 \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} I & W_{11} \\ 0 & W_{21} \end{bmatrix}. \tag{16}$$

Obviously, the equation $V\Pi_2 = \Pi_1$ holds.

Denote $T_1 = \mathrm{diag}\{\Pi_2, \Pi_2, I, I, \Pi_2\}$, $\overline{P} = \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{12}^{\mathrm{T}} & \overline{P}_{22} \end{bmatrix} = \Pi_2^{\mathrm{T}} P \Pi_2$. Let inequality (11) be pre- and postmultiplied by T_1^{T} and T_1, respectively, and substitute the expression of the matrix variables $\widetilde{A}_{ij}$, $\widetilde{B}_{ij}$, $\widetilde{C}_{ij}$, and $\widetilde{D}_{ij}$. The following matrix inequality can be obtained:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & 0 & V_{11}^{\mathrm{T}} & \Xi_{18} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & B_j & 0 & I & W_{11} \\ * & * & -\overline{P}_{11} & -\overline{P}_{12} & 0 & \Xi_{36} & 0 & 0 \\ * & * & * & -\overline{P}_{22} & 0 & \Xi_{46} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & -D_j^{\mathrm{T}} D_{fi}^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\overline{P}_{11} & -\overline{P}_{12} \\ * & * & * & * & * & * & * & -\overline{P}_{22} \end{bmatrix} < 0, \tag{17}$$

where

$$\Xi_{11} = -V_{11} - V_{11}^{\mathrm{T}}, \quad \Xi_{12} = -I - V_{11}^{\mathrm{T}} W_{11} - V_{21}^{\mathrm{T}} W_{21},$$
$$\Xi_{13} = \overline{P}_{11} + V_{11}^{\mathrm{T}}\left(A_j + \Delta A_j\right) + V_{21}^{\mathrm{T}} B_{fi}\left(C_j + \Delta C_j\right),$$
$$\Xi_{14} = \overline{P}_{12} + V_{11}^{\mathrm{T}}\left(A_j + \Delta A_j\right) W_{11} + V_{21}^{\mathrm{T}} B_{fi}\left(C_j + \Delta C_j\right) W_{11} + V_{21}^{\mathrm{T}} A_{fi} W_{21},$$
$$\Xi_{15} = V_{11}^{\mathrm{T}} B_j + V_{21}^{\mathrm{T}} B_{fi} D_j,$$
$$\Xi_{18} = V_{11}^{\mathrm{T}} W_{11} + V_{21}^{\mathrm{T}} W_{21}, \quad \Xi_{22} = -W_{11} - W_{11}^{\mathrm{T}},$$
$$\Xi_{23} = \overline{P}_{12}^{\mathrm{T}} + A_j + \Delta A_j, \quad \Xi_{24} = \overline{P}_{22} + \left(A_j + \Delta A_j\right) W_{11},$$
$$\Xi_{36} = L_j^{\mathrm{T}} - \left(C_j + \Delta C_j\right)^{\mathrm{T}} D_{fi}^{\mathrm{T}},$$
$$\Xi_{46} = W_{11}^{\mathrm{T}} L_j^{\mathrm{T}} - W_{11}^{\mathrm{T}}\left(C_j + \Delta C_j\right)^{\mathrm{T}} D_{fi}^{\mathrm{T}} - W_{21}^{\mathrm{T}} C_{fi}^{\mathrm{T}}. \tag{18}$$

Moreover, denote $T_2 = \mathrm{diag}\{I, W_{11}^{-1}, I, W_{11}^{-1}, I, I, I, W_{11}^{-1}\}$. Similarly, multiply inequality (17) by T_2^{T} on the left and by T_2 on the right. At the same time, let

$$\widetilde{P} = \begin{bmatrix} \widetilde{P}_{11} & \widetilde{P}_{12} \\ \widetilde{P}_{12}^{\mathrm{T}} & \widetilde{P}_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & W_{11}^{-1} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{12}^{\mathrm{T}} & \overline{P}_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_{11}^{-1} \end{bmatrix},$$
$$Q = W_{11}^{-1}, \qquad R = V_{21}^{\mathrm{T}} W_{21} Q,$$
$$X_i = V_{21}^{\mathrm{T}} A_{fi} W_{21} Q,$$
$$Y_i = V_{21}^{\mathrm{T}} B_{fi}, \qquad Z_i = C_{fi} W_{21} Q. \tag{19}$$

Then inequality (17) can be equivalent to the following form:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} \Xi_{11} & \widetilde{\Xi}_{12} & \widetilde{\Xi}_{13} & \widetilde{\Xi}_{14} & \widetilde{\Xi}_{15} & 0 & V_{11}^{\mathrm{T}} & V_{11}^{\mathrm{T}} + R \\ * & \widetilde{\Xi}_{22} & \widetilde{\Xi}_{23} & \widetilde{\Xi}_{24} & Q^{\mathrm{T}} B_j & 0 & Q^{\mathrm{T}} & Q^{\mathrm{T}} \\ * & * & -\widetilde{P}_{11} & -\widetilde{P}_{12} & 0 & \Xi_{36} & 0 & 0 \\ * & * & * & -\widetilde{P}_{22} & 0 & \widetilde{\Xi}_{46} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & -D_j^{\mathrm{T}} D_{fi}^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\widetilde{P}_{11} & -\widetilde{P}_{12} \\ * & * & * & * & * & * & * & -\widetilde{P}_{22} \end{bmatrix} < 0, \tag{20}$$

where

$$\begin{aligned}
\widetilde{\Xi}_{12} &= -Q - V_{11}^{\mathrm{T}} - R, \\
\widetilde{\Xi}_{13} &= \widetilde{P}_{11} + V_{11}^{\mathrm{T}} \left(A_j + \Delta A_j\right) + Y_i \left(C_j + \Delta C_j\right), \\
\widetilde{\Xi}_{14} &= \widetilde{P}_{12} + V_{11}^{\mathrm{T}} \left(A_j + \Delta A_j\right) + Y_i \left(C_j + \Delta C_j\right) + X_i, \\
\widetilde{\Xi}_{15} &= V_{11}^{\mathrm{T}} B_j + Y_i D_j, \qquad \widetilde{\Xi}_{22} = -Q - Q^{\mathrm{T}} \\
\widetilde{\Xi}_{23} &= \widetilde{P}_{12}^{\mathrm{T}} + Q^{\mathrm{T}} \left(A_j + \Delta A_j\right), \\
\widetilde{\Xi}_{24} &= \widetilde{P}_{22} + Q^{\mathrm{T}} \left(A_j + \Delta A_j\right), \\
\widetilde{\Xi}_{46} &= L_j^{\mathrm{T}} - \left(C_j + \Delta C_j\right)^{\mathrm{T}} D_{fi}^{\mathrm{T}} - Z_i^{\mathrm{T}}.
\end{aligned} \tag{21}$$

In the following, by substituting expression (2) of the uncertain matrices $\Delta A_i(t)$ and $\Delta C_i(t)$ into the matrix inequality (20), it can be obtained that

$$\begin{aligned}
\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \big[& \widehat{\Xi}_{ij} + E_{1j} F(t) H_{1j} + H_{1j}^{\mathrm{T}} F^{\mathrm{T}}(t) E_{1j}^{\mathrm{T}} \\
& + E_{2ij} F(t) H_{2j} + H_{2j}^{\mathrm{T}} F^{\mathrm{T}}(t) E_{2ij}^{\mathrm{T}} \big] < 0,
\end{aligned} \tag{22}$$

where

$$\widehat{\Xi}_{ij} = \begin{bmatrix} \Xi_{11} & \widetilde{\Xi}_{12} & \widehat{\Xi}_{13} & \widehat{\Xi}_{14} & \widetilde{\Xi}_{15} & 0 & V_{11}^{\mathrm{T}} & V_{11}^{\mathrm{T}} + R \\ * & \widetilde{\Xi}_{22} & \widehat{\Xi}_{23} & \widehat{\Xi}_{24} & Q^{\mathrm{T}} B_j & 0 & Q^{\mathrm{T}} & Q^{\mathrm{T}} \\ * & * & -\widetilde{P}_{11} & -\widetilde{P}_{12} & 0 & \Xi_{36} & 0 & 0 \\ * & * & * & -\widetilde{P}_{22} & 0 & \widehat{\Xi}_{46} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & -D_j^{\mathrm{T}} D_{fi}^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\widetilde{P}_{11} & -\widetilde{P}_{12} \\ * & * & * & * & * & * & * & -\widetilde{P}_{22} \end{bmatrix},$$

$$\begin{aligned}
\widehat{\Xi}_{13} &= \widetilde{P}_{11} + V_{11}^{\mathrm{T}} A_j + Y_i C_j, \\
\widehat{\Xi}_{23} &= \widetilde{P}_{12}^{\mathrm{T}} + Q^{\mathrm{T}} A_j, \qquad \widetilde{\Xi}_{14} = \widetilde{P}_{12} + V_{11}^{\mathrm{T}} A_j + Y_i C_j + X_i, \\
\widehat{\Xi}_{24} &= \widetilde{P}_{22} + Q^{\mathrm{T}} A_j, \qquad \widehat{\Xi}_{46} = L_j^{\mathrm{T}} - C_j^{\mathrm{T}} D_{fi}^{\mathrm{T}} - Z_i^{\mathrm{T}}, \\
E_{1j}^{\mathrm{T}} &= \left[E_{aj}^{\mathrm{T}} V_{11} \;\; E_{aj}^{\mathrm{T}} Q \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right], \\
H_{1j} &= \left[0 \;\; 0 \;\; H_{aj} \;\; H_{aj} \;\; 0 \;\; 0 \;\; 0 \;\; 0\right], \\
E_{2ij}^{\mathrm{T}} &= \left[E_{cj}^{\mathrm{T}} Y_i^{\mathrm{T}} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -E_{cj}^{\mathrm{T}} D_{fi}^{\mathrm{T}} \;\; 0 \;\; 0\right], \\
H_{2j} &= \left[0 \;\; 0 \;\; H_{cj} \;\; H_{cj} \;\; 0 \;\; 0 \;\; 0 \;\; 0\right].
\end{aligned} \tag{23}$$

According to Lemma 2, the matrix inequality (22) holds for all admissible uncertainty matrices $F(t)$ satisfying condition (3), if and only if there exist constants $\varepsilon_{1ij} > 0$ and

$\varepsilon_{2ij} > 0$, $i, j = 1, 2, \ldots, r$, such that the following matrix inequality holds:

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left[\widehat{\Xi}_{ij} + \varepsilon_{1ij} H_{1j}^{\mathrm{T}} H_{1j} + \varepsilon_{1ij}^{-1} E_{1j} E_{1j}^{\mathrm{T}} \right. \\
&\qquad \left. + \varepsilon_{2ij} H_{2j}^{\mathrm{T}} H_{2j} + \varepsilon_{2ij}^{-1} E_{2ij} E_{2ij}^{\mathrm{T}}\right] \\
&= \sum_{i=1}^{r} h_i^2 \left[\widehat{\Xi}_{ii} + \varepsilon_{1ii} H_{1i}^{\mathrm{T}} H_{1i} + \varepsilon_{1ii}^{-1} E_{1i} E_{1i}^{\mathrm{T}} \right. \\
&\qquad \left. + \varepsilon_{2ii} H_{2i}^{\mathrm{T}} H_{2i} + \varepsilon_{2ii}^{-1} E_{2ii} E_{2ii}^{\mathrm{T}}\right] \\
&\quad + \sum_{i=1}^{r}\sum_{i<j}^{r} h_i h_j \left[\widehat{\Xi}_{ij} + \varepsilon_{1ij} H_{1j}^{\mathrm{T}} H_{1j} + \varepsilon_{1ij}^{-1} E_{1j} E_{1j}^{\mathrm{T}} \right. \\
&\qquad \left. + \varepsilon_{2ij} H_{2j}^{\mathrm{T}} H_{2j} + \varepsilon_{2ij}^{-1} E_{2ij} E_{2ij}^{\mathrm{T}} + \widehat{\Xi}_{ji} \right. \\
&\qquad \left. + \varepsilon_{1ji} H_{1i}^{\mathrm{T}} H_{1i} + \varepsilon_{1ji}^{-1} E_{1i} E_{1i}^{\mathrm{T}} \right. \\
&\qquad \left. + \varepsilon_{2ji} H_{2i}^{\mathrm{T}} H_{2i} + \varepsilon_{2ji}^{-1} E_{2ji} E_{2ji}^{\mathrm{T}}\right] < 0.
\end{aligned} \tag{24}$$

Applying the Schur complement lemma to the above matrix inequality, the following conclusion can be reached from the above deduction.

Theorem 3. *For a given constant $\gamma > 0$, the filtering error system (8) is robustly asymptotically stable and satisfies $\|H_{\tilde{z}w}(s)\|_\infty < \gamma$, if there exist constant $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$, symmetric positive definite matrix $\widetilde{P}_{11}$, $\widetilde{P}_{22}$, and matrices $\widetilde{P}_{12}$, V_{11}, Q, R, X_i, Y_i, Z_i, D_{fi}, $i, j = 1, 2, \ldots, r$, such that for all admissible uncertainties (3) the following linear matrix inequalities hold:*

$$\begin{gathered}
\begin{bmatrix} \widehat{\Xi}_{ii} + \varepsilon_{1ii} H_{1i}^{\mathrm{T}} H_{1i} + \varepsilon_{2ii} H_{2i}^{\mathrm{T}} H_{2i} & E_{1i} & E_{2ii} \\ E_{1i}^{\mathrm{T}} & -\varepsilon_{1ii} I & 0 \\ E_{2ii}^{\mathrm{T}} & 0 & -\varepsilon_{2ii} I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \\
\begin{bmatrix} \begin{pmatrix} \widehat{\Xi}_{ij} + \widehat{\Xi}_{ji} + \varepsilon_{1ij} H_{1j}^{\mathrm{T}} H_{1j} + \varepsilon_{1ji} H_{1i}^{\mathrm{T}} H_{1i} \\ + \varepsilon_{2ij} H_{2j}^{\mathrm{T}} H_{2j} + \varepsilon_{2ji} H_{2i}^{\mathrm{T}} H_{2i} \end{pmatrix} & E_{1j} & E_{2ij} & E_{1i} & E_{2ji} \\ E_{1j}^{\mathrm{T}} & -\varepsilon_{1ij} I & 0 & 0 & 0 \\ E_{2ij}^{\mathrm{T}} & 0 & -\varepsilon_{2ij} I & 0 & 0 \\ E_{1i}^{\mathrm{T}} & 0 & 0 & -\varepsilon_{1ji} I & 0 \\ E_{2ji}^{\mathrm{T}} & 0 & 0 & 0 & -\varepsilon_{2ji} I \end{bmatrix} < 0, \quad i < j.
\end{gathered} \tag{25}$$

Using the matrix relations of formula (19) and the equivalence of the transfer function, filter parameter matrices are given as follows:

$$\begin{aligned}
&A_{fi} = R^{-1} X_i, \qquad B_{fi} = R^{-1} Y_i, \\
&C_{fi} = Z_i, \qquad D_{fi} = D_{fi}, \\
&\qquad\qquad i = 1, 2, \ldots, r.
\end{aligned} \tag{26}$$

Set $\rho = \gamma^2$, and an optimization problem about robust H_∞ filter can be described in the following:

$$\begin{aligned}
&\min \quad \rho \\
&\text{s.t.} \quad (25).
\end{aligned} \tag{27}$$

Thus, the obtained filter can be called an optimal robust H_∞ filter of the uncertain fuzzy system (4), and the corresponding optimal disturbance attenuation level is $\gamma^* = \sqrt{\rho}$.

4. Numerical Example

In this section, a numerical example will be given to illustrate the effectiveness of robust H_∞ filtering approach developed in the previous section (see Figure 1) [30].

According to the literature [30], Figure 1 can be described by the following state equations:

$$\begin{aligned}
\dot{x}_1(t) &= -0.1 x_1(t) - 0.5 x_1^3(t) + 50 x_2(t), \\
\dot{x}_2(t) &= -x_1(t) - 10 x_2(t) + w(t), \\
y(t) &= x_1(t) + w(t), \\
z(t) &= x_1(t),
\end{aligned} \tag{28}$$

where $x_1(t) = v_C(t)$ is capacitor voltage and $x_2(t) = i_L(t)$ is inductor current.

Assume that the state variable $x_1(t)$ satisfies $|x_1(t)| \le 3$. In order to simplify the calculation, two fuzzy rules will be used to approximate the nonlinear system (28).

Plant Rule 1:

$$\begin{aligned}
&\text{IF } x_1(t) \text{ is } M_1(x_1(t)), \\
&\text{THEN } \dot{x}(t) = (A_1 + \Delta A_1(t)) x(t) + B_1 w(t). \\
&\qquad y(t) = (C_1 + \Delta C_1(t)) x(t) + D_1 w(t), \\
&\qquad z(t) = L_1 x(t).
\end{aligned} \tag{29}$$

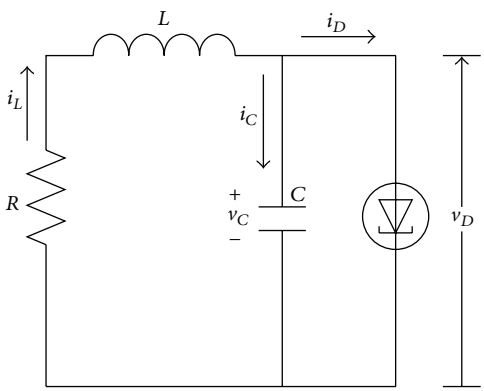

FIGURE 1: Tunnel diode circuit system.

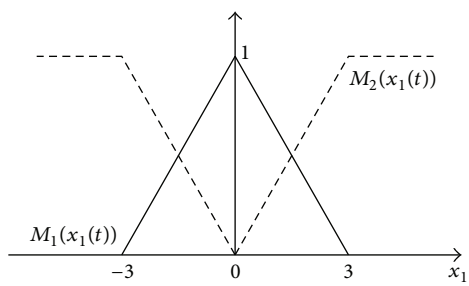

FIGURE 2: Fuzzy membership functions.

Plant Rule 2:

$$\begin{aligned} &\text{IF } x_1(t) \text{ is } M_2(x_1(t)), \\ &\text{THEN } \dot{x}(t) = (A_2 + \Delta A_2(t))\, x(t) + B_2 w(t) \\ &\qquad y(t) = (C_2 + \Delta C_2(t))\, x(t) + D_2 w(t), \\ &\qquad z(t) = L_2 x(t), \end{aligned} \tag{30}$$

where model parameters are given below:

$$\begin{aligned} &A_1 = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix} \\ &B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ &D_1 = D_2 = 1, \qquad L_1 = L_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ &E_{a1} = E_{a2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \qquad H_{a1} = H_{a2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \\ &E_{c1} = E_{c2} = 1, \qquad H_{c1} = H_{c2} = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}. \end{aligned} \tag{31}$$

And the fuzzy membership functions corresponding to the above two rules are given in Figure 2.

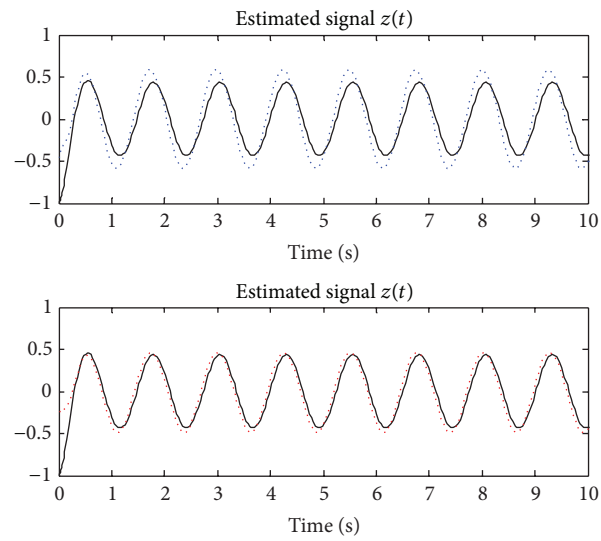

FIGURE 3: Filtering results for $w(t) = 0.5\sin(5t)$.

By giving the H_∞ performance level $\gamma = 1$ and constructing the fuzzy filter (7), by solving linear matrix inequalities (25), the modified filter parameters can be obtained as follows:

$$\begin{aligned} &A_{f1} = \begin{bmatrix} -1.6780 & 41.9421 \\ -1.8564 & -9.1733 \end{bmatrix}, \\ &A_{f2} = \begin{bmatrix} -5.0817 & 43.5742 \\ -1.8467 & -9.1707 \end{bmatrix}, \\ &B_{f1} = \begin{bmatrix} 1.0785 \\ 0.9305 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -0.1095 \\ 0.9196 \end{bmatrix}, \\ &C_{f1} = \begin{bmatrix} 0.6950 & -0.3013 \end{bmatrix}, \\ &C_{f2} = \begin{bmatrix} 0.7422 & -0.3745 \end{bmatrix}, \\ &D_{f1} = 0.2516, \qquad D_{f2} = 0.2566. \end{aligned} \tag{32}$$

Assume that the initial state of system is $x_0 = [-1 \;\; 0]^{\mathrm{T}}$, the initial state of filter is $\hat{x}_0 = [0 \;\; 0]^{\mathrm{T}}$, and the uncertain matrix is selected as $F(t) = \sin(t)$. Apply the above-obtained filter to the system (28) for filtering simulation. When the exogenous interference noise is set as $w(t) = 0.5\sin(5t)$, the simulation results are shown in Figure 3, in which the blue dotted line indicates the case without introducing a free matrix variable, while the red dotted line represents the case with introducing a free matrix variable. Similarly, Figure 4 shows the filtering results when the noise is a random noise with zero mean and variance of 0.01. Obviously, from the simulation results, it can be seen that the filtering results with introducing a free matrix variable are better than those of not introducing, and the former makes the system have a higher error estimation accuracy.

Moreover, by solving the optimization problem (27), the minimum disturbance attenuation level is obtained as $\gamma^* = 5.1 \times 10^{-7}$. By comparison, the result without introducing

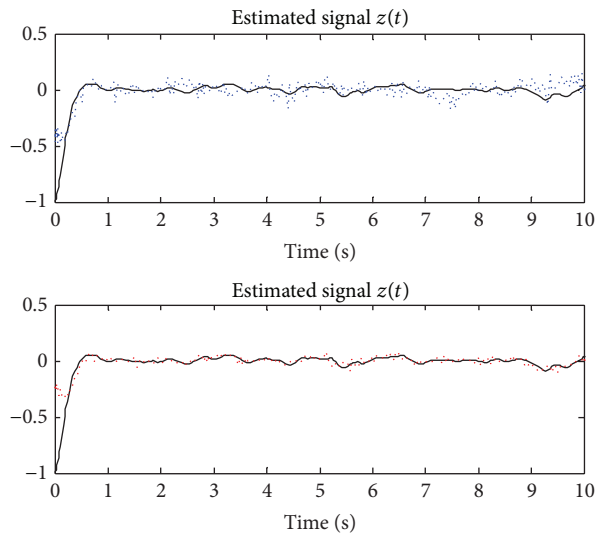

Figure 4: Filtering results for random noise interference.

a free matrix variable is also given as $\gamma^* = 3.3 \times 10^{-6}$. Thus it can be seen that the system can obtain lower disturbance attenuation level by introducing a free matrix variable.

5. Conclusions

This paper successfully extends the ideology of literature [25] to robust H_∞ filter design for a class of uncertain nonlinear systems. By introducing a free matrix variable, this paper gives a new systematic design methodology of robust H_∞ filter. In particular, the filter parameters can be designed independent of the Lyapunov matrix. This method can decouple between the Lyapunov matrix and the system matrix, so it can reduce the conservatism of the system to a certain extent. The solution of filter can be converted into a standard LMI problem. From the simulation results, it can be seen that the improved filter has the lower conservatism and the higher estimation accuracy, which is useful for engineering application.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publishing of this paper.

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