Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 714518, 9 pages
http://dx.doi.org/10.1155/2014/714518

Research Article

Observers of Fuzzy Descriptor Systems with Time-Delays

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Received 25 March 2014; Revised 11 June 2014; Accepted 13 June 2014; Published 6 July 2014

Academic Editor: Elena Braverman

For discrete fuzzy descriptor systems with time-delays, the problem of designing fuzzy observers is investigated in this paper. Based on an equivalent transformation, discrete fuzzy descriptor systems with time-delays are converted into standard discrete systems with time-delays. Then, via linear matrix inequality (LMI) approach, both delay-dependent and delay-independent conditions for the existence of fuzzy state observers are obtained. Finally, two numerical examples are provided to illustrate the proposed method.

1. Introduction

For many practical engineering systems, increased productivity has led to new operating conditions, which are more challenging. Such conditions would affect system's performance. To improve efficiency, a number of methods have been proposed, such as fault-tolerant control [1], fault detection, and isolation [2]. As far as we know, most practical systems are nonlinear, and it has been proved that any smooth nonlinear system can be accurately presented by Takagi-Sugeno (T-S) models, which were firstly presented by Takagi and Sugeno in 1985 [3]. Therefore, a lot of attention has been attracted by them and some important results have been obtained [4, 5]. On the other hand, in many real systems, a nature phenomenon is after-effect. Because of the continuously expanded physical settings and capabilities, the common point-to-point communication form does not work well in modern industry any longer. So it is necessary to seek a new platform. Due to comprehensive diagnosis, low cost, and so on, online communication was introduced, and more and more networks are considered in control loops. However, since the communication media possesses time-sharing, time-delays can not be avoided in the control loops. In addition, time-delays often cause instability and affect system's performance. Therefore, many researchers have investigated time-delayed systems [6–8]. However, there are few papers referring to discrete-time fuzzy systems simultaneously including singularity and time-delays.

Interconnected systems, which are described as different names in [9], have been concerned in many researches. Increasing attention is paid to both theory and practical use of interconnected systems. So its fundamental theory and applications have involved a wide field during recent years. In real world, there are also many interconnected systems, such as large electric networks, electric power systems, and different types of societal systems. One such system consists of a series of independent subsystems, but all subsystems are interrelated with some goals. Recently, for the stability and stabilization of interconnected systems, a number of methods have been used [10, 11]. For a class of uncertain nonlinear descriptor systems with input saturation, [12] considered robust stabilization. Furthermore, with mode-dependent time-varying delays, uncertain discrete-time switched systems were considered to design $l_2 - l_\infty$ filter in [13]. Indeed, T-S fuzzy model is also a kind of interconnected systems, and this paper focuses on studying T-S fuzzy models. When the parameters and structure of a system are not known, a fuzzy model can be used. T-S model, which is usually applied to describe a nonlinear complex system, is one of the most common types of fuzzy models. A T-S fuzzy model includes many fuzzy rules and the consequent part of each rule is in fact a local model. A large number of problems about T-S fuzzy model have been considered.

Utilizing the approach of linear matrix inequality (LMI), [14] investigated D-stability and nonfragile control for T-S fuzzy discrete-time descriptor systems with multiple delays.

Subject to stochastic perturbation and time-varying delay, the passivity and passification problems for T-S fuzzy descriptor systems were studied in [15]. There are also some advanced methods to deal with T-S fuzzy time-delayed systems. For example, a linear lower dimensional model was used to approximate the original discrete-time fuzzy system with time-delays in [16], and the model approximation was casted into a sequential minimization problem with LMI constraints. Distributed fuzzy filters were designed for a class of sensor networks, which were described by discrete-time T-S fuzzy systems with time-varying delays and multiple probabilistic packet losses, in [17]. About stability analysis and stabilization for a class of discrete-time T-S fuzzy systems with time-varying state delay, a novel delay-partitioning method was developed in [18] and a stability condition, which is much less conservative than most existing results, was derived by the new idea. Taking advantage of similar delay-partitioning approach, [19] analyzed dissipativity of T-S fuzzy time-delayed descriptor systems. Also for a class of discrete-time T-S fuzzy time-delay systems, the problem of reliable filter design with strict dissipativity was considered in [20] and a sufficient condition of reliable dissipativity analysis was proposed. Additionally, filter matrices can be obtained by solving a convex optimization problem. This paper focuses on designing observers for discrete-time fuzzy descriptor systems with time-delays via LMI approach, which is often used. Furthermore, we try to investigate the corresponding problems with delay-partitioning method in our next work.

For a system, the control is often based on state feedback. However, all states of a system are not always available. At this time, it seems very important to estimate system states. So many scholars begin to focus on the problem of designing observers and filters. For example, a proportional multiple-integral observer was investigated for fuzzy chaotic model with unknown input in [21]. As for descriptor time-delayed systems with Markovian jump, [22] discussed designing linear memoryless observers. And a method of designing delay-dependent H_∞ filters for singular systems with time-delays was considered in [23]. For fuzzy systems, considerable attention should be paid to fuzzy observers and some achievements have been earned. In [24], a T-S fuzzy system was firstly transformed into a standard form and then a sliding model fuzzy observer could be constructed. With the approach in [24], it is effective to deal with matched and unmatched uncertainties in fuzzy models. In order to estimate the system state and output disturbance at the same time, Gao and his cooperators tried to design a novel fuzzy observer [25], which considered two cases for the output matrices. When each subsystem has the same output matrices, an augmented fuzzy descriptor system could be constructed. When the output matrices of all subsystems are different, a standard T-S fuzzy system was studied. For each case, T-S fuzzy state-space observers were designed, respectively. Moreover, the observer techniques proposed were applied to the fault estimation. In 2007, Marx et al. gave a method of designing decoupling observers for T-S descriptor systems with unknown inputs [26], where the proposed observer could be used to perform fault diagnosis. It is worth noting that only continuous systems are concerned in most existing works. This paper mainly discusses observer design for discrete-time-delayed descriptor systems with T-S fuzzy model.

The paper is organized as follows. Section 2 introduces the problem to be investigated. In Section 3, both delay-independent and delay-dependent sufficient conditions for the existence of fuzzy state observer are given. For each sufficient condition, two numerical examples are shown in Section 4. At last a short conclusion is included in Section 5.

2. Problem Formulation

In this section, we will briefly describe the problem to be studied. Through this paper, the following discrete-time-delayed descriptor system is considered.

Plant form is as follows.

Rule j: IF $\theta_1(k)$ is M_{j1}, $\theta_2(k)$ is M_{j2},..., and $\theta_p(k)$ is M_{jp}, THEN

$$\begin{gathered} Ex(k+1) = A_j x(k) + A_{dj} x(k-d) + B_j u(k) \\ y_j(k) = C_j x(k) \\ x(k) = \phi(k), \quad k = -d, -d+1, \ldots, 0, \end{gathered} \tag{1}$$

where M_{js}, $\theta_s(k)$ $(1 \le s \le p)$ are fuzzy sets and premise variables, respectively. $x(k) \in \mathbb{R}^n$ is the system state, the measurable output is $y(k) \in \mathbb{R}^q$, and $u(k) \in \mathbb{R}^p$ represents the control input. The initial condition is denoted by $\phi(k)$. It is assumed that $E \in \mathbb{R}^{n\times n}$ is singular; that is, rank $E = r < n$. The remaining matrices, A_j, A_{dj}, B_j, and C_j, are known. Besides, $1 \le j \le L$, where L is the number of *IF-THEN* rules. d, L, p are positive integer numbers.

We make the following assumption.

Assumption 1. Consider

$$\operatorname{rank}\begin{bmatrix} E \\ C_j \end{bmatrix} = n, \quad 1 \le j \le L, \tag{2}$$

so there exists a matrix pair $[T_j \ N_j]$ for each $j \in \{1, 2, \ldots, L\}$ such that

$$T_j E + N_j C_j = I_n, \quad 1 \le j \le L. \tag{3}$$

Remark 2. Because of the singularity of matrix E in fuzzy system (1), we give Assumption 1. Condition (2) is the same as that in [27], where another condition is also needed. In this paper, only one is enough.

Now we try to transform system (1) into a form as a standard system. Denote by X^+ the pseudoinverse of a matrix X. Then the general solution to (3), with condition (2), is

$$[T_j \ \ N_j] = \begin{bmatrix} E \\ C_j \end{bmatrix}^+ + \widetilde{Z}\left(I_{n+q} - \begin{bmatrix} E \\ C_j \end{bmatrix}\begin{bmatrix} E \\ C_j \end{bmatrix}^+\right), \tag{4}$$

where matrix T_j $(1 \le j \le L)$ is nonsingular, which can be achieved via designing the arbitrary matrix $\widetilde{Z}$ with appropriate dimension.

According to equality (3), the first equation of fuzzy system (1) is changed into

$$x(k+1) = T_j A_j x(k) + T_j A_{dj} x(k-d) + T_j B_j u(k) + N_j y_j(k+1), \tag{5}$$

where $1 \le j \le L$.

Let

$$h_j(\theta(k)) = \frac{\prod_{s=1}^{p} M_{js}(\theta_j(k))}{\sum_{j=1}^{L}\prod_{s=1}^{p} M_{js}(\theta_j(k))}, \quad 1 \le j \le L, \tag{6}$$

with $\theta(k) = [\theta_1(k)\theta_2(k)\cdots\theta_p(k)]$. $M_{js}(\theta_j(k))$ means the grade of membership of $\theta_j(k)$ in M_{js}. Obviously, $0 \le M_{js}(\theta_j(k)) \le 1$. Therefore, $h_j(\theta(k)) \ge 0$ $(1 \le j \le L)$ and $\sum_{j=1}^{L} h_j(\theta(k)) = 1$ for all k. For fuzzy system (1), the final state and output are given as follows:

$$\begin{aligned} x(k+1) &= \sum_{j=1}^{L} h_j(\theta(k)) \left[T_j A_j x(k) + T_j A_{dj} x(k-d) + T_j B_j u(k) + N_j y_j(k+1)\right], \\ y(k) &= \sum_{j=1}^{L} h_j(\theta(k)) C_j x(k), \\ x(k) &= \phi(k), \quad k = -d, -d+1, \ldots, 0. \end{aligned} \tag{7}$$

Define

$$\begin{aligned} &\left[\widetilde{A}(\theta) \;\; \widetilde{A}_d(\theta) \;\; \widetilde{B}(\theta) \;\; C(\theta)\right] \\ &\triangleq \sum_{j=1}^{L} h_j(\theta(k)) \left[\widetilde{A}_j \;\; \widetilde{A}_{dj} \;\; \widetilde{B}_j \;\; C_j\right], \end{aligned} \tag{8}$$

where $\left[\widetilde{A}_j \;\; \widetilde{A}_{dj} \;\; \widetilde{B}_j\right] = T_j\left[A_j \;\; A_{dj} \;\; B_j\right]$. Then system (7) can be written as

$$\begin{aligned} x(k+1) &= \widetilde{A}(\theta) x(k) + \widetilde{A}_d(\theta) x(k-d) + \widetilde{B}(\theta) u(k) + \sum_{j=1}^{L} h_j(\theta(k)) N_j y_j(k+1), \\ y(k) &= C(\theta) x(k), \\ x(k) &= \phi(k), \quad k = -d, -d+1, \ldots, 0. \end{aligned} \tag{9}$$

Through the above analysis, singular fuzzy system (1) is transformed into the ordinary linear system (9), for which the observer will be designed. In this paper, the state observer as in the following form is considered.

Observer rule j: IF $\theta_1(k)$ is M_{j1}, $\theta_2(k)$ is M_{j2},..., and $\theta_p(k)$ is M_{jp}, THEN

$$\begin{aligned} \widehat{x}(k+1) &= T_j A_j \widehat{x}(k) + T_j A_{dj} \widehat{x}(k-d) + T_j B_j u(k) + G_j\left(\widehat{y}(k) - y(k)\right) + N_j y_j(k+1), \\ \widehat{y}_j(k) &= C_j \widehat{x}(k), \end{aligned} \tag{10}$$

where $1 \le j \le L$, $\widehat{x}(k) \in \mathbb{R}^n$ is the estimate of state $x(k)$, and matrices G_j $(j = 1, 2, \ldots, L)$ are observation error matrices. Denote by $y(k)$ and $\widehat{y}(k)$ the final output of fuzzy system and fuzzy observer, respectively. With the same weight $h_j(\theta(k))$ and notations in system (7) the final estimated state and output of fuzzy observer (10) can be described as

$$\begin{aligned} \widehat{x}(k+1) &= \widetilde{A}(\theta)\widehat{x}(k) + \widetilde{A}_d(\theta)\widehat{x}(k-d) + \widetilde{B}(\theta) u(k) + G(\theta)\left(\widehat{y}(k) - y(k)\right) + \overline{y}(k+1), \\ \widehat{y}(k) &= C(\theta)\widehat{x}(k), \end{aligned} \tag{11}$$

where $\overline{y}(k) = \sum_{j=1}^{L} h_j(\theta(k)) N_j y_j(k)$ and $G(\theta) \triangleq \sum_{j=1}^{L} h_j(\theta(k)) G_j$.

Thus the problem of this paper focuses on finding matrices G_j $(j = 1, 2, \ldots, L)$ such that model (11) is an observer of system (9).

3. Main Results

This section discusses how to design fuzzy state observers for discrete descriptor systems with time-delays. And for existence of fuzzy observer, two different sufficient conditions are derived. We give the Schur complement Lemma first.

Lemma 3 (see [28]). *The LMI*

$$\begin{bmatrix} M & S \\ S^T & R \end{bmatrix} > 0, \tag{12}$$

where $M = M^T$, $R = R^T$ are equivalent to

$$R > 0, \quad M - SR^{-1}S^T > 0. \tag{13}$$

Now a sufficient condition about existence of considered observer, which does not rely on time-delay, is presented by the following theorem.

Theorem 4. *Model* (10) *is a state observer of fuzzy system* (1), *if there exist common matrices $P > 0$ and $Q > 0$ and matrices W_j $(j = 1, 2, \ldots, L)$ such that*

$$\begin{aligned} &\Lambda_{ii} < 0, \quad i = 1, 2, \ldots, L, \\ &\Lambda_{ij} + \Lambda_{ji} < 0, \quad 1 \le i < j \le L, \end{aligned} \tag{14}$$

where

$$\Lambda_{ij} = \begin{bmatrix} Q - P & 0 & \widetilde{A}_i^T P + C_j^T W_i \\ * & -Q & \widetilde{A}_{di}^T P \\ * & * & -P \end{bmatrix}, \tag{15}$$

and $*$ denotes matrix entries implied by the symmetry of a matrix through this paper. Furthermore, the observer gain matrices can be obtained as

$$G_i = P^{-1}W_i^T, \quad 1 \le i \le L. \tag{16}$$

Proof. Define

$$e(k) = x(k) - \hat{x}(k). \tag{17}$$

From system (9) and observer (11), the error dynamic equation of estimation error $e(k)$ can be derived as

$$e(k+1) = \left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]e(k) + \widetilde{A}_d(\theta)e(k-d). \tag{18}$$

Take a Lyapunov function as

$$V(k) = e^T(k)Pe(k) + \sum_{i=1}^{d} e^T(k-i)Qe(k-i), \tag{19}$$

where $P, Q > 0$; then

$$\begin{aligned}
\Delta V(k) &= V(k+1) - V(k) \\
&= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) \\
&\quad + \sum_{i=1}^{d} e^T(k+1-i)Qe(k+1-i) \\
&\quad - \sum_{i=1}^{d} e^T(k-i)Qe(k-i) \\
&= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) \\
&\quad + e^T(k)Qe(k) - e^T(k-d)Qe(k-d) \\
&= \left\{\left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]e(k) + \widetilde{A}_d(\theta)e(k-d)\right\}^T \\
&\quad \times P\left\{\left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]e(k) + \widetilde{A}_d(\theta)e(k-d)\right\} \\
&\quad - e^T(k)Pe(k) + e^T(k)Qe(k) \\
&\quad - e^T(k-d)Qe(k-d).
\end{aligned} \tag{20}$$

Let $\eta(k) = [e^T(k) \;\; e^T(k-d)]^T$; thus

$$\Delta V(k) = \eta^T(k)\Lambda\eta(k), \tag{21}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1(\theta) & \lambda_2(\theta) \\ \lambda_2^T(\theta) & \lambda_3(\theta) \end{bmatrix}, \tag{22}$$

while $\lambda_1(\theta) = [\widetilde{A}(\theta) + G(\theta)C(\theta)]^T P[\widetilde{A}(\theta) + G(\theta)C(\theta)] + Q - P$, $\lambda_2(\theta) = [\widetilde{A}(\theta) + G(\theta)C(\theta)]^T P\widetilde{A}_d(\theta)$, and $\lambda_3(\theta) = \widetilde{A}_d^T(\theta)P\widetilde{A}_d(\theta) - Q$.

The inequality $\Delta V(k) < 0$ holds for all $\eta(k) \ne 0$ if and only if

$$\Lambda < 0. \tag{23}$$

From the Lyapunov stable theory, if inequality (23) holds, the error system (18) would be asymptotically stable.

On the other hand, according to Lemma 3, inequality (23) is equivalent to

$$\begin{bmatrix} Q-P & 0 & \left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]^T P \\ * & -Q & \widetilde{A}_d^T(\theta)P \\ * & * & -P \end{bmatrix} < 0. \tag{24}$$

With the definition of coefficient matrices in system (9), it can be obtained from inequality (24) that

$$\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(\theta(k))h_j(\theta(k)) \begin{bmatrix} Q-P & 0 & \left(\widetilde{A}_i + G_iC_j\right)^T P \\ * & -Q & \widetilde{A}_{di}^T P \\ * & * & -P \end{bmatrix} < 0, \tag{25}$$

which is equivalent to

$$\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(\theta(k))h_j(\theta(k))\Lambda_{ij} < 0. \tag{26}$$

And inequality (26) can be rewritten as

$$\sum_{i=1}^{L} h_i(\theta(k))\Lambda_{ii} + \sum_{i=1}^{L-1}\sum_{j=i+1}^{L} h_i(\theta(k))h_j(\theta(k))\left(\Lambda_{ij} + \Lambda_{ji}\right) < 0. \tag{27}$$

Taking $W_i = G_i^T P$, it follows that conditions (14) are sufficient to guarantee (26) is correct. Inequality (23) holds and the error system (18) is asymptotically stable. □

Obviously, conditions in Theorem 4 are not relevant to delay d. Next, we will give another result which depends on time-delay.

Theorem 5. *For fuzzy system* (1)*, there is a fuzzy state observer in form of* (10) *if there exist common matrices $P > 0$, $U > 0$, and $R > 0$ and matrices W_j $(j = 1, 2, \ldots, L)$ such that*

$$\begin{aligned} \Psi_{ii} &< 0, \quad i = 1, 2, \ldots, L, \\ \Psi_{ij} + \Psi_{ji} &< 0, \quad 1 \le i < j \le L, \end{aligned} \tag{28}$$

where

$$\Psi_{ij} = \begin{bmatrix} U-P & 0 & \widetilde{A}_i^T P + C_j^T W_i & \widetilde{A}_i^T P + C_j^T W_i - P \\ * & -U & \widetilde{A}_{di}^T P & \widetilde{A}_{di}^T P \\ * & * & -P & 0 \\ * & * & * & -\dfrac{1}{d}R \end{bmatrix}. \tag{29}$$

Moreover, the observer gain matrices can be calculated as

$$G_i = P^{-1}W_i^T, \quad 1 \le i \le L. \tag{30}$$

Proof. According to proof of Theorem 4, the dynamic equation of error system is (18). Construct a Lyapunov function as

$$\begin{aligned} V(k) &= e^T(k)Pe(k) + \sum_{l=k-d}^{k-1} e^T(l)Ue(l) \\ &\quad + \sum_{s=-d+1}^{0}\sum_{l=k-1+s}^{k-1} [e(l+1)-e(l)]^T Z [e(l+1)-e(l)], \end{aligned} \tag{31}$$

where $P, U, Z > 0$. Then

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) \\ &\quad + \sum_{l=k-d+1}^{k} e^T(l)Ue(l) - \sum_{l=k-d}^{k-1} e^T(l)Ue(l) \\ &\quad + \sum_{s=-d+1}^{0}\left\{\sum_{l=k+s}^{k} [e(l+1)-e(l)]^T Z [e(l+1)-e(l)] \right. \\ &\qquad - \sum_{l=k-1+s}^{k-1} [e(l+1)-e(l)]^T \\ &\qquad \left. \times Z [e(l+1)-e(l)]\right\} \\ &= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) + e^T(k)Ue(k) \\ &\quad - e^T(k-d)Ue(k-d) \\ &\quad - \sum_{s=-d+1}^{0}\left\{[e(k+s)-e(k-1+s)]^T \right. \\ &\qquad \left. \times Z [e(k+s)-e(k-1+s)]\right\} \\ &\quad + d[e(k+1)-e(k)]^T Z [e(k+1)-e(k)] \\ &\overset{(18)}{=} e^T(k)\omega_1(\theta)e(k) + e^T(k-d)\omega_2(\theta)e(k-d) \\ &\quad + e^T(k)\omega_3(\theta)e(k-d) + e^T(k-d)\omega_3^T(\theta)e(k) \\ &\quad - \sum_{s=-d+1}^{0} [e(k+s)-e(k-1+s)]^T \\ &\qquad \times Z [e(k+s)-e(k-1+s)], \end{aligned} \tag{32}$$

where

$$\begin{aligned} \omega_1(\theta) &= \left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]^T P \left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right] \\ &\quad - P + U + \omega_4(\theta), \\ \omega_2(\theta) &= \widetilde{A}_d^T(\theta)P\widetilde{A}_d(\theta) - U + d\widetilde{A}_d^T(\theta)Z\widetilde{A}_d(\theta), \\ \omega_3(\theta) &= \left[\widetilde{A}(\theta) + G(\theta)C(\theta)\right]^T P\widetilde{A}_d(\theta) \\ &\quad + d\left[\widetilde{A}(\theta) + G(\theta)C(\theta) - I\right]^T Z\widetilde{A}_d(\theta), \\ \omega_4(\theta) &= d\left[\widetilde{A}(\theta) + G(\theta)C(\theta) - I\right]^T \\ &\quad \times Z\left[\widetilde{A}(\theta) + G(\theta)C(\theta) - I\right]. \end{aligned} \tag{33}$$

Define $\eta(k) = [e^T(k) \;\; e^T(k-d)]^T$; then

$$\begin{aligned} \Delta V(k) &= \eta^T(k)\Psi\eta(k) \\ &\quad - \sum_{s=-d+1}^{0} [e(k+s)-e(k-1+s)]^T \\ &\qquad \times Z [e(k+s)-e(k-1+s)] \\ &\le \eta^T(k)\Psi\eta(k), \end{aligned} \tag{34}$$

where

$$\Psi = \begin{bmatrix} \omega_1(\theta) & \omega_3(\theta) \\ \omega_3^T(\theta) & \omega_2(\theta) \end{bmatrix}. \tag{35}$$

If

$$\Psi < 0, \tag{36}$$

then $\Delta V(k) < 0$ and the error system (18) is asymptotically stable.

By Lemma 3, inequality (36) is equivalent to

$$\begin{bmatrix} U-P & 0 & \left[\widetilde{A}(\theta)+G(\theta)C(\theta)\right]^T P & \left[\widetilde{A}(\theta)+G(\theta)C(\theta)-I\right]^T \\ * & -U & \widetilde{A}_d^T(\theta)P & \widetilde{A}_d^T(\theta) \\ * & * & -P & 0 \\ * & * & * & -\frac{1}{d}Z^{-1} \end{bmatrix} < 0. \tag{37}$$

Define $J \triangleq \operatorname{diag}[I, I, I, P]$. Pre- and postmultiplying inequality (37) by matrix J, we can obtain

$$\begin{bmatrix} U-P & 0 & \left[\widetilde{A}(\theta)+G(\theta)C(\theta)\right]^T P & \left[\widetilde{A}(\theta)+G(\theta)C(\theta)-I\right]^T P \\ * & -U & \widetilde{A}_d^T(\theta)P & \widetilde{A}_d^T(\theta)P \\ * & * & -P & 0 \\ * & * & * & -\frac{1}{d}PZ^{-1}P \end{bmatrix} < 0. \tag{38}$$

Taking $R = PZ^{-1}P$ and $W_i = G_i^T P$, it is derived that inequality (38) is equivalent to

$$\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(\theta(k))\, h_j(\theta(k))\, \Psi_{ij} < 0. \tag{39}$$

That is

$$\sum_{i=1}^{L} h_i(\theta(k))\, \Psi_{ii} + \sum_{i=1}^{L-1}\sum_{j=i+1}^{L} h_i(\theta(k))\, h_j(\theta(k)) \left(\Psi_{ij}+\Psi_{ji}\right) < 0. \tag{40}$$

Combining with inequalities (28), we can know that inequality (39) holds and condition (36) is correct. Thus the error system (18) is asymptotically stable. □

Remark 6. It is easy to see that Theorem 5 implies Theorem 4. In fact, with $\varphi_{ij} \triangleq [P\widetilde{A}_i + W_i^T C_j - P \;\; P\widetilde{A}_{di} \;\; 0]^T$ $(i, j = 1, 2, \ldots, L)$, we have

$$\Psi_{ij} = \begin{bmatrix} \Lambda_{ij} & \varphi_{ij} \\ * & -\frac{1}{d}R \end{bmatrix}, \tag{41}$$

where matrices Ψ_{ij} and Λ_{ij} are defined in Theorems 5 and 4, respectively. So, according to Lemma 3, inequalities (28) are equivalent to $\Lambda_{ii} + d\varphi_{ii}R^{-1}\varphi_{ii}^T < 0$ $(i = 1, 2, \ldots, L)$ and $\Lambda_{ij} + \Lambda_{ji} + (d/2)(\varphi_{ij}+\varphi_{ji})R^{-1}(\varphi_{ij}+\varphi_{ji})^T < 0$ $(1 \le i < j \le L)$, respectively. Since $R > 0$, from inequalities (28) we can obtain that $\Lambda_{ii} < 0$ $(i = 1, 2, \ldots, L)$ and $\Lambda_{ij} + \Lambda_{ji} < 0$ $(1 \le i < j \le L)$. Therefore, the conditions of Theorem 5 are stronger than those of Theorem 4. On the other hand, if there do not exist compatible solutions to inequalities (28) for some fuzzy descriptor system, we may design its observer based on Theorem 4.

Remark 7. For fuzzy state-space systems, the results of this paper can also be applied. Choosing $E = I_n$, a discrete fuzzy state-space system can be derived from fuzzy system (1) without time-delays. Let $T_j = I_n$, $N_j = 0$, and $\widetilde{A}_{dj} = 0$ $(1 \le j \le L)$. Then, the presented observer in [4] can also be obtained from model (10).

At the end, we give an algorithm to design observer for fuzzy descriptor system (1).

Algorithm 8. The following steps are introduced to determine observer (10) for fuzzy descriptor system (1) according to Theorem 4.

(1) Verify condition (2). If it holds, then go to step 2. Otherwise, stop.

(2) According to formulation (4), solve (3) and guarantee that matrix T_j $(1 \le j \le L)$ is nonsingular. Then transform fuzzy system (1) into system (9) with solutions to (3).

(3) Solve LMIs (14). If they are solvable, go to step 4. Otherwise, stop.

(4) Give fuzzy observer (10), with $G_i = P^{-1}W_i^T$ $(1 \le i \le L)$.

The steps of designing observer for fuzzy descriptor system (1) based on Theorem 5 are similar to Algorithm 8 and omitted here.

Remark 9. In this paper, we firstly consider observer design for fuzzy discrete systems with singularity and time-delays simultaneously. Compared with existing work [27], the singularity of fuzzy system can be easily eliminated under only one simple assumption. Finally, both delay-dependent and delay-independent conditions are derived for obtaining observer gain matrices.

4. Examples

In this section, two numerical examples are provided. The first one is to design a fuzzy observer according to Theorem 4 and the second one illustrates Theorem 5.

Example 10. Consider a discrete fuzzy descriptor system with time-delays as follows.

Plant Rule 1: IF $x_1(k)$ is $M_1(x_1(k))$, THEN

$$\begin{aligned} Ex(k+1) &= A_1 x(k) + A_{d1} x(k-d) + B_1 u(k), \\ y_1(k) &= C_1 x(k), \\ x(k) &= \phi(k), \quad k = -d, -d+1, \ldots, 0. \end{aligned} \tag{42}$$

Plant Rule 2: IF $x_1(k)$ is $M_2(x_1(k))$, THEN

$$\begin{aligned} Ex(k+1) &= A_2 x(k) + A_{d2} x(k-d) + B_2 u(k), \\ y_2(k) &= C_2 x(k), \\ x(k) &= \phi(k), \quad k = -d, -d+1, \ldots, 0, \end{aligned} \tag{43}$$

where

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}, \\ A_{d1} = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0 & 0.1 \\ -0.2 & 0 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \\ C_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T, \end{gathered} \tag{44}$$

and membership functions are

$$\begin{aligned} M_1(x_1(k)) &\triangleq 1 - \frac{x_1^2(k)}{2.25}, \\ M_2(x_1(k)) &\triangleq \frac{x_1^2(k)}{2.25}. \end{aligned} \tag{45}$$

Orbits of membership functions for Rules 1 and 2 are shown in Figure 1.

Coefficient matrices of fuzzy descriptor system are given, and it can be easily verified that matrices E and C_j $(j = 1, 2)$ satisfy condition (2). Based on formulation (4), a solution to (3) is shown as

$$\begin{gathered} T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad N_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ T_2 = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{gathered} \tag{46}$$

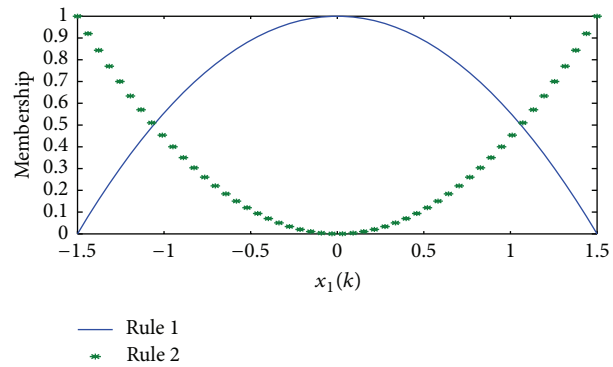

Figure 1: Membership functions.

Then fuzzy descriptor system is converted into the following ordinary system:

$$\begin{aligned} x(k+1) &= \widetilde{A}(k) x(k) + \widetilde{A}_d(k) x(k-d) + \widetilde{B} u(k) \\ &\quad + \sum_{j=1}^{2} M_j(x_1(k)) N_j y_j(k+1), \\ y(k) &= \widetilde{C}(k) x(k), \\ x(k) &= \phi(k), \quad k = -d, -d+1, \ldots, 0, \end{aligned} \tag{47}$$

where

$$\begin{aligned} &\left[\widetilde{A}(k) \;\; \widetilde{A}_d(k) \;\; \widetilde{B}(k) \;\; \widetilde{C}(k)\right] \\ &\quad = \sum_{j=1}^{2} M_j(x_1(k)) \left[T_j A_j \;\; T_j A_{dj} \;\; T_j B_j \;\; C_j\right]. \end{aligned} \tag{48}$$

By using MATLAB LMI Toolbox and solving LMIs in Theorem 4, we obtain

$$\begin{gathered} P = \begin{bmatrix} 227.2273 & -25.4600 \\ -25.4600 & 61.9384 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} -199.6323 \\ 57.1439 \end{bmatrix}^T, \\ Q = \begin{bmatrix} 39.4769 & 3.5449 \\ 3.5449 & 27.3497 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 181.1576 \\ -32.4363 \end{bmatrix}^T, \end{gathered} \tag{49}$$

so

$$G_1 = \begin{bmatrix} -0.8126 \\ 0.5886 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.7742 \\ -0.2054 \end{bmatrix}. \tag{50}$$

The state trajectories of discrete-time-delay fuzzy descriptor system are shown in Figure 2 and their estimations are depicted by Figure 3. On the other hand, from Figure 4 we know that the error system is asymptotically stable.

Example 11. Consider the fuzzy system in Example 10. Assume that time-delay $d = 2$, and take the same solution to

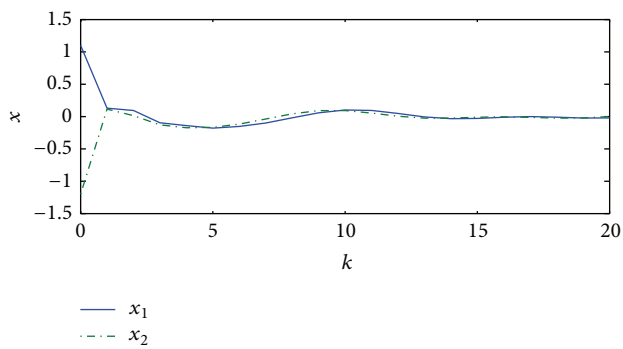

Figure 2: State of the fuzzy descriptor system.

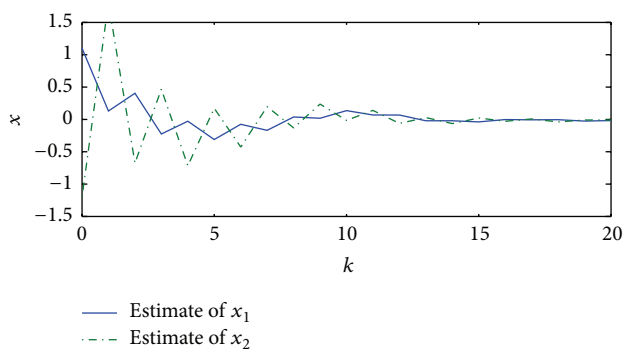

Figure 3: State of the fuzzy observer.

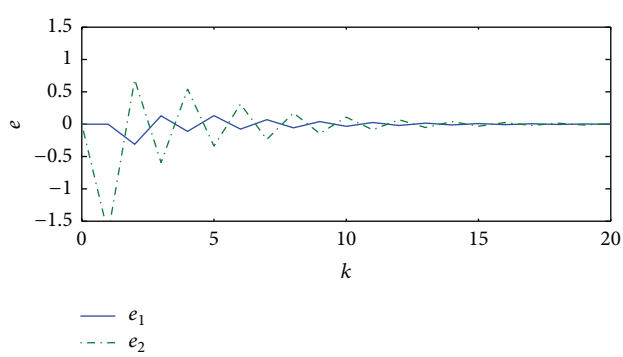

Figure 4: Error system.

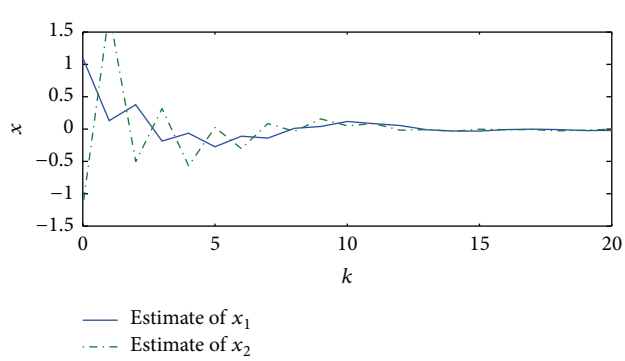

Figure 5: State of the fuzzy observer.

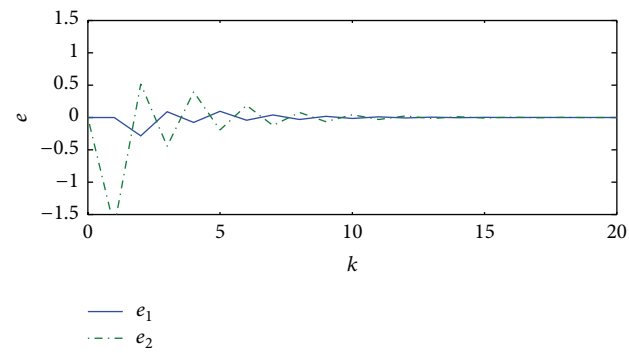

Figure 6: Error system.

(3) as that in Example 10. According to Theorem 5 and using MATLAB LMI Toolbox, we obtain

$$P = \begin{bmatrix} 0.8184 & -0.0655 \\ -0.0655 & 0.1411 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} -0.7240 \\ 0.1516 \end{bmatrix}^T,$$
$$U = \begin{bmatrix} 0.0658 & 0.0075 \\ 0.0075 & 0.0657 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0.7072 \\ -0.0666 \end{bmatrix}^T, \tag{51}$$
$$R = \begin{bmatrix} 14.4443 & -2.7589 \\ -2.7589 & 5.1602 \end{bmatrix}.$$

Based on matrices P and W_j $(j = 1, 2)$, we can derive

$$G_1 = \begin{bmatrix} -0.8295 \\ 0.6896 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.8582 \\ -0.0735 \end{bmatrix}. \tag{52}$$

State trajectories of fuzzy system are the same as those in Figure 2, and Figure 5 is state response of fuzzy observer. Furthermore, from the error system depicted in Figure 6, we know that error converges to zero asymptotically.

Remark 12. From these two examples above, it is obvious that error in Figure 6 converges faster than that in Figure 4. In summary, fuzzy observer designed according to Theorem 5 is better; that is, the convergence time of error is shorter. However, if there is no appropriate observer for some discrete fuzzy descriptor system based on Theorem 5, we may design its observer depending on Theorem 4.

5. Conclusion

This paper has discussed how to design fuzzy observers for discrete descriptor systems with time-delays. According to a simple transformation, the singularity of considered fuzzy systems has been eliminated. Then two sufficient conditions for the existence of fuzzy observer have been derived. Notably, one condition depends on time-delays and the other does not. Finally, different fuzzy observers have been designed for the same discrete-time-delayed descriptor system. By comparison, the fuzzy observer depending on delay condition is better. Additionally, our future work will deal with fuzzy time-delayed descriptor systems by using delay-partitioning method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was partially supported by NNSF of China (61174141, 61374025), Research Awards Young and Middle-Aged Scientists of Shandong Province (BS2011SF009, BS2011DX019), and Excellent Youth Foundation of Shandong Province (JQ201219).

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