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Research Article

Robust Stabilization of Linear Switching Systems with Both Input and Communication Delays

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This work is concerned with stabilization control of a class of linear switching systems with time-delays in both the input and the communication channels. It is observed that the time-delay in communication channel leads to the mismatch between the plant and the controller. Such a phenomenon can be accounted for by reconstructing switching signal for the overall closed-loop system. Therefore, we derive some sufficient stability conditions by using multiple Lyapunov functions approach and, moreover, present a robust controller design methodology. A numerical example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

During the last decade, stability and stabilization problems of switching systems have received considerable attentions. Switching control provides flexibility and robustness. A switching system is generated by a switching signal that orchestrates several plants. Therefore, it can be used to describe practical systems that are subject to abrupt changes. For stability analysis, one of the fundamental problems is to establish the constraint conditions on switching frequency and specify certain classes of switching signal so as to guarantee the stability of the overall system (see, e.g., [1–5]). The associated stabilization problem then is to construct feedback control to make the closed-loop system stable under certain classes of switching signal. Within this context, it is an important assumption that the controller can access the plant's switching signal instantly. However, from practical point of view, it needs certain amount of time to detect the change of subsystems and to respond to it. This phenomenon can be treated as the aftereffect (delay) in the communication channel.

On the other hand, time-delay systems have received renewed interest in recent years since they can be used to model many practical physical systems that are affected by the past information, for example, networked control systems, the Bazykin model [6], and the drugs therapy for HIV infection [7]. Time-delay can significantly influence the dynamic behaviors, especially, the stability behavior, of a practical system. Thus time-delay systems have been studied by many scholars in different aspects. For example, to compensate for the large input delay in a control system and to overcome the shortcoming of the traditional predictor feedback, a new approach referred to as truncated predictor feedback was initially established in [8] for a single input delay and was then extended to more general cases in [9–11]; a nested prediction approach was established in [12] to compensate for arbitrary large input delay for linear systems with both input and state delays; a sampled-date approach was developed in [13] for the stochastic synchronization of Markovian jump neural networks with time-varying delays; a Lyapunov-Krasovskii functional approach was built in [14] for the passivity analysis of discrete-time stochastic Markovian jump neural networks with mixed time-delays; robust stability criteria for uncertain neutral systems with mixed delays were derived in [15] by the Lyapunov-Krasovskii functional approach; and the stability of time-delay neural networks subject to stochastic perturbations was investigated in [16]. For more related work, see [17–29] and the references therein.

The time-delay in the communication channel leads to the fact that the change of controllers fails to be instant and then to keep synchronous with the change of plants. Therefore, in this paper, we are concerned with the feedback stabilization problem of switching linear systems in the presence of time-delays in both the communication and the input channels. For our purpose, the starting point is to investigate the joint effects of such aftereffects on the stability of the overall system.

The time-delay in the communication channel gives rise to a kind of uncertainty in the structure of the overall closed-loop system. In order to characterize the uncertainty, we classify each closed-loop subsystem according to whether the controller and the plant are pairwise matched. Basically, the mismatched subsystems play a negative role in the stability of the overall system (see, e.g., [30, 31]). Thus, we treat the mismatched subsystems as perturbed nominal closed-loop system, which is purely composed of the matched subsystems. Hence, in order to account for the involvement of the mismatched subsystems, we employ the technique of merging switching signals to reconstruct the switching signal for the closed-loop system. Therefore, by using multiple Lyapunov functions approach, it is observed that the effect of the time-delay in the communication channel on the stability can be captured by the ratio of its size to the average dwell-time of the switching signal. Accordingly, we apply the Halanay inequality to derive sufficient stability conditions. With the analysis results in hand, we pose the design strategy in such a way that we solve the controllers according to the matched subsystems and, furthermore, check their robustness with respect to the mismatched subsystems.

The remainder of the paper is organized as follows. In Section 2, we describe the problem and present some preliminary results. Section 3 is devoted to proving the sufficient stability conditions for the closed-loop system. In Section 4, the design methodology for the state-feedback robust control is presented. In Section 5, a numerical example is worked out to demonstrate the effectiveness of the method. Finally, the paper is briefly summarized in Section 6.

2. Problem Formulation and Preliminaries

Consider a switching linear control system as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \quad t \geq 0, \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the state vector and the control input, respectively. The system is generated by the switching signal $\sigma(t)$ that orchestrates the following controlled subsystems:

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i \in \mathfrak{I} := \{1, \ldots, N\}. \tag{2}$$

According to the evolution of switching signal over time, it can be expanded into the following sequential form:

$$\begin{aligned}\Big\{ &(t_0 = 0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k)), \\ &\ldots : \lim_{k\to\infty} t_k = \infty \Big\},\end{aligned} \tag{3}$$

where the sequence $\{t_k\}_{k=0}^{\infty}$ monotonically diverging to infinity constitutes the switching points. It means that the $\sigma(t_k)$th subsystem is activated during the interval $(t_k, t_{k+1}]$. Thus, one way to characterize the evolution of switching signal over time is to represent the density of the switching points distributed within the interval of unit length, which is referred to as *average dwell-time*.

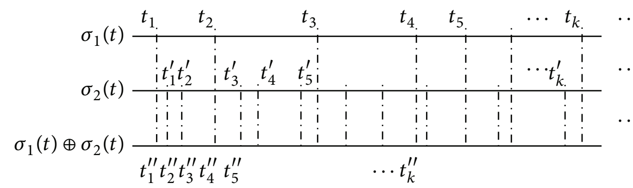

Figure 1: An illustration for the construction of $\sigma_1(t) \oplus \sigma_2(t)$.

Definition 1. Given $\tau > 0$, a switching signal $\sigma(t)$ is said to belong to $\mathcal{S}_\tau$ if it has no more than $(T_2 - T_1)/\tau$ switching points within any interval of time $[T_1, T_2]$. Here, τ is said to be the lower bound of the average dwell-time of the switching signals in $\mathcal{S}_\tau$.

By means of the average dwell-time, categorizing switching signals according to their varying strength and, moreover, defining a merging operation between two switching signals are allowed. To be precise, let the switching signals $\sigma_1(t)$ be in the same index set and, at the same time, let

$$\begin{aligned}\Pi_1 &= \left\{ t_0, t_1, \ldots, t_k, \ldots : \lim_{k\to\infty} t_k = \infty \right\}, \\ \Pi_2 &= \left\{ t_0', t_1', \ldots, t_k', \ldots : \lim_{k\to\infty} t_k' = \infty \right\}\end{aligned} \tag{4}$$

be the collections of their switching points, respectively.

Definition 2. We refer to $\sigma_1(t)\oplus\sigma_2(t)$ as the merging switching signal of $\sigma_1(t)$ and $\sigma_2(t)$ in the sense that the collection of its switching points corresponds to $\Pi_1 \cup \Pi_2$ and, at a switching point, it inherits the corresponding value from $\sigma_1(t)$ and $\sigma_2(t)$.

As illustrated in Figure 1, according to the evolution over time, the ordered switching points of $\sigma_1(t) \oplus \sigma_2(t)$ correspond to the increasing sequence

$$\left\{ t_0'', t_1'', \ldots, t_k'', \ldots : \lim_{k\to\infty} t_k'' = \infty \right\}. \tag{5}$$

Lemma 3 (see [32]). *Given two switching signals $\sigma_1(t) \in \mathcal{S}_{\tau_1}$ and $\sigma_2(t) \in \mathcal{S}_{\tau_2}$ that are assumed in the same index set, one has $\sigma_1(t) \oplus \sigma_2(t) \in \mathcal{S}_\tau$ with $\tau = (1/\tau_1 + 1/\tau_2)^{-1} = (\tau_1 + \tau_2)/\tau_1\tau_2$.*

With respect to certain classes of switching signal, the stabilization problem of system (1) requires us to construct the following state-feedback controller:

$$u(t) = F_i x(t), \quad i \in \mathfrak{I}, \tag{6}$$

in which F_i corresponds to the ith subsystem in (2), such that the overall closed-loop system is asymptotically stable.

Ideally, by supposing that the controller can access the plant's switching signal and can be activated instantly, the resulting closed-loop system

$$\dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)}F_{\sigma(t)}\right)x(t) \tag{7}$$

will keep synchronous with the controlled system in (1). Indeed, it implies that the controller and the plant must be pairwise matched. In practice, however, it needs certain amount of time to detect the change of plants and respond to it by triggering the corresponding controllers into activation. It is thus reasonable to take this aftereffect as well as the aftereffect in the input channel into account. Hence the state-feedback control for system (1) takes the form

$$u(t) = F_{\sigma(t-h_s(t))}x\left(t-h_x(t)\right), \tag{8}$$

where $0 \le h_x(t) \le h_x$ and $0 \le h_s(t) \le h_s$. With this, the task is then to design the controllers as in (8) so that the exponential stability of the closed-loop system is maintained in the presence of time-delays in both the input and the communication channels.

In what follows we will need the following facts.

Lemma 4 (see [20]). *There exists a matrix X such that*

$$\begin{bmatrix} P & Q & X \\ \star & R & V \\ \star & \star & S \end{bmatrix} > 0, \tag{9}$$

if and only if

$$\begin{aligned} &\begin{bmatrix} P & Q \\ \star & R \end{bmatrix} > 0, \\ &\begin{bmatrix} R & V \\ \star & S \end{bmatrix} > 0 \end{aligned} \tag{10}$$

are satisfied.

Lemma 5 (Halanay inequality, see [21]). *Let h, a, b be three given scalars such that $0 \le h$ and $0 \le b < a$. If the positive-definite continuous function $u(t)$ satisfies the following differential inequality:*

$$\dot{u}(t) \le -au(t) + b\sup_{-h\le\theta\le 0} u(t+\theta), \quad \forall t \ge t_0, \tag{11}$$

then one has

$$u(t) \le \left[\sup_{-h\le\theta\le 0} u\left(t_0+\theta\right)\right]e^{-\lambda(t-t_0)}, \quad \forall t \ge t_0, \tag{12}$$

where $\lambda > 0$ is the unique root of the equation $\lambda - a + be^{h\lambda} = 0$.

3. Stability Analysis

It is important to observe that the aftereffect in the communication channel gives rise to the mismatch between plants and controllers. One way to account for the mechanism is to multiply the index set by itself and, at the same time, present the closed-loop subsystems as

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i F_j x\left(t-h_s(t)\right) \\ &= \overline{A}_{ij}x(t) - \overline{D}_{ij}\left(x(t) - x\left(t-h_s(t)\right)\right), \\ &\qquad (i,j) \in \mathfrak{I}\times\mathfrak{I}, \end{aligned} \tag{13}$$

where $\overline{A}_{ij} := A_i + B_iF_j$, $\overline{D}_{ij} := B_iF_j$, and $\mathfrak{I}\times\mathfrak{I} := \{(1,1),\ldots,(1,N),\ldots,(N,1),\ldots,(N,N)\}$. Obviously, the closed-loop subsystem is matched if $i = j$; otherwise it is mismatched.

Correspondingly, the switching signals $\sigma(t)$ and $\sigma_c(t) := \sigma(t-h_s(t))$ are supposed to take values from $\mathfrak{I}\times\mathfrak{I}$. In fact, $\sigma(t)$ takes values of the form (i,i) to represent the switching among the matched subsystems, while $\sigma_c(t)$ takes values of the form (i,j), $i \ne j$, to indicate the activation of the mismatched subsystems. In this way, the overall closed-loop system can be rewritten as

$$\dot{x}(t) = \overline{A}_{\overline{\sigma}(t)}x(t) - \overline{D}_{\overline{\sigma}(t)}\left(x(t) - x\left(t-h_x(t)\right)\right), \tag{14}$$

where

$$\overline{\sigma}(t) := \sigma(t) \oplus \sigma_c(t). \tag{15}$$

It is important to note that $\sigma_c(t)$ belongs to $\mathcal{S}_\tau$ as long as $\sigma(t)$ belongs to $\mathcal{S}_\tau$. Hence, thanks to Lemma 3, we have that $\overline{\sigma}(t)$ belongs to $\mathcal{S}_{\tau/2}$.

We first present a technical lemma whose proof is given in the appendix.

Lemma 6. *For the merging switching signal $\overline{\sigma}(t) = \sigma(t)\oplus\sigma_c(t)$, let the sequence $\{t_k\}_{k=0}^{\infty}$ be the collection of its switching points. Then, for any positive integers $j \le k-1$ and $s \in (t_j, t_{j+1}]$ and $t \in (t_k, t_{k+1}]$, one has*

$$\begin{aligned} \exp&\left(\sum_{i=j}^{k-1}\lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right. \\ &\qquad \left. + \lambda_{\overline{\sigma}(t_j)}\left(t_j - s\right) + (k-j)\ln\chi\right) \\ &\le \exp\left(-\gamma(t-s)\right), \end{aligned} \tag{16}$$

where

$$\gamma := \alpha - \frac{(\alpha+\beta)h_s}{\tau} - \frac{2\ln\chi}{\tau}. \tag{17}$$

We are now in the position to establish some stability conditions for the closed-loop system in (14).

Theorem 7. *The closed-loop switching system* (14) *is exponentially stable for the switching signals belonging to $\mathcal{S}_{\tau/2}$ if there* *exist some positive numbers η_1, η_2, α, β, μ and matrices P_{ij} and X_{ij} with $(i, j) \in \mathfrak{I} \times \mathfrak{I}$ satisfying the following inequalities:*

$$\eta_1 I_n \leq P_{ij} \leq \eta_2 I_n, \tag{18}$$

$$\begin{bmatrix} \overline{A}_{ij}^T P_{ij} + P_{ij}\overline{A}_{ij} - \lambda_{ij} P_{ij} & P_{ij}\overline{D}_{ij} \\ \star & -\dfrac{1}{2h_x} P_{ij} \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} P_{ij} & P_{ij}\overline{D}_{ij} & X_{ij} \\ \star & \mu I_n & A_i^T P_{ij} \\ \star & \star & P_{ij} \end{bmatrix} > 0, \tag{20}$$

$$\alpha - (\alpha + \beta)\frac{h_s}{\tau} - \frac{2\mu h_x}{\eta_1} - \frac{2\ln\chi}{\tau} > 0, \tag{21}$$

where

$$\lambda_{ij} = \begin{cases} -\alpha & i = j \\ \beta & i \neq j, \end{cases} \qquad \chi = \frac{\eta_2}{\eta_1}, \tag{22}$$

and "$\star$" indicates the matrix block indicated by the symmetry.

Proof. From (19), we know

$$\begin{gathered} \overline{A}_{ii}^T P_{ii} + P_{ii}\overline{A}_{ii} + 2h_x P_{ii}\overline{D}_{ii} P_{ii}^{-1}\overline{D}_{ii}^T P_{ii} < -\alpha P_{ii}, \\ \overline{A}_{ij}^T P_{ij} + P_{ij}\overline{A}_{ij} + 2h_x P_{ij}\overline{D}_{ij} P_{ij}^{-1}\overline{D}_{ij}^T P_{ij} < \beta P_{ij}, \quad i \neq j. \end{gathered} \tag{23}$$

Moreover, applying Lemma 4 to (20) gives

$$\begin{gathered} \begin{bmatrix} P_{ij} & P_{ij}\overline{D}_{ij} \\ \star & \mu I_n \end{bmatrix} > 0, \\ \begin{bmatrix} \mu I_n & A_i^T P_{ij} \\ \star & P_{ij} \end{bmatrix} > 0. \end{gathered} \tag{24}$$

We rewrite the closed-loop system in (14) as follows:

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \overline{D}_{ij} x(t - h_x(t)) \\ &= \overline{A}_{ij} x(t) - \overline{D}_{ij} A_i \int_{t-h_x(t)}^{t} x(s)\,\mathrm{d}s \\ &\quad - \overline{D}_{ij}^2 \int_{t-h_x(t)-h_x(t-h_x(t))}^{t-h_x(t)} x(s)\,\mathrm{d}s. \end{aligned} \tag{25}$$

For each subsystem, we construct a Lyapunov function as

$$V_{ij}(x) = x^T P_{ij} x, \quad (i, j) \in \mathfrak{I} \times \mathfrak{I}. \tag{26}$$

By (23) and (24), we can deduce that

$$\begin{aligned} \dot{V}_{ij}(x) &= 2\Bigg[\overline{A}_{ij} x(t) - \overline{D}_{ij} A_i \int_{t-h_x(t)}^{t} x(s)\,\mathrm{d}s \\ &\qquad - \overline{D}_{ij}^2 \int_{t-h_x(t)-h_x(t-h_x(t))}^{t-h_x(t)} x(s)\mathrm{d}s\Bigg]^T P_{ij} x(t) \\ &= 2x^T(t) P_{ij}\overline{A}_{ij} x(t) - 2x^T(t) P_{ij}\overline{D}_{ij} A_i \int_{t-h_x(t)}^{t} x(s)\,\mathrm{d}s \\ &\quad - 2x^T(t) P_{ij}\overline{D}_{ij}^2 \int_{t-h_x(t)}^{t} x(s)\,\mathrm{d}s \\ &\leq 2x^T(t) P_{ij}\overline{A}_{ij} x(t) + \int_{t-h_x(t)}^{t} x^T(s) A_i^T P_{ij} A_i x(s)\,\mathrm{d}s \\ &\quad + h_x x^T(t) P_{ij}\overline{D}_{ij} P_{ij}^{-1}\overline{D}_{ij}^T P_{ij} x(t) \\ &\quad + \int_{t-2h_x(t)}^{t-h_x(t)} x^T(s)\overline{D}_{ij}^T P_{ij}\overline{D}_{ij} x(s)\,\mathrm{d}s \\ &\quad + h_x x^T(t) P_{ij}\overline{D}_{ij} P_{ij}^{-1}\overline{D}_{ij}^T P_{ij} x(t) \\ &= x^T(t)\left(\overline{A}_{ij}^T P_{ij} + P_{ij}\overline{A}_{ij} + 2h_x P_{ij}\overline{D}_{ij} P_{ij}^{-1}\overline{D}_{ij}^T P_{ij}\right) x(t) \\ &\quad + \int_{t-h_x(t)}^{t} x^T(s) A_i^T P_{ij} A_i x(s)\,\mathrm{d}s \\ &\quad + \int_{t-2h_x(t)}^{t-h_x(t)} x^T(s)\overline{D}_{ij}^T P_{ij}\overline{D}_{ij} x(s)\,\mathrm{d}s \\ &\leq \lambda_{ij} V_{ij}(x(t)) + 2\mu h_x \Gamma(t), \end{aligned} \tag{27}$$

where

$$\Gamma(t) := \sup_{-2h_x \leq s \leq 0} |x(t+s)|^2. \tag{28}$$

Let the sequence $\{t_k\}_{k=0}^{+\infty}$ be the switching points of $\overline{\sigma}(t)$. Therefore, for any $t \in (t_k, t_{k+1}]$, we have

$$\begin{aligned} V_{\overline{\sigma}(t_k)}(t) &\leq \exp\left(\lambda_{\overline{\sigma}(t_k)}(t - t_k)\right) V_{\overline{\sigma}(t_k)}(t_k) \\ &\quad + 2\mu h_x \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}(t - s)\right)\Gamma(s)\,\mathrm{d}s \\ &\leq \chi \exp\left(\lambda_{\overline{\sigma}(t_k)}(t - t_k)\right) V_{\overline{\sigma}(t_{k-1})}(t_k) \\ &\quad + 2\mu h_x \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}(t - s)\right)\Gamma(s)\,\mathrm{d}s \end{aligned}$$

$$\begin{aligned}
&\leq \chi \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \left[\exp\left(\lambda_{\overline{\sigma}(t_{k-1})}\left(t_k-t_{k-1}\right)\right) V_{\overline{\sigma}(t_{k-1})}\left(t_{k-1}\right)\right. \\
&\quad\quad \left. + 2\mu h_x \int_{t_{k-1}}^{t_k} \exp\left(\lambda_{\overline{\sigma}(t_{k-1})}\left(t_k-s\right)\right) \Gamma(s)\, ds\right] \\
&\quad + 2\mu h_x \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-s\right)\right) \Gamma(s)\, ds.
\end{aligned} \tag{29}$$

At the same time, from (18) it is seen that

$$\begin{aligned}
V_{\overline{\sigma}(t_l)}\left(t_l\right) &\leq \chi V_{\overline{\sigma}(t_{l-1})}\left(t_l\right) \\
&\leq \chi \exp\left(\lambda_{\overline{\sigma}(t_{l-1})}\left(t_l-t_{l-1}\right)\right) V_{\overline{\sigma}(t_{l-1})}\left(t_{l-1}\right) \\
&\quad + \chi\left(2\mu h_x \int_{t_{l-1}}^{t_l} \exp\left(\lambda_{\overline{\sigma}(t_{l-1})}\left(t_l-s\right)\right) \Gamma(s)\, ds\right), \\
&\qquad l \geq 1.
\end{aligned} \tag{30}$$

Repetitively using the relation in (30) gives

$$\begin{aligned}
V_{\overline{\sigma}(t_k)}(t) &\leq \chi^k \exp\left(\sum_{i=0}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) V_{\overline{\sigma}(t_0)}\left(t_0\right) \\
&\quad + 2\mu h_x \chi^k \exp\left(\sum_{i=1}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_0}^{t_1} \exp\left(\lambda_{\overline{\sigma}(t_0)}\left(t_1-s\right)\right) \Gamma(s)\, ds \\
&\quad + 2\mu h_x \chi^{k-1} \exp\left(\sum_{i=2}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_1}^{t_2} \exp\left(\lambda_{\overline{\sigma}(t_1)}\left(t_2-s\right)\right) \Gamma(s)\, ds \\
&\quad \vdots \\
&\quad + 2\mu h_x \chi \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_{k-1}}^{t_k} \exp\left(\lambda_{\overline{\sigma}(t_{k-1})}\left(t_k-s\right)\right) \Gamma(s)\, ds \\
&\quad + 2\mu h_x \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-s\right)\right) \Gamma(s)\, ds \\
&= \chi^k \exp\left(\sum_{i=0}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) V_{\overline{\sigma}(t_0)}\left(t_0\right) \\
&\quad + 2\mu h_x \sum_{j=0}^{k-1} \chi^{k-j} \exp\left(\sum_{i=j+1}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_j}^{t_{j+1}} \exp\left(\lambda_{\overline{\sigma}(t_j)}\left(t_{j+1}-s\right)\right) \Gamma(s)\, ds \\
&\quad + 2\mu h_x \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-s\right)\right) \Gamma(s)\, ds \\
&= \chi^k \exp\left(\sum_{i=0}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) V_{\overline{\sigma}(t_0)}\left(t_0\right) \\
&\quad + 2\mu h_x \sum_{j=0}^{k-1} \chi^{k-j} \exp\left(\sum_{i=j}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_j}^{t_{j+1}} \exp\left(\lambda_{\overline{\sigma}(t_j)}\left(t_j-s\right)\right) \Gamma(s)\, ds \\
&\quad + 2\mu h_x \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t-t_k\right)\right) \\
&\quad \times \int_{t_k}^{t} \exp\left(\lambda_{\overline{\sigma}(t_k)}\left(t_k-s\right)\right) \Gamma(s)\, ds.
\end{aligned} \tag{31}$$

Since t is free to vary within the interval $(t_k, t_{k+1}]$, without loss of generality, we replace t_{k+1} with t and then put the last inequality in (31) into the following condensed form:

$$\begin{aligned}
V_{\overline{\sigma}(t_k)}\left(t_{k+1}\right) &\leq \chi^k \exp\left(\sum_{i=0}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right) + \lambda_{\overline{\sigma}(t_k)}\left(t_{k+1}-t_k\right)\right) V_{\overline{\sigma}(t_0)}\left(t_0\right) \\
&\quad + 2\mu h_x \sum_{j=0}^{k} \chi^{k-j} \exp\left(\sum_{i=j}^{k-1} \lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right)\right.
\end{aligned}$$

$$+\lambda_{\overline{\sigma}(t_k)}\left(t_{k+1}-t_k\right)\Bigg)$$
$$\times\int_{t_j}^{t_{j+1}}\exp\left(\lambda_{\overline{\sigma}(t_j)}\left(t_j-s\right)\right)\Gamma(s)\,\mathrm{d}s. \tag{32}$$

Therefore, we arrive at

$$\begin{aligned}
&V_{\overline{\sigma}(t_k)}\left(t_{k+1}\right)\\
&\leq\exp\Bigg(\sum_{i=0}^{k-1}\lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right)+\lambda_{\overline{\sigma}(t_k)}\left(t_{k+1}-t_k\right)\\
&\qquad+(k-j)\ln\chi\Bigg)V_{\overline{\sigma}(t_0)}\left(t_0\right)\\
&\quad+2\mu h_x\sum_{j=0}^{k}\int_{t_j}^{t_{j+1}}\exp\Bigg(\sum_{i=j}^{k-1}\lambda_{\overline{\sigma}(t_i)}\left(t_{i+1}-t_i\right)+\lambda_{\overline{\sigma}(t_k)}\left(t_{k+1}-t_k\right)\\
&\qquad+\lambda_{\overline{\sigma}(t_j)}\left(t_j-s\right)\\
&\qquad+(k-j)\ln\chi\Bigg)\Gamma(s)\,\mathrm{d}s.
\end{aligned}\tag{33}$$

Now, in view of Lemma 6, from (33) it follows that

$$\begin{aligned}
&V_{\overline{\sigma}(t_k)}\left(t_{k+1}\right)\\
&\quad\leq\exp\left(-\gamma\left(t-t_0\right)\right)V_{\overline{\sigma}(t_0)}\left(t_0\right)\\
&\qquad+2\mu h_x\sum_{j=0}^{k}\int_{t_j}^{t_{j+1}}\exp\left(-\gamma\left(t_{k+1}-s\right)\right)\Gamma(s)\,\mathrm{d}s\\
&\quad\leq\eta xp\left(-\gamma\left(t-t_0\right)\right)V_{\overline{\sigma}(t_0)}\left(t_0\right)\\
&\qquad+2\mu h_x\int_{t_0}^{t_{k+1}}\exp\left(-\gamma\left(t_{k+1}-s\right)\right)\Gamma(s)\,\mathrm{d}s.
\end{aligned}\tag{34}$$

Furthermore, by the arbitrariness of t_{k+1}, we conclude that

$$\begin{aligned}
&\eta_1|x(t)|^2\\
&\quad\leq\eta_2\exp\left(-\gamma\left(t-t_0\right)\right)\left|x\left(t_0\right)\right|^2\\
&\qquad+2\mu h_x\int_{t_0}^{t}\exp\left(-\gamma(t-s)\right)\Gamma(s)\,\mathrm{d}s\\
&\quad=e^{-\gamma(t-t_0)}\left[\eta_2\left|x\left(t_0\right)\right|^2+2\mu h_x e^{-\gamma t_0}\int_{t_0}^{t}e^{\gamma s}\Gamma(s)\,\mathrm{d}s\right],\\
&\qquad\qquad t\geq t_0.
\end{aligned}\tag{35}$$

Now, define the following continuous function:

$$u(t)=\begin{cases}\eta_2|x(t)|^2 & t_0-2h_x\leq t\leq t_0\\ e^{-\gamma(t-t_0)}\left[\eta_2\left|x\left(t_0\right)\right|^2+2\mu h_x e^{-\gamma t_0}\int_{t_0}^{t}e^{\gamma s}\Gamma(s)\,\mathrm{d}s\right] & t\geq t_0.\end{cases}\tag{36}$$

As a result, it is easy to see that

$$\begin{gathered}
u\left(t_0+s\right)=\eta_2\left|x\left(t_0+s\right)\right|^2,\quad t_0-2h_x\leq s\leq t_0,\\
u(t)\geq\eta_1|x(t)|^2,\quad t\geq t_0-2h_x.
\end{gathered}\tag{37}$$

Recalling (28) and computing the time-derivative of $u(t)$ yield

$$\begin{aligned}
\dot{u}(t)&=-\gamma e^{-\gamma(t-t_0)}\left[\eta_2\left|x\left(t_0\right)\right|^2+2\mu h_x e^{-\gamma t_0}\int_{t_0}^{t}e^{\gamma s}\Gamma(s)\,\mathrm{d}s\right]\\
&\quad+2\mu h_x\sup_{-2h_x\leq s\leq 0}|x(t+s)|^2\leq-\gamma u(t)\\
&\quad+\frac{2\mu h_x}{\eta_1}\sup_{-2h_x\leq s\leq 0}u(t+s),\quad t\geq t_0.
\end{aligned}\tag{38}$$

Thus, by applying Lemma 5 to (38) and noting (37), we assert that

$$\begin{aligned}
|x(t)|^2&\leq\frac{1}{\eta_1}e^{-\lambda(t-t_0)}\sup_{-2h_x\leq s\leq 0}u\left(t_0+s\right)\\
&=\chi e^{-\lambda(t-t_0)}\sup_{-2h_x\leq s\leq 0}\left|x\left(t_0+s\right)\right|^2,\\
&\qquad t\geq t_0,
\end{aligned}\tag{39}$$

where $\lambda>0$ is the unique root of the following equation:

$$\lambda-\gamma+\frac{2\mu h_x}{\eta_1}e^{2\lambda h_x}=0.\tag{40}$$

Comparing (17) and (40) with the hypothesis of Lemma 5, we know that it requires

$$\frac{2\mu h_x}{\eta_1}<\alpha-(\alpha+\beta)\frac{h_s}{\tau}-\frac{2\ln\chi}{\tau},\tag{41}$$

to guarantee the exponential stability of system (38). It turns out to be the inequality in (21). The proof is thus completed. □

Remark 8. It is seen from (21) that the effect of the time-delay in the communication channel on stability can be captured by the ratio of its size to the average dwell-time of the switching signal, namely, h_s/τ. If $h_s/\tau\geq 1$, then the inequality in (21) must fail. It means that it is necessary to require the time-delay in the communication channel to be less than the average dwell-time of the switching signal. Physically, this necessity makes sense because once it is violated there would be no matched subsystem involved.

Remark 9. It is worthy of noting that when we neglect the aftereffects in both the input and the communication channels, namely, $h_x = h_s = 0$, the constraint relation in (21) reduces to

$$\frac{\ln \chi}{\alpha} < \frac{\tau}{2}. \tag{42}$$

Formally, it coincides with the existing results such as that in [3], where one would have $\ln \chi/\alpha < \tau$. Indeed, merging switching signals doubles the switching frequency of the closed-loop system and the influence remains even when h_s tends to 0. In nature, the existence of h_s permanently changes the structure of the overall system no matter how small it is.

4. Stabilizing Controllers Design

For the robust design problem, we first solve the feedback controllers via the constraint conditions on the matched subsystems but leave the constraint conditions on mismatched subsystems to be checked. We now summarize it in the following procedure.

Step 1. Specify a set of parameters α, β, μ, η_1, η_2 subjected to the inequality in (21).

Step 2. Test the existence of the controllers by verifying the conditions in Theorem 7 for the matched subsystems.

Step 3. Examine the robustness of the controllers by checking if the mismatched systems can satisfy the conditions in Theorem 7; if not then go back to Step 1.

To realize the above algorithm, we first need to specify a family of positive scalars α, β, η_1, η_2, and μ that satisfy (21). In the sequel, to design the controllers, it requires that there exist matrices W_i and Q_{ii}, X_{ii} such that the following matrix inequalities

$$\frac{1}{\eta_2} I_n \leq Q_{ii} \leq \frac{1}{\eta_1} I_n, \tag{43}$$

$$\begin{bmatrix} Q_{ii}A_i^T + A_iQ_{ii} + B_iW_i + W_i^TB_i^T + \alpha Q_{ii} & B_iW_i \\ \star & -\dfrac{1}{2h_x}Q_{ii} \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} Q_{ii} & B_iW_i & X_{ii} \\ \star & \dfrac{\mu}{\eta_2}Q_{ii} & Q_{ii}A_i^T \\ \star & \star & Q_{ii} \end{bmatrix} > 0 \tag{45}$$

hold true for each $i \in \mathfrak{I}$. Then, by letting

$$F_i = W_iQ_{ii}^{-1}, \tag{46}$$

one can recover the inequalities in (18), (19), and (20) for the matched subsystems. Further, to check the robustness of the controllers given in (46), it requires that there exist matrices X_{ij}, P_{ij}, $i \neq j$ (i.e., for the mismatched subsystems) such that the following inequalities can be satisfied:

$$\eta_1 I_n \leq P_{ij} \leq \eta_2 I_n, \tag{47}$$

$$\begin{bmatrix} \overline{A}_{ij}^TP_{ij} + P_{ij}\overline{A}_{ij} - \beta P_{ij} & P_{ij}\overline{D}_{ij} \\ \star & -\dfrac{1}{2h_x}P_{ij} \end{bmatrix} < 0, \tag{48}$$

$$\begin{bmatrix} P_{ij} & P_{ij}\overline{D}_{ij} & X_{ij} \\ \star & \mu I_n & A_i^TP_{ij} \\ \star & \star & P_{ij} \end{bmatrix} > 0. \tag{49}$$

Indeed, we have to repeatedly adjust the positive scalars involved in (21) and solve the inequalities in (43)–(49) until they are feasible.

5. An Illustrative Example

Consider the switched system composed of two linear control subsystems with the following parameters:

$$A_1 = \begin{bmatrix} -1 & 0.5 \\ 0 & 0.5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.5 & -1 \\ 2 & -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}. \tag{50}$$

These subsystems are governed by the switching signal that belongs to $\mathcal{S}_\tau$ with $\tau = 2.0$ s. The open-loop system is unstable. Besides, the feedback control is subject to the time-delays in both the input and the communication channels with $h_x = 0.02$ s and $h_s = 0.2$ s.

We first choose $\eta_1 = 0.45$, $\eta_2 = 1.2$, $\alpha = 2.0$, $\beta = 2.85$, and $\mu = 6.0$ to meet the inequality in (21). Then, solving the inequalities in (43)–(45) gives the following state-feedback gains:

$$F_1 = \begin{bmatrix} 0.9996 & 2.2637 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 3.8134 & 0.3945 \end{bmatrix}. \tag{51}$$

Hence, the mismatched closed-loop subsystems turn out to be

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -2.9067 & 0.3027 \\ 0 & 0.50 \end{bmatrix} x(t) + \begin{bmatrix} -1.9067 & -0.1973 \\ 0 & 0 \end{bmatrix} x(t - h_x(t)), \\ \dot{x}(t) &= \begin{bmatrix} 0.9998 & 0.1319 \\ 1.0004 & -3.2637 \end{bmatrix} x(t) + \begin{bmatrix} 0.4998 & 1.1319 \\ -0.9996 & -2.2637 \end{bmatrix} x(t - h_x(t)). \end{aligned} \tag{52}$$

Both of these two mismatched subsystems are unstable. However, the inequalities in (47)–(49) are feasible. It means that the obtained controllers can make the closed-loop system exponentially stable in the presence of time-delays in both the input and the communication channels. The state-response of the overall closed-loop system generated by the periodic switching signal with $\tau = 2.0$ s is depicted in Figure 2.

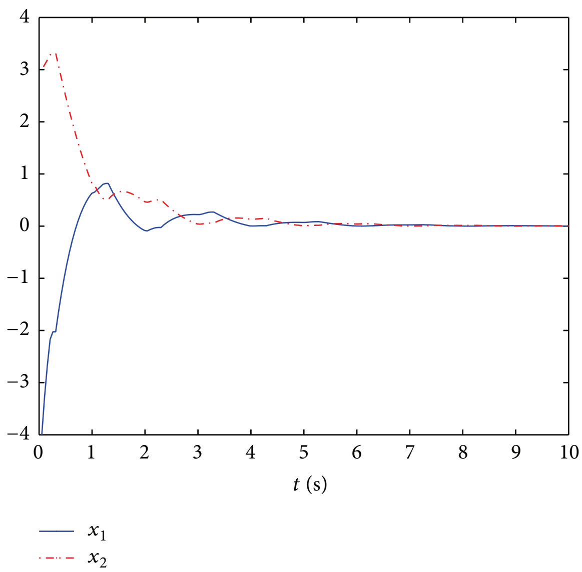

FIGURE 2: State-response of the overall closed-loop system.

6. Conclusion

In this paper we have presented a robust stabilizing controller design methodology for linear switching systems in the presence of time-delays in both the input and the communication channels. We have used the technique of merging switching signals to account for the aftereffect in the communication channel and then reconstructed the switching signal for the overall closed-loop system. Therefore, we established the sufficient stability conditions by using multiple Lyapunov functions approach along with the Halanay inequality. Finally we proposed a strategy to solve the desired robust controllers and worked out an example in detail to illustrate the theoretical results. A possible future research topic along this paper is the robustness issue by considering uncertainties in the system parameters.

Appendix

Proof of Lemma 6

For the sake of generality, let the switching signals $\sigma(t)$ and $\sigma_c(t)$ belong to $\mathcal{S}_{\tau_1}$ and $\mathcal{S}_{\tau_2}$, respectively. For the merging switching signal $\overline{\sigma}(t) = \sigma(t) \oplus \sigma_c(t)$ and $t_0 \leq s < t$, we know that

$$\frac{N_\sigma(s,t)}{N_{\overline{\sigma}}(s,t)} = \frac{(t-s)/\tau_1}{(t-s)/(\tau_1\tau_2/(\tau_1+\tau_2))} = \frac{\tau_2}{\tau_1+\tau_2},$$
$$\frac{N_{\sigma_c}(s,t)}{N_{\overline{\sigma}}(s,t)} = \frac{(t-s)/\tau_2}{(t-s)/(\tau_1\tau_2/(\tau_1+\tau_2))} = \frac{\tau_1}{\tau_1+\tau_2}, \quad (A.1)$$

where $N_\sigma(s,t)$ denotes the number of the switching points of $\sigma(t)$ distributed within the interval $[s,t]$.

Let the increasing sequence $\{t_k\}_{k=0}^{\infty}$ denote the switching points of $\overline{\sigma}(t)$. For positive integers $j \leq k-1$ and $s \in (t_j, t_{j+1}]$ and $t \in (t_k, t_{k+1}]$ we have

$$N_{\overline{\sigma}}(s,t) = k - j \leq \frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s). \quad (A.2)$$

Consequently, we obtain

$$\begin{aligned}
&\exp\Bigg(\sum_{i=j}^{k-1}\lambda_{\overline{\sigma}(t_i)}(t_{i+1}-t_i) + \lambda_{\overline{\sigma}(t_k)}(t-t_k) \\
&\qquad +\lambda_{\overline{\sigma}(t_j)}(t_j-s) + (k-j)\ln\chi\Bigg) \\
&\leq \exp\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\ln\chi\right) \\
&\quad \times \exp\left(-\alpha\left(t-s-N_{\sigma_c}(s,t)h_s\right) + \beta N_{\sigma_c}(s,t)h_s\right) \\
&\leq \exp\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\ln\chi\right) \\
&\quad \times \exp\Bigg(-\alpha\left(t-s-\frac{\tau_1 N_{\overline{\sigma}}(s,t)h_s}{\tau_1+\tau_2}\right) \\
&\qquad +\beta\frac{\tau_1 N_{\overline{\sigma}}(s,t)h_s}{\tau_1+\tau_2}\Bigg) \\
&= \exp\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\ln\chi\right) \\
&\quad \times \exp(-\alpha(t-s))\exp\left((\alpha+\beta)h_s\frac{\tau_1}{\tau_1+\tau_2}N_{\overline{\sigma}}(s,t)\right) \\
&\leq \exp\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\ln\chi\right) \\
&\quad \times \exp(-\alpha(t-s))\exp\Bigg((\alpha+\beta)h_s \\
&\qquad\qquad \times\frac{\tau_1}{\tau_1+\tau_2}\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\Bigg) \\
&\leq \exp\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}(t-s)\ln\chi\right) \\
&\quad \times \exp(-\alpha(t-s))\exp\left((\alpha+\beta)\frac{h_s}{\tau_2}(t-s)\right) \\
&= \exp\left(-\left(\alpha-(\alpha+\beta)\frac{h_s}{\tau_2}-\frac{\tau_1+\tau_2}{\tau_1\tau_2}\ln\chi\right)(t-s)\right).
\end{aligned} \quad (A.3)$$

Therefore, by noting $\tau_1 = \tau_2 = \tau$, we complete the proof.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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