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Research Article

Delay-Dependent Robust $L_2 - L_\infty$ Filtering for a Class of Fuzzy Stochastic Systems

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This paper is concerned with the $L_2 - L_\infty$ filtering problem for a kind of Takagi-Sugeno (T-S) fuzzy stochastic system with time-varying delay and parameter uncertainties. Parameter uncertainties in the system are assumed to satisfy global Lipschitz conditions. And the attention of this paper is focused on the stochastically mean-square stability of the filtering error system, and the $L_2 - L_\infty$ performance level of the output error with the disturbance input. The method designed for the delay-dependent filter is developed based on linear matrix inequalities. Finally, the effectiveness of the proposed method is substantiated with an illustrative example.

1. Introduction

It is well known that many phenomena in engineering have unavoidable uncertain factors that are modeled by the stochastic differential equation. And in recent years, the stochastic system has been widely studied. A great number of investigations on stochastic systems have been reported in the literature. For example, the adaptive back stepping controller has been addressed in [1, 2] for stochastic nonlinear systems in a strict-feedback form. When the time delay appears, [3, 4] have investigated the stability of the time-delay stochastic neutral networks; controllers under different performance levels have been designed for the stochastic system in [5–7] for the delay-dependent controller, H_∞ output feedback controller, and $L_2 - L_\infty$ controller, respectively. And [8–14] have studied the controlling and filtering problem for stochastic jumping systems. However, the results mentioned above are only suitable for the nonlinear systems which have exact known nonlinear dynamics models. As an efficient technique to linearize the nonlinear differential equations, T-S fuzzy model [15] can offer a good way to represent the nonlinear dynamics models.

By using T-S fuzzy model, nonlinear systems turn into linear input-output relations which could be handled easily by appropriate fuzzy sets. This method can be seen in the stirred tank reactor system in [16] and the truck trailer system in [17]. Nowadays, the researches of T-S fuzzy system have grown into a great number. A lot of results have been reported in the literature. For example, the stability and control problem of T-S fuzzy systems have been investigated in [18–22] and the references therein.

On the other hand, state estimation has been found in many practical applications and it has been extensively studied over decades. It aims at estimating the unavailable state variables or their combination for the given system [23, 24]. As a branch of state estimation theory, the filtering problem has become an important research field. The H_∞ filtering problem for the T-S fuzzy system has been addressed in [25–30]; [31–33] have considered the $L_2 - L_\infty$ filtering problem for delayed T-S fuzzy systems with different method. Moreover, robust filters are investigated in [34–36] for stochastic nonlinear systems.

Following above discussion, T-S fuzzy model could be used to divide the nonlinear stochastic systems into several subsystems. And during the past decade, many problems have been tackled. Reference [37] deals with the robust fault detection problem for T-S fuzzy stochastic systems. And [38, 39] consider the stabilization for the fuzzy stochastic systems with delays. References [40–43] have studied the control problem for fuzzy stochastic systems. An adaptive fuzzy controller has been designed for stochastic nonlinear systems in [44]. Reference [45] addresses the passivity of the stochastic

T-S fuzzy system. Solutions to fuzzy stochastic differential equations with local martingales have been addressed in [46]. Then recognizing the value of state estimating when state variables are unavailable, it is important to research the filtering problem for T-S fuzzy stochastic systems. However, there are few results available to the best of the authors knowledge, especially the results on L_2-L_∞ filtering problem for the fuzzy stochastic systems.

As a consequence, this paper will focus on the robust fuzzy delay-dependent L_2-L_∞ filter design for a T-S fuzzy stochastic system with time-varying delay and norm-bounded parameter uncertainties by using the Lyapunov-Krasovskii functional technique and some useful free-weighting matrices. The obtained sufficient conditions are expressed in terms of linear matrix inequality (LMI) approach. The remainder of this paper is organized as follows. The filter design problem is formulated in Section 2. And Section 3 gives our main results. In Section 4, a numerical example is shown to illustrate the effectiveness of the proposed methods. Finally, we conclude the paper in Section 5.

Notation. The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition. Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. I stands for an identity matrix of appropriate dimension, while $I_n \in \mathbf{R}^n$ denotes a vector of ones. The notation $*$ is used as an ellipsis for terms that are induced by symmetry. diag(...) stands for a block-diagonal matrix. $|\cdot|$ denotes the Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm for matrices. $\mathbf{L}_2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$. $\mathsf{E}(\cdot)$ stands for the mathematical expectation operator. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

Consider the time-delay T-S fuzzy stochastic system with time-varying parameter uncertainties as the following form:

$$\begin{aligned}
(\Sigma) : dx(t) &= \sum_{i=1}^{r}\rho_i(s(t))\{[(A_i+\Delta A_i(t))x(t) \\
&\quad + (A_{di}+\Delta A_{di}(t))x(t-\tau(t))+B_i v(t)]\,dt \\
&\quad + [(H_i+\Delta H_i(t))x(t) \\
&\quad + (H_{di}+\Delta H_{di}(t))x(t-\tau(t))]\,d\omega(t)\}, \\
dy(t) &= \sum_{i=1}^{r}\rho_i(s(t))[C_i x(t)+C_{di}x(t-\tau(t))+D_i v(t)]\,dt, \\
z(t) &= \sum_{i=1}^{r}\rho_i(s(t))[L_i x(t)], \\
x(t) &= \varphi(t), \quad t\in[-h_2,0],
\end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^m$ is the system state; $\varphi(t)$ is a given differential initial function on $[-h_2,0]$; $\omega(t)$ is a scalar zero mean Gaussian white noise process with unit covariance; $y(t) \in \mathbf{R}^n$ is the measured output; $z(t) \in \mathbf{R}^l$ is a signal to be estimated; $v(t) \in \mathbf{R}^s$ is the noise signal which belongs to $\mathscr{L}_2[0,\infty)$; $\tau(t)$ is a continuous differentiable function representing the time-varying delay in $x(t)$, which is assumed to satisfy for all $t \geq 0$,

$$0 \leq h_1 \leq \tau(t) < h_2. \tag{2}$$

In the considered fuzzy stochastic system, A_i, A_{di}, B_i, H_i, H_{di}, C_i, C_{di}, D_i, and L_i are known constant matrices with appropriate dimensions. $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta H_i(t)$, and $\Delta H_{di}(t)$ represent the unknown time-varying parameter uncertainties and are assumed to satisfy

$$\begin{bmatrix}\Delta A_i(t) & \Delta A_{di}(t)\\ \Delta H_i(t) & \Delta H_{di}(t)\end{bmatrix} = \begin{bmatrix}M_{1i}\\ M_{2i}\end{bmatrix} F_i(t)\,[N_{1i}\quad N_{2i}], \tag{3}$$

where M_{1i}, M_{2i}, N_{1i}, and N_{2i} are known real constant matrices and the unknown time-varying matrix function satisfying

$$F_i(t)^T F_i(t) \leq I \quad \forall t. \tag{4}$$

And using the fuzzy theory, there always have for all t,

$$\rho_i(s(t)) \geq 0, \quad i = 1,2,\ldots,r, \quad \sum_{i=1}^{r}\rho_i(s(t)) = 1. \tag{5}$$

The fuzzy filters we considered are as follows:

$$\begin{aligned}
d\hat{x}(t) &= \sum_{i=1}^{r}\rho_i(s(t))\left[A_{fi}\hat{x}(t)\,dt + B_{fi}dy(t)\right], \\
\hat{z}(t) &= \sum_{i=1}^{r}\rho_i(s(t))\left[L_{fi}\hat{x}(t)\right],
\end{aligned} \tag{6}$$

in which the fuzzy rules have the same representations as in (1). $\hat{x}(t) \in \mathbf{R}^n$ and $\hat{z}(t) \in \mathbf{R}^l$. A_{fi}, B_{fi}, and L_{fi} are the filters needed to be determined.

Remark 1. It is worth to mention that there are two approaches for the filter design in fuzzy systems. The implementation of the filter could be chosen to depend on or not depend on the fuzzy rules when the fuzzy model is available or not. And it is obvious to see that the former filter related to the fuzzy rules is less conserve and more complex. So we assume that the fuzzy is known here, which means the fuzzy-rule-dependent filter is investigated in this paper as in (6).

Let $\xi(t) = [x(t)^T \quad \hat{x}(t)^T]^T$ and $e(t) = z(t) - \hat{z}(t)$. And the filtering error dynamic system can be written as

$$\begin{aligned}
(\tilde{\Sigma}) : d\xi(t) &= \left[(\tilde{A}+\Delta\tilde{A}(t))\xi(t) + (\tilde{A}_d+\Delta\tilde{A}_d(t))K\xi(t-\tau(t)) \right. \\
&\quad \left. + \tilde{B}v(t)\right]dt \\
&\quad + \left[(\tilde{H}+\Delta\tilde{H}(t))\xi(t) + (\tilde{H}_d+\Delta\tilde{H}_d(t)) \right. \\
&\quad \left. \times K\xi(t-\tau(t))\right]d\omega(t), \\
e(t) &= \tilde{L}\xi(t),
\end{aligned} \tag{7}$$

where

$$\widetilde{A} = \begin{bmatrix} \overline{A} & 0 \\ \overline{B}_f\overline{C} & \overline{A}_f \end{bmatrix}, \qquad \widetilde{A}_d = \begin{bmatrix} \overline{A}_d \\ \overline{B}_f\overline{C}_{d1} \end{bmatrix},$$

$$\widetilde{H} = \begin{bmatrix} \overline{H} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \Delta\widetilde{A}(t) = \begin{bmatrix} \Delta\overline{A}(t) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Delta\widetilde{A}_d(t) = \begin{bmatrix} \Delta\overline{A}_d(t) \\ 0 \end{bmatrix}, \qquad \widetilde{B} = \begin{bmatrix} \overline{B} \\ \overline{B}_f\overline{D} \end{bmatrix},$$

$$\Delta\widetilde{H}(t) = \begin{bmatrix} \Delta\overline{H}(t) & 0 \\ 0 & 0 \end{bmatrix}, \qquad \Delta\widetilde{H}_d(t) = \begin{bmatrix} \Delta\overline{H}_d(t) \\ 0 \end{bmatrix},$$

$$\widetilde{H}_d = \begin{bmatrix} \overline{H}_d \\ 0 \end{bmatrix}, \qquad \overline{A} = \sum_{i=1}^{r}\rho_i(s(t))A_i,$$

$$\overline{A}_d = \sum_{i=1}^{r}\rho_i(s(t))A_{di}, \qquad \overline{C} = \sum_{i=1}^{r}\rho_i(s(t))C_i,$$

$$\overline{H} = \sum_{i=1}^{r}\rho_i(s(t))H_i, \qquad \overline{H}_d = \sum_{i=1}^{r}\rho_i(s(t))H_{di},$$

$$\overline{C}_d = \sum_{i=1}^{r}\rho_i(s(t))C_{di}, \qquad \overline{B} = \sum_{i=1}^{r}\rho_i(s(t))B_i,$$

$$\overline{D} = \sum_{i=1}^{r}\rho_i(s(t))D_i, \qquad \widetilde{L} = \begin{bmatrix} \overline{L} & -\overline{L}_f \end{bmatrix},$$

$$\overline{A}_f = \sum_{i=1}^{r}\rho_i(s(t))A_{fi}, \qquad \overline{B}_f = \sum_{i=1}^{r}\rho_i(s(t))B_{fi},$$

$$\overline{L}_f = \sum_{i=1}^{r}\rho_i(s(t))L_{fi}, \qquad \overline{L} = \sum_{i=1}^{r}\rho_i(s(t))L_i,$$

$$\overline{L}_d = \sum_{i=1}^{r}\rho_i(s(t))L_{di}, \qquad K = \begin{bmatrix} I & 0 \end{bmatrix},$$

$$\Delta\overline{A}(t) = \sum_{i=1}^{r}\rho_i(s(t))\Delta A_i(t),$$

$$\Delta\overline{A}_d(t) = \sum_{i=1}^{r}\rho_i(s(t))\Delta A_{di}(t),$$

$$\Delta\overline{H}(t) = \sum_{i=1}^{r}\rho_i(s(t))\Delta H_i(t),$$

$$\Delta\overline{H}_d(t) = \sum_{i=1}^{r}\rho_i(s(t))\Delta H_{di}(t). \tag{8}$$

We intend to design sets of fuzzy filters in the form of (6) in this paper, such that for any scalar $0 \le h_1 < h_2$ and a prescribed level of noise attenuation $\gamma > 0$, the filtering error system $(\widetilde{\Sigma})$ could be mean square stable. Moreover, the error system $(\widetilde{\Sigma})$ satisfies $L_2 - L_\infty$ performance.

Throughout the paper, we adopt the following definitions and lemmas, which help to complete the proof of the main results.

Definition 2. The system (Σ) is said to be robust stochastic mean-square stable if there exists $\delta(\varepsilon) > 0$ for any $\varepsilon > 0$ such that

$$\mathsf{E}\left(\|x(t)\|^2\right) < \varepsilon, \quad t > 0, \tag{9}$$

when $\sup_{-h\le s\le 0}\mathsf{E}(\|\varphi(s)\|^2) < \delta(\varepsilon)$, for any uncertain variables. And in addition,

$$\lim_{t\to\infty}\mathsf{E}\left(\|x(t)\|^2\right) = 0, \tag{10}$$

for any initial conditions.

Definition 3. The robust stochastic mean-square stable system $(\widetilde{\Sigma})$ is said to satisfy the $L_2 - L_\infty$ performance, for the given scalar $\gamma > 0$ and any nonzero $v(t) \in L_2[0,\infty)$, and the system $(\widetilde{\Sigma})$ satisfies

$$\|e(t)\|_\infty < \gamma\|v(t)\|_2, \tag{11}$$

and for any uncertain variables, where

$$\|e(t)\|_\infty^2 := \sup_t e(t)^T e(t). \tag{12}$$

Lemma 4. *For the given matrices M, N, F with $F^TF \le I$ and positive scalar $\varepsilon > 0$, the following inequality holds:*

$$MFN + (MFN)^T \le \varepsilon MM^T + \varepsilon^{-1}N^TN. \tag{13}$$

3. Robust Stochastic Stabile

First, we define the following variables for convenience:

$$\begin{aligned}\Phi(t) &= \left(\widetilde{A} + \Delta\widetilde{A}(t)\right)\xi(t) + \left(\widetilde{A}_d + \Delta\widetilde{A}_d(t)\right)K\xi(t-\tau(t)) \\ &\quad + \widetilde{B}v(t), \\ g(t) &= \left(\widetilde{H} + \Delta\widetilde{H}(t)\right)\xi(t) + \left(\widetilde{H}_d + \Delta\widetilde{H}_d(t)\right)K\xi(t-\tau(t)).\end{aligned} \tag{14}$$

Theorem 5. *The filtering error system $(\widetilde{\Sigma})$ is robust stochastic mean square stable and* (11) *is satisfied for any time-varying delay $0 \le h_1 \le \tau(t) < h_2$, if there exist matrices $P = P^T > 0$, $R = R^T > 0$, $Q_i = Q_i^T > 0$, $Z_i = Z_i^T > 0$, T_{1i}, T_{2i}, $i = 1, 2$, such that the following matrix inequalities hold:*

$$\begin{bmatrix} P & \widetilde{L}^T \\ \widetilde{L} & \gamma^2 I \end{bmatrix} > 0, \qquad \Psi = \begin{bmatrix} \Omega & \Psi_{12} \\ * & \Psi_{22} \end{bmatrix} < 0, \tag{15}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & 0 & \Omega_{14} & 0 & P\widetilde{B} \\ * & \Omega_{22} & 0 & \Omega_{24} & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned}
\Psi_{12} &= \left[\widetilde{T}_1 \;\; \widetilde{T}_2 \;\; h_{21}\widetilde{T}_1 \;\; h_{21}\widetilde{T}_2 \;\; h_{21}\breve{A}^T K^T Z_1 \;\; h_{21}\breve{H}^T K^T Z_2 \;\; \breve{H}P\right],\\
\Psi_{22} &= \operatorname{diag}\{-Z_2, -Z_2, -h_{21}Z_1, -h_{21}Z_1, -h_{21}Z_1,\\
&\qquad -h_{21}Z_2, -P\},\\
\Omega_{11} &= P\left(\widetilde{A} + \Delta\widetilde{A}(t)\right) + \left(\widetilde{A} + \Delta\widetilde{A}(t)\right)^T P\\
&\quad + K^T\left(Q_1 + Q_2 + (h_2 - h_1)R\right)K,\\
\Omega_{14} &= P\left(\widetilde{A}_d + \Delta\widetilde{A}_d(t)\right),\\
\Omega_{22} &= -Q_1 + T_1 + T_1^T, \qquad \Omega_{24} = -T_1 + T_1,\\
\Omega_{33} &= -Q_2 - T_2 - T_2^T, \qquad \Omega_{34} = T_2 - T_2^T,\\
\Omega_{44} &= -T_1 - T_1^T + T_2 + T_2^T,\\
\Omega_{55} &= \frac{-R}{(h_2 - h_1)},\\
\widetilde{T}_1 &= \left[0 \;\; T_1^T \;\; 0 \;\; T_1^T \;\; 0 \;\; 0\right]^T,\\
\widetilde{T}_2 &= \left[0 \;\; 0 \;\; T_2^T \;\; T_2^T \;\; 0 \;\; 0\right]^T,\\
\breve{A} &= \left[\widetilde{A}^T + \Delta\widetilde{A}^T(t) \;\; 0 \;\; 0 \;\; \widetilde{A}_d^T + \Delta\widetilde{A}_d^T(t) \;\; 0 \;\; \widetilde{B}^T\right]^T,\\
\breve{H} &= \left[\widetilde{H}^T + \Delta\widetilde{H}^T(t) \;\; 0 \;\; 0 \;\; \widetilde{H}_d^T + \Delta\widetilde{H}_d^T(t) \;\; 0 \;\; 0\right]^T,\\
h_{21} &= h_2 - h_1.
\end{aligned} \tag{16}$$

Proof. Define the following Lyapunov-Krasovskii candidate for system $(\widetilde{\Sigma})$:

$$\begin{aligned}
V(\xi(t), t) &= \xi^T(t)P\xi(t) + \int_{t-h_1}^{t} \xi^T(s)K^TQ_1K\xi(s)\,ds\\
&\quad + \int_{t-h_2}^{t} \xi^T(s)K^TQ_2K\xi(s)\,ds\\
&\quad + \int_{-h_2}^{-h_1}\int_{t+\beta}^{t} \Phi^T(s)K^TZ_1K\Phi(s)\,ds\,d\beta\\
&\quad + \int_{-h_2}^{-h_1}\int_{t+\beta}^{t} g^T(s)K^TZ_2Kg(s)\\
&\quad + \int_{-h_2}^{-h_1}\int_{t+\beta}^{t} \xi^T(s)K^TRK\xi(s)\,ds\,d\beta.
\end{aligned} \tag{17}$$

When $v(t) = 0$,

$$dV(\xi(t), t) = \mathbf{L}V(\xi(t), t) + 2\xi^T(t)Pg(t)\,d\omega(t). \tag{18}$$

By using the Newton-Leibnitz formula, the following equations can be got for any matrices T_1, T_2 with appropriate dimensions:

$$\begin{aligned}
&2\eta^T(t)\overline{T}_1K\left[\xi(t-h_1) - \xi(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_1}\Phi(s)\,ds\right.\\
&\qquad\left. - \int_{t-\tau(t)}^{t-h_1} g(s)\,d\omega(s)\right] = 0,\\
&2\eta^T(t)\overline{T}_2K\left[\xi(t-\tau(t)) - \xi(t-h_2) - \int_{t-h_2}^{t-\tau(t)}\Phi(s)\,ds\right.\\
&\qquad\left. - \int_{t-h_2}^{t-\tau(t)} g(s)\,d\omega(s)\right] = 0,\\
&(\tau(t) - h_1)\eta^T(t)\overline{T}_1Z_1^{-1}\overline{T}_1^T\eta(t)\\
&\qquad - \int_{t-\tau(t)}^{t-h_1}\eta^T(t)\overline{T}_1Z_1^{-1}\overline{T}_1^T\eta(t)\,ds = 0,\\
&(h_2 - \tau(t))\eta^T(t)\overline{T}_2Z_1^{-1}\overline{T}_2^T\eta(t)\\
&\qquad - \int_{t-h_2}^{t-\tau(t)}\eta^T(t)\overline{T}_2Z_1^{-1}\overline{T}_2^T\eta(t)\,ds = 0,
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\overline{T}_1 &= \left[0 \;\; T_1^T \;\; 0 \;\; T_1^T \;\; 0\right]^T,\\
\overline{T}_2 &= \left[0 \;\; 0 \;\; T_2^T \;\; T_2^T \;\; 0\right]^T.
\end{aligned} \tag{20}$$

And $\eta(t)$ is a new vector defined as follows:

$$\eta^T(t) = \left[\xi^T(t) \;\; \xi^T(t-h_1)K^T \;\; \xi^T(t-h_2)K^T \;\; \xi^T(t-\tau(t))K^T \;\; \left(\int_{t-h_2}^{t-h_1}\xi(s)^T ds\right)K^T\right]. \tag{21}$$

By the above formulas (19) and Lemma 4, we can deduce that

$$\begin{aligned}
&\mathbf{L}V(\xi(t), t)\\
&\quad = 2\xi^T(t)P\Phi(t) + g^T(t)Pg(t) + \xi^T(t)K^TQ_1K\xi(t)\\
&\qquad + \xi^T(t)K^TQ_2K\xi(t) - \xi^T(t-h_1)K^TQ_1K\xi(t-h_1)\\
&\qquad - \xi^T(t-h_2)K^TQ_2K\xi(t-h_2) + h_{21}\Phi^T(t)K^TZ_1K\Phi(t)\\
&\qquad + h_{21}g(t)^TK^TZ_2Kg(t) - \int_{t-h_2}^{t-h_1}\xi^T(s)K^TRK\xi(s)\,ds\\
&\qquad - \int_{t-h_2}^{t-h_1}\Phi^T(s)K^TZ_1K\Phi(s)\,ds\\
&\qquad - \int_{t-h_2}^{t-h_1} g^T(s)K^TZ_2Kg(s)\,ds + h_{21}\Phi^T(t)K^TRK\Phi(t)\\
&\quad \le \eta^T(t)\left[\overline{\Omega} + h_{21}\overline{T}_1Z_1^{-1}\overline{T}_1^T + \widehat{H}\left(K^T(h_2 - h_1)Z_2K + P\right)\widehat{H}\right.\\
&\qquad + \widehat{A}K^Th_{21}Z_1K\widehat{A}^T + h_{21}\overline{T}_2Z_1^{-1}\overline{T}_2^T\\
&\qquad \left. + \overline{T}_1Z_2^{-1}\overline{T}_1^T + \overline{T}_2Z_2^{-1}\overline{T}_2^T\right]\eta(t)
\end{aligned}$$

$$
\begin{aligned}
&-\int_{t-\tau(t)}^{t-h_1}\left[\eta^T(t)\overline{T}_1+\Phi^T(s)K^TZ_1\right]Z_1^{-1}\\
&\qquad\times\left[Z_1K\Phi(s)+\overline{T}_1^T\eta(t)\right]ds\\
&-\int_{t-h_2}^{t-\tau(t)}\left[\eta^T(t)\overline{T}_2+\Phi^T(s)K^TZ_1\right]Z_1^{-1}\\
&\qquad\times\left[Z_1K\Phi(s)+\overline{T}_2^T\eta(t)\right]ds\\
&+\left(\int_{t-\tau(t)}^{t-h_1}g(s)d\omega(s)\right)^TK^TZ_2K\left(\int_{t-\tau(t)}^{t-h_1}g(s)\,d\omega(s)\right)\\
&+\left(\int_{t-h_2}^{t-\tau(t)}g(s)d\omega(s)\right)^TK^TZ_2K\left(\int_{t-h_2}^{t-\tau(t)}g(s)\,d\omega(s)\right)\\
&-\int_{t-\tau(t)}^{t-h_1}g^T(s)K^TZ_2Kg(s)\,ds\\
&-\int_{t-h_2}^{t-\tau(t)}g^T(s)K^TZ_2Kg(s)\,ds,
\end{aligned}
\tag{22}
$$

where

$$
\overline{\Omega}=\begin{bmatrix}\Omega_{11}&0&0&\Omega_{14}&0\\ *&\Omega_{22}&0&\Omega_{24}&0\\ *&*&\Omega_{33}&\Omega_{34}&0\\ *&*&*&\Omega_{44}&0\\ *&*&*&*&\Omega_{55}\end{bmatrix}
\tag{23}
$$

$$
\begin{aligned}
\widehat{A}&=\left[\left(\widetilde{A}+\Delta\widetilde{A}(t)\right)^T\quad 0\quad 0\quad\left(\widetilde{A}_d+\Delta\widetilde{A}_d(t)\right)^T\quad 0\right]^T,\\
\widehat{H}&=\left[\left(\widetilde{H}+\Delta\widetilde{H}(t)\right)^T\quad 0\quad 0\quad\left(\widetilde{H}_d+\Delta\widetilde{H}_d(t)\right)^T\quad 0\right]^T.
\end{aligned}
$$

During the analysis, it can be seen that

$$
\begin{aligned}
&\left(\int_{t-\tau(t)}^{t-h_1}g(s)d\omega(s)\right)^TK^TZ_2K\left(\int_{t-\tau(t)}^{t-h_1}g(s)\,d\omega(s)\right)\\
&+\left(\int_{t-h_2}^{t-\tau(t)}g(s)\,d\omega(s)\right)^TK^TZ_2K\left(\int_{t-h_2}^{t-\tau(t)}g(s)\,d\omega(s)\right)\\
&-\int_{t-\tau(t)}^{t-h_1}g^T(s)K^TZ_2Kg(s)\,ds\\
&-\int_{t-h_2}^{t-\tau(t)}g^T(s)K^TZ_2Kg(s)\,ds=0,
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&-\int_{t-\tau(t)}^{t-h_1}\left[\eta^T(t)\overline{T}_1+\Phi^T(s)K^TZ_1\right]Z_1^{-1}\\
&\qquad\times\left[Z_1K\Phi(s)+\overline{T}_1^T\eta(t)\right]ds\\
&-\int_{t-h_2}^{t-\tau(t)}\left[\eta^T(t)\overline{T}_2+\Phi^T(s)K^TZ_1\right]Z_1^{-1}\\
&\qquad\times\left[Z_1K\Phi(s)+\overline{T}_2^T\eta(t)\right]ds<0.
\end{aligned}
\tag{25}
$$

And applying the Schur complement to (15), we can derive the following inequality with $v(t)=0$:

$$
\begin{aligned}
&\overline{\Omega}+h_{21}\overline{T}_1Z_1^{-1}\overline{T}_1^T+\widehat{H}\left(K^Th_{21}Z_2K+P\right)\widehat{H}+\overline{T}_1Z_2^{-1}\overline{T}_1^T\\
&\quad+\widehat{A}K^Th_{21}Z_1K\widehat{A}^T+h_{21}\overline{T}_2Z_1^{-1}\overline{T}_2^T+\overline{T}_2Z_2^{-1}\overline{T}_2^T<0.
\end{aligned}
\tag{26}
$$

From (22)–(26), we can get that

$$
\mathbf{L}V(\xi(t),t)<0,
\tag{27}
$$

which ensures that system $(\widetilde{\Sigma})$ with $v(t)=0$ is robustly stochastically stable according to Definition 2 and [47]. By Itô's formula, it is easy to derive

$$
\mathsf{E}(V(\xi(t),t))=\mathsf{E}\left(\int_0^t\mathsf{L}V(\xi(s),s)\,ds\right).
\tag{28}
$$

Now we establish the L_2-L_∞ performance of the filtering error system $(\widetilde{\Sigma})$. It is easy to obtain

$$
\begin{aligned}
&\mathbf{L}V(\xi(t),t)-\omega(t)^T\omega(t)\\
&\quad\le\overline{\eta}^T(t)\left[\Omega+h_{21}\widetilde{T}_1Z_1^{-1}\widetilde{T}_1^T+\check{H}\left(K^Th_{21}Z_2K+P\right)\check{H}\right.\\
&\qquad+\check{A}K^Th_{21}Z_1K\check{A}^T+h_{21}\widetilde{T}_2Z_1^{-1}\widetilde{T}_2^T\\
&\qquad\left.+\widetilde{T}_1Z_2^{-1}\widetilde{T}_1^T+\widetilde{T}_2Z_2^{-1}\widetilde{T}_2^T\right]\overline{\eta}(t).
\end{aligned}
\tag{29}
$$

Then applying the Schur complement formula to (15), we can get

$$
\begin{aligned}
&\overline{\eta}^T(t)\left[\Omega+(h_2-h_1)\widetilde{T}_1Z_1^{-1}\widetilde{T}_1^T+\check{H}\left(K^Th_{21}Z_2K+P\right)\check{H}\right.\\
&\qquad+\widetilde{T}_2Z_2^{-1}\widetilde{T}_2^T+\check{A}K^Th_{21}Z_1K\check{A}^T+h_{21}\widetilde{T}_2Z_1^{-1}\widetilde{T}_2^T\\
&\qquad\left.+\widetilde{T}_1Z_2^{-1}\widetilde{T}_1^T\right]\overline{\eta}(t)<0,
\end{aligned}
\tag{30}
$$

for all $t>0$, where

$$
\overline{\eta}^T(t)=\left[\xi^T(t)\quad\xi^T(t-h_1)K^T\quad\xi^T(t-h_2)K^T\quad\xi^T(t-\tau(t))K^T\quad\left(\int_{t-h_2}^{t-h_1}\xi(s)^Tds\right)K^T\quad v(t)\right].
\tag{31}
$$

Therefore, for all $\overline{\eta}(t) \neq 0$, $\mathbf{L}V(\xi(t),t) - \omega(t)^T\omega(t) < 0$, which means

$$\xi^T(t)P\xi(t) \leq V(\xi(t),t) < \int_0^t \omega(s)^T\omega(s)\,ds. \tag{32}$$

Then using the Schur complement to the first formula in (15), we have $\widetilde{L}^T\widetilde{L} < \gamma^2 P$, which guarantees

$$\begin{aligned} &e(t)^T e(t) - \xi^T(t)\widetilde{L}^T\widetilde{L}\xi(t) \\ &\quad < \gamma^2\xi^T(t)P\xi(t) < \gamma^2\int_0^t \omega(s)^T\omega(s)\,ds \\ &\quad \leq \gamma^2\int_0^\infty \omega(s)^T\omega(s)\,ds. \end{aligned} \tag{33}$$

Therefore, $\|e\|_\infty < \gamma\|\omega\|_2$ for any zero mean Gaussian white noise process $\omega(t)$ with unit covariance. □

Remark 6. The system we studied is a time-varying delay system containing the information of both the lower bound and the upper bound of time delay. By such a consideration, delay-dependent result is more reliable and approaches to reality that not all the delays begin with 0 moment.

Remark 7. It is worth mentioning that Theorem 5 can be easily extended to investigate the robust H_∞ filtering design problem for the systems $(\widetilde{\Sigma})$ with parameter uncertainties.

Now we are in a position to present a sufficient condition for the solvability of robust $L_2 - L_\infty$ filtering problem.

Theorem 8. *Consider the uncertain T-S fuzzy stochastic time-varying delay system (Σ) and a constant scalar $\gamma > 0$. The robust $L_2 - L_\infty$ filtering problem is solvable if there exist scalars $\varepsilon_i > 0$ and matrices $W > 0$, $X > 0$, $R > 0$, $Q_i > 0$, $Z_i > 0$, T_{1i}, T_{2i}, $i = 1,2$; Φ_{1i}, Φ_{2i}, Φ_{3i}, Φ_{4i}, $1 \leq i \leq r$, $\{\Upsilon_i = \Upsilon_i^T, 1 \leq i \leq r\}$, $\{\Delta_{ij}, 1 \leq i < j \leq r\}$, and such that the following LMIs hold:*

$$X - W > 0, \tag{34}$$

$$\begin{bmatrix} \Upsilon_1 & \Delta_{12} & \cdots & \Delta_{1r} \\ * & \Upsilon_2 & \cdots & \Delta_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Upsilon_r \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} \Gamma_{ii} - \Upsilon_i + \varepsilon_i\Xi_i\Xi_i^T & \Theta_i \\ * & -\varepsilon_i \end{bmatrix} < 0, \quad (1 \leq i \leq r), \tag{36}$$

$$\begin{bmatrix} \Gamma_{ij} + \Gamma_{ji} - \Delta_{ij} - \Delta_{ij}^T + \varepsilon_i\Xi_i\Xi_i^T + \varepsilon_j\Xi_j\Xi_j^T & \Theta_i & \Theta_j \\ * & -\varepsilon_i & 0 \\ * & * & -\varepsilon_i \end{bmatrix} < 0, \quad (1 \leq i < j \leq r), \tag{37}$$

$$\begin{bmatrix} W & W & L_j^T - \Phi_{3i}^T \\ * & X & L_j^T \\ * & * & \gamma^2 I \end{bmatrix} > 0, \quad (1 \leq i \leq r,\ 1 \leq j \leq r), \tag{38}$$

where

$$\begin{gathered} \Theta_i^T = \begin{bmatrix} M_{1i}^T W & M_{1i}^T X & \mathbf{0}_{1*9} & h_{21}M_{1i}^T Z_1 & h_{21}M_{2i}^T Z_2 & M_{2i}^T X & M_{2i}^T \end{bmatrix}, \\ \Xi_i^T = \begin{bmatrix} N_{1i}^T & N_{1i}^T & 0 & 0 & N_{2i}^T & \mathbf{0}_{10*1} \end{bmatrix}, \\ \Gamma_{ij} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & \mathbf{0}_{4*4} \\ * & * & \Gamma_{33} \end{bmatrix}, \\ \Gamma_{11} = \begin{bmatrix} G_{11} & G_{12} & 0 & 0 & G_{15} & 0 & G_{17} \\ * & G_{22} & 0 & 0 & G_{25} & 0 & G_{27} \\ * & * & G_{33} & 0 & G_{35} & 0 & 0 \\ * & * & * & G_{44} & G_{45} & 0 & 0 \\ * & * & * & * & G_{55} & 0 & 0 \\ * & * & * & * & * & -\dfrac{R}{h_{21}} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}, \\ G_{11} = WA_i + A_i^T W + Q_1 + Q_2 + h_{21}R, \\ G_{12} = WA_i + A_i^T X + C_j^T\Phi_{1i}^T + \Phi_{2i}^T + Q_1 + Q_2 + h_{21}R, \\ G_{15} = WA_{di}, \qquad G_{17} = WB_i, \\ G_{22} = X^T A_i + \Phi_{1i}C_j + C_j^T\Phi_{1i}^T + A_i^T X + Q_1 + Q_2 + h_{21}R, \\ G_{25} = X^T A_{di} + \Phi_{1i}C_{dj}, \\ G_{27} = X^T B_i + \Phi_{1i}D_j, \\ G_{33} = -Q_1 + T_1 + T_1^T, \qquad G_{35} = -T_1 + T_1^T, \\ G_{44} = -Q_2 - T_2 - T_2^T, \qquad G_{45} = T_2 - T_2^T, \\ G_{55} = -T_1 - T_1^T + T_2 + T_2^T, \\ \Gamma_{12} = \begin{bmatrix} \widetilde{T}_1 & \widetilde{T}_2 & h_{21}\widetilde{T}_1 & h_{21}\widetilde{T}_2 \end{bmatrix}, \\ \Gamma_{22} = \operatorname{diag}\{-Z_2, -Z_2, -h_{21}Z_1, -h_{21}Z_1\}, \\ \Gamma_{13} = \begin{bmatrix} h_{21}A_i^T Z_1 & h_{21}H_i^T Z_2 & H_i^T X & H_i^T \\ h_{21}A_i^T Z_1 & h_{21}H_i^T Z_2 & H_i^T X & H_i^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ h_{21}A_{di}^T Z_1 & h_{21}H_{di}^T Z_2 & H_{di}^T X & H_{di}^T \\ 0 & 0 & 0 & 0 \\ h_{21}B_i^T Z_1 & 0 & 0 & 0 \end{bmatrix}, \\ \Gamma_{33} = \operatorname{diag}\left\{-h_{21}Z_1, -h_{21}Z_2, \begin{bmatrix} -X & -I \\ * & -\Phi_{4i} \end{bmatrix}\right\}. \end{gathered} \tag{39}$$

When the LMIs (34)–(38) are feasible, the time-dependent filter we desired here can be chosen as

$$\begin{aligned} &A_{fi} = \sigma^{-1}\Phi_{2i}W^{-1}\beta^{-T}, \qquad B_{fi} = \sigma^{-1}\Phi_{1i}, \\ &L_{fi} = \Phi_{3i}W^{-1}\beta^{-T}, \quad i = 1,\ldots,r, \end{aligned} \tag{40}$$

where σ and β are nonsingular matrices satisfying $\sigma\beta^T = I - XW^{-1}$.

Proof. Similar to [33], we know that $I - XW^{-1}$ is nonsingular. Therefore, there always exist nonsingular matrices σ and β

such that $\sigma\beta^T = I - XW^{-1}$ holds. Then we define the nonsingular matrices Λ_1 and Λ_2 as follows:

$$\Lambda_1 = \begin{bmatrix} W^{-1} & I \\ \beta^T & o \end{bmatrix}; \qquad \Lambda_2 = \begin{bmatrix} I & X \\ 0 & \sigma^T \end{bmatrix}. \tag{41}$$

Define $U = \Lambda_2\Lambda_1^{-1}$. Then there is

$$U = \begin{bmatrix} X & \sigma \\ \sigma^T & \beta^{-1}W^{-1}(X_W)W^{-1}\beta^{-T} \end{bmatrix} > 0. \tag{42}$$

Now using Lemma 4 and recalling (36), we can deduce that

$$\begin{aligned}
\Lambda &= \sum_{i=1}^{r}\rho_i^2(s(t))\left[\Gamma_{ii} + \Theta_i F_i(t)\Xi_i^T + \Xi_i F_i(t)\Theta_i^T\right] \\
&\quad + \sum_{i=1}^{r}\sum_{j>i}^{r}\rho_i(s(t))\rho_j(s(t)) \\
&\qquad \times \left[\Gamma_{ij} + \Theta_i F_i(t)\Xi_i^T + \Xi_i F_i(t)\Theta_i^T + \Gamma_{ji}\right. \\
&\qquad\quad \left. + \Theta_i F_j(t)\Xi_j^T + \Xi_j F_j(t)\Theta_j^T\right] \\
&< \sum_{i=1}^{r}\rho_i^2(s(t))\left[\Gamma_{ii} + \varepsilon_i^{-1}\Theta_i\Theta_i^T + \varepsilon_i\Xi_i\Xi_i^T\right] \\
&\quad + \sum_{i=1}^{r}\sum_{j>i}^{r}\rho_i(s(t))\rho_j(s(t)) \\
&\qquad \times \left[\Gamma ij + \varepsilon_i^{-1}\Theta\Theta_i^T + \varepsilon_i\Xi_i\Xi_i^T + \Gamma_{ji} + \varepsilon_j^{-1}\Theta\Theta_j^T\right. \\
&\qquad\quad \left. + \varepsilon_j\Xi_j\Xi_j^T\right] \\
&< \sum_{i=1}^{r}\rho_i^2(s(t))\Upsilon_i + \sum_{i=1}^{r}\sum_{j>i}^{r}\rho_i(s(t))\rho_j(s(t))\left[\Delta_{ij} + \Delta_{ji}^T\right] \\
&= \begin{bmatrix} \rho_1(s(t))I \\ \rho_2(s(t))I \\ \vdots \\ \rho_r(s(t))I \end{bmatrix}^T \begin{bmatrix} \Upsilon_1 & \Delta_{12} & \cdots & \Delta_{1r} \\ * & \Delta_2 & \cdots & \Delta_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Upsilon_r \end{bmatrix} \begin{bmatrix} \rho_1(s(t))I \\ \rho_2(s(t))I \\ \vdots \\ \rho_r(s(t))I \end{bmatrix} < 0.
\end{aligned} \tag{43}$$

We can deduce that

$$\begin{aligned}
&\left\{\operatorname{diag}\left(\Lambda_2^{-T}\begin{bmatrix} W^{-1} & 0 \\ 0 & I \end{bmatrix}, I, \ldots, \begin{bmatrix} \sigma^{-T} & 0 \\ 0 & I \end{bmatrix}\right)\right\}\Lambda \\
&\left\{\operatorname{diag}\left(\Lambda_2^{-T}\begin{bmatrix} W^{-1} & 0 \\ 0 & I \end{bmatrix}, I, \ldots, \begin{bmatrix} \sigma^{-1} & 0 \\ 0 & I \end{bmatrix}\right)\right\} \\
&\quad = \begin{bmatrix} \Omega & \Psi_{12} \\ * & \Psi_{22} \end{bmatrix} < 0,
\end{aligned} \tag{44}$$

which is equivalent to (15). Therefore, it is easy to see that the condition in Theorem 5 and the LMIs in (34)–(37) are equivalent. Finally, it can be concluded that the filtering error system $(\widetilde{\Sigma})$ is stochastically stable with $L_2 - L_\infty$ performance level γ. □

Remark 9. The desired $L_2 - L_\infty$ filters can be constructed by solving the LMIs in (34)–(38), which can be implemented by using standard numerical algorithms, and no tuning of parameters will be involved.

Remark 10. In the proof of above Theorem, we adopt (25), (26), and Newton-Leibnitz formula to reduce the conservatism. Moreover, the results obtained in Theorems 5 and 8 can be further extended based on fuzzy or piecewise Lyapunov-Krasovskii function.

4. Numerical Example

In this section, a numerical example is provided to show the effectiveness of the results obtained in the previous section.

Example 1. Consider the T-S fuzzy stochastic system $(\widetilde{\Sigma})$ with model parameters given as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -2.3 & 0 \\ 0.2 & -1.1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.2 & 0.2 \\ -0.16 & -0.18 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -2.1 & 0.1 \\ 0.1 & -1.4 \end{bmatrix}, \\
H_1 &= \begin{bmatrix} -0.4 & 0.1 \\ 0.3 & -0.5 \end{bmatrix}, \qquad H_{d1} = \begin{bmatrix} -0.01 & 0.02 \\ 0.01 & -0.05 \end{bmatrix}, \\
H_2 &= \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.5 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & -0.4 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} -0.4 & -0.1 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} -0.2 & 0.4 \end{bmatrix}, \qquad C_{d2} = \begin{bmatrix} -0.4 & 0.5 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} 1.5 & -0.6 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -0.3 & 0.2 \end{bmatrix}, \\
D_1 &= 0.2, \qquad D_2 = -0.2, \\
B_1 &= \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.3 \\ -0.1 \end{bmatrix}, \\
A_{d2} &= \begin{bmatrix} -0.18 & 0 \\ -0.22 & -0.24 \end{bmatrix}, \qquad H_{d2} = \begin{bmatrix} -0.05 & 0.01 \\ 0.03 & -0.04 \end{bmatrix}.
\end{aligned} \tag{45}$$

And the parameter uncertainties are shown as:

$$\begin{aligned}
M_{11} &= \begin{bmatrix} 0.1 & 0.2 \\ -0.5 & 0.1 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} -0.2 & 0.1 \\ 0.3 & -0.1 \end{bmatrix}, \\
M_{21} &= \begin{bmatrix} 0.8 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}, \\
N_{11} &= \begin{bmatrix} 0 & -0.3 \\ 0.1 - 0.2 & \end{bmatrix}, \qquad N_{21} = \begin{bmatrix} -0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \\
M_{22} &= \begin{bmatrix} -0.1 & 0.2 \\ 0.4 & -0.2 \end{bmatrix}, \\
N_{12} &= \begin{bmatrix} -0.5 & 0 \\ 0.2 & -0.3 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 0 & -0.2 \\ 0 & 0.1 \end{bmatrix}.
\end{aligned} \tag{46}$$

The membership functions are

$$\begin{aligned}
h_1(x_1(t)) &= \left(1 - \frac{3x_1}{1 + \exp(6x_1(t) + 2)}\right), \\
h_2(x_1(t)) &= 1 - h_1(x_1(t)).
\end{aligned} \tag{47}$$

By using the Matlab LMI Control Toolbox, we have the robust $L_2 - L_\infty$ filtering problem which is solvable to Theorem 8. It can be calculated that for any $0 < h_1(t) \leq 3$,

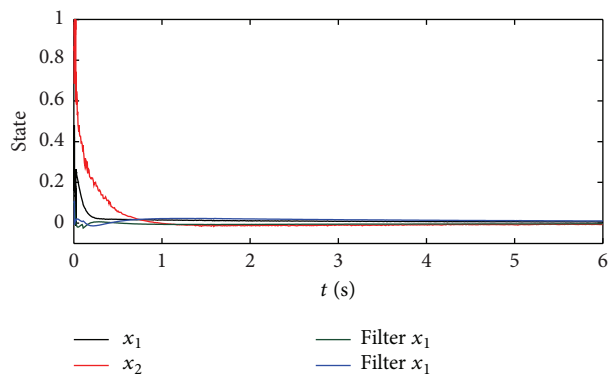

FIGURE 1: State responses of $x(t)$ and $\hat{x}(t)$.

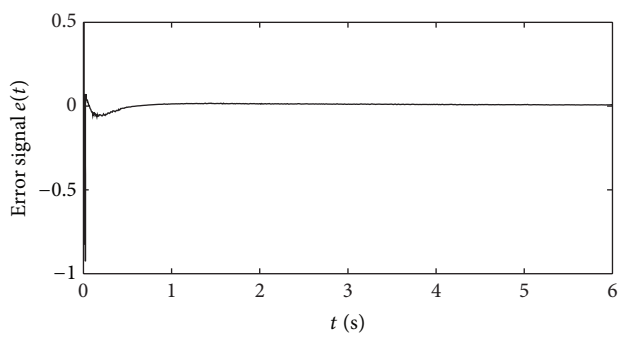

FIGURE 2: Responses of the error signal $e(t)$.

$0 < h_2(t) \leq 8$, $\gamma = 0.42$ the robust $L_2 - L_\infty$ filtering problem can be solved. A desired fuzzy filter can be constructed as in the form of (6) with

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -5.4320 & 0.4511 \\ 1.8159 & -1.5495 \end{bmatrix}, \\ A_{f2} &= \begin{bmatrix} -8.1142 & 3.4902 \\ 2.9687 & -5.9058 \end{bmatrix}, \\ B_{f1} &= \begin{bmatrix} -1.0301 \\ 0.1040 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -1.0171 \\ 0.0415 \end{bmatrix}, \\ L_{f1} &= \begin{bmatrix} -0.3063 & -0.0422 \end{bmatrix}, \\ L_{f2} &= \begin{bmatrix} -0.2667 & -0.0422 \end{bmatrix}. \end{aligned} \tag{48}$$

The simulation results of the state response of the plant and the filter are given in Figure 1, where the initial condition is $x_0(t) = [0.4 \quad 2.5]^T$, $\hat{x}_0(t) = [0.1 \quad 0.1]^T$. Figure 2 shows the simulation results of the signal $e(t)$, and the exogenous disturbance input $v(t)$ is given by $v(t) = 12/(5 + 2t)$, $t \geq 0$, which belongs to $\mathscr{L}_2[0, \infty)$.

5. Conclusion

This paper considers the robust $L_2 - L_\infty$ filter design problem for the uncertain T-S fuzzy stochastic system with time-varying delay. An LMI approach has been developed to design the fuzzy filter ensuring not only the robust stochastic mean-square stability but also a prescribed $L_2 - L_\infty$ performance level of the filtering error system for all admissible uncertainties. A numerical example has been provided to show the effectiveness of the proposed filter design methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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