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Research Article

Positive Solutions for Third-Order p-Laplacian Functional Dynamic Equations on Time Scales

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We study the following third-order p-Laplacian functional dynamic equation on time scales: $[\Phi_p(u^{\Delta\nabla}(t))]^{\nabla}+a(t)f(u(t),u(\mu(t)))=0$, $t\in(0,T)_{\mathbf{T}}$, $u(t)=\varphi(t)$, $t\in[-r,0]_{\mathbf{T}}$, $u^{\Delta}(0)=u^{\Delta\nabla}(T)=0$, and $u(T)+B_0(u^{\Delta}(\eta))=0$. By applying the Five-Functional Fixed Point Theorem, the existence criteria of three positive solutions are established.

1. Introduction

Recently, much attention has been paid to the existence of positive solutions for the boundary value problems with p-Laplacian operator on time scales; for example, see [1–22] and the references therein. But, to the best of our knowledge, there is not much concerning p-Laplacian functional dynamic equations on time scales [6, 12–14, 19, 21, 22], especially for the third-order p-Laplacian functional dynamic equations on time scales [14, 22].

In [14], Song and Gao were concerned with the existence of positive solutions for the p-Laplacian functional dynamic equation on time scales:

$$\left[\Phi_p\left(u^{\Delta\nabla}(t)\right)\right]^{\nabla}+a(t)f(u(t),u(\mu(t)))=0,\quad t\in(0,T)_{\mathbf{T}},$$
$$u(t)=\varphi(t),\quad t\in[-r,0]_{\mathbf{T}},$$
$$u^{\Delta}(0)=u^{\Delta\nabla}(T)=0,\qquad u(T)+B_0\left(u^{\Delta}(\eta)\right)=0, \tag{1}$$

where $\eta\in(0,\rho(T))_{\mathbf{T}}$ and $\Phi_p(s)$ is p-Laplacian operator; that is, $\Phi_p(s)=|s|^{p-2}s$, $p>1$, $(\Phi_p)^{-1}=\Phi_q$, $1/p+1/q=1$, and

(C_1) $f:(\mathbb{R}^+)^2\to\mathbb{R}^+$ is continuous;

(C_2) $a:\mathbf{T}\to\mathbb{R}^+$ is left dense continuous (i.e., $a\in C_{\mathbf{ld}}(\mathbf{T},\mathbb{R}^+)$) and does not vanish identically on any closed subinterval of $[0,T]$, where $C_{\mathbf{ld}}(\mathbf{T},\mathbb{R}^+)$ denotes the set of all left dense continuous functions from $\mathbf{T}$ to $\mathbb{R}^+$;

(C_3) $\varphi:[-r,0]_{\mathbf{T}}\to\mathbb{R}^+$ is continuous and $r>0$;

(C_4) $\mu:[0,T]_{\mathbf{T}}\to[-r,T]_{\mathbf{T}}$ is continuous, $\mu(t)\le 0$ for all t;

(C_5) $B_0:\mathbb{R}\to\mathbb{R}$ is continuous and satisfies the condition that there are $A\ge B\ge 0$ such that

$$Bv\le B_0(v)\le Av,\quad \forall\mathbb{R}. \tag{2}$$

The existence of two positive solutions to problem (1) was obtained by using a double fixed point theorem due to Avery et al. [23] in a cone.

In [22], Wang and Guan considered the existence of positive solutions to problem (1) by applying the well-known Leggett-Williams Fixed Point Theorem.

Motivated by [14, 22], we will show that problem (1) has at least three positive solutions by means of the Five-Functional Fixed Point Theorem [24] (which is a generalization of the Leggett-Williams Fixed Point Theorem [25]). It is worth noting that the Five-Functional Fixed Point Theorem is used extensively in yielding three solutions for BVPs of differential equations, difference equations, and/or dynamic equations on time scales; see [6, 26, 27] and references therein.

Throughout this work we assume knowledge of time scales and time-scale notation, first introduced by Hilger [28].

For more on time scales, please see the texts by Bohner and Peterson [29, 30].

In the remainder of this section, we state the following theorem, which is crucial to our proof.

Let γ, β, θ be nonnegative, continuous, and convex functionals on P and let α, ψ be nonnegative, continuous, and concave functionals on P. Then, for nonnegative real numbers h, a, b, d, and c, we define the convex sets

$$\begin{gathered} P(\gamma, c) = \{x \in P : \gamma(x) < c\}, \\ P(\gamma, \alpha, a, c) = \{x \in P : a \le \alpha(x), \gamma(x) \le c\}, \\ Q(\gamma, \beta, d, c) = \{x \in P : \beta(x) \le d, \gamma(x) \le c\}, \\ P(\gamma, \theta, \alpha, a, b, c) = \{x \in P : a \le \alpha(x), \theta(x) \le b, \gamma(x) \le c\}, \\ Q(\gamma, \beta, \psi, h, d, c) \\ = \{x \in P : h \le \psi(x), \beta(x) \le d, \gamma(x) \le c\}. \end{gathered} \tag{3}$$

Theorem 1 (see [24]). *Let P be a cone in a real Banach space E. Suppose there exist positive numbers c and M; nonnegative, continuous, and concave functionals α and ψ on P; and nonnegative, continuous, and convex functionals γ, β, and θ on P, with*

$$\alpha(x) \le \beta(x), \qquad \|x\| \le M\gamma(x) \tag{4}$$

for all $x \in \overline{P(\gamma, c)}$. Suppose

$$F : \overline{P(\gamma, c)} \longrightarrow \overline{P(\gamma, c)} \tag{5}$$

is completely continuous and there exist nonnegative numbers h, a, k, b, with $0 < a < b$ such that

(i) *$\{x \in P(\gamma, \theta, \alpha, b, k, c) : \alpha(x) > b\} \ne \varnothing$ and $\alpha(Fx) > b$ for $x \in P(\gamma, \theta, \alpha, b, k, c)$;*

(ii) *$\{x \in Q(\gamma, \beta, \psi, h, a, c) : \beta(x) < a\} \ne \varnothing$ and $\beta(Fx) < a$ for $x \in Q(\gamma, \beta, \psi, h, a, c)$;*

(iii) *$\alpha(Fx) > b$ for $x \in P(\gamma, \alpha, b, c)$ with $\theta(Fx) > k$;*

(iv) *$\beta(Fx) < a$ for $x \in Q(\gamma, \beta, a, c)$ with $\psi(Fx) < h$.*

Then F has at least three fixed points $x_1, x_2, x_3 \in \overline{P(\gamma, c)}$ such that

$$\begin{gathered} \beta(x_1) < a, \qquad b < \alpha(x_2), \\ a < \beta(x_3) \quad \text{with } \alpha(x_3) < b. \end{gathered} \tag{6}$$

2. Existence of Three Positive Solutions

We note that $u(t)$ is a solution of BVP (1) if and only if

$$u(t) = \begin{cases} \displaystyle\int_0^T (T-s)\,\Phi_q\left(\int_0^s a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s \\ \displaystyle\quad - B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s\right) \\ \displaystyle\quad + \int_0^t (t-s)\,\Phi_q\left(\int_0^s -a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s, & t \in [0,T]_{\mathbf{T}}, \\ \varphi(t), & t \in [-r, 0]_{\mathbf{T}}. \end{cases} \tag{7}$$

Let $E = C_{\mathrm{ld}}([0,T]_{\mathbf{T}}, \mathbb{R})$ be endowed with $\|u\| = \sup_{t\in[0,T]_{\mathbf{T}}} |u(t)|$, so E is a Banach space. Define cone $P \subset E$ by

$$P = \left\{u \in E : u \text{ is concave and nonnegative valued on } [0,T]_{\mathbf{T}},\ u^{\Delta}(0) = 0\right\}. \tag{8}$$

For each $u \in E$, extend $u(t)$ to $[-r, T]_{\mathbf{T}}$ with $u(t) = \varphi(t)$ for $t \in [-r, 0]_{\mathbf{T}}$.

Define $F : P \to E$ by

$$\begin{aligned} (Fu)(t) &= \int_0^T (T-s)\,\Phi_q \\ &\quad \times \left(\int_0^s a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s \\ &\quad - B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s\right) \\ &\quad + \int_0^t (t-s)\,\Phi_q\left(\int_0^s -a(r) f(u(r), u(\mu(r)))\nabla r\right)\nabla s, \\ &\qquad t \in [0,T]_{\mathbf{T}}. \end{aligned} \tag{9}$$

We seek a point, u_1, of F in the cone P. Define

$$u(t) = \begin{cases} u_1(t), & t \in [0,T]_{\mathbf{T}}, \\ \varphi(t), & t \in [-r, 0]_{\mathbf{T}}. \end{cases} \tag{10}$$

Then $u(t)$ denotes a positive solution of BVP (1).

We have the following results.

Lemma 2. *Let $u \in P$, and then*

(1) *$F : P \to P$ is completely continuous;*

(2) *$u(t) \ge ((T-t)/T)\|u\|$ for $t \in [0,T]_{\mathbf{T}}$;*

(3) *$u(t)$ is decreasing $[0,T]_{\mathbf{T}}$;*

(4) *$(T-\varsigma)u(\tau) \le (T-\tau)u(\varsigma)$ for $0 < \tau < \varsigma < T$ and $\tau, \varsigma \in \mathbf{T}$.*

Proof. (1)–(3) are Lemma 3.1 of [14]. It is easy to conclude that (4) is satisfied by the concavity of u.

Let $l \in \mathbf{T}$ be fixed such that $0 < l < \eta < T$, and set

$$Y_1 = \{t \in [0,T]_{\mathbf{T}} : \mu(t) < 0\};$$
$$Y_2 = \{t \in [0,T]_{\mathbf{T}} : \mu(t) \geq 0\}; \quad (11)$$
$$Y_3 = Y_1 \cap [0,l]_{\mathbf{T}}.$$

Throughout this paper, we assume $Y_3 \neq \varnothing$ and $\int_{Y_3} a(r)\nabla r > 0$.

We define the nonnegative, continuous, and concave functionals α, ψ and the nonnegative, continuous, and convex functionals β, θ, γ on the cone P, respectively, as

$$\gamma(u) = \theta(u) = \max_{t\in[\eta,T]_{\mathbf{T}}} u(t) = u(\eta),$$
$$\alpha(u) = \min_{t\in[0,l]_{\mathbf{T}}} u(t) = u(l),$$
$$\beta(u) = \max_{t\in[l,T]_{\mathbf{T}}} u(t) = u(l), \quad (12)$$
$$\psi(u) = \min_{t\in[0,\eta]_{\mathbf{T}}} u(t) = u(\eta).$$

We observe that $\alpha(u) = \beta(u)$ for each $u \in P$.

In addition, by Lemma 2, we have $\gamma(u) = u(\eta) \geq ((T-\eta)/T)\|u\|$. Hence $\|u\| \leq (T/(T-\eta))\gamma(u)$ for all $u \in P$.

For convenience, we define

$$\mu = T(T+\eta+B)\Phi_q\left(\int_0^T a(r)\nabla r\right),$$
$$\delta = A\int_{Y_3}\Phi_q\left(\int_0^s a(r)\nabla r\right)\nabla s, \quad (13)$$
$$\lambda = T(T+l+B)\Phi_q\left(\int_0^T a(r)\nabla r\right).$$

□

We now state growth conditions on f so that BVP (1) has at least three positive solutions.

Theorem 3. *Let $0 < a < ((T-l)/T)b < ((T-\eta)(T-l)/T^2)c$, $\mu b < \delta c$, and suppose that f satisfies the following conditions:*

(H_1) *$f(u,\varphi(s)) < \Phi_p(c/\mu)$, if $0 \leq u \leq (T/(T-\eta))c$, uniformly in $s \in [-r,0]_{\mathbf{T}}$, and $f(u_1,u_2) < \Phi_p(c/\mu)$, if $0 \leq u_i \leq (T/(T-\eta))c$, $i = 1,2$;*

(H_2) *$f(u,\varphi(s)) > \Phi_p(b/\delta)$, if $b \leq u \leq (T/(T-\eta))^2 b$, uniformly in $s \in [-r,0]_{\mathbf{T}}$;*

(H_3) *$f(u,\varphi(s)) < \Phi_p(a/\lambda)$, if $0 \leq u \leq (T/(T-l))a$, uniformly in $s \in [-r,0]_{\mathbf{T}}$, and $f(u_1,u_2) < \Phi_p(a/\lambda)$, if $0 \leq u_i \leq (T/(T-l))a$, $i = 1,2$.*

Then BVP (1) *has at least three positive solutions of the form*

$$u(t) = \begin{cases} u_i(t), & t \in [0,T]_{\mathbf{T}},\ i = 1,2,3, \\ \varphi(t), & t \in [-r,0]_{\mathbf{T}}, \end{cases} \quad (14)$$

where $\max_{t\in[l,T]_{\mathbf{T}}} u_1(t) < a$, $\min_{t\in[0,l]_{\mathbf{T}}} u_2(t) > b$, and $a < \max_{t\in[l,T]_{\mathbf{T}}} u_3(t)$ with $\min_{t\in[0,l]_{\mathbf{T}}} u_3(t) < b$.

Proof. Let $u \in \overline{P(\gamma,c)}$, and then $\gamma(u) = \max_{t\in[\eta,T]_{\mathbf{T}}} u(t) = u(\eta) \leq c$, and consequently, $0 \leq u(t) \leq c$ for $t \in [\eta,T]_{\mathbf{T}}$. Since $u(\eta) \geq ((T-\eta)/T)u(0)$, so $\|u\| = u(0) \leq (T/(T-\eta))u(\eta) \leq (T/(T-\eta))c$, and this implies

$$0 \leq u(t) \leq \frac{T}{T-\eta}c, \quad \text{for } t \in [0,T]_{\mathbf{T}}. \quad (15)$$

From (H_1), we have

$$\begin{aligned}
\gamma(Fu) &= (Fu)(\eta) \\
&= \int_0^T (T-s)\Phi_q\left(\int_0^s a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad - B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s\right) \\
&\quad + \int_0^\eta (\eta-s)\Phi_q\left(\int_0^s -a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\leq \int_0^T T\Phi_q\left(\int_0^s a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad + B\int_0^T \Phi_q\left(\int_0^s a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad + \int_0^\eta T\Phi_q\left(\int_0^T a(r)f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&= T(T+\eta+B)\Phi_q\left[\int_{Y_1} a(r)f(u(r),\varphi(\mu(r)))\nabla r + \int_{Y_2} a(r)f(u(r),u(\mu(r)))\nabla r\right] \\
&< T(T+\eta+B)\Phi_q\left(\int_0^T a(r)\nabla r\right)\frac{c}{\mu} = c.
\end{aligned} \quad (16)$$

Therefore

$$Fu \in \overline{P(\gamma,c)}. \quad (17)$$

We now turn to property (i) of Theorem 1. Choosing $u \equiv (T/(T-\eta))b$, $k = (T/(T-\eta))b$, it follows that

$$\alpha(u) = u(l) = \frac{T}{T-\eta}b > b,$$
$$\theta(u) = u(\eta) = \frac{T}{T-\eta}b = k, \quad (18)$$
$$\gamma(u) = u(\eta) = \frac{T}{T-\eta}b < c,$$

which shows that $\{u \in P(\gamma,\theta,\alpha,b,k,c) : \alpha(u) > b\} \neq \varnothing$, and, for $u \in P(\gamma,\theta,\alpha,b,(T/(T-\eta))b,c)$, we have

$$b \leq u(t) \leq \left(\frac{T}{T-\eta}\right)^2 b, \quad \text{for } t \in [0,l]_{\mathbf{T}}. \tag{19}$$

From (H_2), we have

$$\begin{aligned}
&\alpha(Fu) \\
&\quad = (Fu)(l) \\
&\quad = \int_0^T (T-s)\,\Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\qquad - B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s\right) \\
&\qquad + \int_0^l (t-s)\,\Phi_q\left(\int_0^s -a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad \geq -B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s\right) \\
&\quad \geq A\int_0^\eta \Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad \geq A\int_0^l \Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad \geq A\int_{Y_3} \Phi_q\left(\int_0^s a(r) f(u(r),\varphi(\mu(r)))\nabla r\right)\nabla s \\
&\quad > A\int_{Y_3} \Phi_q\left(\int_0^s a(r)\nabla r\right)\nabla s \frac{b}{\delta} = b.
\end{aligned} \tag{20}$$

We conclude that (i) of Theorem 1 is satisfied.

We next address (ii) of Theorem 1. If we take $u \equiv ((T-\eta)/T)a$, $h = ((T-\eta)/T)a$, then

$$\begin{aligned}
\gamma(u) &= u(\eta) = \frac{T-\eta}{T}a < c, \\
\psi(u) &= u(\eta) = \frac{T-\eta}{T}a = h, \\
\beta(u) &= u(l) = \frac{T-\eta}{T}a < a.
\end{aligned} \tag{21}$$

From this we know that $\{u \in Q(\gamma,\beta,\psi,h,a,c) : \beta(u) < a\} \neq \varnothing$. If $u \in Q(\gamma,\beta,\psi,((T-\eta)/T)a,a,c)$, then

$$0 \leq u(t) \leq \frac{T}{T-l}a, \quad \text{for } t \in [0,T]_{\mathbf{T}}. \tag{22}$$

From (H_3), we have

$$\begin{aligned}
&\beta(Fu) \\
&\quad = (Fu)(l) \\
&\quad = \int_0^T (T-s)\,\Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\qquad - B_0\left(\int_0^\eta \Phi_q\left(\int_0^s -a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s\right) \\
&\qquad + \int_0^l (t-s)\,\Phi_q\left(\int_0^s -a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad \leq \int_0^T T\Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\qquad + B\int_0^T \Phi_q\left(\int_0^s a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\qquad + \int_0^l T\Phi_q\left(\int_0^T a(r) f(u(r),u(\mu(r)))\nabla r\right)\nabla s \\
&\quad = T(T+l+B)\,\Phi_q\left[\int_{Y_1} a(r) f(u(r),\varphi(\mu(r)))\nabla r\right. \\
&\qquad\qquad \left. + \int_{Y_2} a(r) f(u(r),u(\mu(r)))\nabla r\right] \\
&\quad < T(T+l+B)\,\Phi_q\left(\int_0^T a(r)\nabla r\right)\frac{a}{\lambda} = a.
\end{aligned} \tag{23}$$

Now we show that (iii) of Theorem 1 is satisfied. If $u \in P(\gamma,\alpha,b,c)$ and $\theta(Fu) = Fu(\eta) > (T/(T-\eta))b$, then

$$\begin{aligned}
\alpha(Fu) &\geq (Fu)(l) = \frac{T-l}{T}Fu(l) \geq \frac{T-l}{T}Fu(\eta) \\
&> \frac{T-l}{T-\eta}b > b.
\end{aligned} \tag{24}$$

Finally, if $u \in Q(\gamma,\beta,a,c)$ and $\psi(Fu) = Fu(\eta) < ((T-\eta)/T)a$, then from (4) of Lemma 2 we have

$$\beta(Fu) = Fu(l) \leq \frac{T}{T-l}Fu(l) \leq \frac{T}{T-\eta}Fu(\eta) < a, \tag{25}$$

which shows that condition (iv) of Theorem 1 is fulfilled.

Thus, all the conditions of Theorem 1 are satisfied. Hence, F has at least three fixed points u_1, u_2, u_3 satisfying

$$\begin{aligned}
\beta(u_1) &< a, \qquad b < \alpha(u_2), \\
a &< \beta(u_3) \quad \text{with } \alpha(u_3) < b.
\end{aligned} \tag{26}$$

Let

$$u(t) = \begin{cases} u_i(t), & t \in [0,T]_{\mathbf{T}},\ i = 1,2,3, \\ \varphi(t), & t \in [-r,0]_{\mathbf{T}}, \end{cases} \tag{27}$$

which are three positive solutions of BVP (1). □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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