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Research Article

Equalities and Inequalities for Norms of Block Imaginary Circulant Operator Matrices

Xiaoyu Jiang and Kicheon Hong

Department of Information and Telecommunications Engineering, The University of Suwon, Wau-ri, Bongdam-eup, Hwaseong-si, Gyeonggi-do 445-743, Republic of Korea

Correspondence should be addressed to Kicheon Hong; kchong@suwon.ac.kr

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Circulant, block circulant-type matrices and operator norms have become effective tools in solving networked systems. In this paper, the block imaginary circulant operator matrices are discussed. By utilizing the special structure of such matrices, several norm equalities and inequalities are presented. The norm τ in consideration is the weakly unitarily invariant norm, which satisfies $\tau(\mathscr{A}) = \tau(U\mathscr{A}V)$. The usual operator norm and Schatten p-norm are included. Furthermore, some special cases and examples are given.

1. Introduction

Circulant-type matrices have significant applications in network systems. For example, Noual et al. [1] presented some results on the dynamical behaviours of some specific non-monotone Boolean automata networks called XOR circulant networks. In [2], the authors proposed a special class of the feedback delay networks using circulant matrices. Based on the circulant adjacency matrices of the networks induced by these interior symmetries, Aguiar and Ruan [3] analyzed the impact of interior symmetries on the multiplicity of the eigenvalues of the Jacobian matrix at a fully synchronous equilibrium for the coupled cell systems associated with homogeneous networks. Jing and Jafarkhani [4] proposed distributed differential space-time codes that work for networks with any number of relays using circulant matrices. In [5], the authors showed a structure for the decoupling of circulant symmetric arrays of more than four elements.

The well-known circulant, block circulant-type matrices and operator norms have set up the strong basis with the work in [6–19].

In this paper, let $W(H)$ denote the imaginary circulant algebra of all bounded linear operators on a complex separable Hilbert space H. The direct sum of n copies of H is denoted by $H^{(n)} = \oplus_{n\,\text{copies}} H$. If A_{jk}, $j,k = 1,2,\ldots,n$, are operators in $W(H)$, then the operator matrix (or the partitioned operator) $\mathscr{A} = [A_{jk}]$ can be considered as an operator in $W(H^{(n)})$, which is defined by $\mathscr{A}x = (\sum_{k=1}^{n} A_{1k}x_k, \ldots, \sum_{k=1}^{n} A_{nk}x_k)^T$ for every vector $x = (x_1,\ldots,x_n)^T \in H^{(n)}$.

Recall that a norm τ on $W(H)$ is called weakly unitarily invariant if $\tau(\mathscr{A}) = \tau(U\mathscr{A}U^*)$ for all $\mathscr{A} \in W(H)$ and for all unitary operators $U \in W(H)$.

The Schatten p-norms $\|\cdot\|_p$, $1 \le p < \infty$, are important examples of unitarily invariant norms, which are defined on the Schatten p-classes.

If $V_1, V_2, \ldots, V_n$ are operators in $W(H)$, we write the direct sum $\oplus_{j=1}^{n} V_j$ for the $n \times n$ block-diagonal operator matrix $\begin{pmatrix} V_1 & & 0 \\ & \ddots & \\ 0 & & V_n \end{pmatrix}$, regarded as an operator on $H^{(n)}$. Thus, $\|\oplus_{j=1}^{n} V_j\| = \max\{\|V_j\| : j = 1,2,\ldots,n\}$ and $\|\oplus_{j=1}^{n} V_j\|_p = (\sum_{j=1}^{n} \|V_j\|_p^p)^{1/p}$ for $1 \le p < \infty$. In particular, $\|\oplus_{j=1}^{n} V\| = n^{1/p}\|V\|_p$ for $1 \le p < \infty$.

The pinching inequality asserts that if $\mathscr{A} = [A_{jk}]$, then

$$\tau\left(\oplus_{j=1}^{n} A_{jj}\right) \le \tau(A). \tag{1}$$

For the operator norm and the Schatten p-norms, the inequality (1) states that

$$\max\left\{\left\|A_{jj}\right\| : j = 1, 2, \ldots, n\right\} \leq \|\mathscr{A}\|, \quad (2)$$

$$\left(\sum_{j=1}^{n}\left\|A_{jj}\right\|_p^p\right)^{1/p} \leq \|\mathscr{A}\|_p \quad (3)$$

for $1 \leq p < \infty$. It is known [18] that for $1 < p < \infty$, equality in (3) holds if and only if $\mathscr{A}$ is block-diagonal, that is, if and only if $A_{jk} = 0$, for $j \neq k$.

2. Equalities for the Norm of Imaginary Circulant Operator Matrices

In this section, we present block imaginary circulant operator matrix. By combining the special properties of block imaginary circulant operator matrix with unitarily invariant norm, we prove an equality in the following theorem.

If $A_1, A_2, \ldots, A_n$ are imaginary circulant operators in $W(H)$, the block imaginary circulant operator matrix $A = \mathrm{circ}_i(A_1, A_2, \ldots, A_n)$ is the $n \times n$ matrix whose first row has entries $A_1, A_2, \ldots, A_n$ and the other rows are obtained by successive cyclic permutations of these entries; that is,

$$\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right) = \begin{pmatrix} A_1 & A_2 & A_3 & \cdots & A_n \\ iA_n & A_1 & A_2 & \cdots & A_{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ iA_3 & iA_4 & iA_5 & \cdots & A_2 \\ iA_2 & iA_3 & iA_4 & \cdots & A_1 \end{pmatrix}, \quad (4)$$

where $i = \sqrt{-1}$.

It is known that $\mathrm{circ}_i(A_1, A_2, \ldots, A_n) = T\mathrm{circ}(A_1, \kappa A_2, \ldots, \kappa^{n-1}A_n)T^*$, where

$$T = \begin{pmatrix} I & & & 0 \\ & \kappa I & & \\ & & \ddots & \\ 0 & & & \kappa^{n-1}I \end{pmatrix}, \quad \text{with } \kappa = e^{\pi i/2n}. \quad (5)$$

Thus, every imaginary circulant operator matrix is unitarily equivalent to a circulant operator matrix.

Theorem 1. *Let $A_1, A_2, \ldots, A_n$ be any operators in $W(H)$. Then, for every weakly unitarily invariant norm, one has*

$$\tau\left(\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right)\right) = \tau\left(\oplus_{k=0}^{n-1}\sum_{j=1}^{n}\left(\kappa\omega^k\right)^{j-1}A_j\right), \quad (6)$$

where $i = \sqrt{-1}$, $\kappa = e^{\pi i/2n}$, and $\omega = e^{2\pi i/n}$.

Proof. The n roots of $z^n = i$ are $\kappa, \kappa\omega, \kappa\omega^2, \ldots, \kappa\omega^{n-1}$.

Now, let $U = U_n \otimes I$, where

$$U_n = \frac{1}{\sqrt{n}} \times \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ \kappa & \kappa\omega & \kappa\omega^2 & \cdots & \kappa\omega^{n-1} \\ \kappa^2 & (\kappa\omega)^2 & \left(\kappa\omega^2\right)^2 & \cdots & \left(\kappa\omega^{n-1}\right)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa^{n-1} & (\kappa\omega)^{n-1} & \left(\kappa\omega^2\right)^{n-1} & \cdots & \left(\kappa\omega^{n-1}\right)^{n-1} \end{pmatrix}_{n\times n}. \quad (7)$$

Then it is easy to prove that U is a unitary operator in $W(H)$ and

$$U^*\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right)U = \left(\oplus_{k=0}^{n-1}\sum_{j=1}^{n}\left(\kappa\omega^k\right)^{j-1}A_j\right). \quad (8)$$

By the invariance property of weakly unitarily invariant norms, we obtain

$$\tau\left(\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right)\right) = \tau\left(\oplus_{k=0}^{n-1}\sum_{j=1}^{n}\left(\kappa\omega^k\right)^{j-1}A_j\right). \quad (9)$$

□

Synthesizing the norm equality in Theorem 1 to the usual operator norm and to the Schatten p-norms, we obtain the corollary and remark as follows.

Corollary 2. *Let $A_1, A_2, \ldots, A_n$ be any operators in $W(H)$. Then one has*

$$\begin{aligned} &\left\|\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right)\right\| \\ &\quad = \max\left\{\left\|\sum_{j=1}^{n}\left(\kappa\omega^k\right)^{j-1}A_j\right\| : k = 0, 1, \ldots, n-1\right\}, \\ &\left\|\mathrm{circ}_i\left(A_1, A_2, \ldots, A_n\right)\right\|_p \\ &\quad = \left(\sum_{k=0}^{n-1}\left\|\sum_{j=1}^{n}\left(\kappa\omega^k\right)^{j-1}A_j\right\|_p^p\right)^{1/p} \end{aligned} \quad (10)$$

for $1 \leq p < \infty$.

In particular, let $n = 2$; one has

$$\begin{aligned} \left\|\mathrm{circ}_i\left(A_1, A_2\right)\right\| &= \max\left(\left\|A_1 + e^{\pi i/4}A_2\right\|, \left\|A_1 - e^{\pi i/4}A_2\right\|\right), \\ \left\|\mathrm{circ}_i\left(A_1, A_2\right)\right\|_p &= \left(\left\|A_1 + e^{\pi i/4}A_2\right\|_p^p + \left\|A_1 - e^{\pi i/4}A_2\right\|_p^p\right)^{1/p} \end{aligned} \quad (11)$$

for $1 \leq p < \infty$.

Remark 3. Here we give some special cases of Corollary 2.

(i) If $A \in W(H)$, then

$$\left\| \begin{pmatrix} \mathscr{A} & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \end{pmatrix}_{n\times n} \right\| = \frac{1-i}{1-\kappa} \|\mathscr{A}\|,$$

$$\left\| \begin{pmatrix} \mathscr{A} & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \end{pmatrix}_{n\times n} \right\|_p \tag{12}$$

$$= \left[\sum_{k=0}^{n-1} \left(\frac{1-i}{1-\kappa\omega^k} \right)^p \right]^{1/p} \|\mathscr{A}\|_p$$

for $1 \le p < \infty$.

(ii) If $\mathscr{A} \in W(H)$, then

$$\left\| \begin{pmatrix} 0 & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & 0 & \mathscr{A} & \cdots & \mathscr{A} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & 0 \end{pmatrix}_{n\times n} \right\| = \left| \frac{\kappa - i}{1-\kappa} \right| \|\mathscr{A}\|,$$

$$\left\| \begin{pmatrix} 0 & \mathscr{A} & \mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & 0 & \mathscr{A} & \cdots & \mathscr{A} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & \mathscr{A} \\ i\mathscr{A} & i\mathscr{A} & i\mathscr{A} & \cdots & 0 \end{pmatrix}_{n\times n} \right\| \tag{13}$$

$$= \left[\sum_{k=0}^{n-1} \left| \frac{\kappa\omega^k - i}{1-\kappa\omega^k} \right|^p \right]^{1/p} \|\mathscr{A}\|_p$$

for $1 \le p < \infty$.

3. Pinching-Type Inequalities for Imaginary Circulant Operator Matrices

In this section, for imaginary circulant operator matrices, we obtain pinching-type inequalities by the triangle inequality and the invariance property of unitarily invariant norms.

Theorem 4. *Let $\mathscr{A} = [A_{jk}]$ be an operator matrices in $W(H^{(n)})$. Then, for every weakly unitarily invariant norm, one has*

$$\frac{1}{n}\tau\left(\oplus_{k=0}^{n-1} \sum_{j=1}^{n} \left(\kappa\omega^k\right)^{j-1} V_j \right) \le \tau(\mathscr{A}), \tag{14}$$

where

$$V_1 = \sum_{j=1}^{n} A_{jj}, \qquad V_2 = \sum_{j=2}^{n} A_{j-1,j} + e^{-i(\pi/2)} A_{n1},$$
$$V_3 = \sum_{j=1}^{n-2} A_{j,j+2} + e^{-i(\pi/2)} \sum_{j=n-1}^{n} A_{j,j-(n-2)}, \ldots,$$
$$V_{n-1} = A_{1,n-1} + A_{2n} + e^{-i(\pi/2)} \sum_{j=2}^{n} A_{j-1,j},$$
$$V_n = A_{1n} + e^{-i(\pi/2)} \sum_{j=2}^{n} A_{j,j-1}. \tag{15}$$

Proof. Let $L_{k,n-k} = [l_{rs}]$ be the $n \times n$ operator with

$$l_{rs} = \begin{cases} I, & \text{if } s-r=k; \\ e^{i(\pi/2)}I, & \text{if } r-s=n-k; \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

We can prove easily that $L_{k,n-k} = [l_{rs}]$ is a unitary operator for all $k = 1, 2, 3, \ldots, n$ and

$$\sum_{k=1}^{n} L_{k,n-k} \mathscr{A} L_{k,n-k}^{*} = \operatorname{circ}_i (V_1, V_2, \ldots, V_n) = V, \tag{17}$$

where

$$V_1 = \sum_{j=1}^{n} A_{jj}, \qquad V_2 = \sum_{j=2}^{n} A_{j-1,j} + e^{-i(\pi/2)} A_{n1},$$
$$V_3 = \sum_{j=1}^{n-2} A_{j,j+2} + e^{-i(\pi/2)} \sum_{j=n-1}^{n} A_{j,j-(n-2)}, \ldots,$$
$$V_{n-1} = A_{1,n-1} + A_{2n} + e^{-i(\pi/2)} \sum_{j=2}^{n} A_{j-1,j},$$
$$V_n = A_{1n} + e^{-i(\pi/2)} \sum_{j=2}^{n} A_{j,j-1}. \tag{18}$$

Now let

$$U = \frac{1}{\sqrt{n}} \times \begin{pmatrix} I & I & I & \cdots & I \\ \kappa I & \kappa\omega I & \kappa\omega^2 I & \cdots & \kappa\omega^{n-1} I \\ \kappa^2 I & (\kappa\omega)^2 I & \left(\kappa\omega^2\right)^2 I & \cdots & \left(\kappa\omega^{n-1}\right)^2 I \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa^{n-1} I & (\kappa\omega)^{n-1} I & \left(\kappa\omega^2\right)^{n-1} I & \cdots & \left(\kappa\omega^{n-1}\right)^{n-1} I \end{pmatrix}_{n\times n}. \tag{19}$$

Then, from Theorem 1, we have

$$U^* V U = \oplus_{k=0}^{n-1} \sum_{j=1}^{n} \left(\kappa\omega^k\right)^{j-1} V_j. \tag{20}$$

Thus, by the invariance property of unitarily invariant norms and the triangle inequality, we get

$$\frac{1}{n}\tau\left(\oplus_{k=0}^{n-1}\sum_{j=1}^{n}\left(\kappa\omega^{k}\right)^{j-1}V_j\right)\leq\tau(\mathscr{A}). \tag{21}$$

□

Substituting the norm inequality (14) to the usual operator norm and to the Schatten p-norms, we obtain the following corollary.

Corollary 5. *Let* $\mathscr{A}=[A_{jk}]$ *be an operator matrix in* $W(H^{(n)})$. *Then*

$$\begin{gathered}\frac{1}{n}\max\left\{\left\|\sum_{j=1}^{n}\left(\kappa\omega^{k}\right)^{j-1}V_j\right\|:k=0,1,\ldots,n-1\right\}\leq\|\mathscr{A}\|,\\ \left(\frac{1}{n}\sum_{k=0}^{n-1}\left\|\sum_{j=1}^{n}\left(\kappa\omega^{k}\right)^{j-1}V_j\right\|_p^p\right)^{1/p}\leq\|\mathscr{A}\|_p\end{gathered} \tag{22}$$

for $1\leq p<\infty$, *where* V_j *is given in (15).*

As a special case when $n=2$, Corollary 5 asserts that

$$\begin{aligned}&\frac{1}{2}\max\left(\left\|A_{11}+A_{22}+e^{\pi i/4}\left(A_{12}+e^{-(\pi/2)i}A_{21}\right)\right\|,\right.\\&\qquad\left.\left\|A_{11}+A_{22}-e^{\pi i/4}\left(A_{12}+e^{-(\pi/2)i}A_{21}\right)\right\|\right)\\&\quad\leq\left\|\begin{pmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{pmatrix}\right\|,\\&\frac{1}{2}\left(\left\|A_{11}+A_{22}+e^{\pi i/4}\left(A_{12}+e^{-(\pi/2)i}A_{21}\right)\right\|_p^p\right.\\&\qquad\left.+\left\|A_{11}+A_{22}-e^{\pi i/4}\left(A_{12}+e^{-(\pi/2)i}A_{21}\right)\right\|_p^p\right)^{1/p}\\&\quad\leq\left\|\begin{pmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{pmatrix}\right\|_p\end{aligned} \tag{23}$$

for $1\leq p<\infty$.

It should be mentioned here that the norm inequalities in Theorems 1 and 4 are sharp. This is demonstrated in the following proposition.

Proposition 6. *Let* $A_1,A_2,\ldots,A_n$ *be some operators in* $W(H)$. *If* $\mathscr{A}=\operatorname{circ}_i(A_1,A_2,\ldots,A_n)$, *then the inequality in Theorem 4 becomes an equality.*

Proof. Let $\mathscr{A}=\operatorname{circ}_i(A_1,A_2,\ldots,A_n)$. Then it follows from Theorem 1 that

$$\tau\left(\operatorname{circ}_i\left(A_1,A_2,\ldots,A_n\right)\right)=\tau\left(\oplus_{k=0}^{n-1}\sum_{j=1}^{n}\left(\kappa\omega^{k}\right)^{j-1}A_j\right). \tag{24}$$

Since $V_1=nA_1, V_2=nA_n,\ldots,V_{n-1}=nA_3$, and $V_n=nA_2$, it follows that

$$\frac{1}{n}\tau\left(\oplus_{k=0}^{n-1}D_k\right)=\tau\left(\operatorname{circ}_i\left(A_1,A_2,\ldots,A_n\right)\right), \tag{25}$$

where

$$\begin{aligned}D_0&=\sum_{j=1}^{n}\kappa^{j-1}V_j=n\left[A_1+\kappa A_2+\cdots+\kappa^{n-1}A_n\right],\\D_1&=\sum_{j=1}^{n}(\kappa\omega)^{j-1}V_j=n\left[A_1+(\kappa\omega)A_2+\cdots+(\kappa\omega)^{n-1}A_n\right],\\&\vdots\\D_{n-1}&=\sum_{j=1}^{n}\left(\kappa\omega^{n-1}\right)^{j-1}V_j\\&=n\left[A_1+\left(\kappa\omega^{n-1}\right)A_2+\cdots+\left(\kappa\omega^{n-1}\right)^{n-1}A_n\right].\end{aligned} \tag{26}$$

□

By Proposition 6, it is easy to see that equality holds in the inequality (14) if and only if $\mathscr{A}$ is imaginary circulant for $0<p<\infty$. Furthermore, we obtain the following proposition by using the general Clarkson inequalities which can be seen in Proposition 1 of [19].

Proposition 7. *Let* $\mathscr{A}=[A_{jk}]$ *be an operator matrix in* $W(H^n)$, *and let* $1<p<\infty$. *Then* $\|\mathscr{A}\|_p=(1/n)\|\oplus_{k=0}^{n-1}\sum_{j=1}^{n}(\kappa\omega^k)^{j-1}V_j\|_p$ *if and only if* $\mathscr{A}$ *is imaginary circulant.*

Proof. In view of Proposition 1, it if sufficient to prove the "only if" part. Let $L_{k,n-k}$ be as in the proof of Theorem 1. If $\|\mathscr{A}\|_p=(1/n)\|\oplus_{k=0}^{n-1}\sum_{j=1}^{n}(\kappa\omega^k)^{j-1}V_j\|_p$, then it follows from the proof of Theorem 1 that

$$\begin{aligned}&\left\|L_{1,n-1}\mathscr{A}L_{1,n-1}^{*}\right\|_p\\&\quad=\left\|L_{2,n-2}\mathscr{A}L_{2,n-2}^{*}\right\|_p=\cdots=\left\|L_{1,n-1}\mathscr{A}L_{1,n-1}^{*}\right\|_p=\|\mathscr{A}\|_p,\\&\qquad\sum_{k=0}^{n-1}\left\|L_{k,n-k}\mathscr{A}L_{k,n-k}^{*}\right\|_p=n\|\mathscr{A}\|_p.\end{aligned} \tag{27}$$

Now invoking Clarkson inequalities for several operators, it follows that

$$L_{1,n-1}\mathscr{A}L_{1,n-1}^{*}=L_{2,n-2}\mathscr{A}L_{2,n-2}^{*}=\cdots=L_{1,n-1}\mathscr{A}L_{1,n-1}^{*}. \tag{28}$$

Consequently, $\mathscr{A}$ is imaginary circulant matrix. □

4. Conclusion

By utilizing the special structure of imaginary circulant matrices, we obtain several norm equalities and inequalities, where the norm τ under consideration is the weakly unitarily invariant norm. Based on the existing problems in [20–23], we will exploit solving these problems by circulant matrices technique.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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