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Research Article

Robust Finite-Time H_∞ Control for Nonlinear Markovian Jump Systems with Time Delay under Partially Known Transition Probabilities

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This paper is concerned with the problem of robust finite-time H_∞ control for a class of nonlinear Markovian jump systems with time delay under partially known transition probabilities. Firstly, for the nominal nonlinear Markovian jump systems, sufficient conditions are proposed to ensure finite-time boundedness, H_∞ finite-time boundedness, and finite-time H_∞ state feedback stabilization, respectively. Then, a robust finite-time H_∞ state feedback controller is designed, which, for all admissible uncertainties, guarantees the H_∞ finite-time boundedness of the corresponding closed-loop system. All the conditions are presented in terms of strict linear matrix inequalities. Finally a numerical example is provided to demonstrate the effectiveness of all the results.

1. Introduction

Markovian jump systems, a class of hybrid dynamical systems, which consists of an indexed family of continuous or discrete-time subsystems and a set of Markovian chain that orchestrates the switching between them at stochastic time instants, have received extensive attention over the past few decades [1, 2]. Many real world processes, such as economic systems [3], manufacturing systems [4], electric power systems [5], and communication systems [6], may be modeled as Markovian jump systems when any malfunction of sensors or actuators cause a jump behavior in process performance. Recently, nonlinear Markovian jump systems have been extensively applied and developed in various disciplines of science and engineering, and a great number of excellent works have been developed [7–9].

Generally speaking, the behavior of nonlinear Markovian jump systems is determined by the transition probabilities in the jumping process. Usually, it is assumed that the information on transition probabilities was completely known. However, transition probabilities may be partially known for some real systems. For example, the networked control systems can be modeled by nonlinear Markovian jump systems with partially known transition probabilities when the packet dropouts or channel delays occur [10]. In addition, there are few results about the known bounds of transition probability rates or the fixed connection weighting matrices [11, 12]. Therefore, it is reasonable to study Markovian jump systems with partially known transition probabilities, especially, when it is difficult to measure the bounds of transition probability rates. It stimulates the research interests of the author.

Uncertainties and time delay frequently occur in various engineering systems, which usually is a source of instability and often causes undesirable performance and even makes the system out of control [14, 15]. Therefore, time delay systems with robustness have received an increasing attention among the control community [16–18]. On the other hand, one may be interested in not only system stability but also a bound of system trajectories over a fixed short time [19]. For instance, for the problem of robot arm control [7], when the robot works under different environmental conditions with changing payloads, it requests that the angle position of the arm should not exceed some threshold in a prescribed time interval. Meanwhile, the scholars attach more importance to the H_∞ control problem, which is to find a stable controller such that the disturbance attenuation level γ is below a prescribed level. There are a great number of useful and interesting results about H_∞ control problem for linear and nonlinear Markovian jump systems in the literature [20–25]. To the best of our knowledge, the synthesis issue of

robust finite-time H_∞ control for nonlinear Markovian jump systems with time delay under partially known transition probabilities has not been fully investigated until now, which motivates us to carry out the present study.

In this paper, we investigate the problem of robust finite-time H_∞ control for nonlinear Markovian jump systems with time delay under partially known transition probabilities. The main contributions lie in the fact that some tractable sufficient conditions are provided to ensure H_∞ finite-time boundedness or finite-time H_∞ state feedback stabilization. A robust finite-time H_∞ state feedback controller is designed, which guarantees the H_∞ finite-time boundedness of the closed-loop system. Seeking computational convenience, all the conditions are cast in the format of linear matrix inequalities. Finally, a numerical example is provided to demonstrate the effectiveness of the main results.

Notations. Throughout this paper, the notations used are fairly standard. For real symmetric matrices A and B, the notation $A \geq B$ (resp., $A > B$) means that the matrix A-B is positive semi-definite (resp., positive definite). A^T represents the transpose matrix of A, and A^{-1} represents the inverse matrix of A. $\lambda_{\max}(B)$ ($\lambda_{\min}(B)$) is the maximum (resp., minimum) eigenvalue of a matrix B. $\operatorname{diag}\{A \quad B\}$ represents the block diagonal matrix of A and B. I is the unit matrix with appropriate dimensions, and the term of symmetry is stated by the asterisk $*$ in a matrix. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and $\mathscr{M} = \{1, 2, \ldots, N\}$ means a set of positive numbers. $\| * \|$ denotes the Euclidean norm of vectors. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector. $L_2^n[0, +\infty)$ is the space of n-dimensional square integrable function vector over $[0, +\infty)$.

2. Problem Formulation and Preliminaries

Give a probability space $(\Omega, \mathscr{F}, \mathscr{P})$, where Ω is the sample space, $\mathscr{F}$ is the algebra of events, and $\mathscr{P}$ is the probability measure defined on $\mathscr{F}$. The random process $\{r_t, t \geq 0\}$ is a Markovian stochastic process taking values in a finite set $\mathscr{M} = \{1, 2, \ldots, N\}$ with the transition probability rate matrix $\Pi = \{\pi_{ij}\}$, $i, j \in \mathscr{M}$, and the transition probability from mode i at time t to mode j at time $t + \Delta t$ is expressed as

$$P\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{1}$$

with the transition probability rates $\pi_{ij} \geq 0$, for $i, j \in \mathscr{M}$, $i \neq j$, and $\sum_{j=1, i\neq j}^{N} \pi_{ij} = -\pi_{ii}$, where $\Delta t > 0$, and $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$.

Consider the following nonlinear Markovian jump system with time delay in the probability space $(\Omega, \mathscr{F}, \mathscr{P})$:

$$\begin{aligned} \dot{x}(t) = {} & (A(r_t) + \Delta A(r_t))\, x(t) \\ & + (A_d(r_t) + \Delta A_d(r_t))\, x(t-\tau) \\ & + (B(r_t) + \Delta B(r_t))\, u(t) + G(r_t)\, w(t) \\ & + f(r_t, x(t), x(t-\tau)), \\ z(t) = {} & C(r_t)\, x(t) + C_d(r_t)\, x(t-\tau) + D(r_t)\, u(t) \\ & + E(r_t)\, w(t), \\ x(t) = {} & \varphi(t), \quad t \in [-\tau \quad 0], \end{aligned} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in L_2^n[0, +\infty)$ is an arbitrary external disturbance, $z(t) \in \mathbb{R}^l$ is the control output, $\varphi(t)$ represents a vector-valued initial function, and $\tau \in \mathbb{R}^+$ is the constant delay. $f(\cdot,\cdot,\cdot)$: $\mathscr{M} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m$ is an unknown nonlinear function. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $G(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r_t)$, and $E(r_t)$ are known mode-dependent constant matrices with appropriate dimensions. $\Delta A(r_t)$, $\Delta A_d(r_t)$, and $\Delta B(r_t)$ are unknown matrices, denoting the uncertainties in the system, and the uncertainties are time-varying but norm bounded uncertainties satisfying

$$\begin{aligned} \Delta A(r_t) &= M_1(r_t)\, F(t, r_t)\, N_1(r_t), \\ \Delta B(r_t) &= M_2(r_t)\, F(t, r_t)\, N_2(r_t), \\ \Delta A_d(r_t) &= M_3(r_t)\, F(t, r_t)\, N_3(r_t), \end{aligned} \tag{3}$$

where $M_1(r_t)$, $N_1(r_t)$, $M_2(r_t)$, $N_2(r_t)$, $M_3(r_t)$, and $N_3(r_t)$ are known mode-dependent matrices with appropriate dimensions and $F(t, r_t)$ is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying

$$F(t, r_t)^T F(t, r_t) \leq I. \tag{4}$$

Consider the following state feedback controller:

$$u(t) = K(r_t)\, x(t) + K_d(r_t)\, x(t-\tau), \tag{5}$$

where $K(r_t)$ and $K_d(r_t)$ are the state feedback gains to be designed. Then the closed-loop system is as follows:

$$\begin{aligned} \dot{x}(t) = {} & (A(r_t) + \Delta A(r_t) + B(r_t)\, K(r_t) \\ & + \Delta B(r_t)\, K(r_t))\, x(t) \\ & + (A_d(r_t) + \Delta A_d(r_t) + B(r_t)\, K_d(r_t) \\ & + \Delta B(r_t)\, K_d(r_t))\, x(t-\tau) \\ & + G(r_t)\, w(t) + f(r_t, x(t), x(t-\tau)), \\ z(t) = {} & (C(r_t) + D(r_t)\, K(r_t))\, x(t) \\ & + (C_d(r_t) + D(r_t)\, K_d(r_t))\, x(t-\tau) \\ & + E(r_t)\, w(t), \\ x(t) = {} & \varphi(t), \quad t \in [-\tau \quad 0]. \end{aligned} \tag{6}$$

For notational simplicity, when $r(t) = i, i \in \mathscr{M}$, $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $G(r_t)$, $K(r_t)$, $K_d(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r_t)$, $E(r_t)$, $\Delta A(r_t)$, $\Delta B(r_t)$, $M_1(r_t)$, $N_1(r_t)$, $M_2(r_t)$, $N_2(r_t)$, $M_3(r_t)$, $N_3(r_t)$, and $f(r_t, x(t), x(t-\tau))$ are, respectively, denoted as A_i, A_{di}, B_i, G_i, K_i, K_{di}, C_i, C_{di}, D_i, E_i, ΔA_i, ΔB_i, M_{1i}, N_{1i}, M_{2i}, N_{2i}, M_{3i}, N_{3i}, and $f_i(x(t), x(t-\tau))$.

In addition, the transition probability rates are considered to be partially known; that is, some elements in matrix $\Pi = \{\pi_{ij}\}$ are unknown. For instance, for system (2) with four subsystems, the transition probability rate matrix Π may be as

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & ? & ? \\ ? & ? & \pi_{23} & \pi_{24} \\ \pi_{31} & ? & \pi_{33} & ? \\ \pi_{41} & ? & ? & ? \end{bmatrix}, \tag{7}$$

where "?" represents the unknown transition probability rate. $\forall i \in \mathscr{M}$, we denote $\mathscr{M} = L_k^i \cup L_{uk}^i$, and

$$\begin{aligned} L_k^i &\triangleq \left\{ j : \pi_{ij} \text{ is known, for } j \in \mathscr{M} \right\}, \\ L_{uk}^i &\triangleq \left\{ j : \pi_{ij} \text{ is unknown, for } j \in \mathscr{M} \right\}. \end{aligned} \tag{8}$$

Moreover, if $L_k^i \neq \mathbf{0}$, it is further described as

$$L_k^i = \left\{ k_1^i, k_2^i, \ldots k_m^i \right\}, \quad 1 \le m \le \mathscr{M}, \tag{9}$$

where $k_m^i \in \mathscr{M}$ represents the mth known transition probability rate of the set L_k^i in the ith row of the transition probability rate matrix Π.

Remark 1. When $L_{uk}^i = \mathbf{0}$, $L_k^i = \mathscr{M}$, it is reduced to the case where the transition probability rates of the Markovian jump process $\{r_t, t \ge 0\}$ are completely known. When $L_k^i = \mathbf{0}$, $L_{uk}^i = \mathscr{M}$, it means that the transition probability rates of the Markovian jump process $\{r_t, t \ge 0\}$ are completely unknown. Mixing the above two aspects, here, a general form is considered.

In this paper, the following assumptions, definitions, and lemmas play an important role in our later development.

Assumption 2. The external disturbance $w(t)$ is varying and satisfies the constraint condition:

$$\int_{t_0}^{T} w^T(s)\, w(s)\, ds \le d, \quad d \ge 0. \tag{10}$$

Assumption 3. $\forall i \in \mathscr{M}$, $f_i(0,0) = 0$, and $f_i(x(t), x(t-\tau))$ satisfies the following inequality

$$\begin{aligned} &\left\| f_i(x(t), x(t-\tau)) \right\|^2 \\ &\quad \le \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}^T \begin{bmatrix} F_{11}^i & F_{12}^i \\ * & F_{22}^i \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}, \end{aligned} \tag{11}$$

where

$$F_i := \begin{bmatrix} F_{11}^i & F_{12}^i \\ * & F_{22}^i \end{bmatrix} \ge 0. \tag{12}$$

Definition 4 (*finite-time stability*). For a given time constant $T > 0$, system (2) ($u(t) = 0$, $w(t) = 0$) is said to be finite-time stable with respect to (c_1, c_2, T, H_i), if

$$\mathbb{E}\left\{ x_0^T H_i x_0 \right\} \le c_1 \Longrightarrow \mathbb{E}\left\{ x(t)^T H_i x(t) \right\} \le c_2, \quad \forall t \in [0, T], \tag{13}$$

where $0 < c_1 < c_2$, $H_i > 0$.

Definition 5 (*finite-time boundedness*). For a given time constant $T > 0$, system (2) ($u(t) = 0$) is said to be finite-time bounded with respect to (c_1, c_2, T, H_i, d), if the condition (13) holds, where $0 < c_1 < c_2$, $H_i > 0$.

Definition 6 (H_∞ *finite-time boundedness*). For a given time constant $T > 0$, system (2) ($u(t) = 0$) is said to be H_∞ finite-time bounded with respect to (c_1, c_2, T, H_i, d), if there exists a positive constant γ, such that the following two conditions are true:

(1) system (2) is finite-time bounded with respect to (c_1, c_2, T, H_i, d);

(2) under zero initial condition ($x(t_0) = 0, t_0 = 0$), for any external disturbance $w(t) \neq 0$ satisfying condition (10), the control output $z(t)$ of system (2) satisfies

$$\mathbb{E}\left\{ \int_0^T z^T(t)\, z(t)\, dt \right\} \le \gamma^2 \int_0^T w^T(t)\, w(t)\, dt. \tag{14}$$

Definition 7 (*finite-time* H_∞ *state feedback stabilization*). The system (2) is said to be finite-time H_∞ state feedback stabilizable with respect to (c_1, c_2, T, H_i, d), if there exist a positive constant γ and a state feedback controller in the form of (5), such that the closed-loop system (6) is H_∞ finite-time bounded.

Definition 8 (see [26]). In the Euclidean space $\{\mathbb{R}^n \times \mathscr{M} \times \mathbb{R}^+\}$, introduce the stochastic Lyapunov function for system (2) as $V(x(t), i)$, and the weak infinitesimal operator satisfies

$$\begin{aligned} &\mathscr{L}V(x(t), i) \\ &\quad = \lim_{\Delta_t \to 0} \frac{1}{\Delta_t} \left[\mathbb{E}\left\{ V\left(x(t+\Delta_t)\, r(t+\Delta_t) \right) \right\} - V(x(t), i) \right] \\ &\quad = \frac{\partial}{\partial t} V(x(t), i) + \frac{\partial}{\partial x} V(x(t), i)\, \dot{x}(t) + \sum_{j=1}^{N} \pi_{ij} V(x(t), j). \end{aligned} \tag{15}$$

Remark 9. It easily follows from (12) that $F_{11}^i \ge 0$, $F_{22}^i \ge 0$. So F_{11}^i and F_{22}^i can be decomposed as

$$F_{11}^i = \left(F_{11}^i\right)^{1/2} \left(F_{11}^i\right)^{1/2}, \qquad F_{22}^i = \left(F_{22}^i\right)^{1/2} \left(F_{22}^i\right)^{1/2}. \tag{16}$$

Remark 10. It is noticed that finite-time stability can be regarded as a particular case of finite-time boundedness by setting $w(t) = 0$. That is, finite-time boundedness implies finite-time stability, but the converse is not true.

Lemma 11 (see [27]). *Let T, M, F, and N be real matrices of appropriate dimensions with $F^T F \le I$; then for a positive scalar $\varepsilon > 0$, there holds:*

$$T + MFN + N^T F^T M^T \le T + \varepsilon M M^T + \varepsilon^{-1} N^T N. \tag{17}$$

The aim in this paper is to find a tractable solution to the problem of finite-time H_∞ state feedback stabilization.

3. Main Results

3.1. Finite-Time Boundedness Analysis. In this subsection, we will consider the problem of finite-time boundedness for the nominal system of nonlinear Markovian jump system (2) with $F(t, r_t) = 0$ for all $t \geq 0$; that is,

$$\begin{aligned} \dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-\tau) + B(r_t)u(t) \\ &\quad + G(r_t)w(t) + f(r_t, x(t), x(t-\tau)), \\ z(t) &= C(r_t)x(t) + C_d(r_t)x(t-\tau) + D(r_t)u(t) \\ &\quad + E(r_t)w(t), \\ x(t) &= \varphi(t), \quad t \in [-\tau \quad 0]. \end{aligned} \tag{18}$$

Under the controller (5), the closed-loop system is

$$\begin{aligned} \dot{x}(t) &= (A(r_t) + B(r_t)K(r_t))x(t) \\ &\quad + (A_d(r_t) + B(r_t)K_d(r_t))x(t-\tau) \\ &\quad + G(r_t)w(t) + f(r_t, x(t), x(t-\tau)), \\ z(t) &= (C(r_t) + D(r_t)K(r_t))x(t) \\ &\quad + (C_d(r_t) + D(r_t)K_d(r_t))x(t-\tau) \\ &\quad + E(r_t)w(t), \\ x(t) &= \varphi(t), \quad t \in [-\tau \quad 0]. \end{aligned} \tag{19}$$

Theorem 12. *Given $T > 0$, if there exist positive constants α and ε_{fi}, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{q\times q}$ and $S \in \mathbb{R}^{p\times p}$, and symmetric matrices $W_i \in \mathbb{R}^{n\times n}$, such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix} \Lambda_{1i} & P_iA_{di} + \varepsilon_{fi}F_{12}^i & P_iG_i \\ * & -Q + \varepsilon_{fi}F_{22}^i & 0 \\ * & * & -\alpha S \end{bmatrix} < 0, \tag{20}$$

$$P_j - W_i \leq 0, \quad j \in L_{uk}^i, \ j \neq i, \tag{21}$$

$$P_j - W_i \geq 0, \quad j \in L_{uk}^i, \ j = i, \tag{22}$$

$$\frac{c_1\left[\lambda_{\max}(\widetilde{P}_i) + \tau\lambda_{\max}(\widetilde{Q}_i)\right] + d\lambda_{\max}(S)\left(1 - e^{-\alpha T}\right)}{\lambda_{\min}(\widetilde{P}_i)} < e^{-\alpha T}c_2, \tag{23}$$

then system (18) ($u = 0$) under partially known transition probabilities is finite-time bounded with respect to (c_1, c_2, T, H_i, d), where

$$\begin{aligned} \Lambda_{1i} &= A_i^TP_i + P_iA_i + Q \\ &\quad + \sum_{j\in L_k^i}\pi_{ij}(P_j - W_i) + \varepsilon_{fi}^{-1}P_iP_i + \varepsilon_{fi}F_{11}^i - \alpha P_i, \\ \widetilde{P}_i &= H_i^{-1/2}P_iH_i^{-1/2}, \qquad \widetilde{Q}_i = H_i^{-1/2}QH_i^{-1/2}. \end{aligned} \tag{24}$$

Proof. For system (18) ($u = 0$), choose a Lyapunov function candidate

$$\begin{aligned} V(x(t), i) &= V_1(x(t), i) + V_2(x(t), i) \\ &= x(t)^TP_ix(t) + \int_{t-\tau}^{t} x^T(\xi)Qx(\xi)\,d\xi, \end{aligned} \tag{25}$$

where $P_i > 0$. Then by Definition 8, we get

$$\begin{aligned} \mathscr{L}V_1(x(t), i) &= x^T(t)\left[A_i^TP_i + P_iA_i + \sum_{j=1}^{N}\pi_{ij}P_j\right]x(t) \\ &\quad + x^T(t)P_iA_{di}x(t-\tau) + x^T(t)P_iG_iw(t) \\ &\quad + x^T(t)P_if_i + x^T(t-\tau)A_{di}^TP_ix(t) \\ &\quad + w^T(t)G_i^TP_ix(t) + f_i^TP_ix(t). \end{aligned} \tag{26}$$

Based on Lemma 11, there exist scalars ε_{fi} such that

$$\begin{aligned} &x^T(t)P_if_i + f_i^TP_ix(t) \\ &\quad \leq \varepsilon_{fi}f_i^Tf_i + \varepsilon_{fi}^{-1}x^T(t)P_iP_ix(t) \\ &\quad \leq \varepsilon_{fi}\left[x^T(t)F_{11}^ix(t) + x^T(t)F_{12}^ix(t-\tau)\right. \\ &\qquad \left. + x^T(t-\tau)F_{21}^ix(t) + x^T(t-\tau)F_{22}^ix(t-\tau)\right] \\ &\qquad + \varepsilon_{fi}^{-1}x^T(t)P_iP_ix(t). \end{aligned} \tag{27}$$

Substituting (27) into (26) yields

$$\begin{aligned} &\mathscr{L}V_1(x(t), i) \\ &\quad \leq x^T(t)\left[A_i^TP_i + P_iA_i + \varepsilon_{fi}^{-1}P_iP_i + \sum_{j=1}^{N}\pi_{ij}P_j + \varepsilon_{fi}F_{11}^i\right]x(t) \\ &\qquad + x^T(t)P_iG_iw(t) + x^T(t)\left[P_iA_{di} + \varepsilon_{fi}F_{12}^i\right]x(t-\tau) \\ &\qquad + x^T(t-\tau)\left[A_{di}^TP_i + \varepsilon_{fi}F_{21}^i\right]x(t) + w^T(t)G_i^TP_ix(t) \\ &\qquad + x^T(t-\tau)\varepsilon_{fi}F_{22}^ix(t-\tau). \end{aligned} \tag{28}$$

It is easy to obtain that

$$\mathscr{L}V_2(x(t), i) = x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau). \tag{29}$$

From (28) and (29), the following holds:

$$\begin{aligned}
&\mathscr{L}V(x(t),i)\\
&= \mathscr{L}V_1(x(t),i) + \mathscr{L}V_2(x(t),i)\\
&\le x^T(t)\Bigg[A_i^T P_i + P_i A_i + \varepsilon_{fi}^{-1} P_i P_i\\
&\qquad + \sum_{j=1}^{N} \pi_{ij} P_j + \varepsilon_{fi} F_{11}^i + Q\Bigg] x(t)\\
&\quad + x^T(t) P_i G_i w(t) + x^T(t)\left[P_i A_{di} + \varepsilon_{fi} F_{12}^i\right] x(t-\tau)\\
&\quad + w^T(t) G_i^T P_i x(t) + x^T(t-\tau)\left[A_{di}^T P_i + \varepsilon_{fi} F_{21}^i\right] x(t)\\
&\quad + x^T(t-\tau)\left[\varepsilon_{fi} F_{22}^i - Q\right] x(t-\tau).
\end{aligned} \tag{30}$$

Due to the fact that $\sum_{j=1}^{N} \pi_{ij} W_i = 0$ for arbitrary symmetric matrices W_i, (30) can be written as

$$\begin{aligned}
&\mathscr{L}V(x(t),i)\\
&\le x^T(t)\Bigg[A_i^T P_i + P_i A_i + \varepsilon_{fi}^{-1} P_i P_i\\
&\qquad + \sum_{j=1}^{N} \pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi} F_{11}^i + Q\Bigg] x(t)\\
&\quad + x^T(t) P_i G_i w(t) + x^T(t)\left[P_i A_{di} + \varepsilon_{fi} F_{12}^i\right] x(t-\tau)\\
&\quad + w^T(t) G_i^T P_i x(t) + x^T(t-\tau)\left[A_{di}^T P_i + \varepsilon_{fi} F_{21}^i\right] x(t)\\
&\quad + x^T(t-\tau)\left[\varepsilon_{fi} F_{22}^i - Q\right] x(t-\tau)\\
&= x^T(t)\Bigg[A_i^T P_i + P_i A_i + \varepsilon_{fi}^{-1} P_i P_i\\
&\qquad + \sum_{j\in L_k^i} \pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi} F_{11}^i + Q\Bigg] x(t)\\
&\quad + x^T(t) P_i G_i w(t) + x^T(t)\left[P_i A_{di} + \varepsilon_{fi} F_{12}^i\right] x(t-\tau)\\
&\quad + w^T(t) G_i^T P_i x(t) + x^T(t-\tau)\left[A_{di}^T P_i + \varepsilon_{fi} F_{21}^i\right] x(t)\\
&\quad + x^T(t-\tau)\left[\varepsilon_{fi} F_{22}^i - Q\right] x(t-\tau)\\
&\quad + x^T(t) \sum_{j\in L_{uk}^i} \pi_{ij}\left(P_j - W_i\right) x(t).
\end{aligned} \tag{31}$$

Noticing that $\pi_{ij} \ge 0$ for all $i \ne j$ and $\pi_{ii} = -\sum_{j=1, i\ne j}^{N} \pi_{ij} < 0$ for all $i \in \mathscr{M}$, if $i \in L_k^i$ (the elements of the diagonal are known), by inequalities (20) and (21), the following inequalities hold:

$$\begin{aligned}
\mathscr{L}V(x(t),i) &< \alpha x(t)^T P_i x(t) + \alpha w^T(t) S w(t)\\
&< \alpha x(t)^T P_i x(t) + \alpha \int_{t-\tau}^{t} x^T(\xi) Q x(\xi)\, d\xi\\
&\quad + \alpha w^T(t) S w(t)\\
&= \alpha V(x(t),i) + \alpha w^T(t) S w(t).
\end{aligned} \tag{32}$$

If $i \in L_{uk}^i$ (the elements of the diagonal are unknown), according to the inequalities (20)–(22), inequality (32) holds. Multiplying (32) by $e^{-\alpha t}$ yields

$$\mathscr{L}\left(e^{-\alpha t} V(x(t),i)\right) < \alpha e^{-\alpha t} w^T(t) S w(t). \tag{33}$$

Applying Dynkin's formula for (33), we obtain

$$e^{-\alpha t} V(x(t),i) - V(x_0,t_0) < \alpha \int_0^t e^{-\alpha s} w^T(s) S w(s)\, ds, \tag{34}$$

which shows

$$\begin{aligned}
V(x(t),i) &< e^{\alpha t} V(x_0,t_0) + \alpha e^{\alpha t} \int_0^t e^{-\alpha s} w^T(s) S w(s)\, ds\\
&< e^{\alpha t} V(x_0,t_0) + \alpha d \lambda_{\max}(S) e^{\alpha t} \int_0^t e^{-\alpha s} ds\\
&= e^{\alpha t}\left[V(x_0,t_0) + \alpha d \lambda_{\max}(S) \frac{1-e^{-\alpha t}}{\alpha}\right].
\end{aligned} \tag{35}$$

This together with $\widetilde{P}_i = H_i^{-1/2} P_i H_i^{-1/2}$ and $\widetilde{Q}_i = H_i^{-1/2} Q H_i^{-1/2}$ gives rise to

$$\begin{aligned}
V(x(t),i) < e^{\alpha t}\Big[&c_1\left(\lambda_{\max}\left(\widetilde{P}_i\right) + \tau \lambda_{\max}\left(\widetilde{Q}_i\right)\right)\\
&+ d\lambda_{\max}(S)\left(1 - e^{-\alpha t}\right)\Big].
\end{aligned} \tag{36}$$

Considering that

$$V(x(t),i) \ge x^T(t) P_i x(t) \ge \lambda_{\min}\left(\widetilde{P}_i\right) x^T(t) H_i x(t) \tag{37}$$

and combining (36) and (37), it follows that

$$\begin{aligned}
&\mathbb{E}\left\{x^T(t) H_i x(t)\right\}\\
&< \frac{e^{\alpha t}\left[c_1\left(\lambda_{\max}\left(\widetilde{P}_i\right) + \tau \lambda_{\max}\left(\widetilde{Q}_i\right)\right) + d\lambda_{\max}(S)\left(1 - e^{-\alpha t}\right)\right]}{\lambda_{\min}\left(\widetilde{P}_i\right)}\\
&< c_2.
\end{aligned} \tag{38}$$

Condition (38) implies that, for $t \in [0 \quad T]$, $\mathbb{E}\{x^T(t) H_i x(t)\} < c_2$.

The proof is complete. □

Corollary 13. *Given $T > 0$, if there exist positive constants α, ε_{fi}, and γ, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, and $Q \in \mathbb{R}^{q\times q}$, and symmetric matrices $W_i \in \mathbb{R}^{n\times n}$, such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix} \Lambda_{1i} & P_iA_{di} + \varepsilon_{fi}F_{12}^i & P_iG_i \\ * & -Q + \varepsilon_{fi}F_{22}^i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{39}$$

$$P_j - W_i \le 0, \quad j \in L_{uk}^i, \ j \ne i, \tag{40}$$

$$P_j - W_i \ge 0, \quad j \in L_{uk}^i, \ j = i, \tag{41}$$

$$\begin{aligned} &c_1\left[\lambda_{\max}\left(\widetilde{P}_i\right) + \tau\lambda_{\max}\left(\widetilde{Q}_i\right)\right] + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) \\ &\quad < \lambda_{\min}\left(\widetilde{P}_i\right)e^{-\alpha T}c_2, \end{aligned} \tag{42}$$

then system (18) ($u = 0$) *under partially known transition probabilities is finite-time bounded with respect to (c_1, c_2, T, H_i, d), where*

$$\begin{aligned} \Lambda_{1i} &= A_i^T P_i + P_i A_i + Q \\ &\quad + \sum_{j\in L_k^i} \pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi}^{-1}P_iP_i + \varepsilon_{fi}F_{11}^i - \alpha P_i, \end{aligned} \tag{43}$$

$$\widetilde{P}_i = H_i^{-1/2}P_iH_i^{-1/2}, \qquad \widetilde{Q}_i = H_i^{-1/2}QH_i^{-1/2}.$$

3.2. Finite-Time H_∞ Performance Analysis. In this subsection, based on Corollary 13, some sufficient conditions will be provided ensuring the H_∞ finite-time boundedness of system (18) and the H_∞ finite-time stabilization of system (19).

Theorem 14. *Given $T > 0$ and $w(t)$ satisfying* (10), *system* (18) ($u = 0$) *under partially known transition probabilities is H_∞ finite-time bounded with respect to (c_1, c_2, T, H_i, d), if there exist positive constants α, ε_{fi}, and γ, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{q\times q}$, and symmetric matrices $W_i \in \mathbb{R}^{n\times n}$, such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix} \Lambda_{1i} + C_i^TC_i & P_iA_{di} + \varepsilon_{fi}F_{12}^i + C_i^TC_{di} & P_iG_i + C_i^TE_i \\ * & -Q + \varepsilon_{fi}F_{22}^i + C_{di}^TC_{di} & C_{di}^TE_i \\ * & * & -\gamma^2 I + E_i^TE_i \end{bmatrix} < 0, \tag{44}$$

$$P_j - W_i \le 0, \quad j \in L_{uk}^i, \ j \ne i, \tag{45}$$

$$P_j - W_i \ge 0, \quad j \in L_{uk}^i, \ j = i, \tag{46}$$

$$\begin{aligned} &c_1\left[\lambda_{\max}\left(\widetilde{P}_i\right) + \tau\lambda_{\max}\left(\widetilde{Q}_i\right)\right] + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) \\ &\quad < \lambda_{\min}\left(\widetilde{P}_i\right)e^{-\alpha T}c_2, \end{aligned} \tag{47}$$

where

$$\begin{aligned} \Lambda_{1i} &= A_i^T P_i + P_i A_i + Q \\ &\quad + \sum_{j\in L_k^i} \pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi}^{-1}P_iP_i + \varepsilon_{fi}F_{11}^i - \alpha P_i, \end{aligned} \tag{48}$$

$$\widetilde{P}_i = H_i^{-1/2}P_iH_i^{-1/2}, \qquad \widetilde{Q}_i = H_i^{-1/2}QH_i^{-1/2}.$$

Proof. From (44), the following inequality holds:

$$\begin{aligned} &\begin{bmatrix} \Lambda_{1i} + C_i^TC_i & P_iA_{di} + \varepsilon_{fi}F_{12}^i + C_i^TC_{di} & P_iG_i + C_i^TE_i \\ * & -Q + \varepsilon_{fi}F_{22}^i + C_{di}^TC_{di} & C_{di}^TE_i \\ * & * & -\gamma^2 I + E_i^TE_i \end{bmatrix} \\ &= \begin{bmatrix} \Lambda_{1i} & P_iA_{di} + \varepsilon_{fi}F_{12}^i & P_iG_i \\ * & -Q + \varepsilon_{fi}F_{22}^i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C_i^T \\ C_{di}^T \\ E_i^T \end{bmatrix}\begin{bmatrix} C_i & C_{di}^T & E_i \end{bmatrix} \\ &< 0. \end{aligned} \tag{49}$$

This together with (49) implies (39). Then based on (39)–(42), system (18) is finite-time bounded.

Then, let us prove that inequality (14) is satisfied for any external disturbance $w(t) \ne 0$ under zero initial condition. For system (18), choosing a Lyapunov function candidate (25), we have

$$\begin{aligned} &\mathscr{L}V(x(t), i) \\ &\quad \le x^T(t)\left[A_i^TP_i + P_iA_i + \varepsilon_{fi}^{-1}P_iP_i \right. \\ &\qquad \left. + \sum_{j\in L_k^i}\pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi}F_{11}^i + Q\right]x(t) \\ &\qquad + x^T(t)P_iG_iw(t) + x^T(t)\left[P_iA_{di} + \varepsilon_{fi}F_{12}^i\right]x(t-\tau) \\ &\qquad + w^T(t)G_i^TP_ix(t) + x^T(t-\tau)\left[A_{di}^TP_i + \varepsilon_{fi}F_{21}^i\right]x(t) \\ &\qquad + x^T(t-\tau)\left[\varepsilon_{fi}F_{22}^i - Q\right]x(t-\tau) \\ &\qquad + x^T(t)\sum_{j\in L_{uk}^i}\pi_{ij}\left(P_j - W_i\right)x(t), \end{aligned} \tag{50}$$

for any symmetric matrices W_i.

According to inequality (44), (45), and (46), we derive

$$\mathscr{L}V(x(t),i) < \alpha V(x(t),i) + \gamma^2 w^T(t)w(t) - z^T(t)z(t),$$
$$\mathscr{L}\left[e^{-\alpha t}V(x(t),i)\right] < e^{-\alpha t}\left[\gamma^2 w^T(t)w(t) - z^T(t)z(t)\right]. \tag{51}$$

Under zero initial condition, using Dynkin's formula yields

$$\begin{aligned} &e^{-\alpha t}V(x(t),i) \\ &\quad < \int_0^t e^{-\alpha s}\left[\gamma^2 w^T(s)w(s) - z^T(s)z(s)\right]ds, \\ &\mathbb{E}\int_0^t e^{-\alpha s}z^T(s)z(s)\,ds < \int_0^t e^{-\alpha s}\gamma^2 w^T(s)w(s)\,ds. \end{aligned} \tag{52}$$

Further, it implies that

$$\mathbb{E}\int_0^T z^T(s)z(s)\,ds < \gamma^2 e^{\alpha T}\int_0^t w^T(s)w(s)\,ds. \tag{53}$$

Therefore expression (14) holds with $\overline{\gamma} = \sqrt{e^{\alpha T}}\gamma$.

The proof is complete. □

Corollary 15. *Given $T > 0$ and $w(t)$ satisfying (10), system (19) under partially known transition probabilities is finite-time H_∞ state feedback stabilizable via a state feedback controller (5) with respect to (c_1, c_2, T, H_i, d), if there exist positive constants α, ε_{fi}, and γ, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{q\times q}$, and symmetric matrices $W_i \in \mathbb{R}^{n\times n}$, such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix} \widetilde{\Lambda}_{1i} + \widetilde{C}_i^T\widetilde{C}_i & P_i\widetilde{A}_{di} + \varepsilon_{fi}F_{12}^i + \widetilde{C}_i^T\widetilde{C}_{di} & P_iG_i + \widetilde{C}_i^T E_i \\ * & -Q + \varepsilon_{fi}F_{22}^i + \widetilde{C}_{di}^T\widetilde{C}_{di} & \widetilde{C}_{di}^T E_i \\ * & * & -\gamma^2 I + E_i^T E_i \end{bmatrix} < 0, \tag{54}$$

$$P_j - W_i \le 0, \quad j \in L_{uk}^i,\ j \ne i, \tag{55}$$

$$P_j - W_i \ge 0, \quad j \in L_{uk}^i,\ j = i, \tag{56}$$

$$\begin{aligned} &c_1\left[\lambda_{\max}\left(\widetilde{P}_i\right) + \tau\lambda_{\max}\left(\widetilde{Q}_i\right)\right] + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) \\ &\quad < \lambda_{\min}\left(\widetilde{P}_i\right)e^{-\alpha T}c_2, \end{aligned} \tag{57}$$

where

$$\begin{aligned} \widetilde{\Lambda}_{1i} &= \widetilde{A}_i^T P_i + P_i\widetilde{A}_i + Q \\ &\quad + \sum_{j\in L_k^i}\pi_{ij}\left(P_j - W_i\right) + \varepsilon_{fi}^{-1}P_iP_i + \varepsilon_{fi}F_{11}^i - \alpha P_i, \\ \widetilde{A}_i &= A_i + B_iK_i, \qquad \widetilde{A}_{di} = A_{di} + B_iK_{di}, \\ \widetilde{C}_i^T &= C_i + D_iK_i, \qquad \widetilde{C}_{di} = C_{di} + D_iK_{di}, \\ \widetilde{P}_i &= H_i^{-1/2}P_iH_i^{-1/2}, \qquad \widetilde{Q}_i = H_i^{-1/2}QH_i^{-1/2}. \end{aligned} \tag{58}$$

It is clear that (54) is a nonlinear matrix inequality due to the existence of the nonlinear terms $K_i^T B_i^T P_i$, $P_iB_iK_i$, $K_{di}^T B_i^T P_i$, and $P_iB_iK_{di}$. In order to solve the desired controller K_i, we give the following result.

Theorem 16. *Given $T > 0$, system (18) under partially known transition probabilities is finite-time H_∞ state feedback stabilizable via a state feedback controller with respect to (c_1, c_2, T, H_i, d), if there exist positive scalars α, γ, ε_{fi}, λ_1, and λ_2, symmetric positive definite matrices $X_i \in \mathbb{R}^{n\times n}$, symmetric matrices $\mathscr{W}_i \in \mathbb{R}^{n\times n}$, and matrices $Y_i \in \mathbb{R}^{m\times n}$ and $K_{di} \in \mathbb{R}^{n\times m}$ such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix} \Pi_{11i}^1 & \Pi_{12i} & G_i & \Pi_{14i} & I & \varepsilon_{fi}X_i\left(F_{11}^i\right)^{1/2} & S_{1i}(x) \\ * & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & * & -M_{1i}(x) \end{bmatrix} < 0, \quad i \in L_k^i, \tag{59}$$

$$\begin{bmatrix} \Pi_{11i}^2 & \Pi_{12i} & G_i & \Pi_{14i} & I & \varepsilon_{fi}X_i\left(F_{11}^i\right)^{1/2} & S_{2i}(x) \\ * & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & * & -M_{2i}(x) \end{bmatrix} < 0, \quad i \in L_{uk}^i, \tag{60}$$

$$\begin{bmatrix} -\mathscr{W}_i & X_i \\ * & -X_j \end{bmatrix} < 0, \quad j \in L_{uk}^i,\ j \ne i, \tag{61}$$

$$X_j - \mathscr{W}_j > 0, \quad j \in L_{uk}^i,\ j = i, \tag{62}$$

$$\begin{bmatrix} -e^{-\alpha T}c_2 + c_1\tau\lambda_2 + \dfrac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) & \sqrt{c_1} \\ \sqrt{c_1} & -\lambda_1 \end{bmatrix} < 0, \tag{63}$$

$$\lambda_1 H_i^{-1} < X_i < H_i^{-1}, \qquad 0 < Q < \lambda_2 H_i, \tag{64}$$

where

$$\begin{aligned} \Pi_{11i}^1 &= X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + Q_i \\ &\quad - \sum_{j\in L_k^i}\pi_{ij}\mathscr{W}_i + \pi_{ii}X_i - \alpha X_i, \\ \Pi_{11i}^2 &= X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + Q_i \\ &\quad - \sum_{j\in L_k^i}\pi_{ij}\mathscr{W}_i - \alpha X_i, \\ \Pi_{12i} &= A_{di} + B_iK_{di} + \varepsilon_{fi}X_iF_{12}^i, \\ \Pi_{22i} &= -Q + \varepsilon_{fi}F_{22}^i, \\ \Pi_{14i} &= X_iC_i^T + Y_i^TD_i^T, \\ \Pi_{24i} &= C_{di}^T + K_{di}^TD_i^T, \end{aligned}$$

$$S_{1i}(x) = \left[\sqrt{\pi_{ik_1^i}}X_i, \ldots, \sqrt{\pi_{ik_{r-1}^i}}X_i, \sqrt{\pi_{ik_{r+1}^i}}X_i, \ldots, \sqrt{\pi_{ik_m^i}}X_i\right],$$
$$M_{1i}(x) = \operatorname{diag}\left\{X_{k_1^i}, \ldots, X_{k_{r-1}^i}, X_{k_{r+1}^i}, \ldots, X_{k_m^i}\right\},$$
$$S_{2i}(x) = \left[\sqrt{\pi_{ik_1^i}}X_i, \ldots, \sqrt{\pi_{ik_m^i}}X_i\right],$$
$$M_{2i}(x) = \operatorname{diag}\left\{X_{k_1^i}, \ldots, X_{k_m^i}\right\}, \tag{65}$$

with $k_1^i, k_2^i, \ldots k_m^i$ described in (9) *and $k_r^i = i$. Moreover, the finite-time H_∞ state feedback controller gains in* (5) *are given by $K_i = Y_i X_i^{-1}$.*

Proof. It is clear that system (18) is finite-time H_∞ state feedback stabilizable if the conditions (54)–(57) are satisfied. Notice that inequality (54) is equivalent to the following condition:

$$\Sigma_i = \begin{bmatrix} \widetilde{\Lambda}_{1i} & P_i\widetilde{A}_{di} + \varepsilon_{fi}F_{12}^i & P_iG_i & \widetilde{C}_i^T & P_i & \varepsilon_{fi}\left(F_{11}^i\right)^{1/2} \\ * & -Q + \varepsilon_{fi}F_{22}^i & 0 & \widetilde{C}_{di}^T & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I \end{bmatrix} < 0. \tag{66}$$

Pre- and postmultiplying inequality (66) by block diagonal matrix diag $\{P_i^{-1} \;\; I \;\; I \;\; I \;\; I \;\; I\}$, letting $X_i = P_i^{-1}$, $Y_i = K_iX_i$, and $\mathscr{W}_i = P_i^{-1}W_iP_i^{-1}$, we have

$$\begin{bmatrix} \Xi_{1i} & \Pi_{12i} & G_i & \Pi_{14i} & I & \varepsilon_{fi}X_i\left(F_{11}^i\right)^{1/2} \\ * & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I \end{bmatrix} < 0, \tag{67}$$

where

$$\Xi_{1i} = X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + Q_i + \sum_{j\in L_k^i}\pi_{ij}X_iX_j^{-1}X_i - \sum_{j\in L_k^i}\pi_{ij}\mathscr{W}_i - \alpha X_i. \tag{68}$$

Since $\pi_{ii} < 0$, $\forall i \in \mathscr{M}$, inequality (67) is discussed in the following two cases.

Case 1. When $i \in L_k^i$, the left side of (67) becomes

$$\begin{bmatrix} \Xi_{2i} & \Pi_{12i} & G_i & \Pi_{14i} & I & \varepsilon_{fi}X_i\left(F_{11}^i\right)^{1/2} \\ * & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I \end{bmatrix} + \begin{bmatrix} \sum_{j\in L_k^i, j\neq i}\pi_{ij}X_iX_j^{-1}X_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{69}$$

where

$$\Xi_{2i} = X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + Q_i - \sum_{j\in L_k^i}\pi_{ij}\mathscr{W}_i - \alpha X_i. \tag{70}$$

Applying Schur complement lemma to (69), then (59) easily follows.

Case 2. When $i \in L_{uk}^i$, the inequality (69) turns into

$$\begin{bmatrix} \Xi_{3i} & \Pi_{12i} & G_i & \Pi_{14i} & I & \varepsilon_{fi}X_i\left(F_{11}^i\right)^{1/2} \\ * & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\ * & * & -\gamma^2 I & E_i^T & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon_{fi}I & 0 \\ * & * & * & * & * & -\varepsilon_{fi}I \end{bmatrix} + \begin{bmatrix} \sum_{j\in L_k^i, j\neq i}\pi_{ij}X_iX_j^{-1}X_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{71}$$

where

$$\Xi_{3i} = X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + Q_i - \sum_{j\in L_k^i}\pi_{ij}\mathscr{W}_i. \tag{72}$$

Similar to the proving process of the case one, we can prove that (60) is true.

Pre- and postmultiplying inequalities (55) and (56) by P_i^{-1}, respectively, and letting $X_i = P_i^{-1}$, $Y_i = K_iX_i$,

and $\mathscr{W}_i = P_i^{-1} W_i P_i^{-1}$, we have

$$X_i X_j^{-1} X_i - R_i < 0, \quad j \in L_{uk}^i, \ j \neq i, \tag{73}$$

$$X_j - R_j > 0, \quad j \in L_{uk}^i, \ j = i. \tag{74}$$

Inequality (73) is equivalent to LMI (61). Denoting $\widetilde{X}_i = \widetilde{P}_i^{-1} = H_i^{1/2} X_i H_i^{1/2}$ and taking $\lambda_{\max}(\widetilde{X}_i) = 1/\lambda_{\min}(\widetilde{P}_i)$ into consideration, we conclude that condition (57) holds. Hence, the following conditions

$$\lambda_1 < \lambda_{\min}\left(\widetilde{X}_i\right), \qquad \lambda_{\max}\left(\widetilde{X}_i\right) < 1, \qquad 0 < \lambda_{\min}(Q), \qquad \lambda_{\max}(Q) < \lambda_2, \tag{75}$$

guarantee that

$$\frac{c_1}{\lambda_1} + c_1 \tau \lambda_2 + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha t}\right) < e^{-\alpha t} c_2. \tag{76}$$

It should be easily observed that condition (76) implies LMI (63) and (75) is equivalent to (64). Therefore if LMIs (59)–(64) hold, the closed-loop system (19) is H_∞ finite-time bounded, and then system (18) can be stabilized via the state feedback controller (5).

This completes the proof of Theorem 16. □

3.3. Robust Finite-Time H_∞ Control. In this subsection, a robust finite-time H_∞ state feedback controller is designed to guarantee the finite-time H_∞ state feedback stabilization of system (2).

Theorem 17. *Given $T > 0$, the problem of robust finite-time H_∞ state feedback stabilizable for system (2) under partly known transition probabilities is solvable, if there exist positive scalars α, γ, ε_{fi}, ε_{1i}, ε_{2i}, ε_{3i}, ε_{4i}, λ_1, and λ_2, symmetric positive definite matrices $X_i \in \mathbb{R}^{n\times n}$, symmetric matrices $\mathscr{W}_i \in \mathbb{R}^{n\times n}$, and matrices $Y_i \in \mathbb{R}^{m\times n}$ and $K_{di} \in \mathbb{R}^{n\times m}$ such that for all $i \in \mathscr{M}$*

$$\begin{bmatrix}
\widetilde{\Pi}_{11i}^1 & \Pi_{12i} & G_i & \Pi_{14i} & X_i N_{1i}^T & Y_i^T N_{2i}^T & 0 & 0 & I & \varepsilon_{fi} X_i \left(F_{11}^i\right)^{1/2} & S_{1i}(x) \\
* & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 & N_{3i}^T & K_{di}^T N_{2i}^T & 0 & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\varepsilon_{2i} I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_{3i} I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_{4i} I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon_{fi} I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon_{fi} I & 0 \\
* & * & * & * & * & * & * & * & * & * & -M_{1i}(x)
\end{bmatrix} < 0, \quad i \in L_k^i, \tag{77}$$

$$\begin{bmatrix}
\widetilde{\Pi}_{11i}^2 & \Pi_{12i} & G_i & \Pi_{14i} & X_i N_{1i}^T & Y_i^T N_{2i}^T & 0 & 0 & I & \varepsilon_{fi} X_i \left(F_{11}^i\right)^{1/2} & S_{2i}(x) \\
* & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 & N_{3i}^T & K_{di}^T N_{2i}^T & 0 & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\varepsilon_{2i} I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_{3i} I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_{4i} I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon_{fi} I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon_{fi} I & 0 \\
* & * & * & * & * & * & * & * & * & * & -M_{2i}(x)
\end{bmatrix} < 0, \quad i \in L_{uk}^i, \tag{78}$$

$$\begin{bmatrix} -\mathscr{W}_i & X_i \\ * & -X_j \end{bmatrix} < 0, \quad j \in L_{uk}^i, \ j \neq i, \tag{79}$$

$$X_j - \mathscr{W}_j > 0, \quad j \in L_{uk}^i, \ j = i, \tag{80}$$

$$\begin{bmatrix} -e^{-\alpha T} c_2 + c_1 \tau \lambda_2 + \dfrac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) & \sqrt{c_1} \\ \sqrt{c_1} & -\lambda_1 \end{bmatrix} < 0, \tag{81}$$

$$\lambda_1 H_i^{-1} < X_i < H_i^{-1}, \qquad 0 < Q < \lambda_2 H_i, \tag{82}$$

where

$$\begin{aligned}\widetilde{\Pi}_{11i}^1 &= X_i A_i^T + A_i X_i + Y_i^T B_i^T + B_i Y_i + Q_i \\ &\quad - \sum_{j\in L_k^i} \pi_{ij} \mathscr{W}_i + \varepsilon_{1i} M_{1i} M_{1i}^T + \varepsilon_{2i} M_{2i} M_{2i}^T \\ &\quad + \varepsilon_{3i} M_{3i} M_{3i}^T + \varepsilon_{4i} M_{2i} M_{2i}^T + \pi_{ii} X_i - \alpha X_i,\end{aligned}$$

$$\begin{aligned}\widetilde{\Pi}_{11i}^2 &= X_i A_i^T + A_i X_i + Y_i^T B_i^T + B_i Y_i + Q_i \\ &\quad - \sum_{j\in L_k^i} \pi_{ij} \mathscr{W}_i + \varepsilon_{1i} M_{1i} M_{1i}^T + \varepsilon_{2i} M_{2i} M_{2i}^T \\ &\quad + \varepsilon_{3i} M_{3i} M_{3i}^T + \varepsilon_{4i} M_{2i} M_{2i}^T - \alpha X_i,\end{aligned}$$

$$\begin{aligned}\Pi_{12i} &= A_{di} + B_i K_{di} + \varepsilon_{fi} X_i F_{12}^i, \\ \Pi_{22i} &= -Q + \varepsilon_{fi} F_{22}^i, \\ \Pi_{14i} &= X_i C_i^T + Y_i^T D_i^T, \\ \Pi_{24i} &= C_{di}^T + K_{di}^T D_i^T,\end{aligned}$$

$$\begin{aligned}&S_{1i}(x) \\ &= \left[\sqrt{\pi_{ik_1^i}} X_i, \ldots, \sqrt{\pi_{ik_{r-1}^i}} X_i, \sqrt{\pi_{ik_{r+1}^i}} X_i, \ldots, \sqrt{\pi_{ik_m^i}} X_i\right], \\ &M_{1i}(x) = \operatorname{diag}\left\{X_{k_1^i}, \ldots, X_{k_{r-1}^i}, X_{k_{r+1}^i}, \ldots, X_{k_m^i}\right\}, \\ &S_{2i}(x) = \left[\sqrt{\pi_{ik_1^i}} X_i, \ldots, \sqrt{\pi_{ik_m^i}} X_i\right], \\ &M_{2i}(x) = \operatorname{diag}\left\{X_{k_1^i}, \ldots, X_{k_m^i}\right\},\end{aligned} \tag{83}$$

with $k_1^i, k_2^i, \ldots k_m^i$ described in (9) *and $k_r^i = i$. Moreover, the finite-time H_∞ state feedback controller gains in* (5) *are given by $K_i = Y_i X_i^{-1}$.*

Proof. In (59) and (60), replacing A_i, A_{di}, and B_i with $(A_i + \Delta A_i)$, $(A_{di} + \Delta A_{di})$, and $(B_i + \Delta B_i)$, respectively, the following conditions are obtained:

$$\begin{aligned}\overline{\Pi}_{11i}^1 &= X_i A_i^T + X_i \Delta A_i^T + A_i X_i + \Delta A_i X_i + Y_i^T B_i^T + Y_i^T \Delta B_i^T \\ &\quad + B_i Y_i + \Delta B_i Y_i - \sum_{j\in L_k^i} \pi_{ij} R_i + \pi_{ii} X_i,\end{aligned}$$

$$\begin{aligned}\overline{\Pi}_{21i}^1 &= X_i A_i^T + X_i \Delta A_i^T + A_i X_i + \Delta A_i X_i + Y_i^T B_i^T \\ &\quad + Y_i^T \Delta B_i^T + B_i Y_i + \Delta B_i Y_i - \sum_{j\in L_k^i} \pi_{ij} R_i,\end{aligned}$$

$$\overline{\Pi}_{12i} = A_{di} + \Delta A_{di} + B_i K_{di} + \Delta B_i K_{di} + \varepsilon_{fi} X_i F_{12}^i. \tag{84}$$

Based on Lemma 11, there exist scalars ε_{1i}, ε_{2i}, ε_{3i}, and ε_{4i} such that

$$\begin{aligned}X_i \Delta A_i^T + \Delta A_i X_i &= X_i N_{1i}^T F_i^T(t) M_{1i}^T + M_{1i} F_i(t) N_{1i} X_i \\ &\le \varepsilon_{1i} M_{1i} M_{1i}^T + \varepsilon_{1i}^{-1} X_i N_{1i}^T N_{1i} X_i, \\ Y_i^T \Delta B_i^T + \Delta B_i Y_i &= Y_i^T N_{2i}^T F_i^T(t) M_{2i}^T + M_{2i} F_i(t) N_{2i} Y_i \\ &\le \varepsilon_{2i} M_{2i} M_{2i}^T + \varepsilon_{2i}^{-1} Y_i^T N_{2i}^T N_{2i} Y_i.\end{aligned}$$

$$\begin{aligned}&\begin{bmatrix} 0 & \Delta A_{di} & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & M_{3i} F_i(t) N_{3i} & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} M_{3i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} 0 & N_{3i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ N_{3i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} M_{3i}^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\le \varepsilon_{3i} \begin{bmatrix} M_{3i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} M_{3i}^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\quad + \varepsilon_{3i}^{-1} \begin{bmatrix} 0 \\ N_{3i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & N_{3i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\end{aligned}$$

$$
\begin{bmatrix}
0 & \Delta B_i K_{di} & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
= \begin{bmatrix}
0 & M_{2i}F_i(t)N_{2i}K_{di} & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
= \begin{bmatrix} M_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} 0 & N_{2i}K_{di} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 \\ K_{di}^T N_{2i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} M_{2i}^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\leq \varepsilon_{4i} \begin{bmatrix} M_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} M_{2i}^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
+ \varepsilon_{4i}^{-1} \begin{bmatrix} 0 \\ N_{3i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & N_{3i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{85}
$$

Applying Schur complement lemma to (85), (77) can be obtained. Similar to the above proving process, we can prove that (78) holds. Therefore, if LMIs (77)–(82) hold, the closed-loop system (6) is robust H_∞ finite-time bounded, and further system (18) can be stabilized via the state feedback controller (5).

The proof is complete. □

Remark 18. It should be pointed out that the conditions in Theorems 16 and 17 are not strict linear matrix inequalities such as conditions (20), (39), (44), (54), (59), (60), (77), and (78), due to the product of unknown scalars and matrices. An efficient way to solve this problem is to choose the appropriate values of the unknown scalars and then solve a set of LMIs for the fixed values of these parameters. For example, if α, ε_{fi} are fixed, then conditions (59) and (60) of Theorem 16 can be converted to LMIs conditions.

4. Numerical Examples

This section considers the following four-mode uncertain nonlinear Markovian jump systems with time delay as follows.

Mode 1

$$
A_1 = \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},
$$

$$
G_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix},
$$

$$
D_1 = E_1 = 0.1, \quad M_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},
$$

$$
N_{11} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix},
$$

$$
M_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad N_{21} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},
$$

$$
M_{31} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad N_{31} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}. \tag{86}
$$

Mode 2

$$
A_2 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},
$$

$$
G_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},
$$

$$
C_{d2} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \quad D_2 = E_2 = 0.2, \quad M_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},
$$

$$
N_{12} = \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad M_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad N_{22} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},
$$

$$
M_{32} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}, \quad N_{32} = \begin{bmatrix} 0.02 & 0.03 \\ 0 & 0.02 \end{bmatrix}. \tag{87}
$$

Mode 3

$$
A_3 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} 0.1 & -0.3 \\ -0.2 & 0.3 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix},
$$

$$
G_3 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 3 \end{bmatrix},
$$

Table 1

Case I	1	2	3	4	Case II	1	2	3	4
1	−1.2	0.3	0.5	0.4	1	?	0.3	?	0.4
2	0.2	−1	0.3	0.5	2	?	−1	0.3	?
3	0.8	0.1	−1.3	0.4	3	0.8	?	−1.3	?
4	0.2	0.1	0.5	−0.8	4	0.2	?	?	?
Case III	**1**	**2**	**3**	**4**	**Case VI**	**1**	**2**	**3**	**4**
1	−1.2	?	0.5	?	1	?	?	?	?
2	0.2	?	?	0.5	2	?	?	?	?
3	?	0.1	?	0.4	3	?	?	?	?
4	?	0.1	0.5	−0.8	4	?	?	?	?

Table 2

Case I	Completely known
Controller gains	$K_1 = \begin{bmatrix} -22.2335 & -19.0199 \end{bmatrix}$ $K_{d1} = \begin{bmatrix} -0.9097 & 0.9098 \end{bmatrix}$ $K_2 = \begin{bmatrix} -7.0199 & -2.3824 \end{bmatrix}$ $K_{d2} = \begin{bmatrix} -0.9490 & -0.4701 \end{bmatrix}$ $K_3 = \begin{bmatrix} -6.9528 & -9.8454 \end{bmatrix}$ $K_{d3} = \begin{bmatrix} 0.6466 & -0.3225 \end{bmatrix}$ $K_4 = \begin{bmatrix} -7.9573 & -2.4935 \end{bmatrix}$ $K_{d4} = \begin{bmatrix} -0.4998 & -0.2499 \end{bmatrix}$
Case II	Partially known
Controller gains	$K_1 = \begin{bmatrix} -22.5382 & -18.9685 \end{bmatrix}$ $K_{d1} = \begin{bmatrix} -0.8142 & 0.8142 \end{bmatrix}$ $K_2 = \begin{bmatrix} -7.9189 & -2.8820 \end{bmatrix}$ $K_{d2} = \begin{bmatrix} -0.9007 & -0.4391 \end{bmatrix}$ $K_3 = \begin{bmatrix} -6.9801 & -9.8455 \end{bmatrix}$ $K_{d3} = \begin{bmatrix} 0.6272 & -0.3112 \end{bmatrix}$ $K_4 = \begin{bmatrix} -8.1348 & -2.4949 \end{bmatrix}$ $K_{d4} = \begin{bmatrix} -0.4996 & -0.2498 \end{bmatrix}$
Case III	Partially known
Controller gains	$K_1 = \begin{bmatrix} -20.2412 & -18.1608 \end{bmatrix}$ $K_{d1} = \begin{bmatrix} -0.8925 & 0.9362 \end{bmatrix}$ $K_2 = \begin{bmatrix} -6.2413 & -2.7888 \end{bmatrix}$ $K_{d2} = \begin{bmatrix} -0.9283 & -0.5364 \end{bmatrix}$ $K_3 = \begin{bmatrix} -6.9091 & -9.8876 \end{bmatrix}$ $K_{d3} = \begin{bmatrix} 0.7272 & -0.2134 \end{bmatrix}$ $K_4 = \begin{bmatrix} -8.6329 & -2.4765 \end{bmatrix}$ $K_{d4} = \begin{bmatrix} -0.4998 & -0.2499 \end{bmatrix}$
Case VI	Completely unknown
Controller gains	$K_1 = \begin{bmatrix} -21.8153 & -18.7884 \end{bmatrix}$ $K_{d1} = \begin{bmatrix} -0.8143 & 0.8143 \end{bmatrix}$ $K_2 = \begin{bmatrix} -3.8757 & -0.3739 \end{bmatrix}$ $K_{d2} = \begin{bmatrix} -0.9008 & -0.4391 \end{bmatrix}$ $K_3 = \begin{bmatrix} -6.9153 & -9.8460 \end{bmatrix}$ $K_{d3} = \begin{bmatrix} 0.6272 & -0.3112 \end{bmatrix}$ $K_4 = \begin{bmatrix} -7.9143 & -2.4877 \end{bmatrix}$ $K_{d4} = \begin{bmatrix} -0.4996 & -0.2498 \end{bmatrix}$

$$C_{d3} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}, \qquad D_3 = E_3 = 0.3,$$
$$M_{13} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad N_{13} = \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.5 \end{bmatrix},$$
$$M_{23} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad N_{23} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$
$$M_{33} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \qquad N_{33} = \begin{bmatrix} 0.02 & 0.03 \\ 0 & 0.05 \end{bmatrix}. \tag{88}$$

Mode 4

$$A_4 = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}, \qquad A_{d4} = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}, \qquad B_4 = \begin{bmatrix} 4 \\ 1 \end{bmatrix},$$
$$G_4 = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}, \qquad C_4 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$C_{d4} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}, \qquad D_4 = E_4 = 0.4,$$
$$M_{14} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad N_{14} = \begin{bmatrix} 0.2 & 0.4 \\ 0 & 0.3 \end{bmatrix},$$
$$M_{24} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad N_{24} = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix},$$
$$M_{34} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad N_{34} = \begin{bmatrix} 0.02 & 0.04 \\ 0 & 0.03 \end{bmatrix},$$
$$H_1 = H_2 = H_3 = H_4 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad C_1 = 0.5,$$
$$C_2 = 4, \qquad d = 4, \qquad T = 1.2. \tag{89}$$

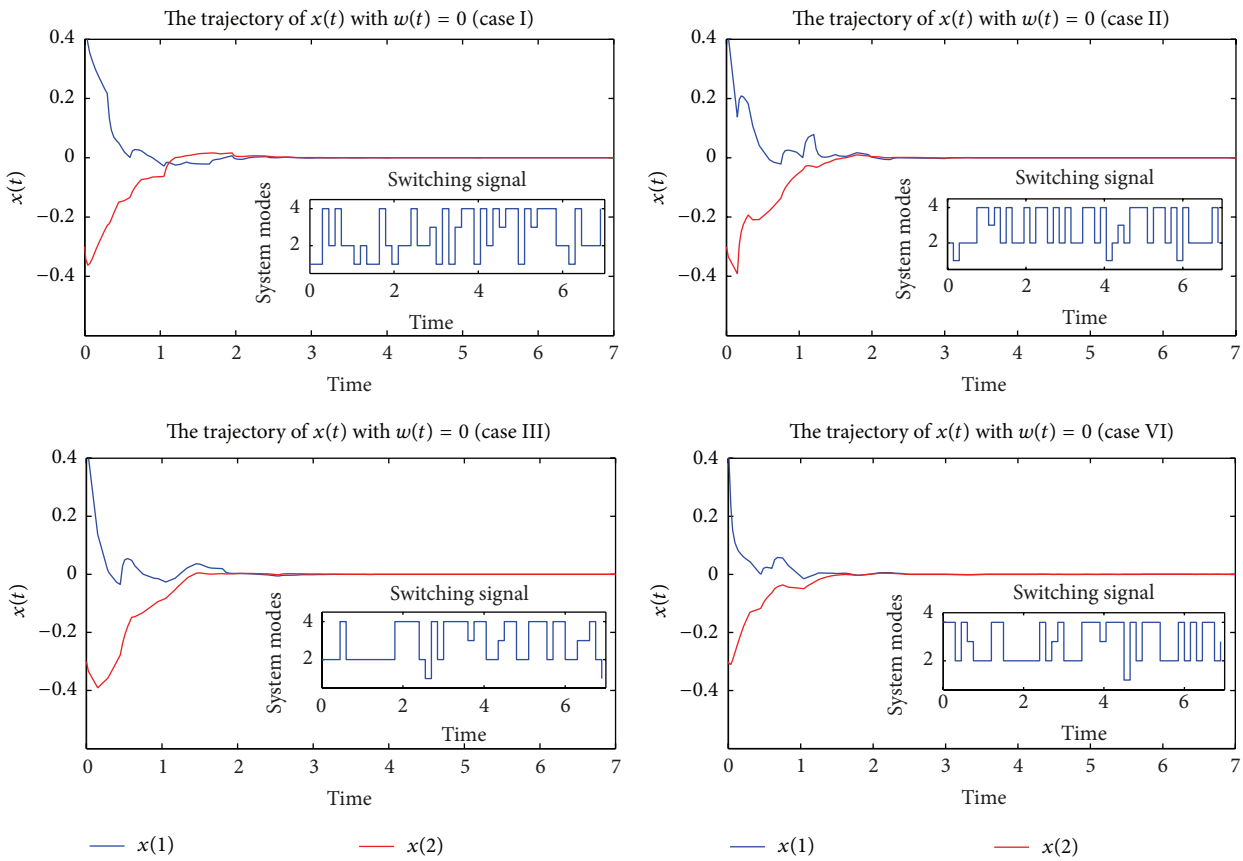

FIGURE 1: The trajectory of $x(t)$.

Choose $\tau = 1$, $\alpha = 0.5$, the exogenous disturbance $w(t) = [1/(5t+1) \quad 1/(t+1)]$, and the nonlinearities

$$f_1(x(t), x(t-\tau)) = \begin{bmatrix} 0.1\sin(x(t)) \\ 0.1\sin(x(t-\tau)) \end{bmatrix},$$

$$f_2(x(t), x(t-\tau)) = \begin{bmatrix} 0.1\sin(x(t-\tau)) \\ -0.15\sin(x(t)) \end{bmatrix},$$

$$f_3(x(t), x(t-\tau)) = \begin{bmatrix} 0.1\sin(x(t)) \\ 0.1\sin(x(t-\tau)) \end{bmatrix},$$

$$f_4(x(t), x(t-\tau)) = \begin{bmatrix} 0.1\sin(x(t-\tau)) \\ -0.15\sin(x(t)) \end{bmatrix},$$

$$F_{11}^1 = \begin{bmatrix} 1.1841 & 0.1704 \\ 0.1562 & 1.1370 \end{bmatrix}, \quad F_{22}^1 = \begin{bmatrix} 0.0606 & 0.1000 \\ 0.1000 & 0.3355 \end{bmatrix},$$

$$F_{11}^2 = \begin{bmatrix} 0.3299 & 0 \\ 0.7999 & 0.5000 \end{bmatrix}, \quad F_{22}^2 = \begin{bmatrix} 0.4000 & 0 \\ 0 & 0.2500 \end{bmatrix},$$

$$F_{11}^3 = \begin{bmatrix} 1.1841 & 0.1704 \\ 0.1562 & 1.1370 \end{bmatrix}, \quad F_{22}^3 = \begin{bmatrix} 0.0606 & 0.1000 \\ 0.1000 & 0.3355 \end{bmatrix},$$

$$F_{11}^4 = \begin{bmatrix} 0.3299 & 0 \\ 0.7999 & 0.5000 \end{bmatrix}, \quad F_{22}^4 = \begin{bmatrix} 0.4000 & 0 \\ 0 & 0.2500 \end{bmatrix},$$

$$F_{12}^1 = F_{12}^2 = F_{12}^3 = F_{12}^4 = 0. \tag{90}$$

The four cases for the transition probability matrix considered in Table 1.

Solving the LMIs (77)–(82) in Theorem 17, the robust finite-time H_∞ state feedback controller gains of K_i are given by Table 2.

Figures 1, 2, and 3 are presented. For every figure, the four different transition probability matrices cases are included, which can be better to demonstrate the effectiveness of the design method. Figure 1 depicts the trajectories of system state $x(t)$ and the corresponding switching signal. It can be seen that system (6) is robust finite-time stable, which implies that system (2) is robust finite-time H_∞ state feedback stabilizable via the designed state feedback controller (5). Figure 2 depicts the trajectories of system state $x(t)$ with $w(t) \neq 0$ and the corresponding switching signal. It can be seen that system (6) is robust finite-time bounded. The trajectory of the output $z(t)$ is described in Figure 3, which further shows the effectiveness of the designed controller (5).

5. Conclusions

In this paper, we have dealt with the problem of robust finite-time H_∞ control for a class of nonlinear Markovian jump systems with time delay under partially known transition probabilities. Based on the free-weighting matrices approach, all sufficient conditions have been firstly proposed to ensure

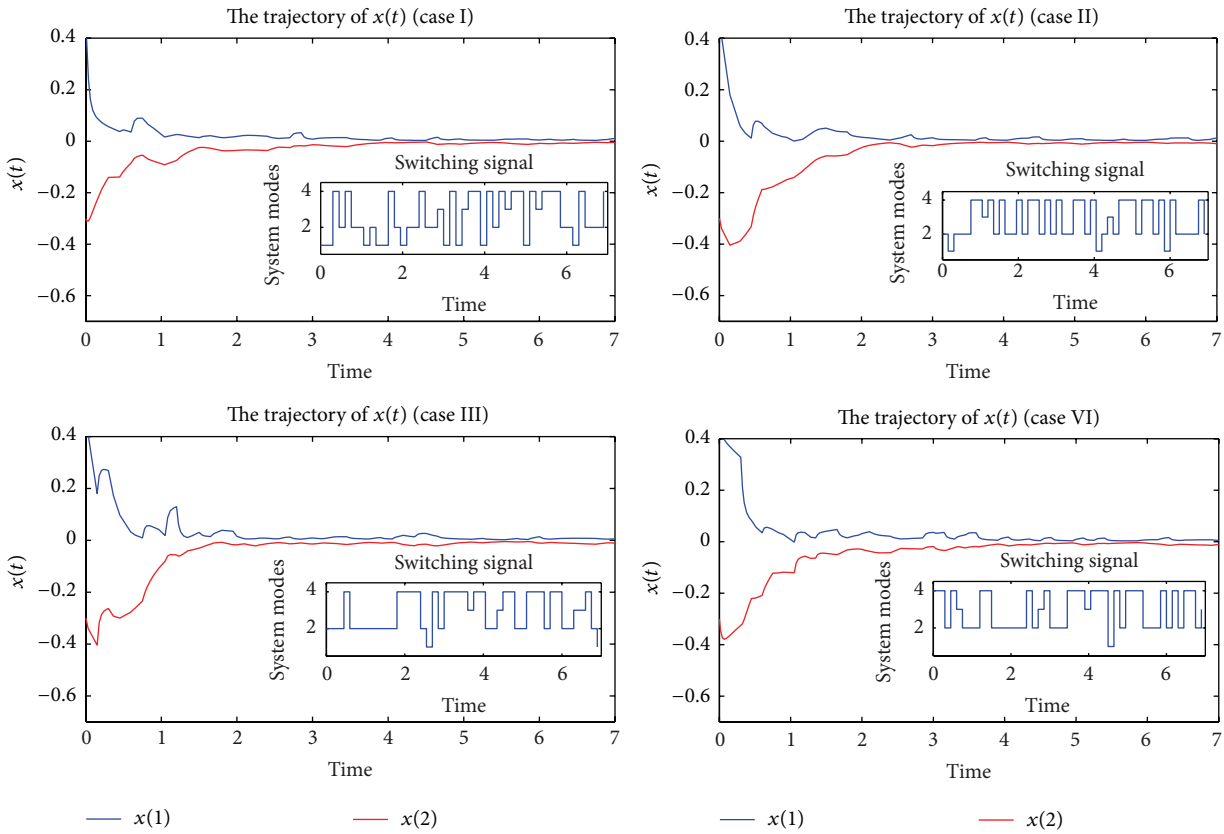

FIGURE 2: The trajectory of $x(t)$ with $w(t) \neq 0$.

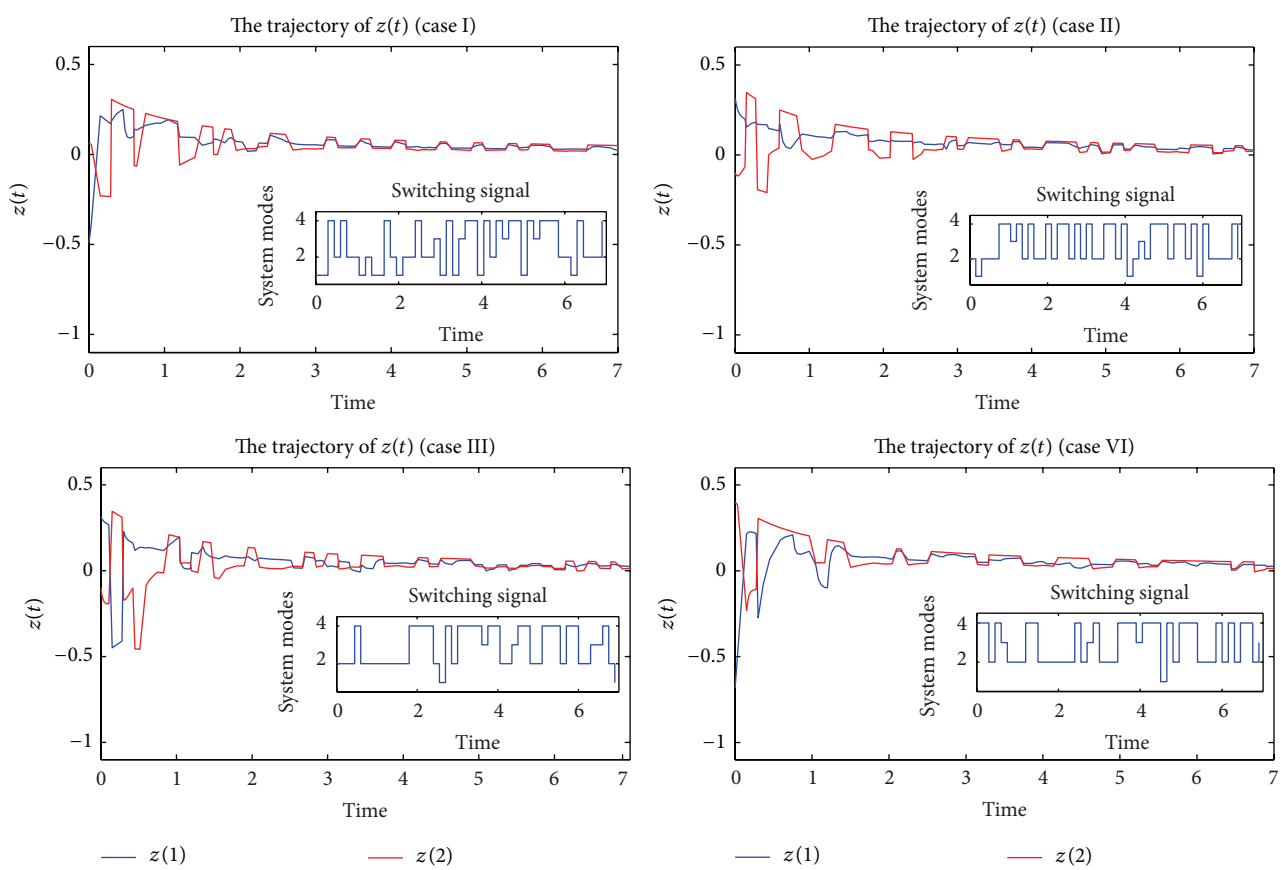

FIGURE 3: The trajectory of $z(t)$.

finite-time boundedness, H_∞ finite-time boundedness, and finite-time H_∞ state feedback stabilization for the given system. We have also designed a robust finite-time H_∞ state feedback controller, which guarantees the H_∞ finite-time boundedness of the closed-loop system. All the conditions have been presented in terms of strict linear matrix inequalities. Finally, a numerical example has been provided to demonstrate the effectiveness of all the results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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