

## Research Article

## Quantized State-Feedback Stabilization for Delayed Markovian Jump Linear Systems with Generally Incomplete Transition Rates

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This paper is concerned with the robust quantized state-feedback controller design problem for a class of continuous-time Markovian jump linear uncertain systems with general uncertain transition rates and input quantization. The uncertainties under consideration emerge in both system parameters and mode transition rates. This new uncertain model is more general than the existing ones and can be applicable to more practical situations because each transition rate can be completely unknown or only its estimate value is known. Based on linear matrix inequalities, the quantized state-feedback controller is formulated to ensure the closed-loop system is stable in mean square. Finally, a numerical example is presented to verify the validity of the developed theoretical results.

#### 1. Introduction

Markovian jump systems have been serving as popular tools for analyzing plants subjected to random abrupt changes, such as random component failures, abrupt environment changes, disturbance, and changes in the interconnections of subsystems; see [1-11] and the references therein. In practice, transition rates are difficult to precisely estimate the transition rates. Therefore, developing the analysis and synthesis method for MJS with uncertain transition rates is of great importance. Bounded uncertain transition rates (BUTRs) are introduced when the precise values of the transition rates are unknown, but their bounds (upper bounds and lower bounds) are known; see, for example, [12-14]. However, in some practical cases, to obtain the bound of every TR is difficult or even impossible. Zhang et al. [15-18] proposed another description for the uncertain TRs, which is partly unknown TRs (PUTRs). In this description, some of the TRs can be unknown. This model also simulates researchers' interests (see, e.g., [19-22]). In this PUTR model, every transition rate is either exactly known or completely unknown, which may be too restrictive in many practical situations. Therefore, Guo and Wang [23] proposed generally uncertain TRs to deal with a more practical situation where the transition rates can be completely unknown or only its bound is known.

On the other hand, in many modern engineering practices, information processing devices, such as analog-todigital and digital-to-analog converters, have been widely used and brought about some advantages, such as lower cost, reduced weight and power, and simple installation and maintenance. However, server deterioration of system performance or even system instability may also be induced. Signal quantization should be fully considered in such cases. Nowadays, the feedback stabilization problem is considered by utilizing dynamic quantizers [24–26] and static quantizers [27–36]. The stabilization problem for single-input discrete Markov jump linear systems via mode dependent quantized state feedback is addressed by Xiao et al. in [37], but the transition rates are assumed to be completely known. Ye et al. [38, 39] considered  $H_2$  control of Markov jump linear systems with unknown transition rates and input quantization. To the best of our knowledge, no result has been presented for control design of delayed continuous-time Markov jump linear uncertain systems with generally unknown transition rates and input signal quantization.

In this paper, robust quantized state-feedback stabilization for delayed Markovian jump linear systems with generally uncertain transition rates is addressed. In Section 2, the considered systems are formulated and the purposes of the paper are stated. In Section 3, the main results are derived via linear matrix inequalities. The structure of the controller consists of two parts. The nonlinear part is provided to eliminate the effect of input quantization. The linear part is obtained by solving LMIs to deal with model uncertainties and unknown transition rates. Section 4 concludes the paper.

*Notation.* In this paper,  $\mathbb{R}^n$  and  $\mathbb{R}^{n\times m}$  denote the *n*-dimensional Euclidean space and the set of all  $n \times m$  real matrices, respectively.  $\mathbb{N}^+$  represents the set of positive integers. The notation P > 0 ( $P \ge 0$ ) means that P is a real symmetric and positive-definite (semi-positive-definite) matrix. For notation  $(\Omega, \mathcal{F}, P), \Omega$  represents the space,  $\mathcal{F}$  is the  $\sigma$ -algebra of subsets of the sample space, and P is the probability measure on  $\mathcal{F}$ .  $E\{\cdot\}$  stands for the mathematical expectation. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

#### 2. Problem Formulation

Consider the following stochastic system with Markovian jump parameters, defined on a complete probability space  $(\Omega, \mathcal{F}, P)$ :

$$\dot{x}(t) = A(r_t) x(t) + A_d(r_t) x(t - \tau) + (B(r_t) + \Delta B(r_t)) q(u(t)), \quad t \ge 0, \quad (1) x_0 = x(0), \quad r_0 = r(0),$$

where  $x(t) \in \mathbb{R}^n$  is the system state. The mode jumping process  $\{r_t, t \ge 0\}$  is a right-continuous Markov process on the probability space taking values in a finite state space  $S = \{1, 2, ..., s\}$  with the mode transition probabilities:

$$Pr\left\{r_{t+\Delta} = j \mid r_t = i\right\} = \begin{cases} \pi_{ij}\Delta + o\left(\Delta\right), & \text{if } i \neq j, \\ 1 + \pi_{ij}\Delta + o\left(\Delta\right), & \text{if } i = j, \end{cases}$$
(2)

where  $\Delta > 0$ ,  $\lim_{\Delta \to 0} (o(\Delta)/\Delta) = 0$ , and  $\pi_{ij} \ge 0$  ( $i, j \in S, j \ne i$ ) is the TR from mode i at time t to mode j at time  $t + \Delta$ , and

$$\pi_{ii} = -\sum_{j=1, j \neq i}^{s} \pi_{ij}.$$
 (3)

for each  $i \in S$ .

The mode TR matrix  $\Pi \triangleq (\pi_{ij})$  is considered to be generally uncertain. For instance, the TR matrix for system (1) with *s* operation modes may be expressed as

$$\begin{bmatrix} \hat{\pi}_{11} + \Delta_{11} & ? & \hat{\pi}_{13} + \Delta_{13} & \cdots & ? \\ ? & ? & \hat{\pi}_{23} + \Delta_{23} & \cdots & \hat{\pi}_{2s} + \Delta_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \hat{\pi}_{s2} + \Delta_{s2} & ? & \cdots & \hat{\pi}_{ss} + \Delta_{ss} \end{bmatrix}, \quad (4)$$

where  $\hat{\pi}_{ij}$  and  $\Delta_{ij} \in [-\delta_{ij}, \delta_{ij}](\delta_{ij} \geq 0)$  represent the estimate value and estimate error of the uncertain TR  $\pi_{ij}$ , respectively, where  $\hat{\pi}_{ij}$  and  $\delta_{ij}$  are known. "?" represents the complete unknown TR, which means its estimate value  $\hat{\pi}_{ij}$  and estimate error bound are unknown. For notational clarity, for all  $i \in \mathbb{S}$ , the set  $U^i$  denotes  $U^i = U^i_k \cup U^i_{uk}$  with  $U^i_k \triangleq \{j :$  the estimate value of  $\lambda_{ij}$  is known for  $j \in \mathbb{S}\}$ ,  $U^i_{uk} \triangleq \{j :$  the estimate value of  $\lambda_{ij}$  is unknown for  $j \in \mathbb{S}\}$ . Moreover, if  $U^i_k \neq \emptyset$ , it is further described as  $U^i_k = \{k^i_1, k^i_2, \ldots, k^i_m\}$ , where  $k^i_m \in \mathbb{N}^+$  represent the *m*th bound-known element with the index  $k^i_m$  in the *i*th row of matrix  $\Pi$ . According to the properties of the TRs (e.g.,  $\lambda_{ij} \geq 0$  ( $\forall i, j \notin \mathbb{S}, i \neq j$ ) and  $\lambda_{ii} = -\sum_{j=1, j \neq i}^N \lambda_{ij}$ ), we assume that the known estimate values of the TRs are well defined. That is, the following assumptions hold.

Assumption 1. If 
$$U_k^i = \mathbb{S}$$
, then  $\widehat{\lambda}_{ij} - \delta_{ij} \ge 0$  ( $\forall j \in \mathbb{S}, j \neq i$ ),  $\widehat{\lambda}_{ii} = -\sum_{j=1, j \neq i}^N \widehat{\lambda}_{ij} \le 0$ , and  $\delta_{ii} = \sum_{j=1, j \neq i}^N \delta_{ij} > 0$ .

Assumption 2. If  $U_k^i \neq \mathbb{S}$ , and  $i \in U_k^i$ , then  $\hat{\lambda}_{ij} - \delta_{ij} \ge 0$  ( $\forall j \in U_k^i, j \neq i$ ),  $\hat{\lambda}_{ii} + \delta_{ii} \le 0$  and  $\sum_{i \in U_k^i} \hat{\lambda}_{ij} \le 0$ .

Assumption 3. If  $U_k^i \neq \mathbb{S}$  and  $i \notin U_k^i$ , then  $\widehat{\lambda}_{ij} - \delta_{ij} \ge 0$  ( $\forall j \in U_k^i$ ).

*Remark 4.* The above assumption is reasonable, since it is the direct result from the properties of the TRs

$$\left(\text{e.g., } \pi_{ij} \ge 0 \left(\forall i, j \notin \mathbb{S}, i \neq j\right), \pi_{ii} = -\sum_{j=1, j \neq i}^{s} \pi_{ij}\right).$$
(5)

The above description about uncertain TRs is more general than either the BUTR or PUTR models. To show this, we rewrite the two uncertain models as follows: BUTR model (see [12–14]):

$$\begin{bmatrix} \hat{\pi}_{11} + \Delta_{11} & \hat{\pi}_{12} + \Delta_{12} & \cdots & \hat{\pi}_{1s} + \Delta_{1s} \\ \hat{\pi}_{21} + \Delta_{21} & \hat{\pi}_{22} + \Delta_{22} & \cdots & \hat{\pi}_{2s} + \Delta_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{s1} + \Delta_{s1} & \hat{\pi}_{s2} + \Delta_{s2} & \cdots & \hat{\pi}_{ss} + \Delta_{ss} \end{bmatrix}$$
(6)

with  $\hat{\pi}_{ij} - \delta_{ij} \ge 0$  ( $\forall j \in \mathbb{S}, j \neq i$ ),  $\hat{\pi}_{ii} = -\sum_{j=1, j \neq i}^{s} \hat{\pi}_{ij}$ , and  $\delta_{ii} = -\sum_{j=1, j \neq i}^{s} \delta_{ij}$ . PUTR model (see [15–22]):

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & \cdots & ? \\ ? & ? & \pi_{23} & \cdots & \pi_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \pi_{s2} & ? & \cdots & \pi_{ss} \end{bmatrix}.$$
 (7)

Obviously, if  $U_k^i \emptyset$ ,  $\forall i \in \mathbb{S}$ , the GUTR model (4) will reduce to the BUTR model (6); if  $\delta_{ij} = 0$ ,  $\forall i \in \mathbb{S}$ ,  $\forall j \in U_k^i$ , the GUTR model (4) will reduce to the PUTR model (7). The GUTR model (4) is more general than the other two models; therefore, it is more practicable. For convenience, the system matrices  $A_i \triangleq A$  ( $r_t = i$ ),  $A_{di} \triangleq A_d$  ( $r_t = i$ ), and  $B_i \triangleq B$  ( $r_t = i$ ),  $\Delta B_i \triangleq \Delta B$  ( $r_t = i$ ),  $i \in \mathbb{S}$ , are known matrix functions of the Markovian process. Then system (1) can be described by

$$\dot{x}(t) = A_{i}x(t) + A_{di}x(t - \tau) + (B_{i} + \Delta B_{i})q(u(t)), \quad t \ge 0,$$
(8)  
$$x_{0} = x(0), \qquad r_{0} = r(0).$$

The following assumptions are assumed to be valid.

Assumption 5.  $\Delta B_i = B_i M_i \Xi_i(t) F_i$  and  $|M_i \Xi_i(t) F_i|_{\infty} \le \psi_i$ , where  $M_i$  and  $F_i$  are known constant matrices with appropriate dimensions,  $\Xi_i(t)$  is time-varying uncertain matrix satisfying  $\Xi_i(t)\Xi_i^T(t) \le I$ , and parameter  $\psi_i$  satisfies  $0 \le \psi_i < 1$ .

In addition, the quantizer  $q(\cdot)$  is defined by an operator function round ( $\cdot$ ) which rounds to the nearest integer; that is,

$$q(u(t)) = \mu \cdot \operatorname{round}\left(\frac{u(t)}{\mu}\right),\tag{9}$$

where  $\mu(> 0)$  is called a quantizing level of the quantizer. In computer-based control systems, the value of  $\mu$  depends on the sampling accuracy and is known a priori.  $q(\cdot)$  is the uniform quantizer with the fixed level  $\mu$ . Define  $e_{\mu} = q(u(t)) - u(t)$ ; since each component of  $e_{\mu}$  is bounded by the half of the quantizing level  $\mu$ , we have  $|e_{\mu}|_{\infty} \leq \mu/2$ .

The objective of this paper is to design a state-feedback control law

$$u(t) = K_i x + u_{ic}, \qquad K_i = K(r_t), \quad \text{when } r_t = i \quad (10)$$

such that the resulting closed-loop system is stochastically stable. The nonlinear part of the controller  $u_{ic}$  is designed against the effect of signal quantization, and the linear part  $K_i x$  is proposed to deal with model uncertainties and unknown transition rates.

**Lemma 6** (Petersen, 1987). Given a symmetric matrix  $\Pi$  and matrices M, N with appropriate dimensions, then  $\Pi + MF(t)N + N^TF^T(t)M^T < 0$  for all F(t) satisfying  $F^T(t)F(t) \le I$ , if and only if there exists a scalar  $\varepsilon > 0$  such that the following inequality holds:

$$\Pi + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0.$$
<sup>(11)</sup>

**Lemma 7.** Given any real number  $\varepsilon$  and any matrix Q, the matrix inequality

$$\varepsilon \left( Q + Q^T \right) \le \varepsilon^2 T + Q T^{-1} Q^T \tag{12}$$

*holds for any matrix* T > 0*.* 

#### 3. Stochastic Stability Analysis

The goal of this section is to develop an analysis result of stability for system (1) with general uncertain TRs.

**Theorem 8.** Consider that uncertain Markovian jump system (8) with a GUTR matrix (4) is stochastically stable if there exist matrices  $P_i > 0$  ( $i \in S$ ),  $T_{ij} > 0$  ( $i \notin U_k^i, j \in U_k^i$ ),  $V_{ijl} >$ 0 ( $i, j \in U_k^i, U_{uk}^i \neq \emptyset, l \in U_{uk}^i$ ),  $W_{ij} > 0$  ( $i, j \in U_k^i, U_{uk}^i = \emptyset$ ), and Q > 0 such that the following LMIs are feasible for i =1,2,...,s. If  $i \notin U_k^i$ ,

$$\begin{bmatrix} \Gamma_{11} & P_i A_{di} & P_{ik_1^i} - P_i & \cdots & P_{ik_m^i} - P_i \\ * & -Q & 0 & \cdots & 0 \\ * & * & -T_{ik_1^i} & \cdots & 0 \\ * & * & * & \ddots & \vdots \\ * & * & * & * & -T_{ik_m^i} \end{bmatrix} < 0$$
(13)  
$$P_i - P_j \ge 0 , \quad \forall j \in U_{uk}^i, \ j \ne i.$$
(14)

If  $i \in U_k^i$  and  $U_{uk}^i \neq \emptyset$ , for one  $l \in U_{uk}^i$ ,

$$\begin{bmatrix} \Theta_{11} & P_i A_{di} & P_{k_1^i} - P_l & \cdots & P_{k_m^i} - P_l \\ * & -Q & 0 & \cdots & 0 \\ * & * & -V_{ik_1^i l} & \cdots & 0 \\ * & * & * & \ddots & \vdots \\ * & * & * & * & -V_{ik_m^i} \end{bmatrix} < 0.$$
(15)

If 
$$i \in U_k^i$$
 and  $U_{uk}^i = \emptyset$ 

$$\begin{bmatrix} \Psi_{11} & P_i A_{di} & P_1 - P_i & \cdots & P_{i-1} - P_i & P_{i+1} - P_i & \cdots & P_s - P_i \\ * & -Q & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & -W_{i1} & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & -W_{i(i-1)} & 0 & \vdots & \vdots \\ * & * & * & * & * & * & -W_{i(i+1)} & \vdots & \vdots \\ * & * & * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & * & * & * & -W_{is} \end{bmatrix} < 0,$$
(16)

where

$$\begin{split} \Gamma_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + \sigma_{i} P_{i} B_{i} M_{i} M_{i}^{T} B_{i}^{T} P_{i} \\ &+ \frac{1}{\sigma_{i}} K_{i}^{T} F_{i}^{T} F_{i} K_{i} + \sum_{j \in U_{k}^{i}} \hat{\pi}_{ij} \left( P_{j} - P_{i} \right) + \sum_{j \in U_{k}^{i}} \frac{\delta_{ij}^{2}}{4} T_{ij} + Q \\ \Theta_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} \\ &+ \sigma_{i} P_{i} B_{i} M_{i} M_{i}^{T} B_{i}^{T} P_{i} + \frac{1}{\sigma_{i}} K_{i}^{T} F_{i}^{T} F_{i} K_{i} \\ &+ \sum_{j \in U_{k}^{i}} \hat{\pi}_{ij} \left( P_{j} - P_{i} \right) + \sum_{j \in U_{k}^{i}} \frac{\delta_{ij}^{2}}{4} V_{ijl} + Q \\ \Psi_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} \\ &+ \sigma_{i} P_{i} B_{i} M_{i} M_{i}^{T} B_{i}^{T} P_{i} + \frac{1}{\sigma_{i}} K_{i}^{T} F_{i}^{T} F_{i} K_{i} \\ &+ \sum_{j \in \mathbb{S}, j \neq i} \hat{\pi}_{ij} \left( P_{j} - P_{i} \right) + \sum_{j \in \mathbb{S}, j \neq i} \frac{\delta_{ij}^{2}}{4} W_{ij} + Q. \end{split}$$

$$(17)$$

Then the controller designed as

$$u(t) = K_i x + u_{ic} \tag{18}$$

can drive the state trajectory to the origin asymptotically, where  $K_i = Y_i X_i^{-1}, P_i = X_i^{-1}$ , and  $u_{ic} = (1 + \psi)u/(2(1 - \psi)) \operatorname{sign}(x^T P_i B_i)$ .

*Proof.* Take the Lyapunov function candidate  $V = x^T P_i x + \int_{t-\tau}^t x^T(\alpha) Qx(\alpha) d\alpha$ ; then along the system trajectory of plant (8), the weak infinitesimal operator  $\mathfrak{F}_a^x[\cdot]$  of the process  $x(t), r_t, t \ge 0$ , for plant (8) at the point t, x, i is as follows:

$$\begin{split} \mathfrak{J}_{a}^{x}\left[V\right] &= \dot{x}^{T}P_{i}x + x^{T}P_{i}\dot{x} + x^{T}\sum_{j=1}^{\mathbb{N}}\lambda_{ij}P_{j}x\\ &= \left[A_{i}x + A_{di}x\left(t - \tau\right)\right.\\ &\quad \left. + \left(B_{i} + \Delta B_{i}\right)\left(K_{i}x + u_{ic} + e_{\mu}\right)\right]^{T}P_{i}x\\ &\quad \left. + x^{T}P_{i}\left[A_{i}x + A_{di}x\left(t - \tau\right)\right.\\ &\quad \left. + \left(B_{i} + \Delta B_{i}\right)\left(K_{i}x + u_{ic} + e_{\mu}\right)\right]\\ &\quad \left. + x^{T}\sum_{j=1}^{\mathbb{N}}\lambda_{ij}P_{j}x\right.\\ &= x^{T}\left[A_{i}^{T}P_{i} + P_{i}A_{i} + P_{i}\left(B_{i} + \Delta B_{i}\right)K_{i}\right] \end{split}$$

$$+ K_{i}^{T} (B_{i} + \Delta B_{i})^{T} P_{i} + Q ] x$$

$$+ x^{T} (t - \tau) A_{di}^{T} P_{i} x + x^{T} P_{i} A_{di} x (t - \tau)$$

$$- x^{T} (t - \tau) Q x (t - \tau)$$

$$+ 2x^{T} P_{i} (B_{i} + \Delta B_{i}) (u_{ic} + e_{\mu}) + x^{T} \sum_{j=1}^{\mathbb{N}} \lambda_{ij} P_{j} x.$$
(19)

According to Assumption 2,  $u_{ic}$  in Theorem 8, and Lemma 6, one can obtain that

$$2x^{T}P_{i}(B_{i} + \Delta B_{i})(u_{ic} + e_{\mu})$$

$$= 2x^{T}P_{i}B_{i}u_{ic} + 2x^{T}P_{i}\Delta B_{i}(u_{ic} + e_{\mu}) + 2x^{T}P_{i}B_{i}e_{\mu}$$

$$= 2x^{T}P_{i}B_{i}u_{ic} + 2x^{T}P_{i}B_{i}M_{i}\Xi_{i}(t)F_{i}(u_{ic} + e_{\mu}) + 2x^{T}P_{i}B_{i}e_{\mu}$$

$$\leq 2x^{T}P_{i}B_{i}u_{ic} + 2|x^{T}P_{i}B_{i}|_{1}|M_{i}\Xi_{i}(t)F_{i}|_{\infty}\left(|u_{ic}|_{\infty} + \frac{\mu}{2}\right)$$

$$+ 2|x^{T}P_{i}B_{i}|_{1}\frac{\mu}{2}$$

$$= 2x^{T}P_{i}B_{i}u_{ic} + 2|x^{T}P_{i}B_{i}B_{i}|_{1}\psi\left(|u_{ic}|_{\infty} + \frac{\mu}{2}\right)$$

$$+ |x^{T}P_{i}B_{i}|_{1}\mu$$

$$\leq 2x^{T}P_{i}B_{i}u_{ic} + (1 + \psi)|x^{T}P_{i}B_{i}|_{1}\mu$$

$$+ 2\psi|x^{T}P_{i}B_{i}|_{1}|u_{ic}|_{\infty}$$

$$= -\frac{(1 + \psi)\mu}{1 - \psi}|x^{T}P_{i}B_{i}|_{1} + (1 + \psi)|x^{T}P_{i}B_{i}|_{1}\mu$$

$$+ \frac{\psi(1 + \psi)\mu}{1 - \psi}|x^{T}P_{i}B_{i}|_{1} = 0.$$
(20)

It follows from (19) and (20) that

$$\mathfrak{J}_{a}^{x}[V] \leq x^{T} \left[ A_{i}^{T}P_{i} + P_{i}A_{i} + P_{i}\left(B_{i} + \Delta B_{i}\right)K_{i} + K_{i}^{T}\left(B_{i} + \Delta B_{i}\right)^{T}P_{i} + Q \right] x + x^{T}\left(t - \tau\right)A_{di}^{T}P_{i}x + x^{T}P_{i}A_{di}x\left(t - \tau\right) \qquad (21)$$

$$- x^{T}\left(t - \tau\right)Qx\left(t - \tau\right) + x^{T}\sum_{j=1}^{\mathbb{N}}\lambda_{ij}P_{j}x.$$
Since  $\Delta B_{i} = B_{i}M_{i}\Xi_{i}(t)E_{i}$ , we introduce

$$\Phi_{i} \triangleq x^{T} \left[ A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + P_{i} B_{i} M_{i} \Xi_{i} (t) F_{i} K_{i} + K_{i}^{T} F_{i}^{T} \Xi_{i}^{T} (t) M_{i}^{T} B_{i}^{T} P_{i} + Q \right] x$$

$$+ x^{T} (t - \tau) A_{di}^{T} P_{i} x + x^{T} P_{i} A_{di} x (t - \tau)$$

$$- x^{T} (t - \tau) Qx (t - \tau). \qquad (22)$$

Then, the above inequality can be rewritten as

$$\mathfrak{F}_{a}^{x}\left[V\right] \leq \Phi_{i} + x^{T} \left[\sum_{j=1}^{\mathbb{N}} \lambda_{ij} P_{j}\right] x.$$
(23)

Three cases should be considered.

*Case I*  $(i \notin U_k^i)$ . In this case, note that  $\sum_{j \in U_{uk}^i, j \neq i} \pi_{ij} = -\pi_{ii} - \sum_{j \in U_k^i} \pi_{ij}$  and  $\pi_{ij} \ge 0, j \in U_{uk}^i, j \neq i$ ; then from (23), we have

$$\begin{aligned} \mathfrak{F}_{a}^{x}\left[V\right] &\leq \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \pi_{ij}P_{j} + \pi_{ii}P_{i} + \sum_{j \in U_{uk}^{i}, j \neq i} \pi_{ij}P_{i}\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \pi_{ij}P_{j} + \pi_{ii}P_{i} + \left(-\pi_{ii} - \sum_{j \in U_{k}^{i}} \pi_{ij}\right)P_{i}\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \pi_{ij}\left(P_{j} - P_{i}\right)\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \left(\widehat{\pi}_{ij} + \Delta_{ij}\right)\left(P_{j} - P_{i}\right)\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij}\left(P_{j} - P_{i}\right) + \sum_{j \in U_{k}^{i}} \Delta_{ij}\left(P_{j} - P_{i}\right)\right] x. \end{aligned}$$

$$(24)$$

There, note that  $\pi_{ij} = \hat{\pi}_{ij} + \Delta_{ij}$  for  $j \in U_k^i$ . On the other hand, in view of Lemma 7, we have

$$\sum_{j \in U_{k}^{i}} \Delta_{ij} \left( P_{j} - P_{i} \right)$$

$$= \sum_{j \in U_{k}^{i}} \left[ \frac{1}{2} \Delta_{ij} \left( P_{j} - P_{i} \right) + \frac{1}{2} \Delta_{ij} \left( P_{j} - P_{i} \right) \right]$$

$$\leq \sum_{j \in U_{k}^{i}} \left[ \left( \frac{1}{2} \Delta_{ij} \right)^{2} T_{ij} + \left( P_{j} - P_{i} \right) T_{ij}^{-1} \left( P_{j} - P_{i} \right) \right]$$

$$\leq \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} T_{ij} + \left( P_{j} - P_{i} \right) T_{ij}^{-1} \left( P_{j} - P_{i} \right) \right].$$
(25)

From (24) and (25), we have

$$\begin{aligned} \mathfrak{F}_{a}^{x}\left[V\right] &\leq \Phi_{i} + x^{T} \\ &\times \left\{ \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left(P_{j} - P_{i}\right) \right. \\ &\left. + \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} T_{ij} + \left(P_{j} - P_{i}\right) T_{ij}^{-1} \left(P_{j} - P_{i}\right) \right] \right\} x. \end{aligned}$$

$$(26)$$

Hence,  $\mathfrak{F}_a^x[V] < 0$  holds if

$$\mathcal{H} = \begin{bmatrix} \mathcal{A}_{11} & P_i A_{di} \\ * & -Q \end{bmatrix} < 0, \tag{27}$$

where

$$\mathcal{A}_{11} = A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + P_{i} B_{i} M_{i} \Xi_{i} (t) F_{i} K_{i} + K_{i}^{T} F_{i}^{T} \Xi_{i}^{T} (t) M_{i}^{T} B_{i}^{T} P_{i} + Q + \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{i} \right) + \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} T_{ij} + \left( P_{j} - P_{i} \right) T_{ij}^{-1} \left( P_{j} - P_{i} \right) \right].$$
(28)

Let

$$\overline{\mathscr{A}}_{11} = A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + Q 
+ \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{i} \right) 
+ \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} T_{ij} + \left( P_{j} - P_{i} \right) T_{ij}^{-1} \left( P_{j} - P_{i} \right) \right];$$
(29)

then we have

$$\begin{aligned} \mathscr{H} &= \begin{bmatrix} \overline{\mathscr{A}}_{11} & P_i A_{di} \\ * & -Q \end{bmatrix} + \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix} \Xi_i (t) \begin{bmatrix} F_i K_i & 0 \end{bmatrix} \\ &+ \begin{bmatrix} F_i K_i & 0 \end{bmatrix}^T \Xi_i^T (t) \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix}^T < 0, \end{aligned}$$
(30)

which is equivalent to (13) by Schur complement.

*Case II*  $(i \in U_k^i \text{ and } U_{uk}^i \neq \emptyset)$ . There must be an  $l \in U_{uk}^i$  such that  $P(l) - P(j) \ge 0, \forall j \in U_{uk}^i$ . We define

$$\mathfrak{F}_{a}^{x}[V] \leq \Phi_{i} + x^{T} \left[ \sum_{j \in U_{k}^{i}} \lambda_{ij} P(j) + \sum_{j \in U_{uk}^{i}} \lambda_{ij} P(l) \right] x$$
$$= \Phi_{i} + x^{T} \left[ \sum_{j \in U_{k}^{i}} \pi_{ij} P_{j} - \left( \sum_{j \in U_{k}^{i}} \pi_{ij} \right) P_{l} \right] x$$

$$= \Phi_{i} + x^{T} \left[ \sum_{j \in U_{k}^{i}} \pi_{ij} \left( P_{j} - P_{l} \right) \right] x$$

$$= \Phi_{i} + x^{T} \left[ \sum_{j \in U_{k}^{i}} \left( \widehat{\pi}_{ij} + \Delta_{ij} \right) \left( P_{j} - P_{l} \right) \right] x$$

$$= \Phi_{i} + x^{T} \left[ \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{l} \right) + \sum_{j \in U_{k}^{i}} \Delta_{ij} \left( P_{j} - P_{l} \right) \right] x.$$
(31)

By using Lemma 7 again, we have

$$\begin{split} \sum_{j \in U_k^i} \Delta_{ij} \left( P_j - P_l \right) \\ &= \sum_{j \in U_k^i} \left[ \frac{1}{2} \Delta_{ij} \left( P_j - P_l \right) + \frac{1}{2} \Delta_{ij} \left( P_j - P_l \right) \right] \\ &\leq \sum_{j \in U_k^i} \left[ \left( \frac{1}{2} \Delta_{ij} \right)^2 V_{ijl} + \left( P_j - P_l \right) V_{ijl}^{-1} \left( P_j - P_l \right) \right] \\ &\leq \sum_{j \in U_k^i} \left[ \frac{\delta_{ij}^2}{4} V_{ijl} + \left( P_j - P_l \right) V_{ijl}^{-1} \left( P_j - P_l \right) \right]. \end{split}$$
(32)

From (31) and (32), we have

$$\begin{aligned} \mathfrak{F}_{a}^{x}\left[V\right] &\leq \Phi_{i} + x^{T} \\ &\times \left\{ \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left(P_{j} - P_{l}\right) \right. \\ &\left. + \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} V_{ijl} + \left(P_{j} - P_{l}\right) V_{ijl}^{-1} \left(P_{j} - P_{l}\right) \right] \right\} x. \end{aligned}$$

$$(33)$$

Hence,  $\mathfrak{J}_a^x[V] < 0$  holds if

$$\mathfrak{A} = \begin{bmatrix} \mathfrak{A}_{11} & P_i A_{di} \\ \ast & -Q \end{bmatrix} < 0, \tag{34}$$

where

$$\begin{aligned} \mathfrak{A}_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + P_{i} B_{i} M_{i} \Xi_{i} (t) F_{i} K_{i} \\ &+ K_{i}^{T} F_{i}^{T} \Xi_{i}^{T} (t) M_{i}^{T} B_{i}^{T} P_{i} + Q + \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{l} \right) \\ &+ \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} V_{ijl} + \left( P_{j} - P_{l} \right) V_{ijl}^{-1} \left( P_{j} - P_{l} \right) \right]. \end{aligned}$$

$$(35)$$

Let

$$\begin{aligned} \overline{\mathfrak{A}}_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + Q \\ &+ \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{l} \right) \\ &+ \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} V_{ijl} + \left( P_{j} - P_{l} \right) V_{ijl}^{-1} \left( P_{j} - P_{l} \right) \right]; \end{aligned}$$
(36)

then we have

$$\begin{aligned} \mathfrak{A} &= \begin{bmatrix} \overline{\mathfrak{A}}_{11} & P_i A_{di} \\ * & -Q \end{bmatrix} + \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix} \Xi_i (t) \begin{bmatrix} F_i K_i & 0 \end{bmatrix} \\ &+ \begin{bmatrix} F_i K_i & 0 \end{bmatrix}^T \Xi_i^T (t) \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix}^T < 0, \end{aligned}$$
(37)

which is equivalent to (14) by Schur complement.

Case III ( $i \in U_k^i$  and  $U_{uk}^i = \emptyset$ ). In this case,

$$\begin{aligned} \mathfrak{F}_{a}^{x}\left[V\right] &\leq \Phi_{i} + x^{T} \left[\sum_{j=1, j \neq i}^{N} \lambda_{ij} P\left(j\right) + \lambda_{ii} P\left(i\right)\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \pi_{ij} P_{j} - \left(\sum_{j \in U_{k}^{i}} \pi_{ij}\right) P_{i}\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \pi_{ij}\left(P_{j} - P_{i}\right)\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \left(\widehat{\pi}_{ij} + \Delta_{ij}\right)\left(P_{j} - P_{i}\right)\right] x \\ &= \Phi_{i} + x^{T} \left[\sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij}\left(P_{j} - P_{i}\right) + \sum_{j \in U_{k}^{i}} \Delta_{ij}\left(P_{j} - P_{i}\right)\right] x. \end{aligned}$$

$$(38)$$

In view of Lemma 7, we have

$$\sum_{j=1,j\neq i}^{s} \Delta_{ij} \left( P_{j} - P_{i} \right)$$

$$= \sum_{j=1,j\neq i}^{s} \left[ \frac{1}{2} \Delta_{ij} \left( P_{j} - P_{i} \right) + \frac{1}{2} \Delta_{ij} \left( P_{j} - P_{i} \right) \right]$$

$$\leq \sum_{j=1,j\neq i}^{s} \left[ \left( \frac{1}{2} \Delta_{ij} \right)^{2} W_{ij} + \left( P_{j} - P_{i} \right) W_{ij}^{-1} \left( P_{j} - P_{i} \right) \right]$$

$$\leq \sum_{j=1,j\neq i}^{s} \left[ \frac{\delta_{ij}^{2}}{4} W_{ij} + \left( P_{j} - P_{i} \right) W_{ij}^{-1} \left( P_{j} - P_{i} \right) \right].$$
(39)

From (38) and (39), we have

$$\begin{aligned} \mathfrak{F}_{a}^{x}\left[V\right] &\leq \Phi_{i} + x^{T} \\ &\times \left\{ \sum_{j=1, j \neq i}^{s} \widehat{\pi}_{ij} \left(P_{j} - P_{i}\right) \right. \\ &+ \sum_{j=1, j \neq i}^{s} \left[ \frac{\delta_{ij}^{2}}{4} W_{ij} + \left(P_{j} - P_{i}\right) \right. \\ &\left. \times W_{ij}^{-1} \left(P_{j} - P_{i}\right) \right] \right\} x. \end{aligned}$$

$$(40)$$

Hence,  $\mathfrak{F}_a^x[V] < 0$  holds if

$$\mathscr{R} = \begin{bmatrix} \mathscr{R}_{11} & P_i A_{di} \\ * & -Q \end{bmatrix} < 0, \tag{41}$$

where

$$\begin{aligned} \mathscr{R}_{11} &= A_{i}^{T} P_{i} + P_{i} A_{i} + K_{i}^{T} B_{i}^{T} P_{i} + P_{i} B_{i} K_{i} + P_{i} B_{i} M_{i} \Xi_{i} (t) F_{i} K_{i} \\ &+ K_{i}^{T} F_{i}^{T} \Xi_{i}^{T} (t) M_{i}^{T} B_{i}^{T} P_{i} + Q + \sum_{j \in U_{k}^{i}} \widehat{\pi}_{ij} \left( P_{j} - P_{i} \right) \\ &+ \sum_{j \in U_{k}^{i}} \left[ \frac{\delta_{ij}^{2}}{4} W_{ij} + \left( P_{j} - P_{i} \right) W_{ij}^{-1} \left( P_{j} - P_{i} \right) \right]. \end{aligned}$$

$$(42)$$

Let

$$\begin{aligned} \overline{\mathscr{R}}_{11} &= A_i^T P_i + P_i A_i + K_i^T B_i^T P_i + P_i B_i K_i + Q \\ &+ \sum_{j \in U_k^i} \widehat{\pi}_{ij} \left( P_j - P_i \right) \\ &+ \sum_{j \in U_k^i} \left[ \frac{\delta_{ij}^2}{4} W_{ij} + \left( P_j - P_i \right) W_{ij}^{-1} \left( P_j - P_i \right) \right]; \end{aligned}$$

$$(43)$$

then we have

$$\begin{aligned} \mathscr{R} &= \begin{bmatrix} \overline{\mathscr{R}}_{11} & P_i A_{di} \\ * & -Q \end{bmatrix} + \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix} \Xi_i (t) \begin{bmatrix} F_i K_i & 0 \end{bmatrix} \\ &+ \begin{bmatrix} F_i K_i & 0 \end{bmatrix}^T \Xi_i^T (t) \begin{bmatrix} P_i B_i M_i \\ 0 \end{bmatrix}^T < 0, \end{aligned}$$
(44)

which is equivalent to (15) by Schur complement. The proof is completed.  $\hfill \Box$ 

*Remark* 9. As mentioned in Section 2, if,  $U_{uk}^i = \emptyset \ \forall i \in \mathbb{S}$ , then GUTR matrix reduces to BUTR one. Similarly, if,  $\delta_{ij} = 0 \ \forall j \in U_k^i, \forall i \in \mathbb{S}$ , GUTR matrix reduces to PUTR one. Therefore, Theorem 8 can also be applicable to the MJSs with BUTRs or PUTRs. Because the BUTR methods require the estimate of every rate to be known, such methods cannot be

#### 4. Conclusions

The stability problems for a class of Markovian jump linear systems with generally uncertain transition rates are investigated in this paper. The considered systems are more general than the systems with bounded uncertain transition rates or partly unknown transition rates, which can be viewed as two special cases of the systems we tackled here. The LMI-based stochastic stability condition for the underlying systems is derived. There are some possible directions to extend the proposed model and method. Tracking control and fault-tolerant control are two important research areas due to their wide application in the practical systems [40–46]. However, for the tracking control and fault-tolerant control, no research has focused on the case of general uncertain transition rates. Therefore, it is worth further extending the proposed method to deal with these problems.

#### **Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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