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Research Article

Dynamic Analysis of Nonlinear Impulsive Neutral Nonautonomous Differential Equations with Delays

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A class of neural networks described by nonlinear impulsive neutral nonautonomous differential equations with delays is considered. By means of Lyapunov functionals and differential inequality technique, criteria on global exponential stability of this model are derived. Many adjustable parameters are introduced in criteria to provide flexibility for the design and analysis of the system. The results of this paper are new and they supplement previously known results. An example is given to illustrate the results.

1. Introduction

Many evolution processes in nature exhibit abrupt changes of states at certain moments. That was the reason for the development of the theory of impulsive differential equations and impulsive delay differential equations; see the monographs [1, 2]. But the theory of impulsive neutral differential equations is not well developed due to some theoretical and technical difficulties. For impulsive neutral differential equations, some existence results and oscillation criteria are obtained in [3–5] and some stability conditions are derived in [6]; for neural networks described by impulsive neutral differential equations with delays, the exponential stability results are obtained in [7–11], but their work focuses on the autonomous system. So in this paper, the exponential stability for neural networks described by nonlinear impulsive neutral nonautonomous differential equations with delays is considered.

The purpose of this paper is to study the stability of the following impulsive neural networks with variable coefficients and several time-varying delays:

$$\begin{aligned} \dot{x}_i(t) = {} & -b_i(t)\,x_i(t) + \sum_{j=1}^{n} a_{ij}(t)\, f_{ij}\left(x_j(t)\right) \\ & + \sum_{j=1}^{n} c_{ij}(t)\, g_{ij}\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ & + \sum_{j=1}^{n} d_{ij}(t)\, h_{ij}\left(x_j'\left(t-\widehat{\tau}_{ij}(t)\right)\right) \\ & + k_i(t), \quad \text{a.e. } t>0,\ t\neq t_k, \end{aligned} \tag{1a}$$

$$\begin{aligned} x_i(t^+) = {} & I_{ik}\left(x_i(t)\right) + J_{ik}\left(x_i\left(t-\varsigma_i(t)\right)\right) + K_{ik}(t), \\ & t=t_k, \quad i=1,2,\ldots,n; \quad k=1,2,\ldots, \end{aligned} \tag{1b}$$

where n corresponds to the number of units in a neural network; for $i, j = 1, 2, \ldots, n$, $x_i(t)$ denotes the potential of cell i at time t; $0 \le \tau_{ij}(t), \widehat{\tau}_{ij}(t), \varsigma_i(t) \le \tau$ correspond to the transmission delays. (1a) (called continuous part) describes the continuous evolution processes of the neural networks. For $i, j = 1, 2, \ldots, n$, $a_{ij}(t)$, $c_{ij}(t)$, and $d_{ij}(t)$ denote the strengths of connectivity between cells i and j at time t, respectively; f_{ij}, g_{ij}, h_{ij} show how the ith neuron reacts to the input; $k_i(t)$ is the external bias on the ith at time t. (1b) (called discrete part) describes that the evolution processes experience abrupt change of states at the moments of t_k (called impulsive moments); for $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots$, the fixed moment t_k satisfies $t_1 < t_2 < \cdots < t_k < \cdots$, and $\lim_{k\to\infty} t_k = \infty$; I_{ik} represents impulsive perturbations of ith unit at time t_k; J_{ik} represents impulsive perturbations of ith unit at time t_k, which is caused by the transmission delays; $K_{ik}(t_k)$ represents the external impulsive input at time t_k.

The theory on linear matrix inequality (LMI) or M-Matrix provides effective methods for the analysis of exponential stability of autonomous neural networks. See [7, 9, 10] and the reference therein. But for nonautonomous neural networks, it is invalid. Differential inequalities are important tools for investigating the stability of impulsive differential equations. See [7, 8, 12, 13] and the reference therein. The method in this paper is partially motivated by the work in [7].

In this paper, we will investigate the global exponential stability of the nonautonomous neural networks and focus on the effect of impulse on the dynamic behavior of (1a) and (1b). The results do not require the boundedness of $\{t_k - t_{k-1}\}$ and the differentiability of τ_{ij}. So they are new and complement previously known results.

For a continuous function $a(t)$, we denote

$$a^+(t) = \max\{0, a(t)\}, \qquad a^-(t) = \min\{0, a(t)\},$$
$$a(t^+) = \lim_{s\to t^+} a(s), \qquad a(t^-) = \lim_{s\to t^-} a(s). \tag{2}$$

Define

$$R^+ = [0,\infty), \quad N = \{1,2,\ldots,n\}, \quad N^* = \{1,2,\ldots\},$$
$$C(\Omega, R) = \{\psi : \Omega \longrightarrow R \mid \psi \text{ is continous}, \Omega \subset R\},$$
$$CB(\Omega, R) = \{\psi \in C(\Omega, R) \mid \psi \text{ is bounded}\},$$
$$PC([-\tau,0],R) = \{\psi : [-\tau,0] \longrightarrow R \mid \psi(t^-) = \psi(t), \text{for } t \in [-\tau,0], \psi(t^+) \text{ exists on } R \text{ and } \psi(t^+) = \psi(t) \text{ for all but at most a finite number of points on } [-\tau,0].\},$$
$$PC^1([-\tau,0],R) = \{\psi \in PC([-\tau,0],R) \mid \psi'(t^+) \text{ and } \psi'(t^-) \text{ exist}, \psi'(t) = \psi'(t^-) \text{ for } t \in [-\tau,0] \text{ and } \psi'(t^+) = \psi'(t) \text{ for all but at most a finite number of points on } [-\tau,0].\},$$
$$PC([-\tau,0],R^n) = \{\widehat{\psi} = (\psi_1,\psi_2,\ldots,\psi_n)^T \mid \psi_i \in PC([-\tau,0],R), i \in N.\},$$
$$PC^1([-\tau,0],R^n) = \{\widehat{\psi} = (\psi_1,\psi_2,\ldots,\psi_n)^T \mid \psi_i \in PC^1([-\tau,0],R), i \in N.\}. \tag{3}$$

For any $\phi \in PC([-\tau,0],R)$, $\widehat{\phi} \in PC^1([-\tau,0],R)$, $\psi = (\psi_1, \psi_2,\ldots,\psi_n)^T \in PC([-\tau,0],R^n)$, and $\widehat{\psi} = (\widehat{\psi}_1,\widehat{\psi}_2,\ldots,\widehat{\psi}_n)^T \in PC^1([-\tau,0],R^n)$, define $\|\cdot\|_\tau$, $\|\cdot\|_{1\tau}$, $\|\cdot\|_\tau^n$, and $\|\cdot\|_{1\tau}^n$ as

$$\|\phi\|_\tau = \sup_{-\tau\le s\le 0}|\phi(s)|, \qquad \|\widehat{\phi}\|_{1\tau} = \max\{\|\widehat{\phi}\|_\tau, \|\widehat{\phi}'\|_\tau\},$$
$$\|\psi\|_\tau^n = \max_{1\le i\le n}\|\psi_i\|_\tau, \qquad \|\widehat{\psi}\|_{1\tau}^n = \max_{1\le i\le n}\|\widehat{\psi}_i\|_{1\tau}, \tag{4}$$

respectively.

For convenience, the following conditions are listed.

(H_1) For $i, j \in N$, $b_i \in C(R^+,R^+)$, $a_{ij} \in C(R^+,R)$, and $c_{ij}, d_{ij} \in CB(R^+,R)$, $f_{ij}, g_{ij}, h_{ij} \in C(R,R)$.

(H_2) There are positive constants F_{ij}, G_{ij}, H_{ij}, $i, j \in N$, such that

$$|f_{ij}(u) - f_{ij}(v)| \le F_{ij}|u-v|,$$
$$|g_{ij}(u) - g_{ij}(v)| \le G_{ij}|u-v|, \tag{5}$$
$$|h_{ij}(u) - h_{ij}(v)| \le H_{ij}|u-v|,$$

for all $u, v \in R$.

(H_3) There exist positive constants I_{ik}^* and J_{ik}^*, $i \in N$, $k \in N^*$, such that

$$|I_{ik}(u) - I_{ik}(v)| \le I_{ik}^*|u-v|,$$
$$|J_{ik}(u) - J_{ik}(v)| \le J_{ik}^*|u-v|, \tag{6}$$
$$\max_{i\in N,k\in N^*} I_{ik}^* + \max_{i\in N,k\in N^*} J_{ik}^* < 1,$$

for all $u, v \in R$.

(H_4) There exist positive constants p_i, q_i, $i \in N$ and σ such that

$$p_i b_i(t) - \sum_{j=1}^n p_j F_{ij} a_{ij}^+(t) - \sum_{j=1}^n \left(p_j G_{ij} c_{ij}^+(t) + q_j H_{ij} d_{ij}^+(t)\right) \ge \sigma > 0,$$
$$q_i - p_i b_i(t) - \sum_{j=1}^n p_j F_{ij} a_{ij}^+(t) - \sum_{j=1}^n \left(p_j G_{ij} c_{ij}^+(t) + q_j H_{ij} d_{ij}^+(t)\right) \ge \sigma > 0, \tag{7}$$

for $t \in [0,\infty)$, $i \in N$.

We assume that (1a) and (1b) are with the following initial conditions:

$$x(s) = \phi(s), \qquad s \in [-\tau, 0], \tag{8}$$

where $\phi \in PC([-\tau, 0], R^n)$. According to [13], the initial value problems (1a), (1b), and (8) have the unique solution $x(t, \phi)$ under assumptions (H$_2$) and (H$_3$).

Definition 1. A function $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is said to be a solution of (1a) and (1b) on $[-\tau, \infty)$ if for $i \in N$,

(i) $x_i(t)$ is absolutely continuous on each interval $(0, t_1)$ and (t_k, t_{k+1}), $k \in N^*$;

(ii) for any $t_k, k \in N^*$, $x_i(t_k^+)$ and $x_i(t_k^-)$ exist and $x_i(t_k^-) = x_i(t_k)$;

(iii) $x(t)$ satisfies (1a) for almost everywhere in $[0, \infty)$ and satisfies (1b) for every $t = t_k$, $k \in N^*$.

Obviously, a solution $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of (1a) and (1b) is continuous at $t \neq t_k$ and discontinuous at $t = t_k$. Furthermore, $X'(t) = (x_1'(t), x_2'(t), \ldots, x_n'(t))^T$ has discontinuities of the first kind at the fixed impulsive moments t_k and some moments $\bar{t} \in (t_k, t_{k+1})$, $k \in N^*$. Denote $X'(t_k) = X'(t_k^-)$, $X'(\bar{t}) = X'(\bar{t}^-)$.

Definition 2. Let $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ and $Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ be two solutions of (1a), (1b), and (8) with $\phi = \varphi$ and $\phi = \psi$, respectively, where φ and $\psi \in PC([-\tau, 0], R^n)$. If there exist $\alpha > 0$ and $M > 1$ such that

$$\left|x_i(t) - y_i(t)\right| \le M \|\varphi - \psi\|_{1\tau}^n e^{-\alpha t}, \quad \forall t > 0, \ i \in N, \tag{9}$$

then (1a) and (1b) are said to be globally exponentially stable.

2. The Main Result

To study the exponential stability of (1a) and (1b), we need the following lemma.

Lemma 3. *Assume that (H$_1$) and (H$_4$) hold and there exist nonnegative vector functions $(V_1(t), V_2(t), \ldots, V_n(t))^T$ and $(W_1(t), W_2(t), \ldots, W_n(t))^T \in PC([-\tau, 0], R^n)$, where $V_i(t)$ is continuous at $t \neq t_k$ $(k \in N^*)$, such that*

$$D^- V_i(t^-) \le -b_i(t) V_i(t^-) + \sum_{j=1}^n a_{ij}^+(t) F_{ij} V_j(t^-) + \sum_{j=1}^n c_{ij}^+(t) G_{ij} \left\|V_{jt^-}\right\|_\tau + \sum_{j=1}^n d_{ij}^+(t) H_{ij} \left\|W_{jt^-}\right\|_\tau, \tag{10a}$$

$$W_i(t^+) \le b_i(t) V_i(t^+) + \sum_{j=1}^n a_{ij}^+(t) F_{ij} V_j(t^+) + \sum_{j=1}^n c_{ij}^+(t) G_{ij} \left\|V_{jt^+}\right\|_\tau + \sum_{j=1}^n d_{ij}^+(t) H_{ij} \left\|W_{jt^+}\right\|_\tau, \tag{10b}$$

$$V_i(t_k^+) \le I_{ik}^* V_i(t_k) + J_{ik}^* V_i(t_k - \varsigma_i(t_k)), \tag{10c}$$

for $t > 0$, $i \in N$, $k \in N^*$. Then for all $t \ge 0$ and $i \in N$, there exists a positive constant L such that

$$V_i(t) \le L \sum_{l=1}^n \max\left\{\left\|V_{l0}\right\|_\tau, \left\|W_{l0}\right\|_\tau\right\} e^{-(\lambda^* - \mu)t}, \tag{11}$$

where λ^* and μ are defined, respectively, as

$$\lambda^* = \min\left\{\lambda_i^*, \hat{\lambda}_i^* \mid i \in N\right\}, \qquad \lambda^* + \frac{1}{\tau} \ln \frac{\max_{i \in N, k \in N^*} J_{ik}^*}{1 - \max_{i \in N, k \in N^*} I_{ik}^*} \le \mu \le \lambda^*, \tag{12}$$

$$\begin{aligned} \lambda_i^* = \inf_{t \ge 0} \Big\{ &\lambda(t) > 0, \lambda(t) - \left[b_i(t) - \frac{1}{p_i} \sum_{j=1}^n p_j F_{ij} a_{ij}^+(t)\right] + \frac{1}{p_i} \sum_{j=1}^n \left(p_j G_{ij} c_{ij}^+(t) + q_j H_{ij} d_{ij}^+(t)\right) \times e^{\lambda(t)\tau} = 0 \Big\} > 0, \\ \hat{\lambda}_i^* = \inf_{t \ge 0} \Big\{ &\lambda(t) > 0, - \left[1 - \frac{p_i}{q_i} b_i(t) - \frac{1}{q_i} \sum_{j=1}^n p_j F_{ij} a_{ij}^+(t)\right] + \frac{1}{q_i} \sum_{j=1}^n \left(p_j G_{ij} c_{ij}^+(t) + q_j H_{ij} d_{ij}^+(t)\right) \times e^{\lambda(t)\tau} = 0 \Big\} > 0. \end{aligned} \tag{13}$$

Proof. By the similar analysis in [14, Lemma 4.1], we can deduce that λ_i^* and $\hat{\lambda}_i^*$ exist uniquely and $\lambda_i^* > 0$, $\hat{\lambda}_i^* > 0$ under the assumption of (H$_1$) and (H$_4$). Consequently, $\lambda^* >$

0. Choose a positive constant θ such that $\min\{p_i, q_i \mid i \in N\}\theta > 1$. Let

$$\Phi_i(t) = \max\left\{\frac{1}{p_i}V_i(t), \frac{1}{q_i}W_i(t)\right\},$$
$$\Psi(t) = \theta\sum_{l=1}^{n}\max\{\|V_{l0}\|_\tau, \|W_{l0}\|_\tau\}e^{-(\lambda^*-\mu)t}, \quad i \in N. \tag{14}$$

Then for all $t \in [-\tau, 0]$ and $\gamma > 1$, we have

$$\gamma\Psi(t) = \gamma\theta\sum_{l=1}^{n}\max\{\|V_{l0}\|_\tau, \|W_{l0}\|_\tau\}e^{-(\lambda^*-\mu)t} > \Phi_i(t). \tag{15}$$

Then

$$\Phi_i(t) < \gamma\Psi(t), \quad \forall t \in [0, \infty), \ i \in N. \tag{16}$$

For the sake of contradiction, assume that there exist $i \in N$ and $\bar{t} > 0$ such that

$$\Phi_i(\bar{t}^+) \geq \gamma\Psi(\bar{t}), \quad \Phi_j(t) < \gamma\Psi(t), \quad \text{for } t \in [0, \bar{t}), \quad j \in N. \tag{17}$$

From (17), we have

$$\begin{aligned}\|V_{j\bar{t}}\|_\tau &= p_j \sup_{-\tau\leq\theta\leq 0}\frac{1}{p_j}V_j(\bar{t}+\theta) \\ &\leq p_j \sup_{-\tau\leq\theta\leq 0}\gamma\Psi(\bar{t}+\theta) \leq \gamma p_j\Psi(\bar{t}-\tau);\end{aligned} \tag{18}$$

similarly,

$$\|W_{j\bar{t}}\|_\tau \leq \gamma q_j\Psi(\bar{t}-\tau). \tag{19}$$

Then we have the following cases.

(I) $(1/p_i)V_i(\bar{t}^+) \geq \gamma\Psi(\bar{t})$; then we have the following subcases.

(i) $\bar{t} \neq t_k$, $k \in N^*$. So $V_i(t)$ is continuous at $\bar{t}$. By (17), we have

$$\frac{1}{p_i}V_i(\bar{t}) = \gamma\Psi(\bar{t}), \quad \frac{1}{p_i}D^-V_i(\bar{t}) > \gamma\Psi'(\bar{t}). \tag{20}$$

From (H_4), (17)–(19), and the definition of λ^*, we have

$$\begin{aligned}&\frac{1}{p_i}D^-V_i(\bar{t}) - \gamma\Psi'(\bar{t}) \\ &\quad\leq -\gamma b_i(\bar{t})\Psi(\bar{t}) + \sum_{j=1}^{n}\frac{p_j}{p_i}\gamma a_{ij}^+(\bar{t})F_{ij}\Psi(\bar{t}) \\ &\qquad+ \sum_{j=1}^{n}\gamma\left(\frac{p_j}{p_i}c_{ij}^+(\bar{t})G_{ij} + \frac{q_j}{p_i}d_{ij}^+(\bar{t})H_{ij}\right) \\ &\qquad\times\Psi(\bar{t}-\tau) + \gamma\lambda^*\Psi(\bar{t}) < 0,\end{aligned} \tag{21}$$

which is a contradiction with (20).

(ii) There exists a $k_0 \in N^*$ such that $\bar{t} = t_{k_0}$. By (17), we have

$$\frac{1}{p_i}V_i(\bar{t}) \leq \gamma\Psi(\bar{t}) \leq \frac{1}{p_i}V_i(\bar{t}^+). \tag{22}$$

Noting $(1/p_i)V_i(\bar{t}^+) \neq (1/p_i)V_i(\bar{t}^-)$, we have $(1/p_i)V_i(\bar{t}^-) < \gamma\Psi(\bar{t})$ or $\gamma\Psi(\bar{t}) < (1/p_i)V_i(\bar{t}^+)$. Without loss of generality, we assume that $\gamma\Psi(\bar{t}) < (1/p_i)V_i(\bar{t}^+)$. From (10c) and (22), we get that

$$\gamma\Psi(\bar{t}) < \frac{1}{p_i}V_i(\bar{t}^+) \leq \gamma\left(I_{ik_0}^* + J_{ik_0}^*e^{(\lambda^*-\mu)\tau}\right)\Psi(\bar{t}). \tag{23}$$

Simplifying (23), we obtain $\mu < \lambda^* + (1/\tau)\ln(J_{ik_0}^*/(1 - I_{ik_0}^*))$, which contradict (12).

If (I) does not hold, then

(II)

$$\begin{aligned}&\frac{1}{q_i}W_i(\bar{t}^+) \geq \gamma\Psi(\bar{t}), \quad \frac{1}{q_j}W_j(t) < \gamma\Psi(t), \\ &\frac{1}{p_j}V_j(t) < \gamma\Psi(t) \\ &\text{for } t \in [0, \bar{t}), \quad j \in N.\end{aligned} \tag{24}$$

Then from (10b) and (17)–(19), we have

$$\begin{aligned}0 &\leq -W_i(\bar{t}^+) + b_i(\bar{t})V_i(\bar{t}^+) + \sum_{j=1}^{n}a_{ij}^+(\bar{t})F_{ij}V_j(\bar{t}^+) \\ &\quad+ \sum_{j=1}^{n}c_{ij}^+(\bar{t})G_{ij}\|V_{j\bar{t}^+}\|_\tau + \sum_{j=1}^{n}d_{ij}^+(\bar{t})H_{ij}\|W_{j\bar{t}^+}\|_\tau, \\ &\leq \gamma\Psi(\bar{t})\Bigg[-q_i + p_ib_i(\bar{t}) + \sum_{j=1}^{n}p_ja_{ij}^+(\bar{t})F_{ij} \\ &\qquad+\sum_{j=1}^{n}\left(p_jc_{ij}^+(\bar{t})G_{ij} + q_jd_{ij}^+(\bar{t})H_{ij}\right)e^{\lambda^*\tau}\Bigg] < 0,\end{aligned} \tag{25}$$

which is a contradiction.

From (I) and (II), (16) holds. Letting $\gamma \to 1^+$ in (16), we have

$$\Phi_i(t) \leq \Psi(t), \quad \forall t \in [0, \infty), \ i \in N. \tag{26}$$

So $(1/p_i)V_i(t) \leq \Psi(t)$ for all $t \in [0, \infty)$, $i \in N$. Let $L = \max_{i\in N}\{\theta p_i\}$; then for $t \geq 0$ and $i \in N$, we have

$$V_i(t) \leq L\sum_{l=1}^{n}\max\{\|V_{l0}\|_\tau, \|W_{l0}\|_\tau\}e^{-(\lambda^*-\mu)t}. \tag{27}$$

The proof of Lemma 3 is complete. □

Theorem 4. *Assume that (H_1)–(H_4) hold. Then systems* (1a) *and* (1b) *are globally exponentially stable.*

Proof. Let $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ and $Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ be solutions of (1a), (1b), and (8) with $\phi = \varphi$ and $\phi = \psi$, respectively. Let

$$V_i(t) = |x_i(t) - y_i(t)|, \quad W_i(t) = |x_i'(t) - y_i'(t)|, \quad t \in R^+, \quad i \in N. \tag{28}$$

By (1a) and (1b), for $i \in N$, we have

$$\begin{aligned} D^- V_i(t^-) \le{}& -b_i(t) V_i(t^-) + \sum_{j=1}^{n} a_{ij}^+(t) F_{ij} V_j(t^-) \\ &+ \sum_{j=1}^{n} c_{ij}^+(t) G_{ij} \|V_{jt^-}\|_\tau \\ &+ \sum_{j=1}^{n} d_{ij}^+(t) H_{ij} \|W_{jt^-}\|_\tau, \quad t > 0, \end{aligned} \tag{29}$$

$$\begin{aligned} W_i(t^+) \le{}& b_i(t) V_i(t^+) \\ &+ \sum_{j=1}^{n} a_{ij}^+(t) F_{ij} V_j(t^+) + \sum_{j=1}^{n} c_{ij}^+(t) G_{ij} \|V_{jt^+}\|_\tau \\ &+ \sum_{j=1}^{n} d_{ij}^+(t) H_{ij} \|W_{jt^+}\|_\tau, \quad t > 0. \end{aligned} \tag{30}$$

By (1b) and (H_3), we have

$$V_i(t_k^+) = |x_i(t_k^+) - y_i(t_k^+)| \le I_{ik}^* V_i(t_k) + J_{ik}^* V_i(t_k - \varsigma_i(t_k)). \tag{31}$$

By (29)–(31) and Lemma 3, there exists a positive constant M such that

$$\begin{aligned} V_i(t) &\le M \sum_{l=1}^{n} \max\{\|V_{l0}\|_\tau, \|W_{l0}\|_\tau\} e^{-(\lambda^* - \mu)t} \\ &\le Mn \|\phi - \psi\|_{1\tau}^{n} e^{-(\lambda^* - \mu)t}, \end{aligned} \tag{32}$$

where λ^* and μ are defined in (12). □

Remark 5. For autonomous system, the exponential stability of the zero solution of (1a) with $x_i(t_k^+) = I_{ik}(x_1(t_k), \ldots, x_n(t_k)), k \in N^*$, is considered in [7]. But the results require that $\{t_k - t_{k-1}\}$ is bounded.

When there is no impulse in systems (1a) and (1b), (1a) and (1b) reduce to the following model which has been studied in [9, 10]:

$$\begin{aligned} \dot{x}_i(t) ={}& -b_i(t) x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_{ij}(x_j(t)) \\ &+ \sum_{j=1}^{n} c_{ij}(t) g_{ij}(x_j(t - \tau_{ij}(t))) \\ &+ \sum_{j=1}^{n} d_{ij}(t) h_{ij}(x_j'(t - \hat{\tau}_{ij}(t))) \\ &+ k_i(t), \quad t > 0, \ i \in N. \end{aligned} \tag{33}$$

Corollary 6. *Assume that (H_1), (H_2), and (H_4) hold. (33) is globally exponentially stable.*

Remark 7. For autonomous system, the stability of (33) with $h_{ij}(x) = x$, $f_{ij} = g_{ij}$, is considered in [10]. However, the authors assume that $f_{ij}, i, j = 1, 2, \ldots, n$, are monotonic, bounded and $\tau_{ij}, i, j = 1, 2, \ldots, n$, are constants.

Remark 8. The stability results about the zero solution of $x'(t) = -b(t)x(t) + c(t)x(t - \tau(t)) + d(t)x'(t - \tau(t))$ are obtained by the fixed-point theory in [15]. But the differentiability of τ is needed.

3. An Illustrative Example

To show the effectiveness of Theorem 4, consider the following nonautonomous neural networks with impulse:

$$\begin{aligned} \dot{x}_i(t) ={}& -b_i(t) x_i(t) + \sum_{j=1}^{2} a_{ij}(t) f_{ij}(x_j(t)) \\ &+ \sum_{j=1}^{2} c_{ij}(t) g_{ij}(x_j(t - \tau_{ij}(t))) \\ &+ \sum_{j=1}^{2} d_{ij}(t) h_{ij}(x_j'(t - \hat{\tau}_{ij}(t))) \\ &+ k_i(t), \quad \text{a.e. } t > 0, \end{aligned} \tag{34a}$$

$$\begin{aligned} &x_i(t_k^+) = g_i x_i(t_k) + I_i, \\ &t_k = 5k, \quad i = 1, 2; \quad k = 1, 2, \ldots, \end{aligned} \tag{34b}$$

where

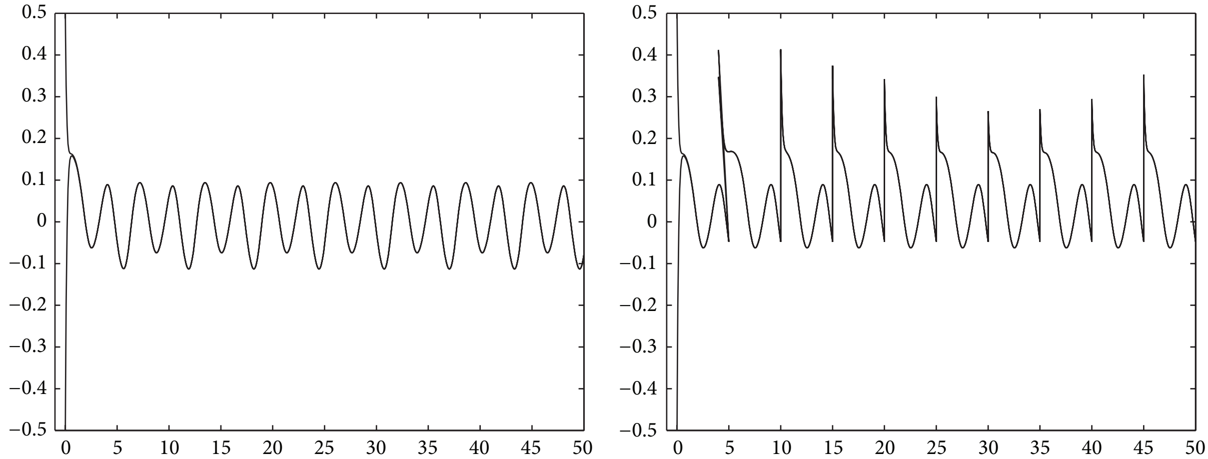
$$\begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} = \begin{pmatrix} 7 + \sin t \\ 5 - \cos t \end{pmatrix}, \qquad \begin{pmatrix} k_1(t) \\ k_2(t) \end{pmatrix} = \begin{pmatrix} e^{-t} \\ e^{-2t} \end{pmatrix},$$

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.3 \end{pmatrix}, \qquad \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0.3 \\ -0.1 \end{pmatrix},$$

$$(a_{ij}(t))_{2\times 2} = \begin{pmatrix} 0, & \frac{1}{3}\cos 3t \\ \frac{\cos 2t}{2}, & 0 \end{pmatrix},$$

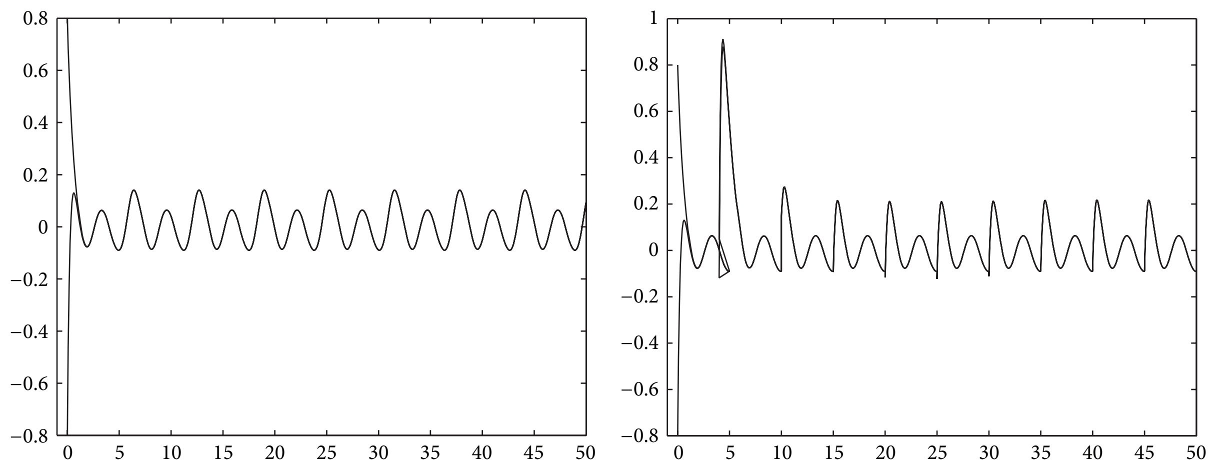
$$(c_{ij}(t))_{2\times 2} = \begin{pmatrix} \sin 2t, & 0 \\ 0, & \frac{\cos t}{2} \end{pmatrix},$$

$$(d_{ij}(t))_{2\times 2} = \begin{pmatrix} \frac{1}{6}\sin 3t, & \frac{1}{8}\sin t \\ \frac{1}{9}\cos t, & \frac{1}{10}\cos 2t \end{pmatrix},$$



(a) (b)

Figure 1: (a) Time response of state variables x_1, u_1 without impulsive effects. (b) Time response of state variables x_1, u_1 with impulsive effects.



(a) (b)

Figure 2: (a) Time response of state variables x_2, u_2 without impulsive effects. (b) Time response of state variables x_2, u_2 with impulsive effects.

$$\left(f_{ij}(x)\right)_{2\times 2} = \begin{pmatrix} 0, & \dfrac{|x+1|-|x-1|}{2} \\ \dfrac{|x+1|+|x-1|}{2}, & 0 \end{pmatrix},$$

$$\left(g_{ij}(x)\right)_{2\times 2} = \begin{pmatrix} \dfrac{|x+1|+|x-1|}{3}, & 0 \\ 0, & \dfrac{|x+1|-|x-1|}{3} \end{pmatrix},$$

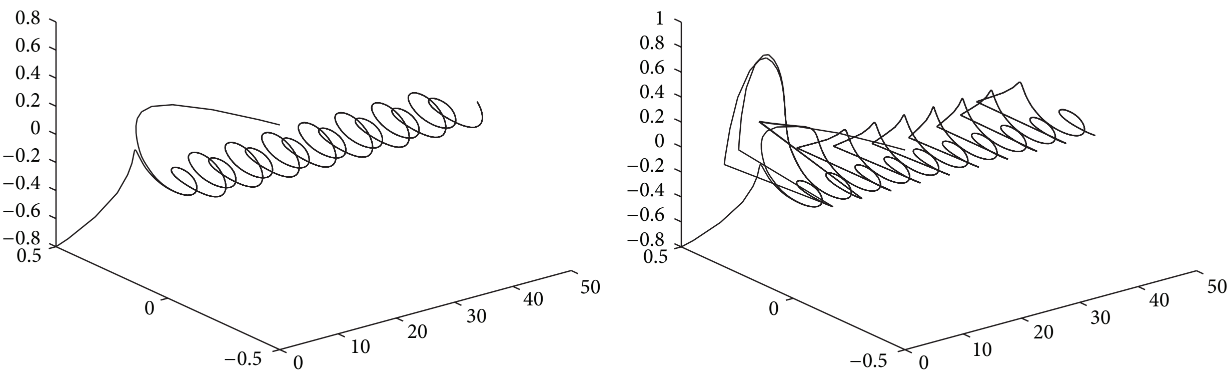
$$\left(h_{ij}(x)\right)_{2\times 2} = \begin{pmatrix} \sin x, & \cos x \\ \cos x, & \sin x \end{pmatrix},$$

$$\left(\tau_{ij}(t)\right)_{2\times 2} = \begin{pmatrix} 2\sin^2 t, & 0 \\ 0, & 2|\cos t| \end{pmatrix},$$

$$\left(\hat{\tau}_{ij}(t)\right)_{2\times 2} = \begin{pmatrix} 0, & \dfrac{1-\sin t}{2} \\ \dfrac{1+\cos t}{2}, & 0 \end{pmatrix}. \tag{35}$$

Obviously, $(F_{ij})_{2\times 2} = \begin{pmatrix} 0, & 1 \\ 1, & 0 \end{pmatrix}$, $(G_{ij})_{2\times 2} = \begin{pmatrix} 2/3, & 0 \\ 0, & 2/3 \end{pmatrix}$, and $(H_{ij})_{2\times 2} = \begin{pmatrix} 1, & 1 \\ 1, & 1 \end{pmatrix}$.

Let $p_1 = p_2 = 1$ and $q_1 = 18$, $q_2 = 10$. From the above assumption, the conditions of Theorem 4 are satisfied. Therefore, (34a) and (34b) are globally exponentially stable. $(x_1(t), x_2(t))^T$ and $(u_1(t), u_2(t))^T$ are the solutions of (34a) and (34b) with $x_1(0) = 0.5$, $x_2(0) = -0.8$ and $u_1(0) = -0.5$, $u_2(0) = 0.8$, respectively. Figures 1(a) and 1(b) depict



(a) (b)

Figure 3: (a) Phase plot in space (t, x_1, x_2), (t, u_1, u_2) without impulsive effects. (b) Phase plot in space (t, x_1, x_2), (t, u_1, u_2) with impulsive effects.

time response of state variables x_1, u_1 without and with impulse effects; Figures 2(a) and 2(b) depict time response of state variables x_2, u_2 without and with impulse effects; Figures 3(a) and 3(b) depict the phase plot in the space (t, x_1, x_2), (t, u_1, u_2) without and with impulse effects.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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