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Research Article

A Local Fractional Integral Inequality on Fractal Space Analogous to Anderson's Inequality

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Anderson's inequality (Anderson, 1958) as well as its improved version given by Fink (2003) is known to provide interesting examples of integral inequalities. In this paper, we establish local fractional integral analogue of Anderson's inequality on fractal space under some suitable conditions. Moreover, we also show that the local fractional integral inequality on fractal space, which we have proved in this paper, is a new generalization of the classical Anderson's inequality.

1. Introduction

In the year 1958, Anderson [1] established the following very interesting result.

Theorem 1. *If $F_i(x)$ is convex increasing on $[0,1]$ and $F_i(0)=0$ for each $i=1,2,\ldots,n$, then*

$$\int_0^1 F_1(x)F_2(x)\cdots F_n(x)\,\mathrm{d}x \geqq \frac{2n}{n+1}\left(\int_0^1 F_1(x)\,\mathrm{d}x\right)\cdots\left(\int_0^1 F_n(x)\,\mathrm{d}x\right). \tag{1}$$

Subsequently, Fink [2] improved Anderson's inequality (1) to the following form.

Theorem 2. *If $F_i(x)/x$ is increasing on $(0,1]$ and $F_i(0)=0$ for each $i=1,2,\ldots,n$, then*

$$\int_0^1 F_1(x)F_2(x)\cdots F_n(x)\,\mathrm{d}x \geqq \frac{2^n}{n+1}\left(\int_0^1 F_1(x)\,\mathrm{d}x\right)\cdots\left(\int_0^1 F_n(x)\,\mathrm{d}x\right). \tag{2}$$

Moreover, Fink [2] also pointed out that the condition $F_i(0)=0$ $(i=1,2,\ldots,n)$ in Theorems 1 and 2 cannot be dropped.

In recent years, the local fractional calculus has received significantly remarkable attention from scientists and engineers. Some of the concepts of the local fractional derivative were established in [3–26]. In particular, the local fractional derivative was introduced in [3–9, 17, 21–26], Jumarie modified the Riemann-Liouville derivative in [10, 11], and the fractal derivative was proposed in [12–16, 18–20]. As a result, the theory of local fractional calculus plays an important

role in applications in several different fields such as theoretical physics [5, 9], the theory of elasticity and fracture mechanics [5], and so on. For example, in [9], the authors proposed the local fractional Fokker-Planck equation. The local fractional Stieltjes transform was established in [27]. The fractal heat conduction problems were presented in [5, 18]. Local fractional improper integral was obtained in [28]. The principles of virtual work and minimum potential and complementary energy in the mechanics of fractal media were investigated in [5]. Local fractional continuous wavelet transform was studied in [29]. Mean-value theorems for local fractional integrals were considered in [30]. In [31], the authors dealt with fractal wave equations. The finite Yang-Laplace transform was introduced in [32]. Local fractional Schrödinger equation was studied in [33]. The local fractional Hilbert transform was given in [34]. The wave equation on Cantor sets was considered in [35]. The diffusion problems in fractal media were reported in [15] (see also several other recent developments on fractional calculus and local fractional calculus presented in [36–41]).

The purpose of this paper is to establish a certain local fractional integral inequality on fractal space, which is analogous to Anderson's inequality asserted by Theorem 1. This paper is divided into the following three sections. In Section 2, we recall some basic facts about local fractional calculus. In Section 3, the main result is presented.

2. Preliminaries

In this section, we would review the basic notions of local fractional calculus (see [3–5]).

2.1. Local Fractional Continuity of Functions. In order to study the local fractional continuity of nondifferentiable functions on fractal sets, we first give the following results on the local fractional continuity of functions.

Lemma 3 (see [5]). *Suppose that $\mathbb{F}$ is a subset of the real line and is a fractal. Suppose also that $f : (\mathbb{F}, d) \to (\Omega', d')$ is a bi-Lipschitz mapping. Then there are two positive constants ρ and τ, and $\mathbb{F} \subset \mathbb{R}$:*

$$\rho^s H^s(\mathbb{F}) \leqq H^s(f(\mathbb{F})) \leqq \tau^s H^s(\mathbb{F}) \quad (\mathbb{F} \subset \mathbb{R}), \tag{3}$$

such that, for all $x_1, x_2 \in \mathbb{F}$,

$$\rho^\alpha |x_1 - x_2|^\alpha \leqq |f(x_1) - f(x_2)| \leqq \tau^\alpha |x_1 - x_2|^\alpha. \tag{4}$$

From Lemma 3, it is easily seen that (see [5])

$$|f(x_1) - f(x_2)| \leqq \tau^\alpha |x_1 - x_2|^\alpha \quad (x_1, x_2 \in \mathbb{F}), \tag{5}$$

so that

$$|f(x_1) - f(x_2)| \leqq \varepsilon^\alpha \quad (x_1, x_2 \in \mathbb{F}), \tag{6}$$

where α is the fractal dimension of $\mathbb{F}$.

Definition 4 (see [3, 5]). Assume that there exists

$$|f(x) - f(x_0)| \leqq \varepsilon^\alpha, \tag{7}$$

with

$$|x - x_0|^\alpha \leqq \delta^\alpha \tag{8}$$

for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in \mathbb{R}$. Then $f(x)$ is said to be local fractional continuous at $x = x_0$, denoted by

$$\lim_{x \to x_0} f(x) = f(x_0). \tag{9}$$

The function $f(x)$ is local fractional continuous on the interval (a, b), denoted by (see [5])

$$f(x) \in C_\alpha(a, b) \tag{10}$$

if (7) holds true for $x \in (a, b)$.

Definition 5 (see [4, 5]). Assume that $f(x)$ is a nondifferentiable function of exponent α $(0 < \alpha \leqq 1)$. Then $f(x)$ is called the Hölder function of exponent α if, for $x, y \in \mathbb{F}$, one has

$$|f(x) - f(y)| \leqq C|x - y|^\alpha \quad (0 < \alpha \leqq 1). \tag{11}$$

Definition 6 (see [4, 5]). A function $f(x)$ is said to be continuous of order α $(0 < \alpha \leqq 1)$ or, equivalently, α-continuous, if

$$|f(x) - f(x_0)| \leqq o\left((x - x_0)^\alpha\right) \quad (0 < \alpha \leqq 1). \tag{12}$$

2.2. Local Fractional Derivatives and Local Fractional Integrals

Definition 7 (see [3–5]). Assume that $f(x) \in C_\alpha(a, b)$. Then a local fractional derivative of $f(x)$ of order α at $x = x_0$ is defined by

$$f^{(\alpha)}(x_0) = \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^\alpha(f(x) - f(x_0))}{(x - x_0)^\alpha}, \tag{13}$$

where

$$\Delta^\alpha(f(x) - f(x_0)) \cong \Gamma(1 + \alpha)\Delta(f(x) - f(x_0)). \tag{14}$$

It follows from Definition 7 that there exists (see [5])

$$f(x) \in D_x^{(\alpha)}(a, b) \tag{15}$$

if

$$f^{(\alpha)}(x) = D_x^{(\alpha)} f(x) \tag{16}$$

for any $x \in (a, b)$.

Definition 8 (see [3, 5]). (a) If $f^{(\alpha)}(x) > 0$ on a given interval, then $f(x)$ is increasing on that interval.

(b) If $f^{(\alpha)}(x) < 0$ on a given interval, then $f(x)$ is decreasing on that interval.

Definition 9 (see [42]). A function $f(x)$ is called α-convex on I if the following inequality holds true:

$$f(\lambda x_1 + (1-\lambda)x_2) \leqq \lambda^\alpha f(x_1) + (1-\lambda)^\alpha f(x_2) \tag{17}$$

for all $x_1, x_2 \in I$ and $0 \leqq \lambda \leqq 1$ such that $\lambda x_1 + (1-\lambda)x_2 \in I$.

Theorem 10 (see [42]). *Assume that $f(x)$ is an α-local differentiable function on I. If $f^{(\alpha)}(x)$ is nondecreasing (nonincreasing) on I, then the function f is α-convex (α-concave) on I.*

Theorem 11 (see [42]). *Assume that $f(x)$ is a local fractional continuous function. Then each of the following assertions holds true*:

(1) *if $f^{(2\alpha)}(x)$ exists on I and $f^{(2\alpha)}(x) \geqq 0$ for all $x \in I$, then f is α-convex on I;*

(2) *if $f^{(2\alpha)}(x)$ exists on I and $f^{(2\alpha)}(x) \leqq 0$ for all $x \in I$, then f is α-concave on I.*

Definition 12 (see [3–5]). Assume that $f(x) \in C_\alpha(a,b)$. A local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is expressed by

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(\mathrm{d}t)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, \end{aligned} \tag{18}$$

where

$$\begin{gathered} \Delta t_j = t_{j+1} - t_j \quad (j = 0, 1, N-1), \\ \Delta t = \max\{\Delta t_1, \Delta t_2, \ldots, \Delta t_j, \ldots\}, \\ [t_j, t_{j+1}] \quad (j = 0, 1, \ldots, N-1) \quad (t_0 = a; t_N = b) \end{gathered} \tag{19}$$

are a partition of the interval $[a,b]$.

It follows from Definition 12 that (see [5])

$$f(x) \in {}_aI_x^{(\alpha)}(a,b), \tag{20}$$

if

$${}_aI_x^{(\alpha)} f(x) \tag{21}$$

for any $x \in (a,b)$.

Remark 13 (see [3–5]). Assume that $f(x) \in D_x^{(\alpha)}(a,b)$ or $f(x) \in C_\alpha(a,b)$; then

$$f(x) \in {}_aI_x^{(\alpha)}(a,b). \tag{22}$$

3. Main Results

Lemma 14. *Let $f(x)$, $g(x) \in C_\alpha(0,1)$ satisfy the constraints that $f(0) = 0$ and $g(x)$ is increasing on $(0,1]$. If the function*

$$\frac{f(x)}{\Gamma(1+2\alpha)x^\alpha/\Gamma(1+\alpha)} \tag{23}$$

is increasing on $(0,1]$, then

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_0^1 f(x) g(x) (\mathrm{d}x)^\alpha \\ &\quad \geqq \frac{1}{\Gamma(1+\alpha)} \int_0^1 f^*(x) g(x) (\mathrm{d}x)^\alpha, \end{aligned} \tag{24}$$

where

$$f^*(x) = \frac{\Gamma(1+2\alpha)x^\alpha}{\Gamma(1+\alpha)} \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha \tag{25}$$

for $x \in [0,1]$.

Proof. Let

$$H(x) = \frac{1}{\Gamma(1+\alpha)} \int_0^x [f^*(t) - f(t)](\mathrm{d}t)^\alpha \quad (x \in [0,1]). \tag{26}$$

Then, clearly, $H(0) = 0$ and

$$\begin{aligned} H(1) &= \frac{1}{\Gamma(1+\alpha)} \int_0^1 [f^*(t) - f(t)](\mathrm{d}t)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^1 f^*(t)(\mathrm{d}t)^\alpha \\ &\quad - \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(t)(\mathrm{d}t)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{\Gamma(1+2\alpha)t^\alpha}{\Gamma(1+\alpha)} \\ &\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha (\mathrm{d}t)^\alpha \\ &\quad - \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{\Gamma(1+2\alpha)t^\alpha}{\Gamma(1+\alpha)} (\mathrm{d}t)^\alpha \\ &\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha \\ &\quad - \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha \\ &= \left(\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)} \cdot \frac{\Gamma(1+\alpha)t^{2\alpha}}{\Gamma(1+2\alpha)} \bigg|_0^1 - 1^\alpha \right) \\ &\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha \\ &= \left(t^{2\alpha}\big|_0^1 - 1^\alpha \right) \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)(\mathrm{d}u)^\alpha = 0. \end{aligned} \tag{27}$$

Moreover, we have

$$\begin{aligned} H^{(\alpha)}(0) &= f^*(0) - f(0) = f^*(0) = 0, \\ H(x) &= f^*(x) - f(x) \\ &= \frac{\Gamma(1+2\alpha)\,x^\alpha}{\Gamma(1+\alpha)} \\ &\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)\,(\mathrm{d}u)^\alpha - f(x) \\ &= \frac{\Gamma(1+2\alpha)\,x^\alpha}{\Gamma(1+\alpha)} \\ &\quad \times \left(\frac{1}{\Gamma(1+\alpha)} \int_0^1 f(u)\,(\mathrm{d}u)^\alpha \right. \\ &\quad \left. - \frac{f(x)}{\Gamma(1+2\alpha)\,x^\alpha/\Gamma(1+\alpha)} \right) \quad (x \in (0,1]). \end{aligned} \tag{28}$$

Since the function

$$\frac{f(x)}{\Gamma(1+2\alpha)\,x^\alpha/\Gamma(1+\alpha)} \tag{29}$$

is increasing on $(0,1]$, we can see that

$$\frac{H^{(\alpha)}(x)}{\Gamma(1+2\alpha)\,x^\alpha/\Gamma(1+\alpha)} \tag{30}$$

is decreasing on $(0,1]$.

Next, we show that $H(x) \geqq 0$ on $[0,1]$. This proof can be divided into the following two parts.

(a) Assume that there exists a point $x_0 \in (0,1)$ such that

$$\frac{H^{(\alpha)}(x_0)}{\Gamma(1+2\alpha)\,x_0^\alpha/\Gamma(1+\alpha)} = 0. \tag{31}$$

Then

$$\begin{aligned} H^{(\alpha)}(x) &\geqq 0 \quad (x \in [0, x_0]), \\ H^{(\alpha)}(x) &\leqq 0 \quad (x \in [x_0, 1]). \end{aligned} \tag{32}$$

Hence we assume that $x \in [0, x_0]$. Then $H(x) \geqq H(0) = 0$. If we assume that $x \in [x_0, 1]$, then $H(x) \geqq H(1) = 0$ on $[x_0, 1]$. Thus $H(x) \geqq 0$ on $[0,1]$.

(b) Suppose that

$$\frac{H^{(\alpha)}(x)}{\Gamma(1+2\alpha)\,x^\alpha/\Gamma(1+\alpha)} > 0 \quad (x \in (0,1)). \tag{33}$$

In this case, $H(x)$ is increasing on $[0,1)$. Hence $H(x) \geqq H(0) = 0$ on $[0,1)$. It follows from $H(1) = 0$ that $H(x) \geqq 0$ on $[0,1]$. Thus, by applying a known result [5, Theorem 2.28], we have

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_0^1 [f(x) - f^*(x)]\, g(x)\,(\mathrm{d}x)^\alpha \\ &\quad = -\frac{1}{\Gamma(1+\alpha)} \int_0^1 H^{(\alpha)}(x)\, g(x)\,(\mathrm{d}x)^\alpha \\ &\quad = -\left(H^{(\alpha)}(x) g(x) \Big|_0^1 \right. \\ &\qquad \left. - \frac{1}{\Gamma(1+\alpha)} \int_0^1 g^{(\alpha)}(x)\, H(x)\,(\mathrm{d}x)^\alpha \right) \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_0^1 g^{(\alpha)}(x)\, H(x)\,(\mathrm{d}x)^\alpha \geqq 0, \end{aligned} \tag{34}$$

because $H(x) \geqq 0$ and $g(x)$ is increasing (and hence $g^{(\alpha)}(x) \geqq 0$). We have thus completed our proof. □

We are in a position to state and prove our main result as follows.

Theorem 15. *Let $f_1(x), f_2(x), \ldots, f_n(x) \in C_\alpha(0,1)$ with $f_i(0) = 0$ and*

$$\frac{f_i(x)}{\Gamma(1+2\alpha)\,x^\alpha/\Gamma(1+\alpha)} \tag{35}$$

increasing on $(0,1]$ for $i = 1, 2, \ldots, n$. If $f_i(x)$ $(i = 1, 2, \ldots, n)$ is increasing on $(0,1]$, then

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_0^1 \prod_{i=1}^n f_i(x)\,(\mathrm{d}x)^\alpha \\ &\quad \geqq \left(\frac{[\Gamma(1+2\alpha)]^n}{[\Gamma(1+\alpha)]^n} \frac{1}{\Gamma(1+\alpha)} \int_0^1 x^{n\alpha} (\mathrm{d}x)^\alpha \right) \\ &\quad \times \left(\prod_{i=1}^n \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_i(u)\,(\mathrm{d}u)^\alpha \right). \end{aligned} \tag{36}$$

Proof. It follows from Lemma 14 and the increasing property of $f_i(x)$ $(i = 1, 2, \ldots, n)$ that, for $f_i^*(x)$ defined as in Lemma 14,

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_0^1 \prod_{i=1}^n f_i(x)\,(\mathrm{d}x)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(x) \prod_{i=1}^{n-1} f_i(x)\,(\mathrm{d}x)^\alpha \\ &\geqq \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n^*(x) \prod_{i=1}^{n-1} f_i(x)\,(\mathrm{d}x)^\alpha \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\Gamma(1+\alpha)} \\
&\quad \times \int_0^1 \left(\frac{\Gamma(1+2\alpha)\, x^{\alpha}}{\Gamma(1+\alpha)} \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \right) \\
&\qquad \times \prod_{i=1}^{n-1} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&= \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{\Gamma(1+2\alpha)\, x^{\alpha}}{\Gamma(1+\alpha)} \prod_{i=1}^{n-1} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&= \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-1}(x) \frac{\Gamma(1+2\alpha)\, x^{\alpha}}{\Gamma(1+\alpha)} \prod_{i=1}^{n-2} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&\geqq \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-1}^{*}(x) \frac{\Gamma(1+2\alpha)\, x^{\alpha}}{\Gamma(1+\alpha)} \prod_{i=1}^{n-2} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&= \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-1}(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{[\Gamma(1+2\alpha)]^2 x^{2\alpha}}{[\Gamma(1+\alpha)]^2} \prod_{i=1}^{n-2} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&= \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-1}(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{[\Gamma(1+2\alpha)]^2 x^{2\alpha}}{[\Gamma(1+\alpha)]^2} f_{n-2}(x) \prod_{i=1}^{n-3} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&\geqq \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_n(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-1}(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_{n-2}(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{[\Gamma(1+2\alpha)]^3 x^{3\alpha}}{[\Gamma(1+\alpha)]^3} f_{n-2}(x) \prod_{i=1}^{n-3} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&\geqq \cdots \geqq \cdots \\
&= \prod_{i=1}^{n} \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_i(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 \frac{[\Gamma(1+2\alpha)]^n x^{n\alpha}}{[\Gamma(1+\alpha)]^n} (\mathrm{d}x)^{\alpha} \\
&= \prod_{i=1}^{n} \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_i(u)\,(\mathrm{d}u)^{\alpha} \\
&\quad \times \frac{[\Gamma(1+2\alpha)]^n}{[\Gamma(1+\alpha)]^n} \cdot \frac{1}{\Gamma(1+\alpha)} \int_0^1 x^{n\alpha} (\mathrm{d}x)^{\alpha}.
\end{aligned}
\tag{37}
$$

which yields

$$
\begin{aligned}
&\frac{1}{\Gamma(1+\alpha)} \int_0^1 \prod_{i=1}^{n} f_i(x)\,(\mathrm{d}x)^{\alpha} \\
&\qquad \geq \left(\frac{[\Gamma(1+2\alpha)]^n}{[\Gamma(1+\alpha)]^n} \frac{1}{\Gamma(1+\alpha)} \int_0^1 x^{n\alpha} (\mathrm{d}x)^{\alpha} \right) \\
&\qquad \times \left(\prod_{i=1}^{n} \frac{1}{\Gamma(1+\alpha)} \int_0^1 f_i(u)\,(\mathrm{d}u)^{\alpha} \right).
\end{aligned}
\tag{38}
$$

The proof of Theorem 15 is thus completed. □

Remark 16. In its special case when $\alpha = 1$, the inequality (36) asserted by Theorem 15 would reduce to the Anderson-Fink inequality (2).

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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